

Rosen, Discrete Mathematics and Its Applications, 6th edition
Extra Examples

Section 1.1—Propositional Logic



— Page references correspond to locations of Extra Examples icons in the textbook.

p.2, icon at Example 1

#1. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

“Portland is the capital of Maine.”

Solution:

This makes a declarative statement, and hence is a proposition. The proposition is false because Augusta, not Portland, is the capital of Maine.

p.2, icon at Example 1

#2. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

“Can Allen come with you?”

Solution:

This is a question, and hence not a proposition.

p.2, icon at Example 1

#3. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

$1 + 2 = 3$ or $2 + 3 = 5$.

Solution:

This is a proposition, and it is true because $1 + 2 = 3$ is true and $2 + 3 = 5$ is true.

p.2, icon at Example 1

#4. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

“Take two aspirin.”

Solution:

This is an imperative sentence. It is not a proposition.

p.2, icon at Example 1

#5. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

$$"x + 4 > 9."$$

Solution:

Because this is true for certain values of x (such as $x = 6$) and false for other values of x (such as $x = 5$), it is not a proposition.

p.3, icon at Example 3

#1. Write the negation of "George Washington was the first president of the United States."

Solution:

The negation is "It is not the case that George Washington was the first president of the U.S." In more straightforward language we say "George Washington was not the first president of the U.S."

p.3, icon at Example 3

#2. Write the negation of " $1 + 5 = 7$."

Solution:

The negation states that $1 + 5$ is not equal to 7: " $1 + 5 \neq 7$."

p.3, icon at Example 3

#3. Write the negation of " $1 + 5 \neq 7$."

Solution:

The negation states that $1 + 5$ is equal to 7: " $1 + 5 = 7$."

p.3, icon at Example 3

#4. Write the negation of "It is hot today."

Solution:

The negation is "It is not that case that it is hot today", or "It is not hot today." Note that the negation is not "It is cold today," because the temperature could be neither hot nor cold, making both statements false. But a statement and its negation must have opposite truth values.

p.3, icon at Example 3

#5. Write the negation of “6 is negative.”

Solution:

The negation states that “It is not the case that 6 is negative.” This means that 6 is greater than or equal to 0; we say “6 is nonnegative.” Note that “6 is positive” is not the negation of the given statement. Saying that a number is not negative means that the number can be either 0 or positive.

p.5, icon at Example 6

#1. The following proposition uses the English connective “or”. Determine from the context whether “or” is intended to be used in the inclusive or exclusive sense.

“Tonight I will stay home or go out to a movie.”

Solution:

Because the one alternative (staying home) precludes the other (going out), “or” is used in the exclusive sense.

p.5, icon at Example 6

#2. The following proposition uses the English connective “or”. Determine from the context whether “or” is intended to be used in the inclusive or exclusive sense.

“If you fail to make a payment on time or fail to pay the amount due, you will incur a penalty.”

Solution:

You might both fail to make a payment on time and your late payment might be for an incorrect amount. Hence the inclusive “or” is used here.

p.5, icon at Example 6

#3. The following proposition uses the English connective “or”. Determine from the context whether “or” is intended to be used in the inclusive or exclusive sense.

“If I can’t schedule the airline flight or if I can’t get a hotel room, then I can’t go on the trip.”

Solution:

If both events happen, you won’t go on the trip. Hence the inclusive “or” is used here.

p.5, icon at Example 6

#4. The following proposition uses the English connective “or”. Determine from the context whether “or” is intended to be used in the inclusive or exclusive sense.

“She has one or two brothers.”

Solution:

The person cannot have both one and two brothers. Therefore, “or” is used in the exclusive sense.

p.5, icon at Example 6

#5. The following proposition uses the English connective “or”. Determine from the context whether “or” is intended to be used in the inclusive or exclusive sense.

“If you do not wear a shirt or do not wear shoes, then you will be denied service in the restaurant.”

Solution:

It is implied that you won’t be served if you fail to wear a shirt and also fail to wear shoes. Therefore, the inclusive “or” is used here.

p.5, icon at Example 6

#6. The following proposition uses the English connective “or”. Determine from the context whether “or” is intended to be used in the inclusive or exclusive sense.

“I will pass or fail the course.”

Solution:

One alternative excludes the other; both cannot be true together. Here “or” must be exclusive.

p.5, icon at Example 6

#7. The following proposition uses the English connective “or”. Determine from the context whether “or” is intended to be used in the inclusive or exclusive sense.

“To register for ENL 499 you must have passed the qualifying exam or be listed as an English major.”

Solution:

Presumably, if you have passed the qualifying exam and are also listed as an English major, you can still register for ENL 499. Therefore, “or” is inclusive.

p.7, icon at Example 7

#1. The following statement is a conditional proposition in one of its many alternate forms. Write it in English in the form “If ... then ...”

“If it rains, I’ll stay home.”

Solution:

The word “then” is often omitted in English, as it is here. Therefore the statement is “If it rains, then I’ll stay home.”

p.7, icon at Example 7

#2. The following statement is a conditional proposition in one of its many alternate forms. Write it in English in the form “If ... then ...”

“I go walking whenever it rains.”

Solution:

The word “whenever” is equivalent to “if”. Therefore we have “If it rains, then I go walking.”

p.7, icon at Example 7

#3. The following statement is a conditional proposition in one of its many alternate forms. Write it in English in the form “If ... then ...”

“To pass the course it is sufficient that you get a high grade on the final exam.”

Solution:

Getting a high grade on the final exam is a sufficient condition for passing the course. That is, getting a high grade on the final exam will guarantee that you pass the course. Hence we have “If you get a high grade on the final exam, then you will pass the course.”

p.7, icon at Example 7

#4. The following statement is a conditional proposition in one of its many alternate forms. Write it in English in the form “If ... then ...”

“To pass the course it is necessary that you get a high grade on the final exam.”

Solution:

We first rephrase the statement as “Getting a high grade on the final exam is a necessary condition for passing the course.”

The word “necessary” and the word “sufficient” give rise to converse implications: “ p is sufficient for q ” is $p \rightarrow q$, while “ p is necessary for q ” is $q \rightarrow p$ (or, equivalently, $\neg p \rightarrow \neg q$). If the given statement had the word “sufficient” instead of “necessary”, the statement would be “If you get a high grade on the final exam, then you will pass the course.” The word “necessary” yields the converse of this statement; that is, “If you will pass the course, then you get a high grade on the final exam.” The equivalent contrapositive sounds better in English: “If you do not get a high grade on the final exam, then you will not pass the course.”

p.7, icon at Example 7

#5. The following statement is a conditional proposition in one of its many alternate forms. Write it in English in the form “If ... then ...”

“I will buy the tickets only if you call.”

Solution:

“ p if q ” and “ p only if q ” are converses; “only” has the effect of reversing the implication arrow. The proposition “ p if q ” is equivalent to $q \rightarrow p$; thus “ p only if q ” is equivalent to $p \rightarrow q$. In English we have “If I buy the tickets, then you call.” Using the equivalent contrapositive, this is “If you do not call, then I won’t buy the tickets.”

p.7, icon at Example 7

#6. The following statement is a conditional proposition in one of its many alternate forms. Write it in English in the form “If ... then ...”

“To be able to go on the trip, it is necessary that you get written permission.”

Solution:

Getting written permission is a necessary condition for going on the trip. If you fail to meet the necessary condition, you can’t go on the trip. Thus, “If you don’t get written permission, then you’re not able to go on the trip.” (Equivalently: “If you go on the trip, then you get written permission.”)

p.7, icon at Example 7

#7. The following sign is at the entrance of a restaurant: “No shoes, no shirt, no service.” Write this sentence as a conditional proposition.

Solution:

The implication is “If you do not wear shoes and a shirt, then you cannot enter the restaurant.”

p.7, icon at Example 7

#8. Write the compound proposition $s \rightarrow v$ in English, using the variables:

v : “I take a vacation”

s : “it is summer”

Solution:

The statement is “If it is summer, then I take a vacation,” or “I take a vacation in the summer.”

p.7, icon at Example 7

#9. Write the compound proposition $s \rightarrow \neg w$ in English, using the variables:

s : “it is summer”

w : “I work”

Solution:

The statement says that “If it is summer, then I do not work,” which we can rephrase as “I don’t work in the summer.”

p.7, icon at Example 7

#10. Write the compound proposition $\neg v \rightarrow w$ in English, using the variables:

v : “I take a vacation”

w : “I work”

Solution:

The statement says that “If I don’t take a vacation, then I work.”

p.7, icon at Example 7

#11. “Tell me what you eat and I will tell you what you are” is a quote by Jean-Anthelme Brillat-Savarin (French gastronome, 1755-1829). Express this as a compound proposition.

Solution:

The statement is not a conjunction. There is a conditional “if” understood. The word “and” functions like the word “then”. Therefore, “If you tell me what you eat, then I will tell you what you are.”

p.7, icon at Example 7

#12. Write the negation of “If it rains, I stay home.”

Solution:

The statement has the form $p \rightarrow q$, with an implied “then”. In order for a statement of the form $p \rightarrow q$ to be false, p must be true and q must be false. Therefore, the negation is “It rains and I do not stay home.”

p.7, icon at Example 7

#13. Find the negation of the statement “If you pay your membership dues, then if you come to the club, you can enter free.”

Solution:

The statement has the form $p \rightarrow (c \rightarrow e)$ where p stands for “you pay your membership dues”, c stands for “you come to the club”, and e stands for “you can enter free”.

We first rewrite the statement in the equivalent form $\neg p \vee (c \rightarrow e)$. To more easily analyze the statement, we use De Morgan's laws and other properties of predicates to rewrite it:

$$\begin{aligned}\neg(\neg p \vee (c \rightarrow e)) &\equiv \neg\neg p \wedge \neg(c \rightarrow e) \\ &\equiv p \wedge \neg(\neg c \vee e) \\ &\equiv p \wedge (\neg\neg c \wedge \neg e) \\ &\equiv p \wedge c \wedge \neg e.\end{aligned}$$

In English the negation says that “You pay your membership dues and you come to the club, but you cannot enter free.”

p.8, icon at Example 9

#1. Write the contrapositive, converse, and inverse of the following proposition:

“If the number is positive, then its square is positive.”

Solution:

The contrapositive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$. Therefore the contrapositive of “If the number is positive, then its square is positive” is the equivalent statement “If the square of a number is not positive, then the number is not positive.”

The converse of $p \rightarrow q$ is $q \rightarrow p$. Thus the converse is “If the square of a number is positive, then the number is positive.”

The inverse of $p \rightarrow q$ is $\neg p \rightarrow \neg q$. Therefore we have the inverse “If the number is not positive, then the square is not positive.”

p.8, icon at Example 9

#2. Write the contrapositive, converse, and inverse of the following proposition:

“I stay home whenever it is stormy.”

Solution:

Remember that the word “whenever” is equivalent to “if”, so the original proposition is “I stay home if it is stormy,” or, equivalently “If it is stormy, then I stay home.” Therefore:

Contrapositive: “If I do not stay home, then it is not stormy.”

Converse: “If I stay home, then it is stormy.”

Inverse: “If it is not stormy, then I do not stay home.”

p.9, icon at Example 10

#1. Write the following proposition in the form “... if and only if ...”

“It rains exactly when I plan a picnic.”

Solution:

The word “exactly” takes the place of “if and only if.” The statement says that “If it rains, then I plan a picnic” and “If I plan a picnic, then it rains.” Putting these two together we have “It rains if and only if I plan a picnic.”

p.9, icon at Example 10

#2. Write the following proposition in the form “... if and only if”

“I attend class when I have a quiz and I have a quiz when I attend class.”

Solution:

Using “if” in place of “when,” we have “I attend class if I have a quiz” and “If I attend class, then I have a quiz.” Therefore we have “I attend class if and only if I have a quiz.”

p.9, icon at Example 10

#3. Write the following proposition in the form “... if and only if”

“I visit the library whenever I have a paper to write, and conversely.”

Solution:

Using v for “I visit the library” and p for “I have a paper to write,” we have both $p \rightarrow v$ and its converse $v \rightarrow p$. Together, these two statements give $p \leftrightarrow v$, or “I visit the library if and only if I have a paper to write.”

p.9, icon at Example 10

#4. The following English statement can be written in the form “if ..., then ...”. Yet in some cases there is an implied “only if”; that is, the converse is implied. Do you think that the following statement has an implied converse?

“If you study hard, then you will pass the course.”

Solution:

Possibly you knew the material prior to the course, so you might pass the course without studying hard. Therefore, this proposition does not have an implied converse.

p.9, icon at Example 10

#5. The following English statement can be written in the form “if ..., then ...”. Yet in some cases there is an implied “only if”; that is, the converse is implied. Do you think that the following statement has an implied converse?

“If you have a red ink cartridge in your printer, then you can use the printer to print the report in red.”

Solution:

If you are able to print in red, you presumably have a red cartridge in the printer. Therefore, this proposition has an implied converse.

p.9, icon at Example 10

#6. The following English statement can be written in the form “if . . . , then . . .”. Yet in some cases there is an implied “only if”; that is, the converse is implied. Do you think that the following statement has an implied converse?

“If you pay the electric bill, then the electric company will turn on your power.”

Solution:

It is possible that the electric company might still turn on your power, even if you do not pay your bill. For example, someone else might pay the bill for you. Therefore, this proposition does not have an implied converse.

p.9, icon at Example 10

#7. The following English statement can be written in the form “if . . . , then . . .”. Yet in some cases there is an implied “only if”; that is, the converse is implied. Do you think that the following statement has an implied converse?

“You must be a resident in order to vote.”

Solution:

The original proposition has the form “If you are not a resident, then you cannot vote.” The converse, “If you cannot vote, then you are not a resident”, does not necessarily follow, because you might not meet an age requirement, for example. This proposition does not have an implied converse.

p.9, icon at Example 10

#8. The following English statement can be written in the form “if . . . , then . . .”. Yet in some cases there is an implied “only if”; that is, the converse is implied. Do you think that the following statement has an implied converse?

“If you have a dollar, then you can buy coffee from the vending machine.”

Solution:

If you don’t have a dollar, then you probably can’t buy coffee from the vending machine (unless the machine accepts a larger bill you might have). This proposition probably has an implied converse.

p.9, icon at Example 10

#9. The following English statement can be written in the form “if . . . , then . . .”. Yet in some cases there is an implied “only if”; that is, the converse is implied. Do you think that the following statement has an implied converse?

“You need a ticket in order to enter the theater.”

Solution:

The statement says that “If you don’t have a ticket, then you can’t enter the theater.” The converse is “If you do have a ticket, then you can enter the theater,” which is also probably true. Therefore the statement is “You can enter the theater if and only if you have a ticket,” so there is an implied converse.

p.11, icon at Example 12

#1. Suppose u represents “you understand the material”, s represents “you study the theory”, and w represents “you work on exercises”. Write the following compound proposition using u , s , w , and appropriate connectives.

“You study the theory and work on exercises, but you don’t understand the material.”

Solution:

This compound statement makes three assertions: you study the theory, you work on exercises, you don’t understand the material. The word “but” acts like “and”. Thus, the proposition is $s \wedge w \wedge \neg u$.

p.11, icon at Example 12

#2. Suppose u represents “you understand the material” and s represents “you study the theory”. Write the following compound proposition using u , s , and appropriate connectives.

“Studying the theory is sufficient for understanding the material.”

Solution:

We can rewrite the statement as “If you study the theory, then you understand the material,” or in symbols $s \rightarrow u$.

p.11, icon at Example 12

#3. Suppose s represents “you study the theory” and w represents “you work on exercises”. Write the following compound proposition using s , w , and appropriate connectives.

“In order to work on exercises, you need to study the theory.”

Solution:

The statement is “If you do not study the theory, then you cannot work on the exercises,” or $\neg s \rightarrow \neg w$. We can also rewrite this as $w \rightarrow s$.

p.11, icon at Example 12

#4. Suppose u represents “you understand the material”, s represents “you study the theory”, and w represents “you work on exercises”. Write the following compound proposition using u , s , w , and appropriate connectives.

“When you study the theory and work on exercises, you understand the material.”

Solution:

Using “if” for “when”, we have $s \wedge w \rightarrow u$.

p.11, icon at Example 12

#5. Suppose u represents “you understand the material”, s represents “you study the theory”, and w represents “you work on exercises”. Write the following compound proposition using u , s , w , and appropriate connectives.

“You don’t understand the material unless you study the theory and work on exercises.”

Solution:

The word “unless” conveys the meaning “if not”. We can rewrite the proposition as “If you do not study the theory and work on exercises, then you don’t understand the material,” which is $\neg(s \wedge w) \rightarrow \neg u$, or, equivalently, $u \rightarrow (s \wedge w)$.

p.12, icon at Example 14

#1. Translate this system specification into symbols:

“The online user is sent a notification of a link error if the network link is down.”

Solution:

The statement is equivalent to “If the network link is down, then the online user is sent a notification of a link error.” Using d for “the network link is down” and s for “the online user is sent a notification of a link error,” the statement becomes $d \rightarrow s$.

p.12, icon at Example 14

#2. Translate this system specification into symbols:

“Whenever the file is locked or the system is in executive clearance mode, the user cannot make changes in the data.”

Solution:

The statement is equivalent to “If the file is locked or the system is in executive clearance mode, the user cannot make changes in the data.” Using l for “the file is locked,” e for “the system is in executive clearance mode,” and u for “the user can make changes in the data,” the statement is $(l \vee e) \rightarrow \neg u$. Note that the parentheses are not necessary because the order of precedence of operations requires that the disjunction be

performed before the implication; thus we can also write $l \vee e \rightarrow \neg u$.

p.12, icon at Example 14

#3. Write these system specifications in symbols using the propositions

v : “The user enters a valid password,”

a : “Access is granted to the user,”

c : “The user has contacted the network administrator,”

and logical connectives. Then determine if the system specifications are consistent.

(i) “The user has contacted the network administrator, but does not enter a valid password.”

(ii) “Access is granted whenever the user has contacted the network administrator or enters a valid password.”

(iii) “Access is denied if the user has not entered a valid password or has not contacted the network administrator.”

Solution:

(i) The word “but” means “and”, so we have $c \wedge \neg v$.

(ii) The statement says that if either of two conditions is satisfied, then access is granted. Therefore we have $(c \vee v) \rightarrow a$.

(iii) In this case, if either of two negations happens, then access is not granted to the user. Therefore we have $(\neg v \vee \neg c) \rightarrow \neg a$.

In symbols, the three propositions are:

$$c \wedge \neg v$$

$$(c \vee v) \rightarrow a$$

$$(\neg v \vee \neg c) \rightarrow \neg a.$$

In order for the first proposition to be true, c must be true and v must be false. Because c is true, $c \vee v$ is true. Therefore, from the second proposition, a must be true.

But because v is false, $\neg v$ is true. Therefore $\neg v \vee \neg c$ is true. Therefore, the implication in the third proposition forces $\neg a$ to be true. Thus, a is false. But this contradicts that fact that a must be true. Therefore the three propositions are not consistent.

p.13, icon at Example 17

#1. How would you do a Boolean search for the appropriate Web pages for each of these:

(a) hotels in New England.

(b) hotels in England.

(c) hotels in England or New England.

Solution:

(a) We need to examine “hotels” and both “New” and “England”; that is, **HOTELS AND (NEW AND ENGLAND)**.

(b) To avoid getting hotels in New England, we use ENGLAND NOT NEW. Therefore we have HOTELS AND (ENGLAND NOT NEW).

(c) The two key words here are “hotels” and “England” (which will include both the country and the part of the United States). Therefore we can search for HOTELS AND ENGLAND.

p.13, icon at Example 18

#1. Suppose you have three cards: one red on both sides (red/red), one green on both sides (green/green), and one red on one side and green on the other side (red/green). The three cards are placed in a row on a table. Explain how to determine the identity of all three cards by selecting one card and turning it over.

Solution:

When the three cards are put in a row, exactly two of the three must have the same color showing — say red. Pick one of these two red cards and turn it over.

If the other side is also red, then you have found the red/red card. The other card with red showing must be the red/green card, and the card with green showing must be the green/green card.

If the card you turn over has green on the other side, you have located the red/green card. The other card with red showing must be the red/red card, and the card with green showing must be the green/green card.

(A similar procedure will determine the identity of the three cards, if two cards have green showing.)

p.13, icon at Example 18

#2. Another of Smullyan’s puzzles poses this problem. You meet two people, *A* and *B*. Each person either always tells the truth (i.e., the person is a knight) or always lies (i.e., the person is a knave). Person *A* tells you, “We are not both truth-tellers.”

Determine, if possible, which type of person each one is.

Solution:

One way to solve the problem is by considering each of the four possible cases: both lie, both tell the truth, *A* lies and *B* tells the truth, *A* tells the truth and *B* lies. We examine each of the four cases separately.

Both lie: That would mean that neither one is a truth-teller. The statement “We are not both truth-tellers” is true because they are not both truth-tellers. Thus *A* is telling the truth, which contradicts the assumption that both are liars. Therefore, this case cannot happen.

Both tell the truth: Therefore the statement “We are not both truth-tellers” is a lie. Thus *A* is a liar, contradicting the assumption that both are truth-tellers. Therefore, this case cannot happen.

A lies and B tells the truth: If *A* is a liar, *A*’s statement “We are not both truth-tellers” must be a lie. Therefore *A* and *B* must both be truth-tellers. But this contradicts the assumption that *A* is a liar. Therefore, this case cannot happen.

A tells the truth and B lies: In this case *A*’s statement “We are not both truth-tellers” is true and no contradiction is obtained. Therefore, this case is not ruled out.

Therefore, the only possibility is that *A* is a truth-teller and *B* is a liar.

Another way to solve the problem is to see where the assumption “ A is a liar” leads us. If we assume that A always lies, then A ’s statement “We are not both truth-tellers” must be false. Therefore both A and B must be truth-tellers. Therefore A ’s statement “We are not both truth-tellers” must be true, a contradiction of the fact that the statement is false.

This says that A is not a liar. Therefore A is a truth-teller. Therefore A ’s statement “We are not both truth-tellers” must be true. Because A is a truth-teller, A ’s statement forces B to be a liar.
