CHAPTER 6

P.P.6.1 $v = \frac{q}{C} = \frac{120x10^{-6}}{4.5x10^{-6}} =$ **26.67 V**

$w = \frac{1}{2}Cv^2 = \frac{1}{2}x4.5x10^{-6}x711.1 =$ **1.6 mJ**.

P.P.6.2 $i(t) = C\frac{dv}{dt} = 10x10^{-6}\frac{d}{dt}(75\sin(2000t))$

= **1.5cos(2000t) A**.

P.P.6.3 $v = \frac{1}{C}\int_0^t idt = \frac{10^{-3}}{0.1x10^{-3}}\int_0^t 50\sin 120\pi t\ dt$ V

$= -\frac{500}{120\pi}\cos 120\pi t\Big|_0^t = \frac{50}{12\pi}(1-\cos 120\pi t)$V

v(t = 1ms) = $\frac{50}{12\pi}(1-\cos 0.12\pi) =$ **93.14 mV**

v(t = 5ms) = $\frac{50}{12\pi}(1-\cos 0.6\pi) =$ **1.736 V**

P.P.6.4 $i(t) = \begin{cases} 50t, & 0 < t < 2 \\ 100, & 2 < t < 6 \end{cases}$

$v = \frac{1}{C}\int idt = \frac{1}{10^{-3}}\int idt \cdot 10^{-3} = \int idt$

For 0< t <2, $v = \frac{1}{C}\int_0^t 50t\ dt = 25t^2 \text{ x } 10^3$

For 2< t <6, $v = \frac{1}{C}\int_2^t 100dt + v(2) = (100t - 0.2 + 0.1)$

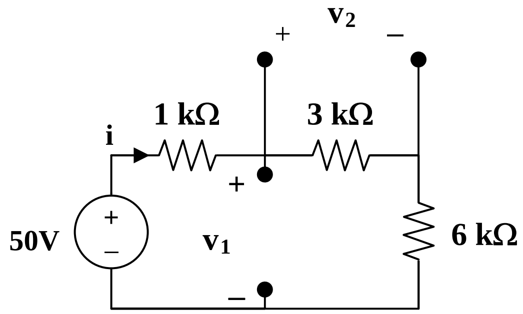
= (100t - 0.1)V

At t = 2ms, v = **100 mV**

At t = 5ms, v = (500 - 100)mV

= **400 mV**

P.P.6.5 Under dc conditions, the capacitors act like open-circuits as shown below:



$$i = \frac{50}{1+3+6} = 5\text{mA}$$

$$v_1 = (3k + 6k)i = 45\text{V}$$

$$v_2 = (3k)i = 15\text{V}$$

$$w_1 = \frac{1}{2}C_1v_1^2 = \frac{1}{2}\left(20x10^{-6}\right)(45)^2 = \mathbf{20.25\ mJ}$$

$$w_2 = \frac{1}{2}C_2v_2^2 = \frac{1}{2}\left(30x10^{-6}\right)(15)^2 = \mathbf{3.375\ mJ}.$$

P.P.6.6 Combining 60 and 120μF in series = $\frac{60x120}{180}$ = 40μF

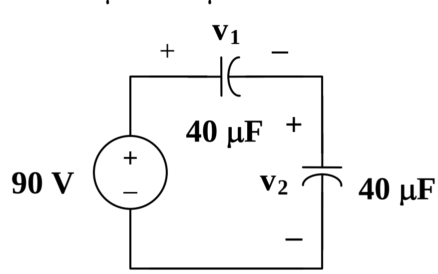
40μF in parallel with 20μF = 40 + 20 = 60μF
50μF in parallel with 70μF = 50 + 70 = 120μF

60μF in series with 120μF = $\frac{60x120}{180}$ = **40μF**

P.P.6.7 Before we solve this, we need to assume that the initial charge on each capacitor is equal to zero.

60μF in series with 30μF = $\frac{60x30}{90}$ = 20μF

20μF in parallel with 20μF = 40μF



From the Figure, $v_1 = v_2$ = 90/2 = **45 V**; $q_1 = 45x40x10^{-6}$ = 1.8 mC; $q_2 = 45x20x10^{-6}$ = 0.9 mC = $q_3 = q_4$ leading to $v_3 = 0.0009/(60x10^{-6})$ = **15 V** and
$v_4 = 0.0009/(30x10^{-6})$ = **30 V**.

P.P.6.8 $v = L\frac{di}{dt} = 10^{-3}\frac{d}{dt}(60\cos(100t))\cdot 10^{-3}$

= **–6sin(100t) mV**

$w = \frac{1}{2}Li^2 = \frac{1}{2}x10^{-3}\left(3600\cos^2(100t)\right)\cdot 10^{-6}$

= **1.8cos ^{2}(100t) μJ**.

P.P.6.9 $i = \frac{1}{L}\int_{t_0}^{t} v(t)dt + i(t_0) = \frac{1}{2}\int_0^t 10(1-t)dt + 2$

$= 5\left(t - \frac{t^2}{2}\right) + 2$

At t = 4, i = 5(4 - 8) + 2 = **–18 A**

p = vi = 10(1 - t) $\left[5t - \frac{5}{2}t^2 + 2\right]$

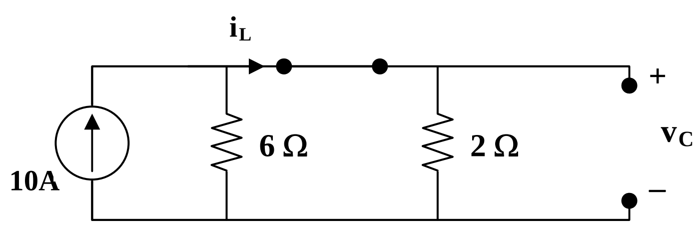
= 20 + 30t - 75t^2 + 25t^3

w = $\int_0^4 p\,dt$ = [20t + 15t^2 - 25t^3 + $25t^4/4$]$\Big|_0^4$

= 80 + 15 x 16 - 1600 + 1600

w = **320 J**

P.P.6.10 Under dc conditions, the circuit is equivalent to that shown below



$i_L = \frac{3}{1+3}(10) = $ **7.5 A**

$v_C = 2i_C$ = **15 V**

$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(4)(15)^2 =$ **450 J**

$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(6)(7.5)^2 =$ **168.75 J**.

P.P.6.11 40mH in series with 20mH = 40 + 20 = 60mH
60mH in parallel with 30mH = 30 x 60/(90) = 20mH
20mH in series with 100mH = 120mH
120mH in parallel with 40mH = 40 x 120/(160) = 30mH
30mH in series with 20mH = 50mH
50mH in parallel with 50mH = 25mH

L_{eq} = **25mH**

P.P.6.12 (a) $i_2 = i - i_1 \longrightarrow i_2(0) = i(0) - i_1(0) = 1.4 - 0.6 =$ **800 mA**

(b) $v_1 = 6\frac{di_1}{dt} = 6(0.6)(-2)e^{-2t} = -7.2e^{-2t}$

$$i_2 = \frac{1}{3}\int_0^t v_1 dt + i_2(0) = \frac{1}{3}\frac{(-7.2)}{(-2)}e^{-2t}\Big|_0^t + 0.8$$

$= \mathbf{(-0.4 + 1.2e^{-2t})\ A}$

$i = i_1 + i_2 = \mathbf{(-0.4 + 1.8e^{-2t})\ A}$

(c) From (b),

$v_1 = \mathbf{-7.2e^{-2t}V}$

$v_2 = 8\frac{di}{dt} = 8(-2)(1.8)e^{-2t} = \mathbf{-28.8e^{-2t}V}$

$v = v_1 + v_2 = \mathbf{-36e^{-2t}V}$

P.P.6.13 RC = 100 x 10^3 x 20 x 10^{-6} = 2

$$v_o = -\frac{1}{RC}\int_o^t v_i(t) + v_o(0) = -\frac{1}{2}\int_o^t 2.5dt \text{ mV} + 0$$

= **–1.25t mV**.

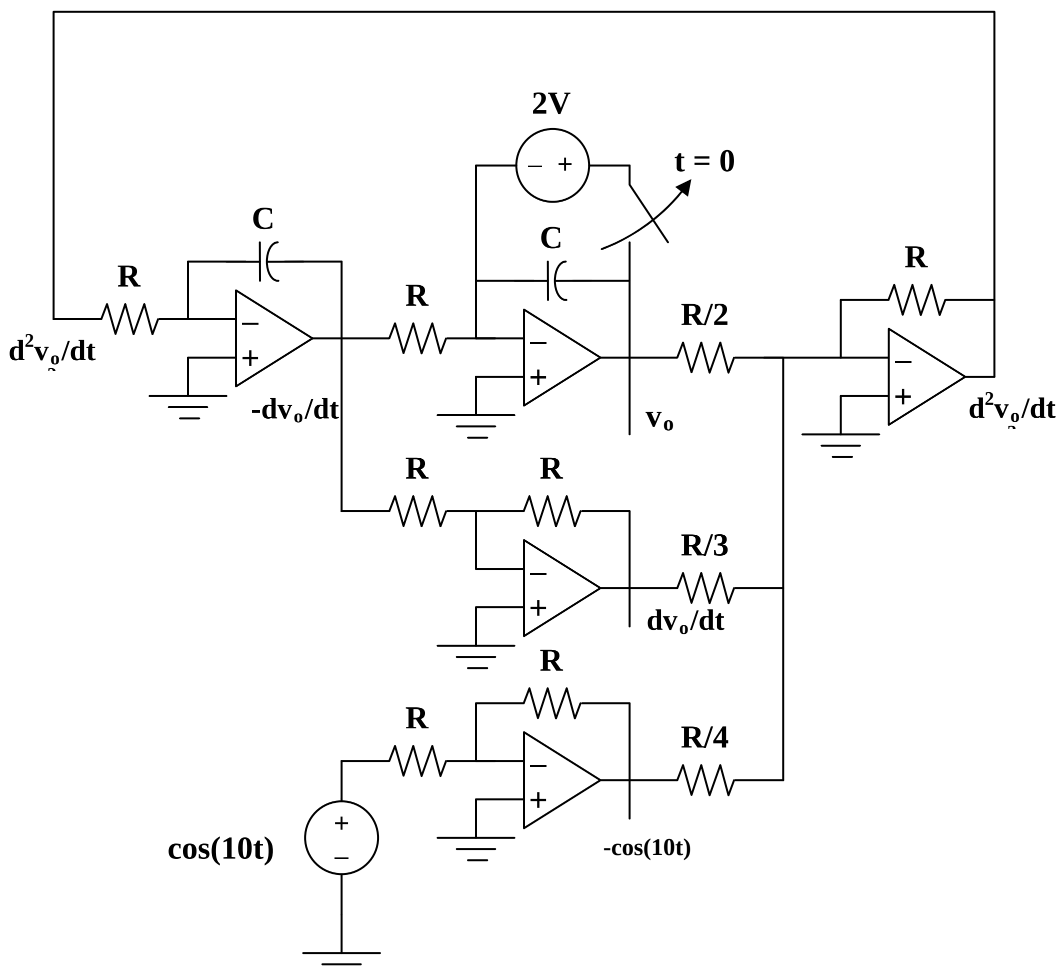
P.P.6.14 RC = 100 x 10^3 x 0.1 x 10^{-6} = 10 x 10^{-3}

$$v_o = -RC\frac{dv_i}{dt} = -10x10^{-3}\frac{d}{dt}(1.25t)$$

v_o = **–12.5 mV**.

P.P.6.15 $$\frac{dv_o^2}{dt^2} = 4\cos 10t - 3\frac{dv_o}{dt} - 2v_o$$

Using this we obtain the analog computer as shown below. We may let RC = 1s.



2V
t = 0
C
C
R
R
R
R/2
d^2v_o/dt
$-dv_o/dt$
v_o
d^2v_o/dt
R
R
R/3
dv_o/dt
R
R
R/4
cos(10t)
-cos(10t)