

Rosen, Discrete Mathematics and Its Applications, 6th edition
Extra Examples

Section 1.5—Rules of Inference

 — Page references correspond to locations of Extra Examples icons in the textbook.

p.67, icon at Example 6

#1. The proposition $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology, as the reader can check. It is the basis for the rule of inference *modus tollens*:

$$\begin{array}{r} \neg q \\ p \rightarrow q \\ \hline \therefore \neg p \end{array}$$

Suppose we are given the propositions: “If the class finishes Chapter 2, then they have a quiz” and “The class does not have a quiz.” Find a conclusion that can be drawn using modus tollens.

Solution:

Let p represent “The class finishes Chapter 2” and q represent “The class has a quiz.” According to modus tollens, because we have $\neg q$ and $p \rightarrow q$, we can conclude $\neg p$, or “The class did not finish Chapter 2.”

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#2. Suppose that “it is snowing” is true and that “it is windy” is true. Using the conjunction rule, what conclusion can be drawn?

Solution:

Using s for “it is snowing” and w for “it is windy,” we are given that s is true and w is true. By the conjunction rule, we can conclude $s \wedge w$, or “it is snowing and windy”.

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#3. Suppose “I have a dime or a quarter in my pocket” and “I do not have a dime in my pocket.” According to the disjunctive syllogism rule, what can we conclude?

Solution:

Using d for “I have a dime in my pocket” and q for “I have a quarter in my pocket”, we are given $d \vee q$ and $\neg d$. According to the disjunctive syllogism rule, we can conclude q , or “I have a quarter in my pocket.”

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#4. Determine whether this argument is valid by using a truth table:

I play golf or tennis.

If it is not Sunday, I play golf and tennis.
 If it is Saturday or Sunday, then I don't play golf.
 Therefore, I don't play golf.

Solution:

Using the variables:

g : I play golf
 t : I play tennis
 s : it is Saturday
 u : it is Sunday,

the argument can be written in symbols as:

$$\frac{\begin{array}{l} g \vee t \\ \neg u \rightarrow (g \wedge t) \\ (s \vee u) \rightarrow \neg g \end{array}}{\therefore \neg g}$$

We construct the truth table:

g	t	s	u	$g \vee t$	$\neg u \rightarrow (g \wedge t)$	$(s \vee u) \rightarrow \neg g$	$\neg g$
T	T	T	T	T	T	F	F
T	T	T	F	T	T	F	F
T	T	F	T	T	T	F	F
T	T	F	F	T	T	T	F
T	F	T	T	T	T	F	F
T	F	T	F	T	F	F	F
T	F	F	T	T	T	F	F
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	T	F	T	F	T	T
F	T	F	T	T	T	T	T
F	T	F	F	T	F	T	T
F	F	T	T	F	T	T	T
F	F	T	F	F	F	T	T
F	F	F	T	F	T	T	T
F	F	F	F	F	F	T	T

In the fourth row the three hypotheses (columns 5, 6, 7) are true and the conclusion is false. Therefore, the argument is not valid.

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#5. Determine whether this argument is valid:

Lynn works part time or full time.
 If Lynn does not play on the team, then she does not work part time.
 If Lynn plays on the team, she is busy.
 Lynn does not work full time.
 Therefore, Lynn is busy.

Solution:

Using the variables:

p : Lynn works part time
 f : Lynn works full time
 t : Lynn plays on the team
 b : Lynn is busy,

the argument can be written in symbols:

$$\begin{array}{l} p \vee f \\ \neg t \rightarrow \neg p \\ t \rightarrow b \\ \neg f \\ \hline \therefore b \end{array}$$

One method to find whether the argument is valid is to construct the truth table:

p	f	t	b	$p \vee f$	$\neg t \rightarrow \neg p$	$t \rightarrow b$	$\neg f$	b
T	T	T	T	T	T	T	F	T
T	T	T	F	T	T	F	F	F
T	T	F	T	T	F	T	F	T
T	T	F	F	T	F	T	F	F
T	F	T	T	T	T	T	T	T
T	F	T	F	T	T	F	T	F
T	F	F	T	T	F	T	T	T
T	F	F	F	T	F	T	T	F
F	T	T	T	T	T	T	F	T
F	T	T	F	T	T	F	F	F
F	T	F	T	T	T	T	F	T
F	T	F	F	T	T	T	F	F
F	F	T	T	F	T	T	T	T
F	F	T	F	F	T	F	T	F
F	F	F	T	F	T	T	T	T
F	F	F	F	F	T	T	T	F

We need to examine all cases where the hypotheses (columns 5, 6, 7, 8) are all true. There is only one case in which all four hypotheses are true (row 5), and in this case the conclusion is also true. Therefore, the argument is valid.

Alternately, rules of logic can be used to give a proof that the argument is valid. We begin with the four hypotheses and show how to derive the conclusion, b .

1. $p \vee f$ premise
2. $\neg t \rightarrow \neg p$ premise
3. $t \rightarrow b$ premise
4. $\neg f$ premise
5. p disjunctive syllogism on (1) and (4)
6. $p \rightarrow t$ contrapositive of (2)
7. t modus ponens on (5) and (6)
8. b modus ponens on (7) and (3)

Note: This method can provide a relatively quick way to verify that an argument is valid. It requires that we be clever enough to be able to chain together valid argument forms that lead from the hypotheses to the conclusion. But suppose we do not have any idea whether the argument is valid. The truth table method will always enable us to determine whether or not the argument is valid — either the conclusion is true whenever the hypotheses are all true (argument is valid), or else there is a case where the hypotheses are true but the

conclusion if false (argument is not valid). However, using the rules of logic cannot tell us that the argument is not valid. If we use this technique on an argument and are unable to reach the conclusion, that does not tell us that the argument is not valid — someone else might still be able to reach the conclusion, which would mean that the argument is valid.

p.68, icon at Example 8

#1. Suppose we have the two propositions (with symbols to represent them):

“It is raining (r) or I work in the yard (w)”

“It is not raining ($\neg r$) or I go to the library (l).”

What conclusion can we draw from these propositions?

Solution:

We can use the resolution rule of inference to draw a conclusion from these propositions. In symbols the two given propositions are $(r \vee w) \wedge (\neg r \vee l)$. From resolution we have $(r \vee w) \wedge (\neg r \vee l) \rightarrow (w \vee l)$. Therefore, we can draw the conclusion “I work in the yard or I go to the library.”

p.71, icon at Example 12

#1. Suppose we have:

“Every student in this class is a Junior.”

“Every Junior in this class passed the final exam.”

“Allen is a student in this class.”

Explain why we can draw the conclusion “Allen passed the final exam.”

Solution:

We will use $S(x)$ to mean “ x is a student in this class”, $J(x)$ to mean “ x is a Junior”, and $P(x)$ to mean “ x passed the final exam”, where the universe for x consists of all people. The proof is:

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|--|---------------------------------------|
| 1. $\forall x (S(x) \rightarrow J(x))$ | premise |
| 2. $\forall x (J(x) \rightarrow P(x))$ | premise |
| 3. $S(\text{Allen}) \rightarrow J(\text{Allen})$ | universal instantiation on (1) |
| 4. $J(\text{Allen}) \rightarrow P(\text{Allen})$ | universal instantiation on (2) |
| 5. $S(\text{Allen}) \rightarrow P(\text{Allen})$ | hypothetical syllogism on (3) and (4) |
| 6. $S(\text{Allen})$ | premise |
| 7. $P(\text{Allen})$ | modus ponens on (5) and (6) |
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