Rosen, Discrete Mathematics and Its Applications, 6th edition
Extra Examples

Section 9.1—Graphs and Graph Models

Extra Examples — *Page references correspond to locations of Extra Examples icons in the textbook.*

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p.592, icon at Example 1

#1. The relative strengths of the teams that compete against each other can be measured using a graph model.

For example, we can set up a graph G that models the National Football League season. The vertices of G are the NFL teams. We draw an edge from vertex x to vertex y if x played a game against y and x beat y. (If these two teams played a second game, we would add a second edge from x to y if x beat y a second time, and from y to x is y beat x.) If a game between x and y ended in a tie, we add an edge from x to y and an edge from y to x. The resulting digraph will have 32 vertices (one for each of the 32 NFL teams) and 256 edges (because each team plays 16 games). (The number of edges would be larger than 256 if tie games occurred during the season.)

We can use this digraph to measure the strength of the teams. For example, suppose teams A and B each have a final win-loss record of 10–6. (In graph theory terms, this means that the outdegree of each of the two vertices is 10 and the indegree is 6.) If team A played team B during the season and A beat B in their game, it might be reasonable to say that A is stronger than B. But suppose that A and B did not play each other. To decide which of the two teams might be considered the stronger team, we might look at the strength of the teams that each of these two teams beat. For example, A might have beaten "strong" teams (i.e., teams with many wins) and B might have beaten "weak" teams (i.e., teams with few wins). In this case we might conclude that team A is stronger than team B because A's ten wins are against stronger teams than those that B beat. In graph theory terms, we are examining paths of the form A–x–y ("A beat x who beat y"); these are "two-step wins" — paths of length 2.

Later in this chapter we will see that this large graph G can be described as a 32×32 matrix (called the adjacency matrix) and the two-step wins can be easily counted by examining terms of the square of this matrix.
