

Rosen, Discrete Mathematics and Its Applications, 6th edition  
Extra Examples

Section 10.5—Minimum Spanning Trees

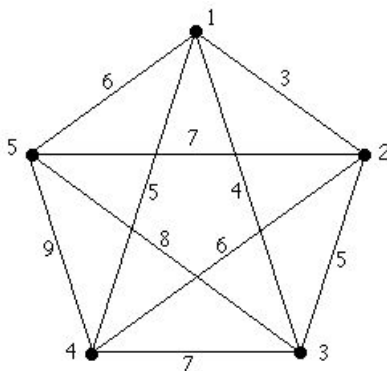


— Page references correspond to locations of Extra Examples icons in the textbook.

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**p.740, icon at Example 3**

#1. Suppose the vertices of  $K_5$  are numbered 1, 2, 3, 4, 5 (in clockwise order) and each edge is assigned a weight equal to the sum of the labels on the endpoints of the edge, as in the following figure. Find a spanning tree of minimum weight for this graph.



**Solution:**

Using either Kruskal's Algorithm or Prim's Algorithm, the edges  $\{1, 2\}$ ,  $\{1, 3\}$ ,  $\{1, 4\}$ , and  $\{1, 5\}$  make up the spanning tree of minimum weight. Its weight is 18.

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**p.740, icon at Example 3**

#2. Suppose the vertices of  $K_n$  are numbered 1, 2, ...,  $n$  (in clockwise order) and each edge is assigned a weight equal to the sum of the labels on the endpoints of the edge. Find a spanning tree of minimum weight for this graph and find the weight of this spanning tree.

**Solution:**

The spanning tree of minimum cost has edges  $\{1, 2\}, \{1, 3\}, \dots, \{1, n\}$ . Using either Kruskal's Algorithm or Prim's Algorithm, the first edges added are  $\{1, 2\}$  and  $\{1, 3\}$ . At the next stage, edges  $\{2, 3\}$  and  $\{1, 4\}$  have the smallest weight, but adding edge  $\{2, 3\}$  would create a circuit. Therefore edges  $\{1, 2\}$ ,  $\{1, 3\}$ , and  $\{1, 4\}$  are inserted into the spanning tree. In general, if edges  $\{1, 2\}, \{1, 3\}, \dots, \{1, k\}$  have been selected, the next edge inserted must be  $\{1, k + 1\}$  (of weight  $k + 2$ ). (Any other edge  $\{i, j\}$  with weight  $\leq k + 2$  would have  $1 < i \leq k$  and  $1 < j \leq k$  and would create a circuit when combined with  $\{1, i\}$  and  $\{1, j\}$ .) Thus, the spanning tree of minimum weight consists of  $\{1, 2\}, \{1, 3\}, \dots, \{1, n\}$ . Its total weight is

$$(1 + 2) + (1 + 3) + \dots + (1 + n) = (n - 2) + \frac{n(n + 1)}{2} = \frac{(n + 4)(n - 1)}{2}.$$

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