CHAPTER 12

TWO-WAY ANALYSIS OF VARIANCE

OBJECTIVES

After completing this chapter, you should

- have an intuitive understanding of two-way, or two-factor, analysis of variance.

CHAPTER REVIEW

The two-way ANOVA is presented in this chapter as a method for analyzing data resulting from the administration of two or more levels of two independent variables. When there is more than one independent variable, the variables are called *factors.*

In a two-factor experiment, the effect of each separate independent variable is called a *main effect.* Thus, a two-factor experiment has two main effects. Also, there is the possibility of *interaction* or the joint effect of two independent variables on behavior. If an interaction effect exists, the effect of one factor depends on the levels of the second factor.

One way to look for interaction or the lack of it is to graph the results of a factorial study. If there is no interaction, the graph will show roughly parallel lines or lines that are approximately equidistant at each data point. Interaction is shown by converging or crossing lines. "Rules" for interpreting graphs of two-factor experiments are as follows:

1. Assuming factor A is shown on the baseline and the levels of factor B are plotted as different lines, if the averages of the points above each level of factor A are unequal, there may be a significant main effect for factor A.
2. If the averages of the points used to plot the lines are unequal, there may be a significant main effect for factor B.
3. If the lines converge, cross, or in some way depart from parallel, there may be a significant interaction.

The main advantage of the two-factor design is that you can test for interaction between the factors. If you manipulated the same variables in two separate one-factor experiments, you would not be able to test for an interaction of the factors. Another advantage of the two-factor design is that it allows a savings in the number of subjects, and probably the time and resources, needed. The two-factor design usually increases the power of the tests on the main effects and permits greater generalizability.

The logic of the tests for two-factor designs is the same as for the one-way ANOVA. Instead of one *F* test, there are three *F* tests for the two-way ANOVA: main effects tests for factors A and B, and a test of the interaction between factors A and B. Each *F* ratio consists of the variance (*MS*) for each effect divided by the variance within treatments. The same error term, MS_w, is used for each *F* ratio. The variance between treatments is assumed to be caused by individual differences, experimental error, and a treatment effect; the variance within treatments is caused by individual differences and experimental error. An *F* ratio close to 1.00 indicates the lack of a treatment effect, and a value much larger than 1.00 signals a valid treatment effect.

Interpretation of the two-way ANOVA depends in large part on whether the interaction is significant. If the interaction is not significant, significant main effects can be analyzed with the post hoc tests discussed in Chapter 11. The first step in interpreting a significant interaction is to plot the group means, with further interpretation requiring individual group comparisons.

TERMS TO DEFINE AND/OR IDENTIFY

two-way ANOVA

factors

factorial design

main effect

interaction

FILL-IN-THE-BLANK ITEMS

Introduction

In Chapter 11, we used the (1) _________________ _________________ to analyze the results of an experiment in which two or more levels of an independent variable were manipulated. The (2) _________________ _________________ is used when another independent variable is added. When more than one independent variable is used, the variables are called (3) _________________. An experiment studying the effects of task difficulty and anxiety in which there are three levels of task difficulty and three levels of anxiety is an example of a (4) _________________ factorial design.

Main Effects and Effects of Interaction

The effect of each independent variable in a two-factor experiment is called a (5) _________________ _________________. Also, there is the possibility of the joint effect of the two independent variables on behavior; this effect is called the (6) _________________. If an interaction exists, the effect of one factor (7) _________________ on the levels of the second factor. If there is no interaction between factors, the graph of the data will show essentially (8) _________________ lines or lines that are approximately equidistant at each data point. Thus, interaction is usually revealed by (9) _________________ lines or (10) _________________ lines. When levels of factor B are plotted as lines above levels of factor A, data can be interpreted from a two-factor experiment with the help of the following "rules":

1. If the averages of the points above each level of factor A are unequal, a significant main effect for factor (11) _________________ is suggested.

2. If the averages of the points used to plot the (12) _________________ are unequal, there may be a significant main effect for factor B.

3. If the lines aren't parallel, there may be a significant (13) _________________.

Advantages of the Two-Factor Design

The main advantage of the two-factor design over two one-factor experiments is the test for (14) _________________. A second advantage is economy. The two-factor design allows a reduction in the number of (15) _________________ required. Statistical tests on the main effects are usually more (16) _________________ with the two-factor design than with two one-factor designs. Finally, the two-factor design tells about more conditions; that is, it allows for greater (17) _________________.

Logic of the Two-Way ANOVA

The two-factor ANOVA results in (18) _________________ *F* ratios. There are tests of the

(19) _________________ _________________ of factors A and B and a test of the

(20) _________________ of factors A and B. The same error term, symbolized by

(21) _________________ is used for each *F* ratio.

Interpretation of Results

Interpretation of the two-way ANOVA depends mainly on whether the (22) _________________ is

significant. If it is not significant, a significant main effect can be analyzed with the

(23) _________________ _________________ tests covered in Chapter 11. Plotting the group means is

often the first step in interpreting a significant (24) _________________.

PROBLEMS

1. For the problems in this chapter, assume that the within-group variability is small enough that any between-group mean differences will result in a significant *F* ratio. For each of the following tables, draw a graph and use it to predict the result of each tested main effect and the interaction.

a.

		Factor B	
		B_1	B_2
Factor A	A_1	10	15
	A_2	30	25

b.

		Factor B	
		B_1	B_2
Factor A	A_1	15	7
	A_2	30	22
	A_3	20	12

c.

			Factor B	
		B_1	B_2	B_3
Factor A	A_1	10	30	20
	A_2	20	20	20
	A_3	30	10	20

2. A pursuit-rotor task is performed by either sinistral or dextral men (left-handed or right-handed, respectively) under three different illumination levels. For each of the hypothetical outcomes, what result would you predict for the *F* ratios? Cell scores are the average times on target in seconds for a 1-minute test; $N_g = 10$.

a.

		Illumination Level		
		Low	Medium	High
Handedness	Sinistrals	25	35	45
	Dextrals	35	45	55

b.

		Illumination Level		
		Low	Medium	High
Handedness	Sinistrals	20	40	20
	Dextrals	40	20	40

c.

		Illumination Level		
		Low	Medium	High
Handedness	Sinistrals	20	30	40
	Dextrals	40	30	20

3. A study examined the effects of task difficulty and anxiety level on problem-solving ability. There were three levels of task difficulty (easy, moderate, hard) and three levels of anxiety as determined by scores on the Taylor Manifest Anxiety Scale (low, medium, high). The dependent variable was the average time to solve 10 problems in minutes. Using the following table, graph the results and indicate the probable outcome of *F* tests. Each cell value is a group average in minutes; $N_g = 20$.

		Anxiety Level		
		Low	Medium	High
Task Difficulty	Easy	8.6	6.2	4.3
	Moderate	10.2	8.4	12.5
	Hard	12.7	10.5	21.8

4. A cognitive psychologist studied the effects of mathematics anxiety on the speed of solution of arithmetic problems. Students were tested on the Mathematics Anxiety Rating Scale and separated into high, medium, and low mathematics anxiety groups. Half of each group then solved easy (one-digit) addition problems and the other half solved hard (two-digit) problems. Each cell contains the average number of seconds taken to solve each problem. Graph the means, and predict the outcome of the significance tests.

		Anxiety Level		
		Low	Medium	High
Problem Difficulty	Easy	2	4	4
	Hard	8	12	20

5. A social psychologist is interested in the effects of thrill-seeking tendency and alcohol dose on performance on a simulated driving task. She administers a test that measures thrill-seeking tendency and divides the subject pool into two levels on the basis of subject scores: low and high. Thirty volunteers from each level are given either no alcohol, 1 ounce of alcohol, or 2 ounces of alcohol. After 15 minutes, each participant is tested on the task; the score is the length of time the car stays on the road in a 2-minute test. Each cell value is the average in seconds for 10 participants. Graph the results, and predict the outcome of significance tests.

		Alcohol Level		
		0 oz	1 oz	2 oz
Thrill Seeking	Low	110	105	95
	High	95	75	45

6. Hypothetical *F* ratios for each part of Problem 2 are shown next. Give the critical value for each *F* ratio and provide an interpretation for each significant test. For example, a significant *F* ratio for handedness would mean that time on target was affected by the hand used. Review of the group means would suggest whether left-handed or right-handed participants performed better.

a.

Source	*F*	*p*
Between groups		
A (handedness)	6.31 ($df = 1, 54$)	
B (illumination)	4.05 ($df = 2, 54$)	
A × B	1.18 ($df = 2, 54$)	

b.

Source	*F*	*p*
Between groups		
A (handedness)	4.52 ($df = 1, 54$)	
B (illumination)	0.67 ($df = 2, 54$)	
A × B	10.91 ($df = 2, 54$)	

c.

Source	F		p
Between groups			
A (handedness)	0.75	(*df* = 1, 54)	
B (illumination)	1.18	(*df* = 2, 54)	
A × B	9.83	(*df* = 2, 54)	

CHECKING YOUR PROGRESS: A SELF-TEST

1. Converging or crossing lines on a graph of the results of a two-factor experiment often signal a significant
   a. main effect for factor A.
   b. main effect for factor B.
   c. interaction.
   d. all of the above

2. Which of the following is *not* an advantage of a two-factor design over two separate one-factor experiments?
   a. a test for interaction
   b. fewer subjects required
   c. greater generalizability
   d. decreased power on the main effects tests

3. An experiment might be done to compare the effects of nicotine and a placebo on hand–eye coordination in smokers and nonsmokers. Different possible outcomes are shown in the following contingency tables. The cell numbers are mean errors on a mirror tracing task. Graph the results, and predict the outcomes of significance tests.

a.

	Placebo	Nicotine
Smoker	15	15
Nonsmoker	8	15

b.

	Placebo	Nicotine
Smoker	15	5
Nonsmoker	5	15

c.

	Placebo	Nicotine
Smoker	15	20
Nonsmoker	5	10