

CHAPTER 14

CHI SQUARE

OBJECTIVES

After completing this chapter, you should

- understand and be able to compute and use the chi-square technique for analyzing nominal scale data.

CHAPTER REVIEW

Previous chapters have detailed the t test and the F test, both of which are parametric methods. *Parametric* tests examine hypotheses about population parameters such as μ and σ , usually assume at least interval scale measurement, and assume a normal distribution of the measured variable in the population from which samples were drawn. The chi-square test is called a *nonparametric* test because population parameters are not estimated. It is also a *distribution-free* test because no particular population distribution is assumed. The chi-square test is particularly useful with nominal scale or categorical data—data in which there are only frequencies of occurrence.

A chi-square test on a single categorical variable is called a *goodness-of-fit* test. The formula for the goodness-of-fit test is

$$\chi^2 = \sum \frac{(O - E)^2}{E},$$

where O is the *observed* frequency and E is the *expected* frequency. If the expected frequencies are assumed to be equally distributed across the levels of the categorical variable, then the total number of observations (N) is divided by the number of categories to determine E for each category. Expected frequencies may also be determined by assigning the frequencies on the basis of percentages obtained in previous research. The χ^2 computed for the goodness-of-fit test is compared with critical values from Table G (see Appendix 2) with $df = K - 1$, where K is the number of levels of the categorical variable. The chi-square goodness-of-fit test can sometimes be used to confirm the research hypothesis.

The chi-square test for different levels of two categorical variables is called the *chi-square test of independence*, the *two-sample chi square*, or the *chi-square test of significance*. The test assumes that the distribution of frequencies across the levels of one variable is the same for all levels of the other variable; that is, the test assumes that the two categorical variables are independent. The formula for the chi-square test of independence is the same as for the goodness-of-fit test. Expected frequencies may be given by theory or previous research. Most often, however, they must be computed. Expected frequencies for a given cell in a frequency table are computed by dividing the product of the marginal totals for the cell by the total number of observations. The computed χ^2 is compared with critical values from Table G with $df = (R - 1)(C - 1)$, where R is the number of rows and C is the number of columns.

The chi-square test can be used only with frequency data. However, any data can be converted to frequency data by dividing the data into several logical categories and counting the number of observations that occur in each category.

A second restriction on chi square is that the individual observations must be independent of one another. Another restriction is that if you are recording whether an event occurs, you must have in the data both the frequency of occurrence and the frequency of nonoccurrence. Also, no expected frequencies should be less than 5, although the restriction may be relaxed if there are more than four cells and only a few have expected frequencies less than 5. Although there is a statistical alternative to χ^2 with a 2×2 table and expected frequencies less than 5, the best approach is to test more subjects to increase the expected frequencies.

SYMBOLS

<u>Symbol</u>	<u>Stands For</u>
χ^2	chi square
O	observed frequency
E	expected frequency
K	number of categories
RT	row total
CT	column total
R	number of rows
C	number of columns

FORMULAS

Formula 14-1. Equation for chi square (used for both the goodness-of-fit test and the chi-square test of independence [significance])

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

O stands for the observed frequencies, and E stands for the expected frequencies.

Formula 14-2. Equation for finding expected frequencies for the chi-square test of independence if there is no other basis for determining them

$$E = \frac{(RT) (CT)}{N}$$

RT stands for the row total for a given cell, and CT stands for the column total for the same cell.

Formula 14-3. Formula for finding degrees of freedom for the chi-square test of independence

$$df = (R - 1)(C - 1)$$

TERMS TO DEFINE AND/OR IDENTIFY

parametric tests

nonparametric tests

distribution-free tests

chi-square test

chi-square goodness-of-fit test

chi-square test of independence

contingency table

FILL-IN-THE-BLANK ITEMS

Introduction

The t test and the F test are examples of (1) _____ tests, or tests designed to test hypotheses about population parameters. Tests that don't test hypotheses about population parameters are called (2) _____ tests. Also, because the tests don't assume any particular distribution, they are often called (3) _____ tests.

The (4) _____ scale is used for labeling only; all we can do with it is to record (5) _____ of occurrence. (6) _____ is an appropriate test with categorical or nominal scale data.

The Chi-Square Goodness-of-Fit Test

A chi-square test on different levels of a single categorical variable is called a (7) _____ test, because it assesses how well the observed data fit expectations. In the equation for the chi-square test, for each cell in a frequency table, the difference between the observed and expected frequencies is (8) _____ and is divided by the expected frequency. If the expected frequencies are assumed to have been (9) _____ across levels of the categorical variable, then each E can be determined by dividing the total number of observations, N , by the number of levels. Expected frequencies may also be based on percentages observed in (10) _____.

The results for each cell are (11) _____ for all cells to give the chi-square statistic. The computed chi square is compared with critical values from Table G with $df =$ (12) _____, where K is the number of (13) _____ of the categorical variable.

Confirming hypotheses with chi square

Sometimes the chi-square goodness-of-fit test null hypothesis is the (14) _____ that the investigator seeks to confirm. If this is the case, then failure to reject H_0 may provide some (15) _____ of the theory under investigation. The degree of confirmation depends on the statistical (16) _____ in the analysis and on whether (17) _____ leads to the same conclusion.

The Chi-Square Test of Independence

The chi-square test based on (18) _____ categorical variables is called the chi-square test of (19) _____, the (20) _____ chi square, or the chi-square test of significance.

The test tries to determine whether the two categorical variables are (21) _____.

The frequency table formed from the observations made under each categorical variable is called a (22) _____ table.

An alternative method for finding expected values

Occasionally, expected frequencies are known because of theory or previous (23) _____.

More often, however, the expected frequencies for a given cell are found by multiplying the (24) _____ totals for the cell and dividing by (25) _____. Many expected frequencies can be determined by (26) _____, because the expected frequencies for a given row or column must sum to the row or column total. df for the chi-square test of independence equals (27) _____, where R is the number of rows and C is the number of columns.

Restrictions on Chi Square

Chi square can be used only with (28) _____ data. Also, the events or observations that make up the data must be (29) _____ of one another. A third restriction on chi square is that we must have in the data both the frequency of (30) _____ and the frequency of (31) _____, if we are recording whether an event occurs. No expected frequency should be less than (32) _____, although this rule may be relaxed if there are more than (33) _____ cells and only a few have small expected frequencies.

Troubleshooting Your Computations

If you're finding E by subtraction, remember that both the (34) _____ frequencies and the observed frequencies must sum to give you a particular row or column total. Be sure you don't get any (35) _____ signs for a cell total.

PROBLEMS

1. Determine the expected frequencies for each of the cells in the following contingency tables. For each problem, state how many of the expected values actually had to be computed and how many could be found by subtraction.

a.

14	16
31	4

b.

23	13	15
25	17	10

c.

21	25	27
10	13	17
11	45	13

2. Compute χ^2 for each of the contingency tables from Problem 1, and test it for significance.

a.

14	16
31	4

b.

23	13	15
25	17	10

c.

21	25	27
10	13	17
11	45	13

3. Left- and right-handed individuals with unilateral (on one side) brain damage were compared to see whether brain injury is more likely to produce aphasia (language difficulties) in one group than in the other. Test the observed result for significance, and tell what your result means in the context of the problem.

	Aphasic	Nonaphasic
Left-Handed	10	24
Right-Handed	62	36

4. Research indicates that some alcoholism is inherited. The families of alcoholic and nonalcoholic adults have been studied to determine whether one or both parents were alcoholic. The results are shown here. Test the observed results for significance, and tell what your result means in the context of the problem.

	Alcoholic	Nonalcoholic
No Parental Alcoholism	17	61
One or Both Parents Alcoholic	43	83

5. A rhesus monkey was trained to make same–different judgments about pairs of objects. For 50 trials, the animal was shown pictures of objects; it made correct judgments on 43 of the trials. Use the chi-square test to determine whether the animal had generalized its learned response from objects to pictures of objects. *Note:* The probability of making a correct response on any trial is .5.

6. A survey was conducted of the product preferences of introverts and extraverts. Does introversion or extraversion make a difference in brand preference?

	Brand A	Brand B	Brand C
Introverts	42	18	20
Extraverts	36	16	28

7. A professor teaching introductory psychology gave 25 As, 35 Bs, 60 Cs, 30 Ds, and 20 Fs to 170 students in her class. According to the normal curve, she should have given about 4% As, 14% Bs, 64% Cs, 14% Ds, and 4% Fs. Use the chi-square test to determine whether the actual grade assignment significantly departed from a normal distribution.

8. A test of risk-taking attitudes was given to 60 students, 27 scoring low and 33 scoring high on self-esteem. A score of 10 or higher indicates a positive risk-taking attitude, and a score less than 10 indicates a more cautious approach to life. Determine whether high- and low-self-esteem students differ on this variable.

	Less Than 10	10 or More
High Self-Esteem	14	19
Low Self-Esteem	18	9

9. A professor taught a course in physiological psychology. On the student–teacher evaluation, the professor scored higher than the departmental average on 23 of 28 items. Is this result significantly better than one might expect?

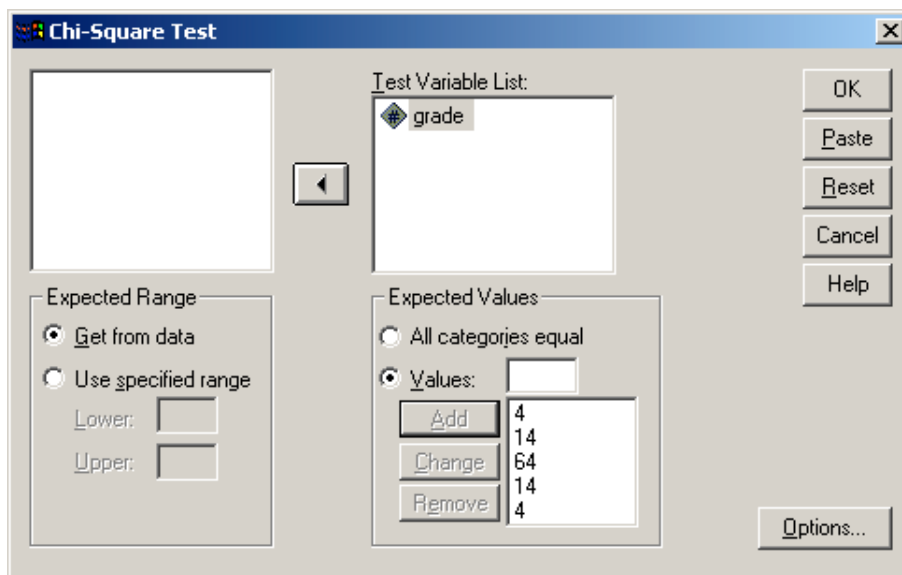
10. In a statistics class, the same professor from Problem 9 scored higher than the departmental average on 10 of 28 items. Is this result significantly different from the expected result?

USING SPSS—EXAMPLES AND EXERCISES

SPSS has procedures for performing the chi-square goodness-of-fit test and the chi-square test of independence between two categorical variables. However, SPSS is designed for use with raw data. The data presented for the exercises in this chapter are in summary form. As a consequence, in order to use SPSS, we must “unsummarize” the data—reconstruct the data as if they came from individual participants. Unfortunately, the process of creating the data file will be somewhat laborious, and you may decide that it’s easier to work the problems using a hand calculator than using SPSS. In spite of the difficulties, you will gain experience that will allow you to perform the chi-square-based tests using SPSS on data in the form you would obtain in an actual research setting.

Example—Chi-Square Goodness-of-Fit Test: To illustrate this technique, we will work Problem 7 using SPSS. The steps are as follows:

1. Start SPSS and name the first variable **grade**. We must simulate the grade data for 170 students. Use the code numbers 4 for A, 3 for B, 2 for C, 1 for D, and 0 for F. Because there were 25 As, 35 Bs, 60 Cs, 30 Ds, and 20 Fs, you should enter 4 twenty-five times, 3 thirty-five times, and so on. (*Hint:* Use the editing features of the Windows environment to help. For example, enter 4 five times, copy the block, and paste it below each successive block until you reach 25. Do the same for the other numbers in blocks of 5 or 10 depending on the totals needed.)
2. Select *Analyze>Nonparametric Tests>Chi-Square*.
3. Move **grade** into the Test Variable List box.
4. We now need to tell SPSS what our expected frequencies ratio is, if it is something other than an equal proportion for each group. In the Expected *Values* box, select *Values* and enter 4 (for 4%) in the entry box, then click *Add*. Repeat this process in order for all five expected value percentages: 4, 14, 64, 14, and 4. The resulting dialog box should appear as follows. Click *OK* and the results should appear in the output Viewer window.



Notes on Reading the Output

1. You should find the output easy to read and understand. The observed frequency, expected frequency, and difference (labeled Residual) are given in the first box and the test statistics in the second box. The results indicate that the actual grade assignment does differ significantly from a normal distribution: $\chi^2(4, N = 170) = 103.11, p < .001$.

```

NPAR TEST
  /CHISQUARE=grade
  /EXPECTED=4 14 64 14 4
  /MISSING ANALYSIS.

```

NPar Tests

Chi-Square Test

Frequencies

GRADE

	Observed N	Expected N	Residual
.00	20	6.8	13.2
1.00	30	23.8	6.2
2.00	60	108.8	-48.8
3.00	35	23.8	11.2
4.00	25	6.8	18.2
Total	170		

Test Statistics

	GRADE
Chi-Square ^a	103.109
df	4
Asymp. Sig.	.000

a. 0 cells (.0%) have expected frequencies less than 5. The minimum expected cell frequency is 6.8.

Example—Chi-Square Test of Independence: We will use SPSS to work Problem 6. Let's shorten the data entry required for this problem by taking half of the frequency for each condition to result in an $N = 80$ rather than 160. The steps are as follows:

1. Start SPSS and name variables **ie** and **pref**. Use a code value of 1 for introverts and 2 for extraverts; use 1 for Brand A, 2 for Brand B, and 3 for Brand C. You will need 21 entries for introverts preferring Brand A (**ie** = 1 and **pref** = 1), and so on. The data entry window should show a pattern like this: 1, 1 for 21 entries; 2, 1 for 18 entries; 1, 2 for 9 entries; 2, 2 for 8 entries; 1, 3 for 10 entries; and 2, 3 for 14 entries.
2. Select *Analyze>Summarize>Crosstabs*.
3. In the dialog box, move **ie** into the Rows box and **pref** into the Columns box. Select the Statistics box, then *Chi-Square>Continue*.
4. Also select the Cells box—in the Counts box, *Observed* should be checked by default; also check *Expected*. In the Percentages box, click *Total*, then *Continue* and *OK*. The results should appear in the output Viewer window.

Notes on Reading the Output

1. The IE*PREF Crosstabulation box shows the frequency (count) for each cell. Note that in each cell, the first number is the frequency, the second is the expected frequency, and the third number is the percentage for that cell based on the cell frequency divided by the total sample size. If you look at the percentages across the brand preference groups, you will see that there is not much difference in the pattern between introverts and extraverts. Both prefer Brand A, then Brand C, and Brand B least.
2. Shown in the Chi-Square Tests box, the Pearson Chi-Square value is 0.956, with $p = .62$. Thus, the results indicate that introversion or extraversion does not make a difference in brand preference:
 $\chi^2(2, N = 80) = 0.956, p = .62$.

CROSSTABS

```

/TABLES=ie BY pref
/FORMAT= AVALUE TABLES
/STATISTIC=CHISQ
/CELLS= COUNT EXPECTED TOTAL .

```

Crosstabs

Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
IE * PREF	80	100.0%	0	.0%	80	100.0%

IE * PREF Crosstabulation

			PREF			Total
			1.00	2.00	3.00	
IE	1.00	Count	21	9	10	40
		Expected Count	19.5	8.5	12.0	40.0
		% of Total	26.3%	11.3%	12.5%	50.0%
	2.00	Count	18	8	14	40
		Expected Count	19.5	8.5	12.0	40.0
		% of Total	22.5%	10.0%	17.5%	50.0%
Total		Count	39	17	24	80
		Expected Count	39.0	17.0	24.0	80.0
		% of Total	48.8%	21.3%	30.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	.956 ^a	2	.620
Likelihood Ratio	.960	2	.619
Linear-by-Linear Association	.804	1	.370
N of Valid Cases	80		

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 8.50.

Exercises Using SPSS

1. Use SPSS to work Problem 9. There should be 28 data entries, with a value of 1 for 23 and 0 for the remaining 5. Chance prediction would be that being above or below the departmental average would be equally likely; thus, the expected frequencies would be equal. This is the default in the Chi-Square Test dialog box. For Expected Values, All Categories Equal already should be checked and is the appropriate setting for this exercise. Write a conclusion.
2. Use SPSS to perform a chi-square test of independence using the data from Problem 8. Name the variables **esteem** and **risk**. Use a code of 1 for low self-esteem and 2 for high self-esteem; use a code of 1 for low risk takers and 2 for high risk takers. You will need to enter data for 60 participants. Write a conclusion.

CHECKING YOUR PROGRESS: A SELF-TEST

1. Which of the following statements are true about the chi-square test?
 - a. Chi square is a nonparametric test.
 - b. Chi square is a parametric test.
 - c. Chi square is useful with interval scale data.
 - d. Chi square is useful with nominal scale data.
 - e. A restriction on chi square is that no expected frequency should be less than 5.
 - f. Chi square is based on the normal curve distribution.
 - g. The chi-square test on a single categorical variable is called the chi-square test of independence.
 - h. Chi square is a distribution-free test.
2. A sample of Republicans and a sample of Democrats in a large city are asked whether they favor increasing spending for entitlement programs for the elderly. The results are shown here. Is there a difference between Republicans and Democrats on this question?

	Favor	Oppose
Republicans	26	78
Democrats	53	44

3. Three groups of 10 rats each are fed diets suspected to affect problem-solving ability. Then they are tested on a series of problems until all subjects have achieved criterion on at least two problems. The total number of problems solved by the rats in each dietary group is as follows: Group A, 33; Group B, 22; Group C, 25. Assuming that problem-solving ability should be evenly distributed across the groups, do the groups differ?