Changes in supply, demand, and equilibrium

In math note 3.3, "Equilibrium price and quantity," we showed that for linear demand and supply curves of the form $P = a - bQ_D$ and $P = c + dQ_S$, the equilibrium quantity and price, respectively, were $Q_E = \frac{a-c}{b+d}$ and $P_E = \frac{ad+bc}{b+d}$. What happens to these values if demand increases? If supply increases?

Perhaps the easiest way to model these changes is to add a term to the right hand side of the demand and supply relationships. For example, consider the equation $P = a' - bQ_D$, where a' differs from a by some amount Δa. That is, $a' = a + \Delta a$. If Δa is positive, we can interpret this as an increase in demand: the price at which consumers demand any given quantity is greater than previously. Likewise, consider the supply curve $P = c' + dQ_S$, where $c' = c - \Delta c$. The minus sign on Δc may be curious, but consider: if Δc is positive, the minus sign in front indicates that the price at which producers supply any given amount is lower than before. In other words, an increase in supply can be modeled by subtracting a positive amount Δc from the supply price.

Inserting these new values of a' and c' into the general solution for equilibrium quantity, we find $Q_E + \Delta Q = Q_E' = \frac{a'-c'}{b+d} = \frac{a+\Delta a-(c-\Delta c)}{b+d}$. Since we know that $Q_E = \frac{a-c}{b+d}$ from before, we can subtract this from both sides to obtain $\Delta Q = \frac{\Delta a + \Delta c}{b+d}$. It is now apparent that any factor that increases demand (a positive Δa) or increases supply (a positive Δc) will increase equilibrium quantity ($\Delta Q > 0$). Likewise, anything that decreases demand or supply (a negative Δa or a negative Δc) will reduce equilibrium quantity ($\Delta Q < 0$). If Δa and Δc are of opposite sign, equilibrium quantity will move in the direction of whichever change is greater. For example, equilibrium quantity will increase if a demand increase is greater than a supply decrease, so that $(\Delta a + \Delta c) > 0$.

What of equilibrium price? Following the same procedure, equilibrium price is found to be: $P_E' = P_E + \Delta P = \frac{a'd+bc'}{b+d} = \frac{(a+\Delta a)d+(c-\Delta c)b}{b+d}$. As before, we note that $P_E = \frac{a+c}{b+d}$, and subtracting this from both sides we conclude that $\Delta P = \frac{d\Delta a - b\Delta c}{b+d}$. Here we see that an increase in demand (Δa positive) or a decrease in supply (Δc negative) will raise the price. If both demand and supply increase (both Δa and Δc positive), the impact on equilibrium price depends on which change is relatively greater, that is, it depends on whether $(d\Delta a - b\Delta c)$ is greater than or less than zero.