

Parallel Circuits

- A parallel circuit is any circuit that provides one common voltage across all components. Each component across the voltage source provides a separate path or branch for current flow. The individual branch currents are calculated as $\frac{V_A}{R}$ where V_A is the applied voltage and R is the individual branch resistance. The total current, I_T , supplied by the applied voltage, must equal the sum of all individual branch currents.

The equivalent resistance of a parallel circuit equals the applied voltage, V_A , divided by the total current, I_T . The term equivalent resistance refers to a single resistance that would draw the same amount of current as all of the parallel connected branches. The equivalent resistance of a parallel circuit is designated R_{EQ} .

This chapter covers all of the characteristics of parallel circuits, including important information about how to troubleshoot a parallel circuit containing a defective component.

Outline

- 5-1 The Applied Voltage V_A Is the Same across Parallel Branches
- 5-2 Each Branch I Equals V_A/R
- 5-3 Kirchhoff's Current Law (KCL)
- 5-4 Resistances in Parallel
- 5-5 Conductances in Parallel
- 5-6 Total Power in Parallel Circuits
- 5-7 Analyzing Parallel Circuits with Random Unknowns
- 5-8 Troubleshooting: Opens and Shorts in Parallel Circuits

Objectives

After studying this chapter you should be able to

- *Explain* why voltage is the same across all branches in a parallel circuit.
- *Calculate* the individual branch currents in a parallel circuit.
- *Calculate* the total current in a parallel circuit using Kirchhoff's current law.
- *Calculate* the equivalent resistance of two or more resistors in parallel.
- *Explain* why the equivalent resistance of a parallel circuit is always less than the smallest branch resistance.
- *Calculate* the total conductance of a parallel circuit.
- *Calculate* the total power in a parallel circuit.
- *Solve* for the voltage, current, power, and resistance in a parallel circuit having random unknowns.
- *Describe* the effects of an open and short in a parallel circuit.
- *Troubleshoot* parallel circuits containing opens and shorts.

Important Terms

equivalent resistance, R_{EQ}
Kirchhoff's current law (KCL)
main line

parallel bank
reciprocal resistance formula

5-1 The Applied Voltage V_A Is the Same across Parallel Branches

GOOD TO KNOW

Components can be connected in parallel even if they are not connected to a voltage source.

A parallel circuit is formed when two or more components are connected across a voltage source, as shown in Fig. 5-1. In this figure, R_1 and R_2 are in parallel with each other and a 1.5-V battery. In Fig. 5-1b, the points A, B, C, and E are equivalent to a direct connection at the positive terminal of the battery because the connecting wires have practically no resistance. Similarly, points H, G, D, and F are the same as a direct connection at the negative battery terminal. Since R_1 and R_2 are directly connected across the two terminals of the battery, both resistances must have the same potential difference as the battery. It follows that the voltage is the same across components connected in parallel. The parallel circuit arrangement is used, therefore, to connect components that require the same voltage.

A common application of parallel circuits is typical house wiring to the power line, with many lights and appliances connected across the 120-V source (Fig. 5-2). The wall receptacle has a potential difference of 120 V across each pair of terminals. Therefore, any resistance connected to an outlet has an applied voltage of 120 V. The lightbulb is connected to one outlet and the toaster to another outlet, but both have the same applied voltage of 120 V. Therefore, each operates independently of any other appliance, with all the individual branch circuits connected across the 120-V line.

MultiSim Figure 5-1 Example of a parallel circuit with two resistors. (a) Wiring diagram. (b) Schematic diagram.

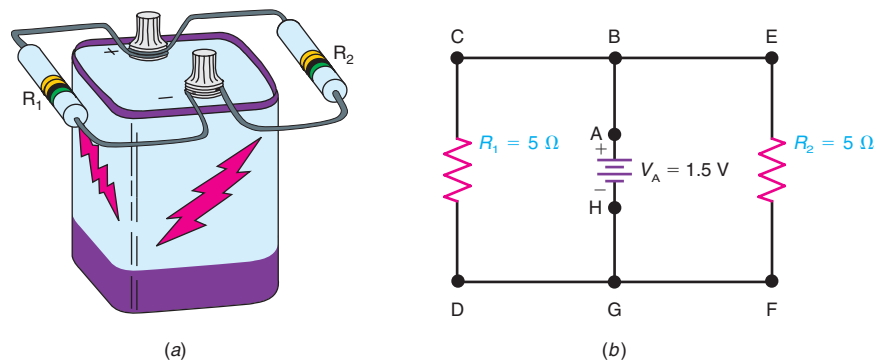
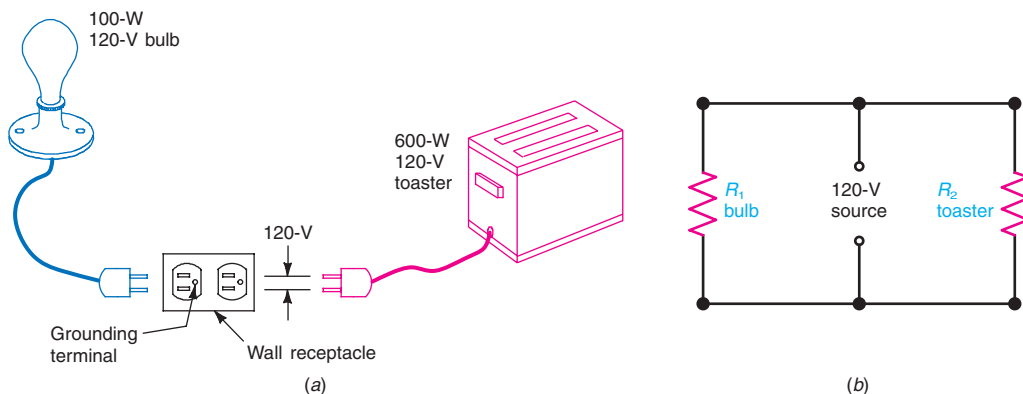


Figure 5-2 Lightbulb and toaster connected in parallel with the 120-V line. (a) Wiring diagram. (b) Schematic diagram.



■ 5-1 Knowledge Check

Answer at end of chapter.

If another $5\text{-}\Omega$ resistor, R_3 , is connected across points E and F in Fig. 5-1b, how much is its voltage?

■ 5-1 Self-Review

Answers at end of chapter.

- In Fig. 5-1, how much is the common voltage across R_1 and R_2 ?
- In Fig. 5-2, how much is the common voltage across the bulb and the toaster?
- How many parallel branch circuits are connected across the voltage source in Figs. 5-1 and 5-2?

GOOD TO KNOW

In a parallel circuit, the branch with the lowest resistance always has the most current. This must be true since each branch current is calculated as $\frac{V_A}{R}$ where V_A is the same across all branches.

5-2 Each Branch / Equals V_A/R

In applying Ohm's law, it is important to note that the current equals the voltage applied across the circuit divided by the resistance between the two points where that voltage is applied. In Fig. 5-3a, 10 V is applied across the $5\text{ }\Omega$ of R_2 , resulting in the current of 2 A between points E and F through R_2 . The battery voltage is also applied across the parallel resistance of R_1 , applying 10 V across $10\text{ }\Omega$. Through R_1 , therefore, the current is 1 A between points C and D. The current has a different value through R_1 , with the same applied voltage, because the resistance is different. These values are calculated as follows:

$$I_1 = \frac{V_A}{R_1} = \frac{10}{10} = 1\text{ A}$$

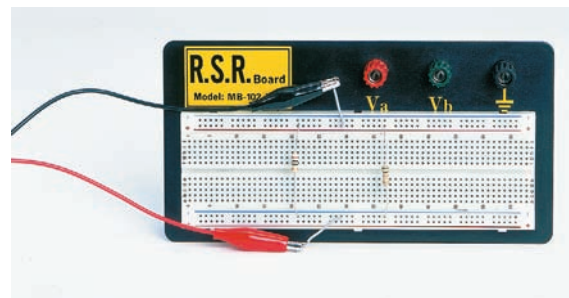
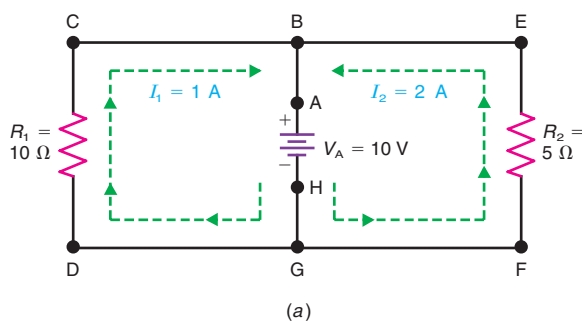
$$I_2 = \frac{V_A}{R_2} = \frac{10}{5} = 2\text{ A}$$

Figure 5-3b shows how to assemble axial-lead resistors on a lab prototype board to form a parallel circuit.

Just as in a circuit with one resistance, any branch that has less R allows more I . If R_1 and R_2 were equal, however, the two branch currents would have the same value. For instance, in Fig. 5-1b each branch has its own current equal to $1.5\text{ V}/5\text{ }\Omega = 0.3\text{ A}$.

The I can be different in parallel circuits that have different R because V is the same across all the branches. Any voltage source generates a potential difference across its two terminals. This voltage does not move. Only I flows around the circuit. The source voltage is available to make electrons move

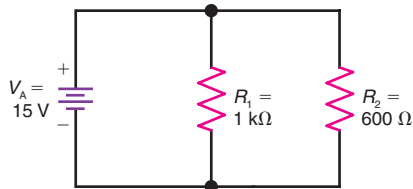
Figure 5-3 Parallel circuit. (a) The current in each parallel branch equals the applied voltage V_A divided by each branch resistance R . (b) Axial-lead resistors assembled on a lab prototype board, forming a parallel circuit.



around any closed path connected to the terminals of the source. The amount of I in each separate path depends on the amount of R in each branch.

Example 5-1

MultiSim Figure 5-4 Circuit for Example 5-1.



In Fig. 5-4, solve for the branch currents I_1 and I_2 .

ANSWER The applied voltage, V_A , of 15 V is across both resistors R_1 and R_2 . Therefore, the branch currents are calculated as $\frac{V_A}{R}$, where V_A is the applied voltage and R is the individual branch resistance.

$$\begin{aligned} I_1 &= \frac{V_A}{R_1} \\ &= \frac{15 \text{ V}}{1 \text{ k}\Omega} \\ &= 15 \text{ mA} \\ I_2 &= \frac{V_A}{R_2} \\ &= \frac{15 \text{ V}}{600 \Omega} \\ &= 25 \text{ mA} \end{aligned}$$

■ 5-2 Knowledge Check

Answer at end of chapter.

If a 2- Ω resistor, R_3 is connected across points B and G in Fig. 5-3a, how much is its branch current?

■ 5-2 Self-Review

Answers at end of chapter.

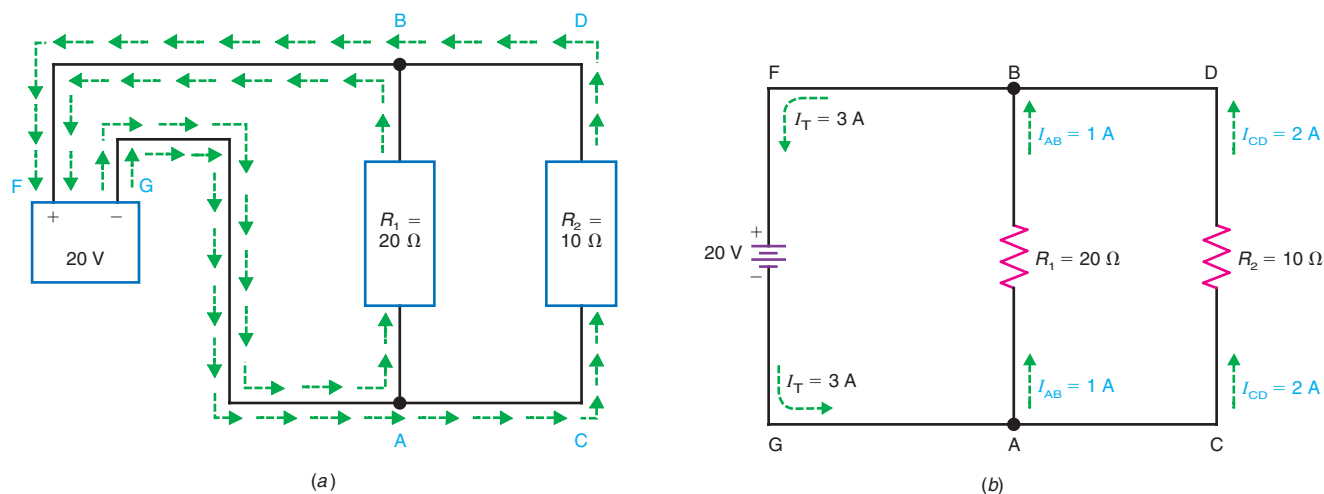
Refer to Fig. 5-3.

- How much is the voltage across R_1 ?
- How much is I_1 through R_1 ?
- How much is the voltage across R_2 ?
- How much is I_2 through R_2 ?

5-3 Kirchhoff's Current Law (KCL)

Components to be connected in parallel are usually wired directly across each other, with the entire parallel combination connected to the voltage source, as illustrated in Fig. 5-5. This circuit is equivalent to wiring each parallel branch directly to the voltage source, as shown in Fig. 5-1, when the connecting wires have essentially zero resistance.

Figure 5–5 The current in the main line equals the sum of the branch currents. Note that from G to A at the bottom of this diagram is the negative side of the main line, and from B to F at the top is the positive side. (a) Wiring diagram. Arrows inside the lines indicate current in the main line for R_1 ; arrows outside indicate current for R_2 . (b) Schematic diagram. I_T is the total line current for both R_1 and R_2 .



The advantage of having only one pair of connecting leads to the source for all the parallel branches is that usually less wire is necessary. The pair of leads connecting all the branches to the terminals of the voltage source is the **main line**. In Fig. 5–5, the wires from G to A on the negative side and from B to F in the return path form the main line.

In Fig. 5–5b, with $20\ \Omega$ of resistance for R_1 connected across the 20-V battery, the current through R_1 must be $20\text{ V}/20\ \Omega = 1\text{ A}$. This current is electron flow from the negative terminal of the source, through R_1 , and back to the positive battery terminal. Similarly, the R_2 branch of $10\ \Omega$ across the battery has its own branch current of $20\text{ V}/10\ \Omega = 2\text{ A}$. This current flows from the negative terminal of the source, through R_2 , and back to the positive terminal, since it is a separate path for electron flow.

All current in the circuit, however, must come from one side of the voltage source and return to the opposite side for a complete path. In the main line, therefore, the amount of current is equal to the total of the branch currents.

For example, in Fig. 5–5b, the total current in the line from point G to point A is 3 A. The total current at branch point A subdivides into its component branch currents for each of the branch resistances. Through the path of R_1 from A to B the current is 1 A. The other branch path ACDB through R_2 has a current of 2 A. At the branch point B, the electron flow from both parallel branches combines, so that the current in the main-line return path from B to F has the same value of 3 A as in the other side of the main line.

Kirchhoff's current law (KCL) states that the total current I_T in the main line in a parallel circuit equals the sum of the individual branch currents. Expressed as an equation, Kirchhoff's current law is:

$$I_T = I_1 + I_2 + I_3 + \cdots + \text{etc.} \quad (5-1)$$

where I_T is the total current and $I_1, I_2, I_3 \dots$ are the individual branch currents. Kirchhoff's current law applies to any number of **parallel branches**, whether the resistances in the branches are equal or unequal.

GOOD TO KNOW

As more branches are added to a parallel circuit the total current, I_T , increases.

Example 5-2

MultiSim

An R_1 of $20\ \Omega$, an R_2 of $40\ \Omega$, and an R_3 of $60\ \Omega$ are connected in parallel across the 120-V power line. Using Kirchhoff's current law, determine the total current I_T .

ANSWER Current I_1 for the R_1 branch is $120/20$ or 6 A. Similarly, I_2 is $120/40$ or 3 A, and I_3 is $120/60$ or 2 A. The total current in the main line is

$$I_T = I_1 + I_2 + I_3 = 6 + 3 + 2$$

$$I_T = 11\ \text{A}$$

Example 5-3

Two branches R_1 and R_2 across the 120-V power line draw a total line current I_T of 15 A. The R_1 branch takes 10 A. How much is the current I_2 in the R_2 branch?

ANSWER $I_2 = I_T - I_1 = 15 - 10$

$$I_2 = 5\ \text{A}$$

With two branch currents, one must equal the difference between I_T and the other branch current.

Example 5-4

Three parallel branch currents are 0.1 A, 500 mA, and $800\ \mu\text{A}$. Using Kirchhoff's current law, calculate I_T .

ANSWER All values must be in the same units to be added. In this case, all units will be converted to milliamperes: $0.1\ \text{A} = 100\ \text{mA}$ and $800\ \mu\text{A} = 0.8\ \text{mA}$. Applying Kirchhoff's current law

$$I_T = 100 + 500 + 0.8$$

$$I_T = 600.8\ \text{mA}$$

You can convert the currents to A, mA, or μA units, as long as the same unit is used for adding all currents.

5-3 Knowledge Check

Answer at end of chapter.

A parallel circuit has the following branch currents: $I_1 = 1.5$ A, $I_2 = 350$ mA, $I_3 = 100$ mA, and $I_4 = 50$ mA. How much is I_T ?

5-3 Self-Review

Answers at end of chapter.

- Branch currents in a parallel circuit are 1 A for I_1 , 2 A for I_2 , and 3 A for I_3 . How much is I_T ?
- Assume $I_T = 6$ A for three branch currents; I_1 is 1 A, and I_2 is 2 A. How much is I_3 ?
- Branch currents in a parallel circuit are 1 A for I_1 and 200 mA for I_2 . How much is I_T ?

5-4 Resistances in Parallel

The combined equivalent resistance across the main line in a parallel circuit can be found by Ohm's law: *Divide the common voltage across the parallel resistances by the total current of all the branches.* Referring to Fig. 5-6a, note that the parallel resistance of R_1 with R_2 , indicated by the equivalent resistance R_{EQ} , is the opposition to the total current in the main line. In this example, V_A/I_T is 60 V/ 3 A = 20 Ω for R_{EQ} .

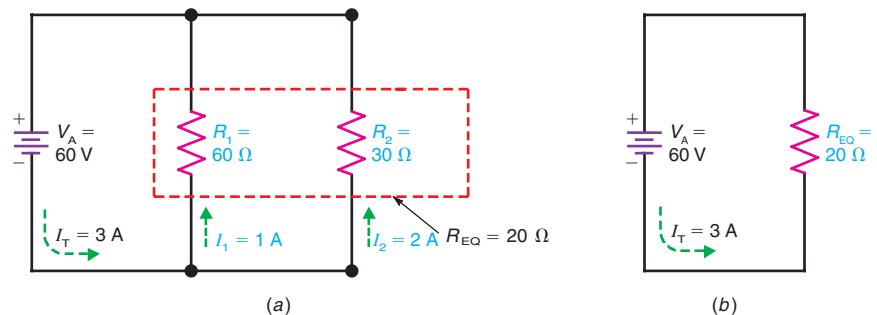
The total load connected to the source voltage is the same as though one equivalent resistance of 20 Ω were connected across the main line. This is illustrated by the equivalent circuit in Fig. 5-6b. For any number of parallel resistances of any value, use the following equation,

$$R_{EQ} = \frac{V_A}{I_T} \quad (5-2)$$

where I_T is the sum of all the branch currents and R_{EQ} is the equivalent resistance of all parallel branches across the applied voltage source V_A .

The first step in solving for R_{EQ} is to add all the parallel branch currents to find the I_T being delivered by the voltage source. The voltage source thinks that it is connected to a single resistance whose value allows I_T to flow in the circuit according to Ohm's law. This single resistance is R_{EQ} . An illustrative example of a circuit with two parallel branches will be used to show how R_{EQ} is calculated.

MultiSim Figure 5-6 Resistances in parallel. (a) Combination of R_1 and R_2 is the total R_{EQ} for the main line. (b) Equivalent circuit showing R_{EQ} drawing the same 3-A I_T as the parallel combination of R_1 and R_2 in (a).



GOOD TO KNOW

The statement "current always takes the path of least resistance" is not always true. If it were, all the current in a parallel circuit would flow in the lowest branch resistance only.

Example 5-5

Two branches, each with a 5-A current, are connected across a 90-V source. How much is the equivalent resistance R_{EQ} ?

ANSWER The total line current I_T is $5 + 5 = 10$ A. Then,

$$R_{EQ} = \frac{V_A}{I_T} = \frac{90}{10}$$
$$R_{EQ} = 9 \Omega$$

Parallel Bank

A combination of parallel branches is often called a **bank**. In Fig. 5-6, the bank consists of the $60\text{-}\Omega$ R_1 and $30\text{-}\Omega$ R_2 in parallel. Their combined parallel resistance R_{EQ} is the bank resistance, equal to 20Ω in this example. A bank can have two or more parallel resistors.

When a circuit has more current with the same applied voltage, this greater value of I corresponds to less R because of their inverse relation. Therefore, the combination of parallel resistances R_{EQ} for the bank is always less than the smallest individual branch resistance. The reason is that I_T must be more than any one branch current.

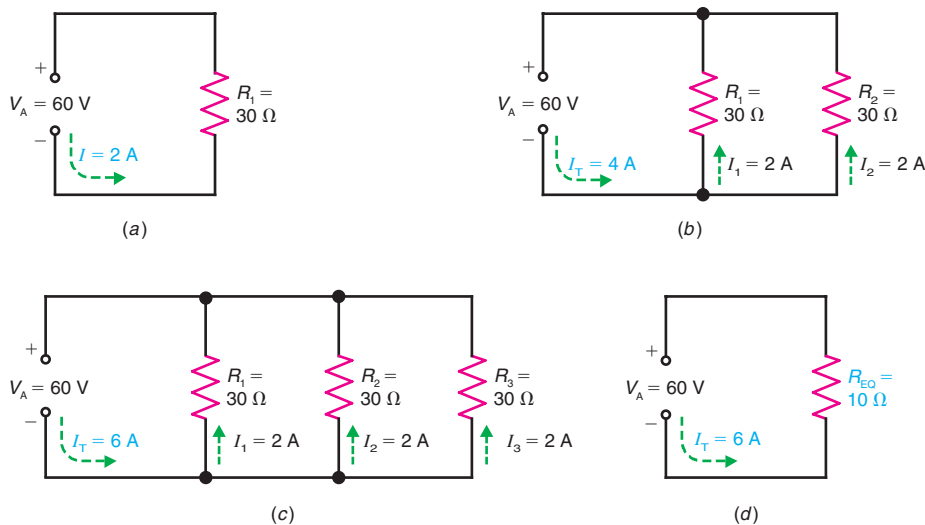
Why R_{EQ} Is Less than Any Branch R

It may seem unusual at first that putting more resistance into a circuit lowers the equivalent resistance. This feature of parallel circuits is illustrated in Fig. 5-7. Note that equal resistances of 30Ω each are added across the source voltage, one branch at a time. The circuit in Fig. 5-7a has just R_1 , which allows 2 A with 60 V applied. In Fig. 5-7b, the R_2 branch is added across the same V_A .

GOOD TO KNOW

Assume two resistors are connected in parallel. If one of the two resistors has a value ten or more times larger than the other, the equivalent resistance, R_{EQ} , is approximately equal to the value of the smaller resistor.

Figure 5-7 How adding parallel branches of resistors increases I_T but decreases R_{EQ} . (a) One resistor. (b) Two branches. (c) Three branches. (d) Equivalent circuit of the three branches in (c).



CALCULATOR

When using the calculator to find a reciprocal such as $1/R$, choose either of two methods. Either divide the number 1 by the value of R , or use the reciprocal key labeled $1/x$. As an example, to find the reciprocal of $R = 20\ \Omega$ by division:

- First punch in the number 1 on the key pad.
- Then press the division \div key.
- Punch in 20 for the value of R .
- Finally, press the equal $=$ key for the quotient of 0.05 on the display.
- To use the reciprocal key, first punch in 20 for R . Then press the $(1/x)$ key. This may be a second function on some calculators, requiring that you push the $(2^{nd}F)$ or $(SHIFT)$ key before pressing $(1/x)$. The reciprocal equal to 0.05 is displayed without the need for the $=$ key.

This branch also has 2 A. Now the parallel circuit has a 4-A total line current because of $I_1 + I_2$. Then the third branch, which also takes 2 A for I_3 , is added in Fig. 5-7c. The combined circuit with three branches, therefore, requires a total load current of 6 A, which is supplied by the voltage source.

The combined resistance across the source, then, is V_A/I_T , which is $60/6$, or $10\ \Omega$. This equivalent resistance R_{EQ} , representing the entire load on the voltage source, is shown in Fig. 5-7d. More resistance branches reduce the combined resistance of the parallel circuit because more current is required from the same voltage source.

Reciprocal Resistance Formula

We can derive the **reciprocal resistance formula** from the fact that I_T is the sum of all the branch currents, or,

$$I_T = I_1 + I_2 + I_3 + \cdots + \text{etc.}$$

However, $I_T = V/R_{EQ}$. Also, each $I = V/R$. Substituting V/R_{EQ} for I_T on the left side of the formula and V/R for each branch I on the right side, the result is

$$\frac{V}{R_{EQ}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots + \text{etc.}$$

Dividing by V because the voltage is the same across all the resistances gives us:

$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \text{etc.}$$

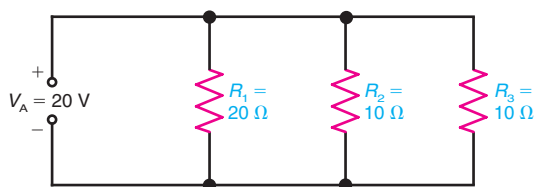
Next, solve for R_{EQ} .

$$R_{EQ} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3 + \cdots + \text{etc.}} \quad (5-3)$$

This reciprocal formula applies to any number of parallel resistances of any value. Using the values in Fig. 5-8a as an example,

$$R_{EQ} = \frac{1}{1/20 + 1/10 + 1/10} = 4\ \Omega$$

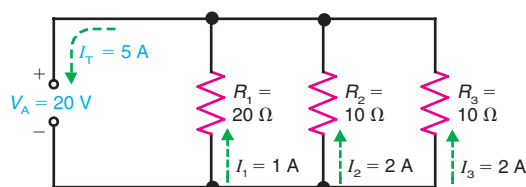
MultiSim **Figure 5-8** Two methods of combining parallel resistances to find R_{EQ} . (a) Using the reciprocal resistance formula to calculate R_{EQ} as $4\ \Omega$. (b) Using the total line current method with an assumed line voltage of 20 V gives the same $4\ \Omega$ for R_{EQ} .



$$R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$R_{EQ} = 4\ \Omega$$

(a)

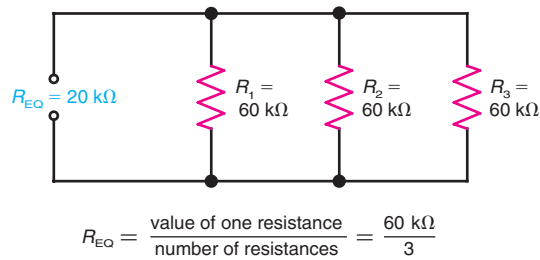


$$R_{EQ} = \frac{V_A}{I_T} = \frac{20\ \text{V}}{5\ \text{A}}$$

$$R_{EQ} = 4\ \Omega$$

(b)

Figure 5–9 For the special case of all branches having the same resistance, just divide R by the number of branches to find R_{EQ} . Here, $R_{EQ} = 60 \text{ k}\Omega/3 = 20 \text{ k}\Omega$.



Total-Current Method

It may be easier to work without fractions. Figure 5–8b shows how this same problem can be calculated in terms of total current instead of by the reciprocal formula. Although the applied voltage is not always known, any convenient value can be assumed because it cancels in the calculations. It is usually simplest to assume an applied voltage of the same numerical value as the highest resistance. Then one assumed branch current will automatically be 1 A and the other branch currents will be more, eliminating fractions less than 1 in the calculations.

In Fig. 5–8b, the highest branch R is 20Ω . Therefore, assume 20 V for the applied voltage. Then the branch currents are 1 A in R_1 , 2 A in R_2 , and 2 A in R_3 . Their sum is $1 + 2 + 2 = 5 \text{ A}$ for I_T . The combined resistance R_{EQ} across the main line is V_A/I_T , or $20 \text{ V}/5 \text{ A} = 4 \Omega$. This is the same value calculated with the reciprocal resistance formula.

Special Case of Equal R in All Branches

If R is equal in all branches, the combined R_{EQ} equals the value of one branch resistance divided by the number of branches.

$$R_{EQ} = \frac{R}{n}$$

where R is the resistance in one branch and n is the number of branches.

This rule is illustrated in Fig. 5–9, where three 60-k Ω resistances in parallel equal 20 k Ω .

The rule applies to any number of parallel resistances, but they must all be equal. As another example, five 60- Ω resistances in parallel have the combined resistance of $60/5$, or 12 Ω . A common application is two equal resistors wired in a parallel bank for R_{EQ} equal to one-half R .

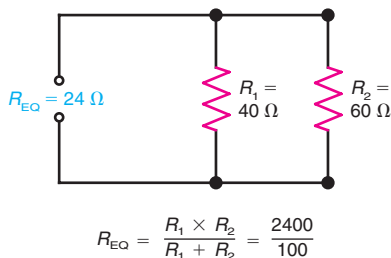
Special Case of Only Two Branches

When there are two parallel resistances and they are not equal, it is usually quicker to calculate the combined resistance by the method shown in Fig. 5–10. This rule says that the combination of two parallel resistances is their product divided by their sum.

$$R_{EQ} = \frac{R_1 \times R_2}{R_1 + R_2} \quad (5-4)$$

where R_{EQ} is in the same units as all the individual resistances. For the example in Fig. 5–10,

Figure 5–10 For the special case of only two branch resistances, of any values, R_{EQ} equals their product divided by the sum. Here, $R_{EQ} = 2400/100 = 24\Omega$.



CALCULATOR

Formula (5-4) states a product over a sum. When using a calculator, group the R values in parentheses before dividing. The reason is that the division bar is a mathematical sign of grouping for terms to be added or subtracted. You must add $R_1 + R_2$ before dividing. By grouping R_1 , and R_2 within parentheses, the addition will be done first before the division. The complete process is as follows.

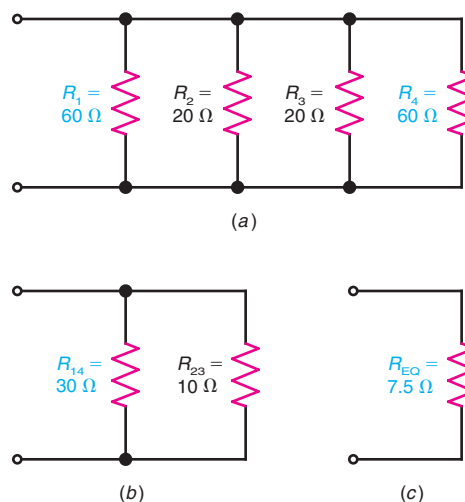
Multiply the R values in the numerator. Press the divide (\div) key and then the left (or open) parentheses ($($) key. Add the R values, $R_1 + R_2$, and press the right (or close) parentheses ($)$ key. Then press the equal ($=$) key for R_{EQ} on the display. Using the values in Fig. 5-10 as an example, multiply 40×60 ; press divide (\div) and left parentheses ($($) then $40 \oplus 60$ and the right parentheses ($)$. Finally, press ($=$) to display 24 as the answer.

GOOD TO KNOW

For more than two resistors connected in parallel, the value of an unknown resistance can be calculated using the following formula:

$$R_X = \frac{1}{1/R_{EQ} - 1/R_1 - 1/R_2 - \dots \text{etc.}}$$

Figure 5-11 An example of parallel resistance calculations with four branches. (a) Original circuit. (b) Resistors combined into two branches. (c) Equivalent circuit reduces to one R_{EQ} for all the branches.



$$R_{EQ} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{40 \times 60}{40 + 60} = \frac{2400}{100}$$

$$R_{EQ} = 24 \Omega$$

Each R can have any value, but there must be only two resistances.

Short-Cut Calculations

Figure 5-11 shows how these special rules can help in reducing parallel branches to a simpler equivalent circuit. In Fig. 5-11a, the $60\text{-}\Omega$ R_1 and R_4 are equal and in parallel. Therefore, they are equivalent to the $30\text{-}\Omega$ R_{14} in Fig. 5-11b. Similarly, the $20\text{-}\Omega$ R_2 and R_3 are equivalent to the $10\text{-}\Omega$ R_{23} . The circuit in Fig. 5-11a is equivalent to the simpler circuit in Fig. 5-11b with just the two parallel resistances of 30 and $10\text{-}\Omega$.

Finally, the combined resistance for these two equals their product divided by their sum, which is $300/40$ or $7.5\text{-}\Omega$, as shown in Fig. 5-11c. This value of R_{EQ} in Fig. 5-11c is equivalent to the combination of the four branches in Fig. 5-11a. If you connect a voltage source across either circuit, the current in the main line will be the same for both cases.

The order of connections for parallel resistances does not matter in determining R_{EQ} . There is no question as to which is first or last because they are all across the same voltage source and receive their current at the same time.

Finding an Unknown Branch Resistance

In some cases with two parallel resistors, it is useful to be able to determine what size R_X to connect in parallel with a known R to obtain a required value of R_{EQ} . Then the factors can be transposed as follows:

$$R_X = \frac{R \times R_{EQ}}{R - R_{EQ}} \quad (5-5)$$

This formula is just another way of writing Formula (5-4).

Example 5-6

What R_X in parallel with $40\ \Omega$ will provide an R_{EQ} of $24\ \Omega$?

ANSWER $R_X = \frac{R \times R_{EQ}}{R - R_{EQ}} = \frac{40 \times 24}{40 - 24} = \frac{960}{16}$
 $R_X = 60\ \Omega$

This problem corresponds to the circuit shown before in Fig. 5-10.

Note that Formula (5-5) for R_X has a product over a difference. The R_{EQ} is subtracted because it is the smallest R . Remember that both Formulas (5-4) and (5-5) can be used with only two parallel branches.

Example 5-7

What R in parallel with $50\ \text{k}\Omega$ will provide an R_{EQ} of $25\ \text{k}\Omega$?

ANSWER $R = 50\ \text{k}\Omega$

Two equal resistances in parallel have R_{EQ} equal to one-half R .

■ 5-4A Knowledge Check

Answer at end of chapter.

A $1.2\text{-k}\Omega$ resistor, R_1 , is in parallel with a $6.8\text{-k}\Omega$ R_2 . How much is R_{EQ} ?

■ 5-4B Knowledge Check

Answer at end of chapter.

The following resistors are in parallel. $R_1 = 150\ \Omega$, $R_2 = 60\ \Omega$, $R_3 = 100\ \Omega$ and $R_4 = 120\ \Omega$. How much is R_{EQ} ?

■ 5-4C Knowledge Check

Answer at end of chapter.

How much resistance, R_X , must be connected in parallel with a $3.3\text{-k}\Omega$ resistor to obtain an equivalent resistance of $1.32\ \text{k}\Omega$?

■ 5-4 Self-Review

Answers at end of chapter.

- Find R_{EQ} for three $4.7\text{-M}\Omega$ resistances in parallel.
- Find R_{EQ} for $3\ \text{M}\Omega$ in parallel with $2\ \text{M}\Omega$.
- Find R_{EQ} for two parallel $20\text{-}\Omega$ resistances in parallel with $10\ \Omega$.

5-5 Conductances in Parallel

Since conductance G is equal to $1/R$, the reciprocal resistance Formula (5-3) can be stated for conductance as $R_{EQ} = \frac{1}{G_T}$ where G_T is calculated as

$$G_T = G_1 + G_2 + G_3 + \cdots + \text{etc.} \quad (5-6)$$

With R in ohms, G is in siemens. For the example in Fig. 5-12, G_1 is $1/20 = 0.05$, G_2 is $1/5 = 0.2$, and G_3 is $1/2 = 0.5$. Then

$$G_T = 0.05 + 0.2 + 0.5 = 0.75 \text{ S}$$

Notice that adding the conductances does not require reciprocals. Each value of G is the reciprocal of R .

The reason why *parallel conductances* are added directly can be illustrated by assuming a 1-V source across all branches. Then calculating the values of $1/R$ for the conductances gives the same values as calculating the branch currents. These values are added for the total I_T or G_T .

Working with G may be more convenient than working with R in parallel circuits, since it avoids the use of the reciprocal formula for R_{EQ} . Each branch current is directly proportional to its conductance. This idea corresponds to the fact that each voltage drop in series circuits is directly proportional to each of the series resistances. An example of the currents for parallel conductances is shown in Fig. 5-13. Note that the branch with G of 4 S has twice as much current as the 2-S branches because the branch conductance is doubled.

■ 5-5A Knowledge Check

Answer at end of chapter.

How much is the equivalent resistance, R_{EQ} , in Fig. 5-12?

■ 5-5B Knowledge Check

Answer at end of chapter.

If a 4- Ω resistor, R_4 , is added in parallel with R_1 , R_2 , and R_3 in Fig. 5-12, how much is G_T ? What about R_{EQ} ?

Figure 5-12 Conductances G_1 , G_2 , and G_3 in parallel are added for the total G_T .

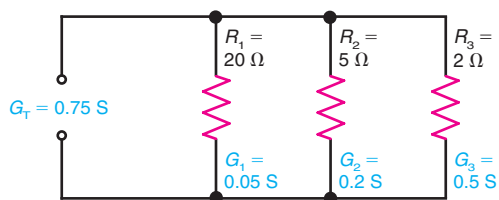
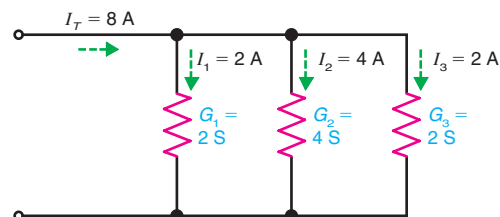


Figure 5-13 Example of how parallel branch currents are directly proportional to each branch conductance G .



■ 5-5 Self-Review

Answers at end of chapter.

- If G_1 is 2 S and G_2 in parallel is 4 S, calculate G_T .
- If G_1 is $0.05\ \mu\text{S}$, G_2 is $0.2\ \mu\text{S}$, and G_3 is $0.5\ \mu\text{S}$, all in parallel, find G_T and its equivalent R_{EQ} .
- If G_T is $4\ \mu\text{S}$ for a parallel circuit, how much is R_{EQ} ?

5-6 Total Power in Parallel Circuits

Since the power dissipated in the branch resistances must come from the voltage source, the **total power** equals the sum of the individual values of power in each branch. This rule is illustrated in Fig. 5-14. We can also use this circuit as an example of applying the rules of current, voltage, and resistance for a parallel circuit.

The applied 10 V is across the $10\text{-}\Omega$ R_1 and $5\text{-}\Omega$ R_2 in Fig. 5-14. The branch current I_1 then is V_A/R_1 or $10/10$, which equals 1 A. Similarly, I_2 is $10/5$, or 2 A. The total I_T is $1 + 2 = 3$ A. If we want to find R_{EQ} , it equals V_A/I_T or $10/3$, which is $3\frac{1}{3}\ \Omega$.

The power dissipated in each branch R is $V_A \times I$. In the R_1 branch, I_1 is $10/10 = 1$ A. Then P_1 is $V_A \times I_1$ or $10 \times 1 = 10$ W.

For the R_2 branch, I_2 is $10/5 = 2$ A. Then P_2 is $V_A \times I_2$ or $10 \times 2 = 20$ W.

Adding P_1 and P_2 , the answer is $10 + 20 = 30$ W. This P_T is the total power dissipated in both branches.

This value of 30 W for P_T is also the total power supplied by the voltage source by means of its total line current I_T . With this method, the total power is $V_A \times I_T$ or $10 \times 3 = 30$ W for P_T . The 30 W of power supplied by the voltage source is dissipated or used up in the branch resistances.

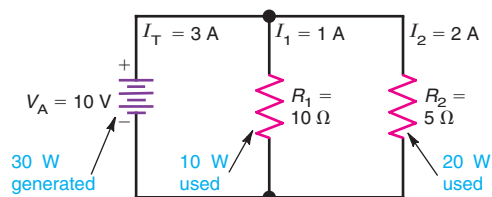
It is interesting to note that in a parallel circuit, the smallest branch resistance will always dissipate the most power. Since $P = \frac{V^2}{R}$ and V is the same across all parallel branches, a smaller value of R in the denominator will result in a larger amount of power dissipation.

Note also that in both parallel and series circuits, the sum of the individual values of power dissipated in the circuit equals the total power generated by the source. This can be stated as a formula

$$P_T = P_1 + P_2 + P_3 + \cdots + \text{etc.} \quad (5-7)$$

The series or parallel connections can alter the distribution of voltage or current, but power is the rate at which energy is supplied. The circuit arrangement cannot change the fact that all the energy in the circuit comes from the source.

Figure 5-14 The sum of the power values P_1 and P_2 used in each branch equals the total power P_T produced by the source.



■ 5-6 Knowledge Check

Answer at end of chapter.

In Fig. 5-14 assume V_A is increased to 20 V. What are the new values for P_1 , P_2 , and P_T ?

■ 5-6 Self-Review

Answers at end of chapter.

- Two parallel branches each have 2 A at 120 V. How much is P_T ?
- Three parallel branches of 10, 20, and 30 Ω have 60 V applied. How much is P_T ?
- Two parallel branches dissipate a power of 15 W each. How much is P_T ?

5-7 Analyzing Parallel Circuits with Random Unknowns

For many types of problems with parallel circuits, it is useful to remember the following points.

- When you know the voltage across one branch, this voltage is across all the branches. There can be only one voltage across branch points with the same potential difference.
- If you know I_T and one of the branch currents I_1 , you can find I_2 by subtracting I_1 from I_T .

The circuit in Fig. 5-15 illustrates these points. The problem is to find the applied voltage V_A and the value of R_3 . Of the three branch resistances, only R_1 and R_2 are known. However, since I_2 is given as 2 A, the $I_2 R_2$ voltage must be $2 \times 60 = 120$ V.

Although the applied voltage is not given, this must also be 120 V. The voltage across all the parallel branches is the same 120 V that is across the R_2 branch.

Now I_1 can be calculated as V_A/R_1 . This is $120/30 = 4$ A for I_1 .

Current I_T is given as 7 A. The two branches take $2 + 4 = 6$ A. The third branch current through R_3 must be $7 - 6 = 1$ A for I_3 .

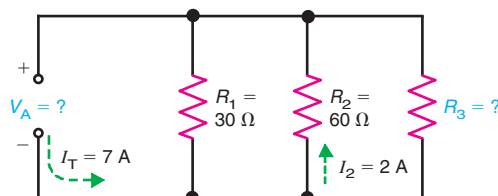
Now R_3 can be calculated as V_A/I_3 . This is $120/1 = 120 \Omega$ for R_3 .

■ 5-7 Knowledge Check

Answer at end of chapter.

In Fig. 5-15, assume $I_2 = 1.5$ A instead of 2 A. I_T , R_1 , and R_2 remain the same. Recalculate V_A , I_1 , I_3 , and R_3 .

Figure 5-15 Analyzing a parallel circuit. What are the values for V_A and R_3 ? See solution in text.



■ 5-7 Self-Review

Answers at end of chapter.

Refer to Fig. 5-15.

- How much is V_2 across R_2 ?
- How much is I_1 through R_1 ?
- How much is I_T ?

5-8 Troubleshooting: Opens and Shorts in Parallel Circuits

In a parallel circuit, the effect of an open or a short is much different from that in a series circuit. For example, if one branch of a parallel circuit opens, the other branch currents remain the same. The reason is that the other branches still have the same applied voltage even though one branch has effectively been removed from the circuit. Also, if one branch of a parallel circuit becomes shorted, all branches are effectively shorted. The result is excessive current in the shorted branch and zero current in all other branches. In most cases, a fuse will be placed in the main line that will burn open (blow) when its current rating is exceeded. When the fuse blows, the applied voltage is removed from each of the parallel-connected branches. The effects of opens and shorts are examined in more detail in the following paragraphs.

The Effect of an Open in a Parallel Circuit

An open in any circuit is an infinite resistance that results in no current. However, in parallel circuits there is a difference between an open circuit in the main line and an open circuit in a parallel branch. These two cases are illustrated in Fig. 5-16. In Fig. 5-16a the open circuit in the main line prevents any electron flow in the line to all the branches. The current is zero in every branch, therefore, and none of the bulbs can light.

However, in Fig. 5-16b the open is in the branch circuit for bulb 1. The **open branch** circuit has no current, then, and this bulb cannot light. The current in all the other parallel branches is normal, though, because each is connected to the voltage source. Therefore, the other bulbs light.

These circuits show the advantage of wiring components in parallel. An open in one component opens only one branch, whereas the other parallel branches have their normal voltage and current.

The Effect of a Short in a Parallel Circuit

A **short circuit** has practically zero resistance. Its effect, therefore, is to allow excessive current in the shorted circuit. Consider the example in Fig. 5-17. In

Figure 5-16 Effect of an open in a parallel circuit. (a) Open path in the main line—no current and no light for all bulbs. (b) Open path in any branch—bulb for that branch does not light, but the other two bulbs operate normally.

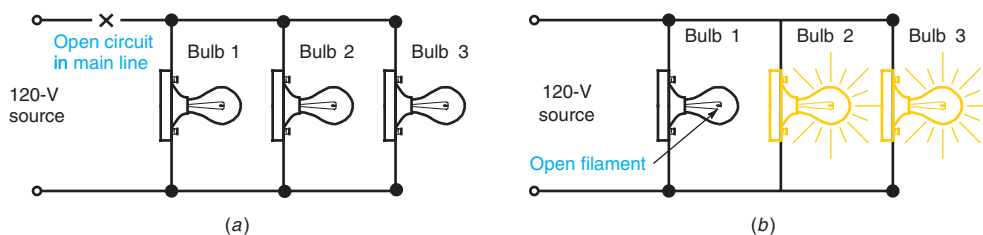


Figure 5-17 Effect of a short circuit across parallel branches. (a) Normal circuit. (b) Short circuit across points G and H shorts out all the branches.

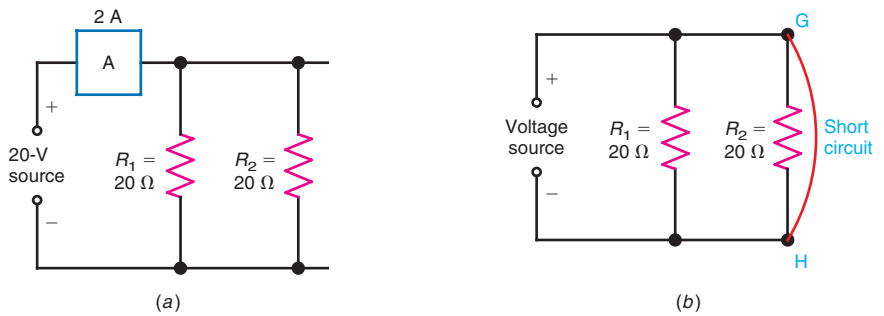


Fig. 5-17a, the circuit is normal, with 1 A in each branch and 2 A for the total line current. However, suppose that the conducting wire at point G accidentally makes contact with the wire at point H, as shown in Fig. 5-17b. Since the wire is an excellent conductor, the short circuit results in practically zero resistance between points G and H. These two points are connected directly across the voltage source. Since the short circuit provides practically no opposition to current, the applied voltage could produce an infinitely high value of current through this current path.

The Short-Circuit Current

Practically, the amount of current is limited by the small resistance of the wire. Also, the source usually cannot maintain its output voltage while supplying much more than its rated load current. Still, the amount of current can be dangerously high. For instance, the short-circuit current might be more than 100 A instead of the normal line current of 2 A in Fig. 5-17a. Because of the short circuit, excessive current flows in the voltage source, in the line to the short circuit at point H, through the short circuit, and in the line returning to the source from G. Because of the large amount of current, the wires can become hot enough to ignite and burn the insulation covering the wire. There should be a fuse that would open if there is too much current in the main line because of a short circuit across any of the branches.

The Short-Circuited Components Have No Current

For the short circuit in Fig. 5-17b, the I is 0 A in the parallel resistors R_1 and R_2 . The reason is that the short circuit is a parallel path with practically zero resistance. Then all the current flows in this path, bypassing the resistors R_1 and R_2 . Therefore R_1 and R_2 are short-circuited or *shorted out* of the circuit. They cannot function without their normal current. If they were filament resistances of light bulbs or heaters, they would not light without any current.

The short-circuited components are not damaged, however. They do not even have any current passing through them. Assuming that the short circuit has not damaged the voltage source and the wiring for the circuit, the components can operate again when the circuit is restored to normal by removing the short circuit.

All Parallel Branches Are Short-Circuited

If there were only one R in Fig. 5-17 or any number of parallel components, they would all be shorted out by the short circuit across points G and H.

Therefore, a short circuit across one branch in a parallel circuit shorts out all parallel branches.

This idea also applies to a short circuit across the voltage source in any type of circuit. Then the entire circuit is shorted out.

Troubleshooting Procedures for Parallel Circuits

When a component fails in a parallel circuit, voltage, current, and resistance measurements can be made to locate the defective component. To begin our analysis, let's refer to the parallel circuit in Fig. 5-18a, which is normal. The individual branch currents I_1 , I_2 , I_3 , and I_4 are calculated as follows:

$$I_1 = \frac{120 \text{ V}}{20 \Omega} = 6 \text{ A}$$

$$I_2 = \frac{120 \text{ V}}{15 \Omega} = 8 \text{ A}$$

$$I_3 = \frac{120 \text{ V}}{30 \Omega} = 4 \text{ A}$$

$$I_4 = \frac{120 \text{ V}}{60 \Omega} = 2 \text{ A}$$

By Kirchhoff's current law, the total current I_T equals $6 \text{ A} + 8 \text{ A} + 4 \text{ A} + 2 \text{ A} = 20 \text{ A}$. The total current I_T of 20 A is indicated by the ammeter M_1 , which is placed in the main line between points J and K. The fuse F_1 between points A and B in the main line can safely carry 20 A , since its maximum rated current is 25 A , as shown.

Now consider the effect of an open branch between points D and I in Fig. 5-18b. With R_2 open, the branch current I_2 is 0 A . Also, the ammeter M_1 shows a total current I_T of 12 A , which is 8 A less than its normal value. This makes sense because I_2 is normally 8 A . Notice that with R_2 open, all other branch currents remain the same. This is because each branch is still connected to the applied voltage of 120 V . It is important to realize that voltage measurements across the individual branches would not help determine which branch is open because even the open branch between points D and I will measure 120 V .

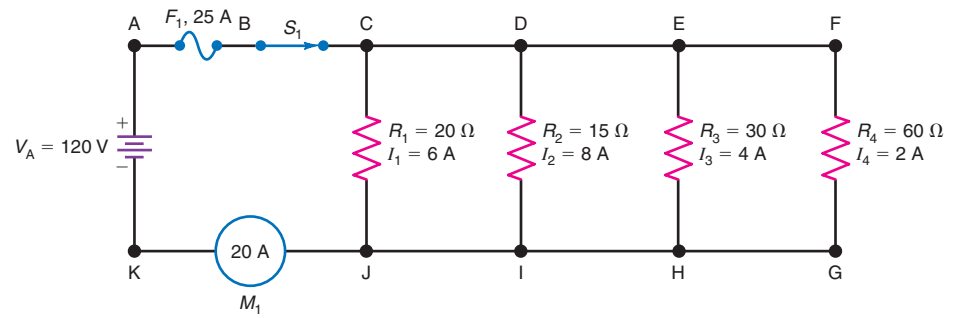
In most cases, the components in a parallel circuit provide a visual indication of failure. If a lamp burns open, it doesn't light. If a motor opens, it stops running. In these cases, the defective component is easy to spot.

In summary, here is the effect of an open branch in a parallel circuit.

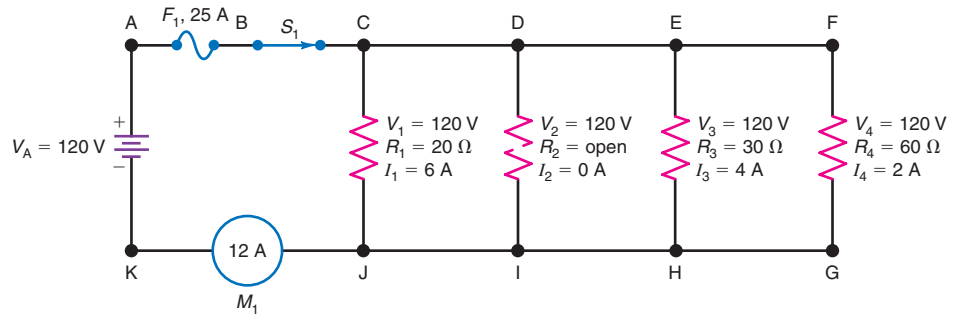
1. The current in the open branch drops to 0 A .
2. The total current I_T decreases by an amount equal to the value normally drawn by the now open branch.
3. The current in all remaining branches remains the same.
4. The applied voltage remains present across all branches whether they are open or not.

Next, let's consider the effect of an open between two branch points such as points D and E in Fig. 5-18c. With an open between these two points, the current through branch resistors R_3 and R_4 will be 0 A . Since $I_3 = 4 \text{ A}$ and $I_4 = 2 \text{ A}$ normally, the total current indicated by M_1 will drop from 20 A to 14 A as shown. The reason that I_3 and I_4 are now 0 A is that the applied voltage has effectively been removed from these two branches. If a voltmeter were placed across either points E and H or F and G, it would read 0 V . A voltmeter placed across points D and E would measure 120 V , however. This is indicated by the voltmeter M_2 as shown. The reason M_2 measures 120 V between points D and E is explained as follows: Notice that the positive (red) lead of M_2 is connected through S_1 and F_1 to the positive side of the applied voltage. Also,

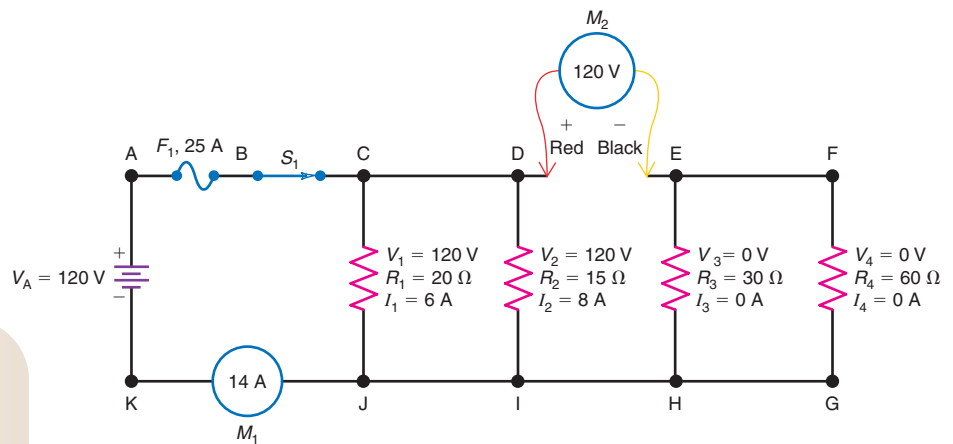
Figure 5-18 Parallel circuit for troubleshooting analysis. (a) Normal circuit values; (b) circuit values with branch R_2 open; (c) circuit values with an open between points D and E; (d) circuit showing the effects of a shorted branch.



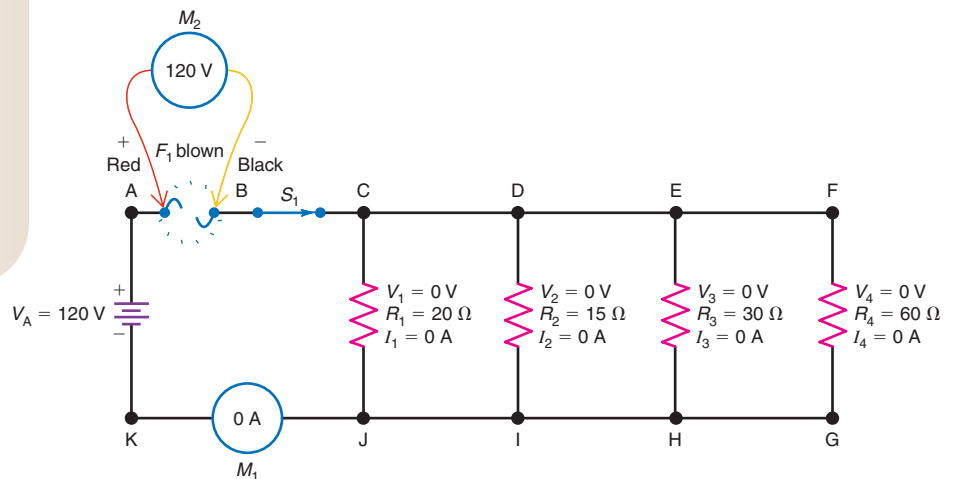
(a)



(b)



(c)



(d)

GOOD TO KNOW

A fuse is a safety device which serves to protect the circuit components and wiring in the event of a short circuit. Excessive current melts the fuse element which blows the fuse. With the fuse blown, there is no voltage across any of the parallel connected branches.

the negative (black) lead of M_2 is connected to the top of resistors R_3 and R_4 . Since the voltage across R_3 and R_4 is 0 V, the negative lead of M_2 is in effect connected to the negative side of the applied voltage. In other words, M_2 is effectively connected directly across the 120-V source.

Example 5-8

In Fig. 5-18a, suppose that the ammeter M_1 reads 16 A instead of 20 A as it should. What could be wrong with the circuit?

ANSWER Notice that the current I_3 is supposed to be 4 A. If R_3 is open, this explains why M_1 reads a current that is 4 A less than its normal value. To confirm that R_3 is open, open S_1 and disconnect the top lead of R_3 from point E. Next place an ammeter between the top of R_3 and point E. Now, close S_1 . If I_3 measures 0 A, you know that R_3 is open. If I_3 measures 4 A, you know that one of the other branches is drawing less current than it should. In this case, the next step would be to measure each of the remaining branch currents to find the defective component.

Consider the circuit in Fig. 5-18d. Notice that the fuse F_1 is blown and the ammeter M_1 reads 0 A. Notice also that the voltage across each branch measures 0 V and the voltage across the blown fuse measures 120 V as indicated by the voltmeter M_2 . What could cause this? The most likely answer is that one of the parallel-connected branches has become short-circuited. This would cause the total current to rise well above the 25-A current rating of the fuse, thus causing it to blow. But how do we go about finding out which branch is shorted? There are at least three different approaches. Here's the first one: Start by opening switch S_1 and replacing the bad fuse. Next, with S_1 still open, disconnect all but one of the four parallel branches. For example, disconnect branch resistors R_1 , R_2 , and R_3 along the top (at points C, D, and E). With R_4 still connected, close S_1 . If the fuse blows, you know R_4 is shorted! If the fuse does not blow, with only R_4 connected, open S_1 and reconnect R_3 to point E. Then, close S_1 and see if the fuse blows.

Repeat this procedure with branch resistors R_1 and R_2 until the shorted branch is identified. The shorted branch will blow the fuse when it is reconnected at the top (along points C, D, E, and F) with S_1 closed. Although this troubleshooting procedure is effective in locating the shorted branch, another fuse has been blown and this will cost you or the customer money.

Here's another approach to finding the shorted branch. Open S_1 and replace the bad fuse. Next, measure the resistance of each branch separately. It is important to remember that when you make resistance measurements in a parallel circuit, one end of each branch must be disconnected from the circuit so that the rest of the circuit does not affect the individual branch measurement. The branch that measures $0\ \Omega$ is obviously the shorted branch. With this approach, another fuse will not get blown.

Here is yet another approach that could be used to locate the shorted branch in Fig. 5-18d. With S_1 open, place an ohmmeter across points C and J.

With a shorted branch, the ohmmeter will measure $0\ \Omega$. To determine which branch is shorted, remove one branch at a time until the ohmmeter shows a value other than $0\ \Omega$. The shorted component is located when removal of a given branch causes the ohmmeter to show a normal resistance.

In summary, here is the effect of a shorted branch in a parallel circuit:

1. The fuse in the main line will blow, resulting in zero current in the main line as well as in each parallel-connected branch.
2. The voltage across each branch will equal $0\ \text{V}$, and the voltage across the blown fuse will equal the applied voltage.
3. With power removed from the circuit, an ohmmeter will measure $0\ \Omega$ across all the branches.

Before leaving the topic of troubleshooting parallel circuits, one more point should be made about the fuse F_1 and the switch S_1 in Fig. 5–18a: The resistance of a good fuse and the resistance across the closed contacts of a switch are practically $0\ \Omega$. Therefore, the voltage drop across a good fuse or a closed switch is approximately $0\ \text{V}$. This can be proven with Ohm's law, since $V = I \times R$. If $R = 0\ \Omega$, then $V = I \times 0\ \Omega = 0\ \text{V}$. When a fuse blows or a switch opens, the resistance increases to such a high value that it is considered infinite. When used in the main line of a parallel circuit, the voltage across an open switch or a blown fuse is the same as the applied voltage. One way to reason this out logically is to treat all parallel branches as a single equivalent resistance R_{EQ} in series with the switch and fuse. The result is a simple series circuit. Then, if either the fuse or the switch opens, apply the rules of an open to a series circuit. As you recall from your study of series circuits, the voltage across an open equals the applied voltage.

■ 5–8A Knowledge Check

Answer at end of chapter.

How much voltage will exist across R_4 in Fig. 5–18a if it is open?

■ 5–8B Knowledge Check

Answer at end of chapter.

How much voltage exists across the fuse, F_1 in Fig. 5–18b?

■ 5–8 Self-Review

Answers at end of chapter.

- a. In Fig. 5–16b, how much voltage is across bulb 1?
- b. In Fig. 5–17b, how much is the resistance across points G and H?
- c. In Fig. 5–18a, how much current will M_1 show if the wire between points C and D is removed?
- d. With reference to Question c, how much voltage would be measured across R_4 ? Across points C and D?
- e. In Fig. 5–18a, how much voltage will be measured across points A and B, assuming the fuse is blown?

Summary

- There is only one voltage V_A across all components in parallel.
- The current in each branch I_b equals the voltage V_A across the branch divided by the branch resistance R_b , or $I_b = V_A/R_b$.
- Kirchhoff's current law states that the total current I_T in a parallel circuit equals the sum of the individual branch currents. Expressed as an equation, Kirchhoff's current law is $I_T = I_1 + I_2 + I_3 + \dots + \text{etc.}$
- The equivalent resistance R_{EQ} of parallel branches is less than the smallest branch resistance, since all the branches must take more current from the source than any one branch.
- For only two parallel resistances of any value, $R_{EQ} = R_1 R_2 / (R_1 + R_2)$.
- For any number of *equal* parallel resistances, R_{EQ} is the value of one resistance divided by the number of resistances.
- For the general case of any number of branches, calculate R_{EQ} as V_A/I_T or use the reciprocal resistance formula:
$$R_{EQ} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3 + \dots + \text{etc.}}$$
- For any number of conductances in parallel, their values are added for G_T , in the same way as parallel branch currents are added.
- The sum of the individual values of power dissipated in parallel resistances equals the total power produced by the source.
- An open circuit in one branch results in no current through that branch, but the other branches can have their normal current. However, an open circuit in the main line results in no current for any of the branches.
- A short circuit has zero resistance, resulting in excessive current. When one branch is short-circuited, all parallel paths are also short-circuited. The entire current is in the short circuit and no current is in the short-circuited branches.
- The voltage across a good fuse and the voltage across a closed switch are approximately 0 V. When the fuse in the main line of a parallel circuit opens, the voltage across the fuse equals the full applied voltage. Likewise, when the switch in the main line of a parallel circuit opens, the voltage across the open switch equals the full applied voltage.
- Table 5-1 compares Series and Parallel Circuits.

Table 5-1 Comparison of Series and Parallel Circuits

Series Circuit	Parallel Circuit
Current the same in all components	Voltage the same across all branches
V across each series R is $I \times R$	I in each branch R is V/R
$V_T = V_1 + V_2 + V_3 + \dots + \text{etc.}$	$I_T = I_1 + I_2 + I_3 + \dots + \text{etc.}$
$R_T = R_1 + R_2 + R_3 + \dots + \text{etc.}$	$G_T = G_1 + G_2 + G_3 + \dots + \text{etc.}$
R_T must be more than the largest individual R	R_{EQ} must be less than the smallest branch R
$P_T = P_1 + P_2 + P_3 + \dots + \text{etc.}$	$P_T = P_1 + P_2 + P_3 + \dots + \text{etc.}$
Applied voltage is divided into IR voltage drops	Main-line current is divided into branch currents
The largest IR drop is across the largest series R	The largest branch I is in the smallest parallel R
Open in one component causes entire circuit to be open	Open in one branch does not prevent I in other branches

Important Terms

- Equivalent Resistance, R_{EQ} - in a parallel circuit, this refers to a single resistance that would draw the same amount of current as all of the parallel connected branches.
- Kirchhoff's Current Law (KCL) - a law which states that the sum of the individual branch currents in a parallel circuit must equal the total current, I_T .
- Main Line - the pair of leads connecting all individual branches in a parallel circuit to the terminals of the applied voltage, V_A . The main line carries the total current, I_T , flowing to and from the terminals of the voltage source.
- Parallel Bank - a combination of parallel connected branches.
- Reciprocal Resistance Formula - a formula which states that the equivalent resistance, R_{EQ} , of a parallel circuit equals the reciprocal of the sum of the reciprocals of the individual branch resistances.

Related Formulas

$$I_1 = \frac{V_A}{R_1}, I_2 = \frac{V_A}{R_2}, I_3 = \frac{V_A}{R_3}$$

$$I_T = I_1 + I_2 + I_3 + \cdots + \text{etc.}$$

$$R_{EQ} = \frac{V_A}{I_T}$$

$$R_{EQ} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3 + \cdots + \text{etc.}}$$

$$R_{EQ} = \frac{R}{N} \text{ (} R_{EQ} \text{ for equal branch resistances)}$$

$$R_{EQ} = \frac{R_1 \times R_2}{R_1 + R_2} \text{ (} R_{EQ} \text{ for only two branch resistances)}$$

$$R_X = \frac{R \times R_{EQ}}{R - R_{EQ}}$$

$$G_T = G_1 + G_2 + G_3 + \cdots + \text{etc.}$$

$$P_T = P_1 + P_2 + P_3 + \cdots + \text{etc.}$$

Self-Test

Answers at back of book.

1. A 120-k Ω resistor, R_1 , and a 180-k Ω resistor, R_2 , are in parallel. How much is the equivalent resistance, R_{EQ} ?
 - a. 72 k Ω
 - b. 300 k Ω
 - c. 360 k Ω
 - d. 90 k Ω
2. A 100- Ω resistor, R_1 , and a 300- Ω resistor, R_2 , are in parallel across a dc voltage source. Which resistor dissipates more power?
 - a. the 300- Ω resistor
 - b. Both resistors dissipate the same amount of power.
 - c. the 100- Ω resistor
 - d. This is impossible to determine.
3. Three 18- Ω resistors are in parallel. How much is the equivalent resistance, R_{EQ} ?
 - a. 54 Ω
 - b. 6 Ω
 - c. 9 Ω
 - d. none of the above
4. Which of the following statements about parallel circuits is false?
 - a. The voltage is the same across all branches in a parallel circuit.
 - b. The equivalent resistance, R_{EQ} , of a parallel circuit is always smaller than the smallest branch resistance.
 - c. In a parallel circuit the total current, I_T , in the main line equals the sum of the individual branch currents.
 - d. The equivalent resistance, R_{EQ} , of a parallel circuit decreases when one or more parallel branches are removed from the circuit.
5. Two resistors, R_1 and R_2 , are in parallel with each other and a dc voltage source. If the total current, I_T , in the main line equals 6A and I_2 through R_2 is 4A, how much is I_1 through R_1 ?
 - a. 6 A
 - b. 2 A
 - c. 4 A
 - d. I_1 cannot be determined.
6. How much resistance must be connected in parallel with a 360- Ω resistor to obtain an equivalent resistance, R_{EQ} , of 120 Ω ?
 - a. 360 Ω
 - b. 480 Ω
 - c. 1.8 k Ω
 - d. 180 Ω
7. If one branch of a parallel circuit becomes open,
 - a. all remaining branch currents increase.
 - b. the voltage across the open branch will be 0 V.
 - c. the remaining branch currents do not change in value.
 - d. the equivalent resistance of the circuit decreases.
8. If a 10- Ω R_1 , 40- Ω R_2 , and 8- Ω R_3 are in parallel, calculate the total conductance, G_T , of the circuit.
 - a. 250 mS
 - b. 58 S
 - c. 4 Ω
 - d. 0.25 μ S

9. Which of the following formulas can be used to determine the total power, P_T , dissipated by a parallel circuit.
- $P_T = V_A \times I_T$
 - $P_T = P_1 + P_2 + P_3 + \dots + \text{etc.}$
 - $P_T = \frac{V_A^2}{R_{EQ}}$
 - all of the above
10. A $20\text{-}\Omega$ R_1 , $50\text{-}\Omega$ R_2 , and $100\text{-}\Omega$ R_3 are connected in parallel. If R_2 is short-circuited, what is the equivalent resistance, R_{EQ} , of the circuit?
- approximately $0\text{ }\Omega$
 - infinite (∞) Ω
 - $12.5\text{ }\Omega$
 - R_{EQ} cannot be determined.
11. If the fuse in the main line of a parallel circuit opens,
- the voltage across each branch will be 0 V .
 - the current in each branch will be zero.
 - the current in each branch will increase to offset the decrease in total current.
 - both a and b above
12. A $100\text{-}\Omega$ R_1 and a $150\text{-}\Omega$ R_2 are in parallel. If the current, I_1 , through R_1 is 24 mA , how much is the total current, I_T ?
- 16 mA
 - 40 mA
 - 9.6 mA
 - I_T cannot be determined.
13. A $2.2\text{-k}\Omega$ R_1 is in parallel with a $3.3\text{-k}\Omega$ R_2 . If these two resistors carry a total current of 7.5 mA , how much is the applied voltage, V_A ?
- 16.5 V
 - 24.75 V
 - 9.9 V
 - 41.25 V
14. How many $120\text{-}\Omega$ resistors must be connected in parallel to obtain an equivalent resistance, R_{EQ} , of $15\text{ }\Omega$?
- 15
 - 8
 - 12
 - 6
15. A $220\text{-}\Omega$ R_1 , $2.2\text{-k}\Omega$ R_2 , and $200\text{-}\Omega$ R_3 are connected across 15 V of applied voltage. What happens to R_{EQ} if the applied voltage is doubled to 30 V ?
- R_{EQ} doubles.
 - R_{EQ} cuts in half.
 - R_{EQ} does not change.
 - R_{EQ} increases but is not double its original value.
16. If one branch of a parallel circuit opens, the total current, I_T ,
- doesn't change.
 - decreases.
 - increases.
 - goes to zero.
17. In a normally operating parallel circuit, the individual branch currents are
- independent of each other.
 - not affected by the value of the applied voltage.
 - larger than the total current, I_T .
 - none of the above
18. If the total conductance, G_T , of a parallel circuit is $200\text{ }\mu\text{S}$, how much is R_{EQ} ?
- $500\text{ }\Omega$
 - $200\text{ k}\Omega$
 - $5\text{ k}\Omega$
 - $500\text{ k}\Omega$
19. If one branch of a parallel circuit is short-circuited,
- the fuse in the main line will blow.
 - the voltage across the short-circuited branch will measure the full value of applied voltage.
 - all the remaining branches are effectively short-circuited as well.
 - both a and c
20. Two lightbulbs in parallel with the 120-V power line are rated at 60 W and 100 W , respectively. What is the equivalent resistance, R_{EQ} , of the bulbs when they are lit?
- $144\text{ }\Omega$
 - $90\text{ }\Omega$
 - $213.3\text{ }\Omega$
 - There is not enough information to calculate R_{EQ} .

Questions

- Draw a wiring diagram showing three resistances connected in parallel across a battery. Indicate each branch and the main line.
- State two rules for the voltage and current values in a parallel circuit.
- Explain briefly why the current is the same in both sides of the main line that connects the voltage source to the parallel branches.
- (a) Show how to connect three equal resistances for a combined equivalent resistance one-third the value of one resistance. (b) Show how to connect three equal resistances for a combined equivalent resistance three times the value of one resistance.
- Why can the current in parallel branches be different when they all have the same applied voltage?
- Why does the current increase in the voltage source as more parallel branches are added to the circuit?
- Show how the formula

$$R_{EQ} = R_1 R_2 / (R_1 + R_2)$$
 is derived from the reciprocal formula

$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2}$$

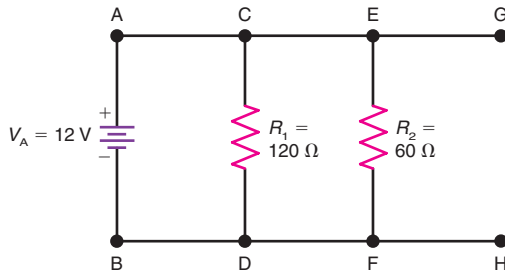
8. Redraw Fig. 5-17 with five parallel resistors R_1 to R_5 and explain why they all would be shorted out with a short circuit across R_3 .
9. State briefly why the total power equals the sum of the individual values of power, whether a series circuit or a parallel circuit is used.
10. Explain why an open in the main line disables all the branches, but an open in one branch affects only that branch current.
11. Give two differences between an open circuit and a short circuit.
12. List as many differences as you can in comparing series circuits with parallel circuits.
13. Why are household appliances connected to the 120-V power line in parallel instead of in series?
14. Give one advantage and one disadvantage of parallel connections.
15. A $5\text{-}\Omega$ and a $10\text{-}\Omega$ resistor are in parallel across a dc voltage source. Which resistor will dissipate more power? Provide proof with your answer.

Problems

SECTION 5-1 THE APPLIED VOLTAGE V_A IS THE SAME ACROSS PARALLEL BRANCHES

- 5-1 **MultiSim** In Fig. 5-19, how much voltage is across points
- a. A and B?
 - b. C and D?
 - c. E and F?
 - d. G and H?

Figure 5-19



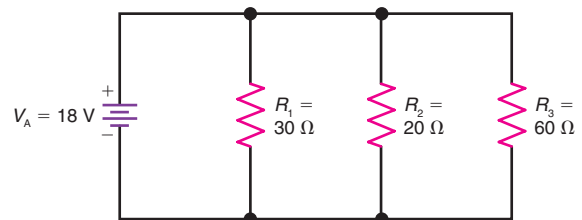
- 5-2 In Fig. 5-19, how much voltage is across
- a. the terminals of the voltage source?
 - b. R_1 ?
 - c. R_2 ?
- 5-3 In Fig. 5-19, how much voltage will be measured across Points C and D if R_1 is removed from the circuit?

SECTION 5-2 EACH BRANCH / EQUALS $\frac{V_A}{R}$

- 5-4 In Fig. 5-19, solve for the branch currents, I_1 and I_2 .
- 5-5 In Fig. 5-19, explain why I_2 is double the value of I_1 .
- 5-6 In Fig. 5-19, assume a $10\text{-}\Omega$ resistor, R_3 , is added across Points G and H.
- a. Calculate the branch current, I_3 .
 - b. Explain how the branch currents, I_1 and I_2 are affected by the addition of R_3 .

- 5-7 In Fig. 5-20, solve for the branch currents I_1 , I_2 , and I_3 .

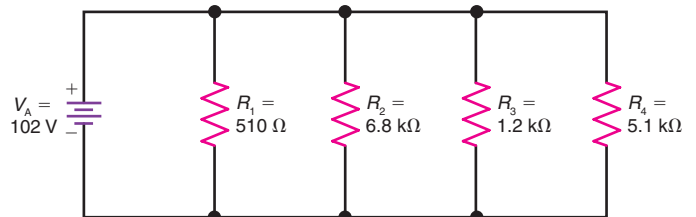
Figure 5-20



- 5-8 In Fig. 5-20, do the branch currents I_1 and I_3 remain the same if R_2 is removed from the circuit? Explain your answer.

- 5-9 In Fig. 5-21, solve for the branch currents I_1 , I_2 , I_3 , and I_4 .

Figure 5-21



- 5-10 Recalculate the values for I_1 , I_2 , I_3 , and I_4 in Fig. 5-21 if the applied voltage, V_A , is reduced to 51V.

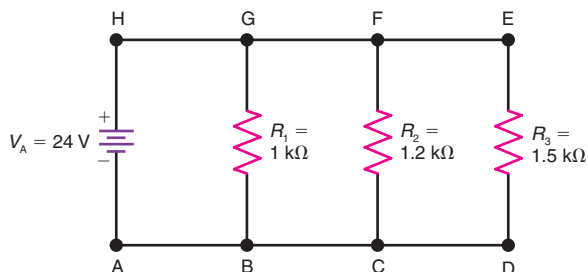
SECTION 5-3 KIRCHHOFF'S CURRENT LAW (KCL)

- 5-11 **MultiSim** In Fig. 5-19, solve for the total current, I_T .
- 5-12 **MultiSim** In Fig. 5-19 re-solve for the total current, I_T , if a $10\text{-}\Omega$ resistor, R_3 , is added across Points G and H.
- 5-13 In Fig. 5-20, solve for the total current, I_T .
- 5-14 In Fig. 5-20, re-solve for the total current, I_T , if R_2 is removed from the circuit.
- 5-15 In Fig. 5-21, solve for the total current, I_T .

5-16 In Fig. 5-21, re-solve for the total current, I_T , if V_A is reduced to 51 V.

5-17 In Fig. 5-22, solve for I_1 , I_2 , I_3 , and I_T .

Figure 5-22



5-18 In Fig. 5-22, how much is the current in the wire between points

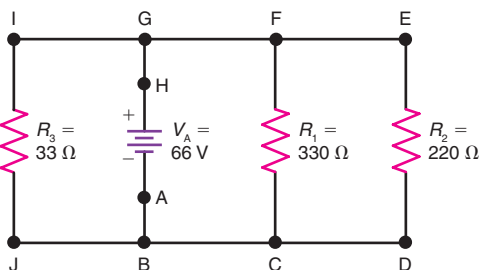
- A and B?
- B and C?
- C and D?
- E and F?
- F and G?
- G and H?

5-19 In Fig. 5-22 assume that a $100\text{-}\Omega$ resistor, R_4 , is added to the right of resistor, R_3 . How much is the current in the wire between points

- A and B?
- B and C?
- C and D?
- E and F?
- F and G?
- G and H?

5-20 In Fig. 5-23, solve for I_1 , I_2 , I_3 , and I_T .

Figure 5-23



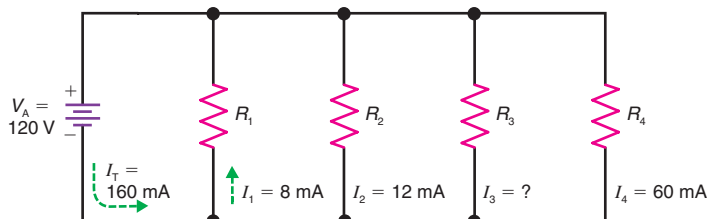
5-21 In Fig. 5-23, how much is the current in the wire between points

- A and B?
- B and C?
- C and D?
- E and F?

- F and G?
- G and H?
- G and I?
- B and J?

5-22 In Fig. 5-24, apply Kirchhoff's Current Law to solve for the unknown current, I_3 .

Figure 5-24



5-23 Two resistors R_1 and R_2 are in parallel with each other and a dc voltage source. How much is I_2 through R_2 if $I_T = 150\text{ mA}$ and I_1 through R_1 is 60 mA ?

SECTION 5-4 RESISTANCES IN PARALLEL

5-24 In Fig. 5-19, solve for R_{EQ} .

5-25 In Fig. 5-19, re-solve for R_{EQ} if a $10\text{-}\Omega$ resistor, R_3 is added across Points G and H.

5-26 In Fig. 5-20, solve for R_{EQ} .

5-27 In Fig. 5-20, re-solve for R_{EQ} if R_2 is removed from the circuit.

5-28 In Fig. 5-21, solve for R_{EQ} .

5-29 In Fig. 5-21, re-solve for R_{EQ} if V_A is reduced to 51 V.

5-30 In Fig. 5-22, solve for R_{EQ} .

5-31 In Fig. 5-23, solve for R_{EQ} .

5-32 In Fig. 5-24, solve for R_{EQ} .


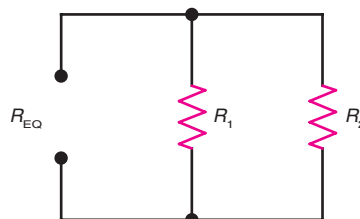

5-33  **MultiSim** In Fig. 5-25, how much is R_{EQ} if $R_1 = 100\text{ }\Omega$ and $R_2 = 25\text{ }\Omega$?

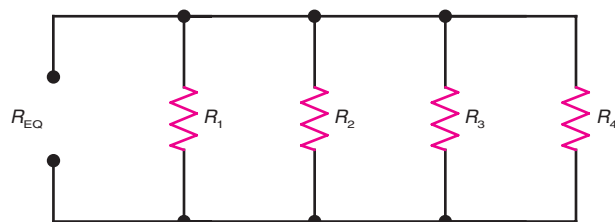
Figure 5-25



5-34  **MultiSim** In Fig. 5-25, how much is R_{EQ} if $R_1 = 1.5\text{ M}\Omega$ and $R_2 = 1\text{ M}\Omega$?

- 5-35 **MultiSim** In Fig. 5-25, how much is R_{EQ} if $R_1 = 2.2\text{ k}\Omega$ and $R_2 = 220\text{ }\Omega$?
- 5-36 In Fig. 5-25, how much is R_{EQ} if $R_1 = R_2 = 10\text{ k}\Omega$?
- 5-37 In Fig. 5-25, how much resistance, R_2 , must be connected in parallel with a $750\text{ }\Omega$ R_1 to obtain an R_{EQ} of $500\text{ }\Omega$?
- 5-38 In Fig. 5-25, how much resistance, R_1 , must be connected in parallel with a $6.8\text{ k}\Omega$ R_2 to obtain an R_{EQ} of $1.02\text{ k}\Omega$?
- 5-39 How much is R_{EQ} in Fig. 5-26 if $R_1 = 1\text{ k}\Omega$, $R_2 = 4\text{ k}\Omega$, $R_3 = 200\text{ }\Omega$, and $R_4 = 240\text{ }\Omega$?

Figure 5-26

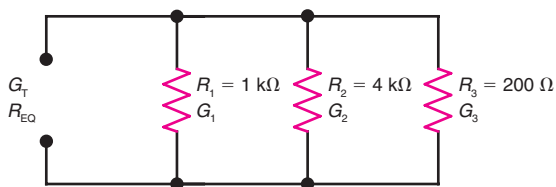


- 5-40 How much is R_{EQ} in Fig. 5-26 if $R_1 = 5.6\text{ k}\Omega$, $R_2 = 4.7\text{ k}\Omega$, $R_3 = 8.2\text{ k}\Omega$, and $R_4 = 2.7\text{ k}\Omega$?
- 5-41 **MultiSim** How much is R_{EQ} in Fig. 5-26 if $R_1 = 1.5\text{ k}\Omega$, $R_2 = 1\text{ k}\Omega$, $R_3 = 1.8\text{ k}\Omega$, and $R_4 = 150\text{ }\Omega$?
- 5-42 How much is R_{EQ} in Fig. 5-26 if $R_1 = R_2 = R_3 = R_4 = 2.2\text{ k}\Omega$?
- 5-43 A technician is using an ohmmeter to measure a variety of different resistor values. Assume the technician has a body resistance of $750\text{ k}\Omega$. How much resistance will the ohmmeter read if the fingers of the technician touch the leads of the ohmmeter when measuring the following resistors:
- $270\text{ }\Omega$.
 - $390\text{ k}\Omega$.
 - $2.2\text{ M}\Omega$.
 - $1.5\text{ k}\Omega$.
 - $10\text{ k}\Omega$.

SECTION 5-5 CONDUCTANCES IN PARALLEL

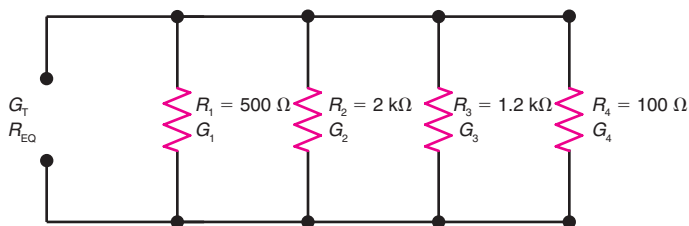
- 5-44 In Fig. 5-27, solve for G_1 , G_2 , G_3 , G_T , and R_{EQ} .

Figure 5-27



- 5-45 In Fig. 5-28, solve for G_1 , G_2 , G_3 , G_4 , G_T , and R_{EQ} .

Figure 5-28



- 5-46 Find the total conductance, G_T for the following branch conductances; $G_1 = 1\text{ mS}$, $G_2 = 200\text{ }\mu\text{S}$, and $G_3 = 1.8\text{ mS}$. How much is R_{EQ} ?
- 5-47 Find the total conductance, G_T for the following branch conductances; $G_1 = 100\text{ mS}$, $G_2 = 66.67\text{ mS}$, $G_3 = 250\text{ mS}$, and $G_4 = 83.33\text{ mS}$. How much is R_{EQ} ?

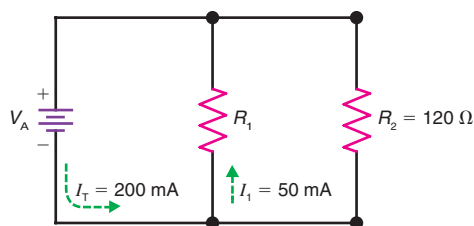
SECTION 5-6 TOTAL POWER IN PARALLEL CIRCUITS

- 5-48 In Fig. 5-20, solve for P_1 , P_2 , P_3 , and P_T .
- 5-49 In Fig. 5-21, solve for P_1 , P_2 , P_3 , P_4 , and P_T .
- 5-50 In Fig. 5-22, solve for P_1 , P_2 , P_3 , and P_T .
- 5-51 In Fig. 5-23, solve for P_1 , P_2 , P_3 , and P_T .
- 5-52 In Fig. 5-24, solve for P_1 , P_2 , P_3 , P_4 , and P_T .

SECTION 5-7 ANALYZING PARALLEL CIRCUITS WITH RANDOM UNKNOWN

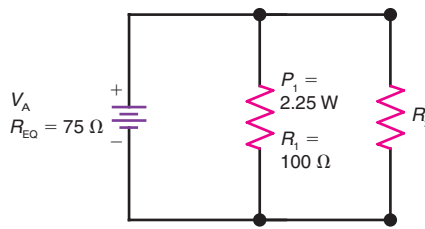
- 5-53 In Fig. 5-29, solve for V_A , R_1 , I_2 , R_{EQ} , P_1 , P_2 , and P_T .

Figure 5-29



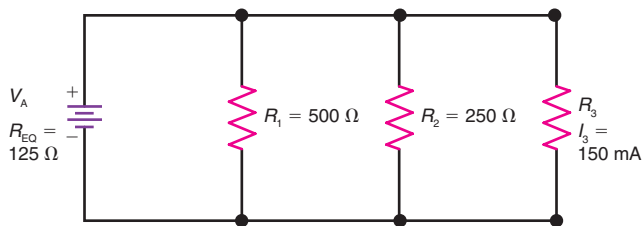
- 5-54 In Fig. 5-30, solve for V_A , I_1 , I_2 , R_2 , I_T , P_2 , and P_T .

Figure 5-30



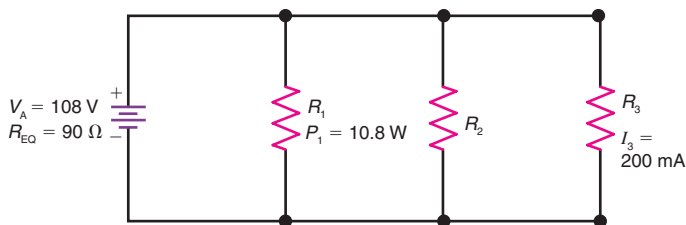
5-55 In Fig. 5-31, solve for R_3 , V_A , I_1 , I_2 , I_T , P_1 , P_2 , P_3 , and P_T .

Figure 5-31



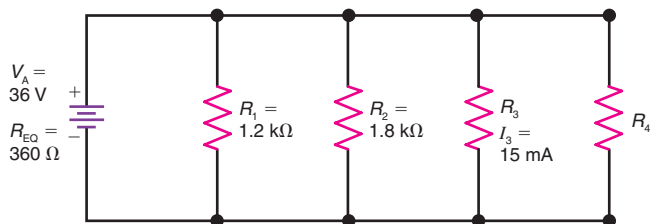
5-56 In Fig. 5-32, solve for I_T , I_1 , I_2 , R_1 , R_2 , R_3 , P_2 , P_3 , and P_T .

Figure 5-32



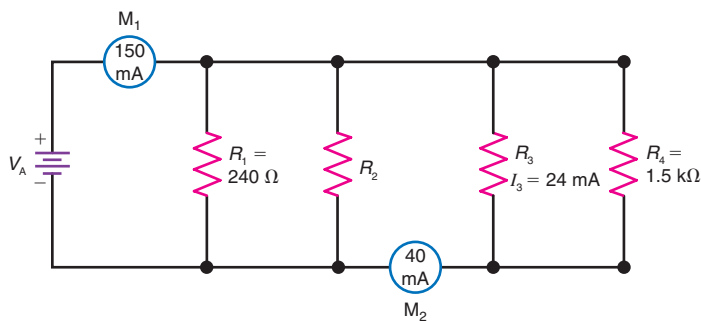
5-57 In Fig. 5-33, solve for I_T , I_1 , I_2 , I_4 , R_3 , R_4 , P_1 , P_2 , P_3 , P_4 , and P_T .

Figure 5-33



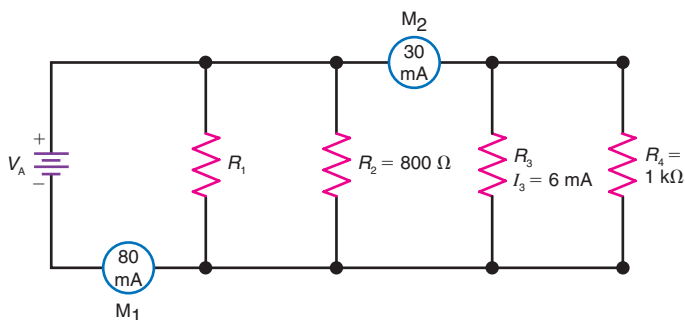
5-58 In Fig. 5-34, solve for V_A , I_1 , I_2 , R_2 , R_3 , I_4 , and R_EQ .

Figure 5-34



5-59 In Fig. 5-35, solve for V_A , I_1 , I_2 , I_4 , R_1 , R_3 , and R_EQ .

Figure 5-35

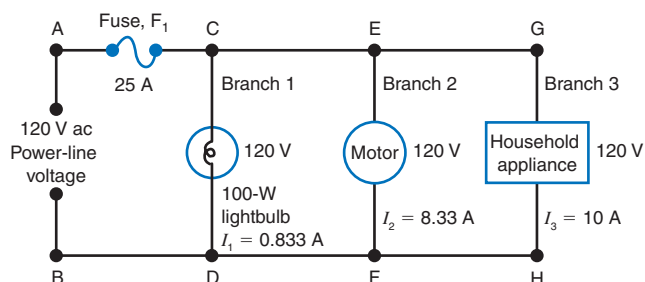


SECTION 5-8 TROUBLESHOOTING: OPENS AND SHORTS IN PARALLEL CIRCUITS

5-60 Figure 5-36 shows a parallel circuit with its normal operating voltages and currents. Notice that the fuse in the main line has a 25 A rating. What happens to the circuit components and their voltages and currents if

- the appliance in Branch 3 shorts?
- the motor in Branch 2 burns out and becomes an open?
- the wire between Points C and E develops an open?
- the motor in Branch 2 develops a problem and begins drawing 16 A of current?

Figure 5-36



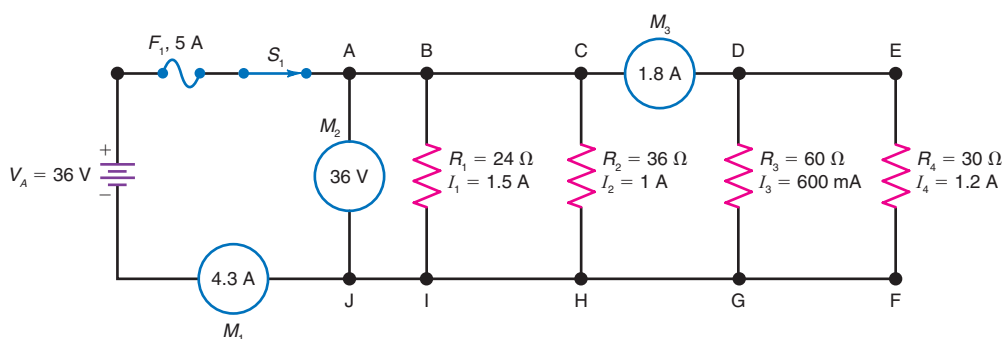
Critical Thinking

- 5-61** A $180\text{-}\Omega$, $\frac{1}{4}\text{-W}$ resistor is in parallel with $1\text{-k}\Omega$, $\frac{1}{2}\text{-W}$ and $12\text{-k}\Omega$, 2-W resistors. What is the maximum total current, I_T , that this parallel combination can have before the wattage rating of any resistor is exceeded?
- 5-62** A $470\text{-}\Omega$, $\frac{1}{8}\text{-W}$ resistor is in parallel with $1\text{-k}\Omega$, $\frac{1}{4}\text{-W}$ and $1.5\text{-k}\Omega$, $\frac{1}{2}\text{-W}$ resistors. What is the maximum voltage, V , that can be applied to this circuit without exceeding the wattage rating of any resistor?
- 5-63** Three resistors in parallel have a combined equivalent resistance R_{EQ} of $1\text{ k}\Omega$. If R_2 is twice the value of R_3 and three times the value of R_1 , what are the values for R_1 , R_2 , and R_3 ?
- 5-64** Three resistors in parallel have a combined equivalent resistance R_{EQ} of 4Ω . If the conductance, G_1 , is one-fourth that of G_2 and one-fifth that of G_3 , what are the values of R_1 , R_2 , and R_3 ?
- 5-65** A voltage source is connected in parallel across four resistors R_1 , R_2 , R_3 , and R_4 . The currents are labeled I_1 , I_2 , I_3 , and I_4 , respectively. If $I_2 = 2I_1$, $I_3 = 2I_2$, and $I_4 = 2I_3$, calculate the values for R_1 , R_2 , R_3 , and R_4 if $R_{EQ} = 1\text{ k}\Omega$.

Troubleshooting Challenge

Figure 5-37 shows a parallel circuit with its normal operating voltages and currents. Notice the placement of the meters M_1 , M_2 , and M_3 in the circuit. M_1 measures the total current I_T , M_2 measures the applied voltage V_A , and M_3 measures the current between points C and D. The following problems deal with troubleshooting the parallel circuit in Fig. 5-37.

Figure 5-37 Circuit diagram for troubleshooting challenge. Normal values for I_1 , I_2 , I_3 , and I_4 are shown on schematic.



- 5-66** If M_1 measures 2.8 A , M_2 measures 36 V , and M_3 measures 1.8 A , which component has most likely failed? How is the component defective?
- 5-67** If M_1 measures 2.5 A , M_2 measures 36 V , and M_3 measures 0 A , what is most likely wrong? How could you isolate the trouble by making voltage measurements?
- 5-68** If M_1 measures 3.3 A , M_2 measures 36 V , and M_3 measures 1.8 A , which component has most likely failed? How is the component defective?
- 5-69** If the fuse F_1 is blown, (a) How much current will be measured by M_1 and M_3 ? (b) How much voltage will be measured by M_2 ? (c) How much voltage will be measured across the blown fuse? (d) What is most likely to have caused the blown fuse? (e) Using resistance measurements, outline a procedure for finding the defective component.
- 5-70** If M_1 and M_3 measure 0 A but M_2 measures 36 V , what is most likely wrong? How could you isolate the trouble by making voltage measurements?
- 5-71** If the fuse F_1 has blown because of a shorted branch, how much resistance would be measured across points B and I? Without using resistance measurements, how could the shorted branch be identified?
- 5-72** If the wire connecting points F and G opens, (a) How much current will M_3 show? (b) How much voltage would be measured across R_4 ? (c) How much voltage would be measured across points D and E? (d) How much voltage would be measured across points F and G?
- 5-73** Assuming that the circuit is operating normally, how much voltage would be measured across, (a) the fuse F_1 , (b) the switch S_1 ?
- 5-74** If the branch resistor R_3 opens, (a) How much voltage would be measured across R_3 ? (b) How much current would be indicated by M_1 and M_3 ?
- 5-75** If the wire between points B and C breaks open, (a) How much current will be measured by M_1 and M_3 ? (b) How much voltage would be measured across points B and C? (c) How much voltage will be measured across points C and H?

Answers to Knowledge Check Problems

5-1 1.5 V

5-2 5 A

5-3 $I_T = 2 \text{ A}$

5-4A $R_{EQ} = 1.02 \text{ k}\Omega$

5-4B $R_{EQ} = 24 \Omega$

5-4C $R_X = 2.2 \text{ k}\Omega$

5-5A $R_{EQ} = 1.33 \Omega$

5-5B $G_T = 1 \text{ S}, R_{EQ} = 1 \Omega$

5-6 $P_1 = 40 \text{ W}$

$P_2 = 80 \text{ W}$

$P_T = 120 \text{ W}$

5-7 $V_A = 90 \text{ V}$

$I_1 = 3 \text{ A}$

$I_3 = 2.5 \text{ A}$

$R_3 = 36 \Omega$

5-8A $V_4 = 120 \text{ V}$

5-8B 0 V

Answers to Self-Reviews

- 5-1 a. 1.5 V
b. 120 V
c. Two each

- 5-2 a. 10 V
b. 1 A
c. 10 V
d. 2 A

- 5-3 a. $I_T = 6 \text{ A}$
b. $I_3 = 3 \text{ A}$
c. $I_T = 1.2 \text{ A}$

- 5-4 a. $R_{EQ} = 1.57 \text{ M}\Omega$
b. $R_{EQ} = 1.2 \text{ M}\Omega$
c. $R_{EQ} = 5 \Omega$

- 5-5 a. $G_T = 6 \text{ S}$
b. $G_T = 0.75 \mu\text{S}$
 $R_{EQ} = 1.33 \text{ M}\Omega$
c. $R_{EQ} = 0.25 \text{ M}\Omega$

- 5-6 a. 480 W
b. 660 W
c. 30 W

- 5-7 a. 120 V
b. $I_1 = 4 \text{ A}$
c. 7 A

- 5-8 a. 120 V
b. 0Ω
c. 6 A
d. $0 \text{ V}, 120 \text{ V}$
e. 120 V

[illegible]