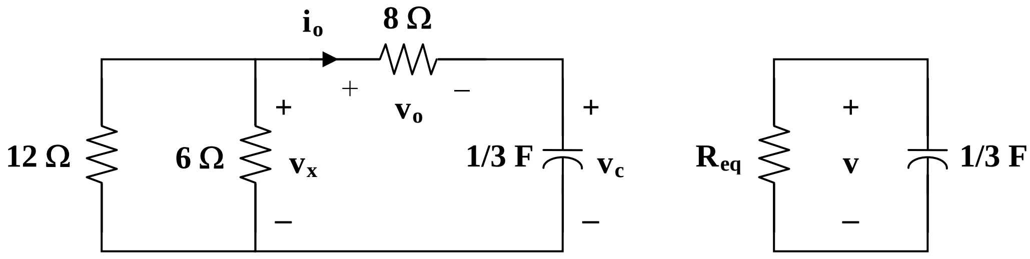
CHAPTER 7

P.P.7.1 The circuit in Fig. (a) is equivalent to the one shown in Fig. (b).



(a) (b)

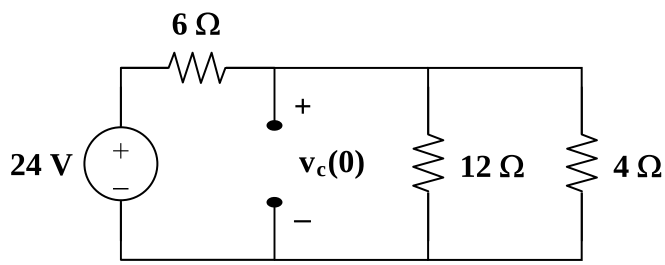
$$R_{eq} = 8 + 12 \| 6 = 12\ \Omega$$
$$\tau = R_{eq}C = (12)(1/3) = 4\text{ s}$$
$$v_c = v_c(0)e^{-t/\tau} = 60e^{-t/4} = \mathbf{60e^{-0.25t}\ V}$$

$$v_x = \frac{4}{4+8}v_c = \mathbf{20e^{-0.25t}\ V}$$

$$v_x = v_o + v_c \longrightarrow v_o = v_x - v_c = -40e^{-0.25t}\ V$$

$$i_o = \frac{v_o}{8} = \mathbf{-5e^{-0.25t}\ A}.$$

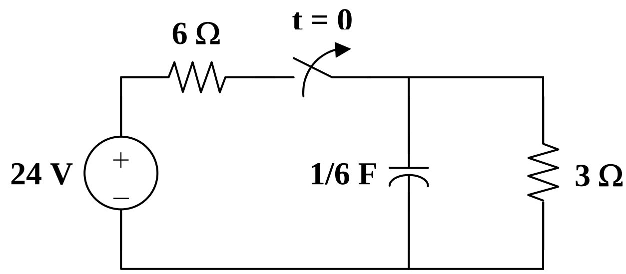
P.P.7.2 When t < 0, the switch is closed as shown in Fig. (a).



(a)

$$R_{eq} = 4 \| 12 = 3\ \Omega \qquad v_c(0) = \frac{3}{3+6}(24) = 8\text{ V}$$

When t > 0, the switch is open as shown in Fig. (b).



(b)

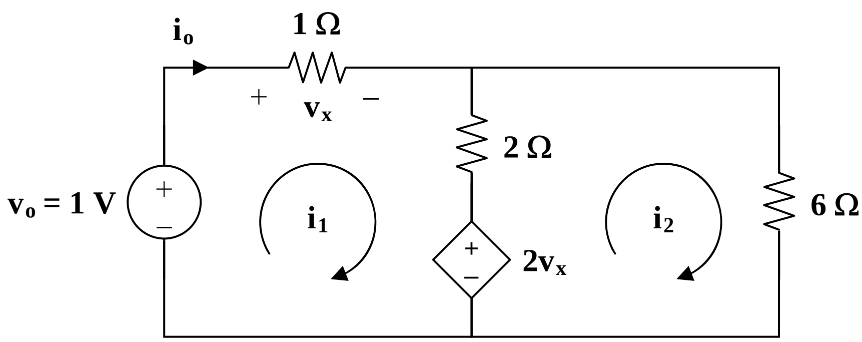
$\tau = R_{eq}C = (3)(1/6) = 1/2$ s

$v(t) = v_c(0)e^{-t/\tau}$ = $\mathbf{8e^{-2t}}$ **V**

$w_c(0) = \frac{1}{2}Cv_c^2(0) = \frac{1}{2} \times \frac{1}{6} \times 64 =$ **5.333J**

P.P.7.3 This can be solved in two ways.

Method 1: Find R_{th} at the inductor terminals by inserting a voltage source.



Applying mesh analysis gives

Loop 1: $-1 + 3i_1 - 2i_2 + 2v_x = 0$, where $v_x = 1i_1$

$5i_1 - 2i_2 = 1$ (1)

Loop 2: $8i_2 - 2i_1 - 2v_x = 0 = 8i_2 - 2i_1 - 2i_1$

$i_2 = \frac{1}{2}i_1$ (2)

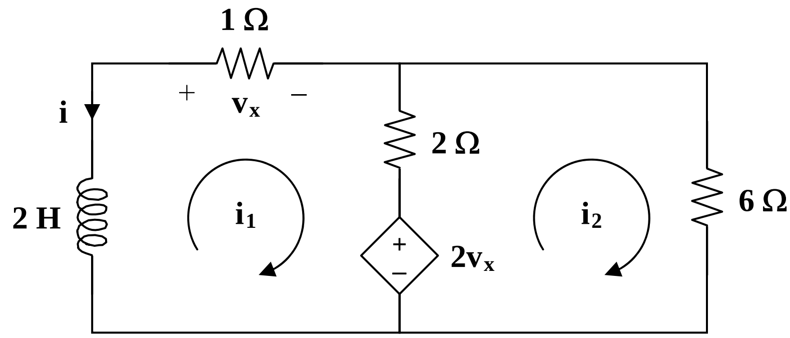
From (1) and (2), $5i_1 - 1i_1 = 1$ or

$i_o = i_1 = (1/4)$ A

$$R_{th} = \frac{v_o}{i_o} = 4\,\Omega, \qquad \tau = \frac{L}{R} = \frac{2}{4} = \frac{1}{2}\,s$$

$$i(t) = 12e^{-2t}\ A$$

Method 2: We can obtain i using mesh analysis.



Applying KVL to the loops, we obtain

Loop 1: $2\frac{di_1}{dt} + 3i_1 - 2i_2 + 2v_x = 0$ where $v_x = 1i_1$

$$2\frac{di_1}{dt} + 5i_1 - 2i_2 = 0 \qquad (3)$$

Loop 2: $8i_2 - 2i_1 - 2v_x = 0$

$$i_2 = \frac{1}{2}i_1 \qquad (4)$$

Substituting (4) into (3) yields

$$2\frac{di_1}{dt} + 5i_1 - 1i_1 = 0$$

or $\frac{di_1}{dt} + 2i_1 = 0$

$$i_1 = Ae^{-2t}$$

$$i = -i_1 = Be^{-2t}$$
$$i(0) = 12 = B$$

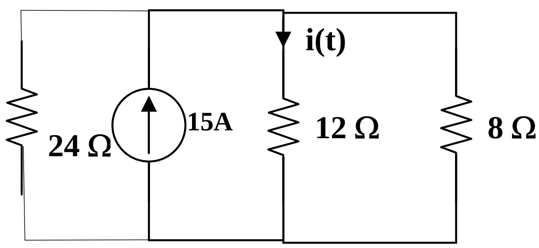
$$i(t) = 12e^{-2t}\ A$$

Therefore,

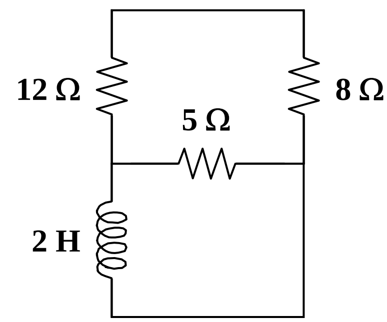
$i(t) = \mathbf{12e^{-2t}\ A}$

and $v_x(t) = -1i(t) = \mathbf{-12e^{-2t}\ V}$ for all $t > 0$.

P.P.7.4 For t < 0, the equivalent circuit is shown in Fig. (a).



(a)



(b)

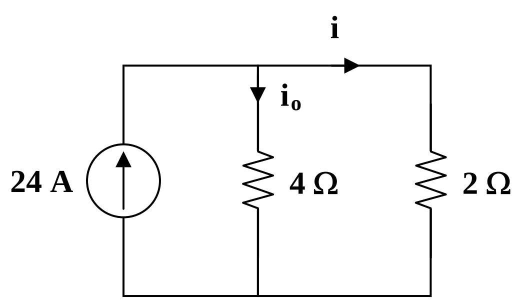
$$i(0) = 15[1/\{(1/24) + (1/12) + (1/8)\}]/12 = (15x24/6)/12 = 5\ A$$

For t > 0, the current source and 24-ohm is cut off and the RL circuit is shown in Fig. (b).

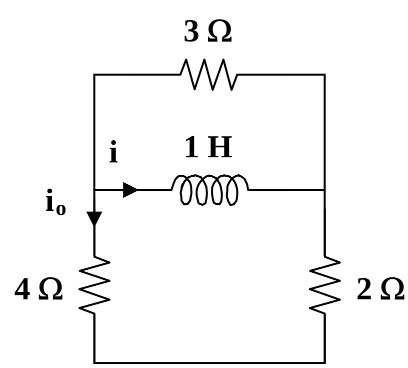
$$R_{eq} = (12 + 8) \,||\, 5 = 20 \,||\, 5 = 4\,\Omega, \qquad \tau = \frac{L}{R_{eq}} = \frac{2}{4} = 0.5$$

$i(t) = i(0)e^{-2t} = \mathbf{5e^{-2t}}$ **amps**, for all t > 0.

P.P.7.5 For t < 0, the switch is closed. The inductor acts like a short so the equivalent circuit is shown in Fig. (a).



(a)



(b)

$$i = \frac{4}{4+2}(24) = 16\ A, \qquad i_o = 24 - 16 = 8\ A, \qquad v_o = 2i = 32\ V$$

For t > 0, the current source is cut off so that the circuit becomes that shown in Fig. (b). The Thevenin equivalent resistance at the inductor terminals is

$$R_{th} = (4+2) \,||\, 3 = 2\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{1}{2}$$

$$i_o = \frac{3(-i)}{6+3} = \frac{-1}{3}i = -5.333e^{-2t}\ A \qquad \text{and} \qquad v_o = -2i_o = 10.667e^{-2t}V$$

Thus,

$$i = \begin{cases} 16\text{ A} & t<0 \\ 16e^{-2t}\text{ A} & t>0 \end{cases} \quad i_o = \begin{cases} 8\text{ A} & t<0 \\ -5.333e^{-2t}\text{ A} & t>0 \end{cases} \quad v_o = \begin{cases} 32\text{ V} & t<0 \\ 10.667e^{-2t}\text{V} & t>0 \end{cases}$$

P.P.7.6 $$i(t) = \begin{cases} 0 & t<0 \\ 10 & 0<t<2 \\ -10 & 2<t<4 \end{cases}$$

$$i(t) = 10[u(t) - u(t-2)] - 10[u(t-2) - u(t-4)]$$
$$i(t) = \mathbf{10[u(t) - 2u(t-2) + u(t-4)]A}$$

Let $I = \int_{-\infty}^{t} i\, dt$.

For $t < 0$, $I = 0$.

For $0 < t < 2$, $I = \int_0^t 10\, dt = 10t$

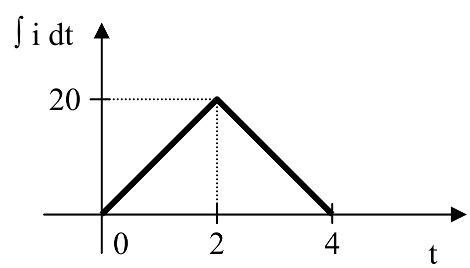
For $2 < t < 4$, $I = \int_0^2 10\, dt - 10\int_2^t dt = 20 - 10t\Big|_2^t = 40 - 10t$

For $t > 4$, $I = 20 - 10t\Big|_2^4 = 0$

Thus,

$$I = \begin{cases} 0 & t<0 \\ 10t & 0<t<2 \\ 40-10t & 2<t<4 \\ 0 & t>4 \end{cases}$$

or $$I = \mathbf{10[r(t) - 2r(t-2) + r(t-4)]\underline{A}}$$
which is sketched below



P.P.7.7 $$i(t) = \begin{cases} 2-2t & 0<t<2 \\ -6+2t & 2<t<3 \\ 0 & \text{otherwise} \end{cases}$$

$$i(t) = (2-2t)[u(t) - u(t-2)] + (-6+2t)[u(t-2) - u(t-3)]$$

$$i(t) = 2\,u(t) - 2t\,u(t) + 4(t-2)\,u(t-2) - 2(t-3)\,u(t-3)$$

$$i(t) = \mathbf{[2\,u(t) - 2\,r(t) + 4\,r(t-2) - 2\,r(t-3)]\,A}$$

Remember the singularity function, r(t), is a ramp function equal to t for all values of t > 0 and equal to zero for all values of t < 0.

P.P.7.8 $h(t) = -4[u(t) - u(t-2)] + (3t-8)[u(t-2) - u(t-6)]$

$h(t) = -4u(t) + 4u(t-2) + 3tu(t-2) - 8u(t-2) - 3tu(t-6) + 8u(t-6)$

$h(t) = -4u(t) + (4-8+6)u(t-2) + 3(t-2)u(t-2) - 3(t-6)u(t-6) + (-18+8)u(t-6)$

$h(t) = -4u(t) + 2u(t-2) + 3(t-2)u(t-2) - 3(t-6)u(t-6) - 10u(t-6)$

$h(t) = \mathbf{-4u(t) + 2u(t-2) + 3r(t-2) - 10u(t-6) - 3r(t-6)}$.

P.P.7.9 (a) $\int_{-\infty}^{\infty}\left(t^3 + 5t^2 + 10\right)\delta(t+3)\,dt = t^3 + 5t^2 + 10\Big|_{t=-3}$

$= -27 + 45 + 10 = \mathbf{28}$

(b) $\int_{0}^{10}\delta(t-\pi)\cos(3t)\,dt = \cos(3\pi) = \mathbf{-1}$

P.P.7.10 For t < 0, the capacitor acts like an open circuit.

$v(0^-) = v(0^+) = v(0) = 15$

For t > 0, $[(v(\infty)-15)/2] + [(v(\infty)-(-7.5))/6] = 0$ or $(4/6)v(\infty) = 7.5-1.25 = 6.25$ or

$v(\infty) = 9.375$ V

$$R_{th} = 2\,||\,6 = \frac{3}{2}\,\Omega, \qquad \tau = R_{th}C = \frac{3}{2}\times\frac{1}{3} = \frac{1}{2}$$

$$v(t) = v(\infty) + \left[\,v(0) - v(\infty)\right]e^{-t/\tau} = 9.375 + (15-9.375)e^{-2t}$$

$$v(t) = \mathbf{(9.375 + 5.625e^{-2t})\ V\ for\ all\ t > 0}$$

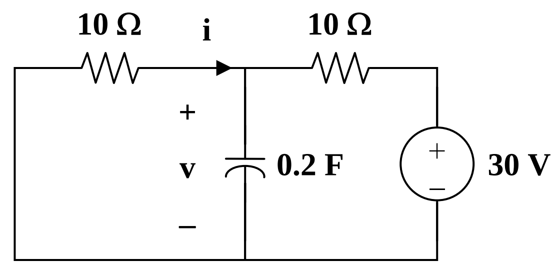
At t = 0.5, $v(0.5) = 6.25 + 3.75e^{-1} = 6.25 + 1.3795 = \mathbf{7.63\ V}$

P.P.7.11 For t < 0, only the left portion of the circuit is operational at steady state.

$v(0^-) = v(0^+) = v(0) = 20$, $i(0) = 0$

For t > 0, $20u(-t) = 0$ so that the voltage source is replaced by a short circuit.

Transforming the current source leads to the circuit below.



$$v(\infty) = \frac{5}{15}(30) = 10$$

$$R_{th} = 5 \,||\, 10 = \frac{10}{3}\,\Omega, \qquad \tau = R_{th}C = \frac{10}{3} \times 0.2 = \frac{2}{3}$$

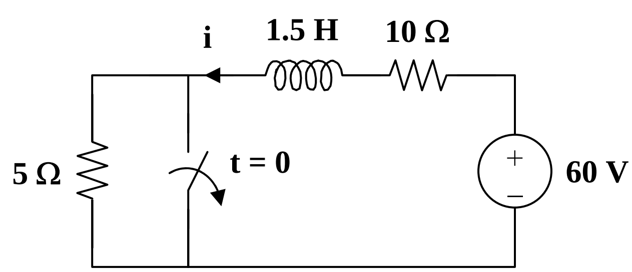
$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right]\, e^{-t/\tau}$$
$$v(t) = 10 + (20 - 10)\, e^{-3t/2}$$
$$v(t) = 10\left(1 + e^{-1.5t}\right)$$

$$i(t) = \frac{-\,v(t)}{5} = -2\left(1 + e^{-1.5t}\right)$$

$$i(t) = \begin{cases} \mathbf{0} & \mathbf{t < 0} \\ \mathbf{-\,2\left(1 + e^{-1.5t}\right) A} & \mathbf{t > 0} \end{cases}$$

$$v(t) = \begin{cases} \mathbf{20\ V} & \mathbf{t < 0} \\ \mathbf{10\left(1 + e^{-1.5t}\right) V} & \mathbf{t > 0} \end{cases}$$

P.P.7.12 Applying source transformation, the circuit is equivalent to the one below.



At t < 0, the switch is closed so that the 5 ohm resistor is short circuited.

$$i(0^-) = i(0) = \frac{60}{10} = 6\ A$$

For t > 0, the switch is open.

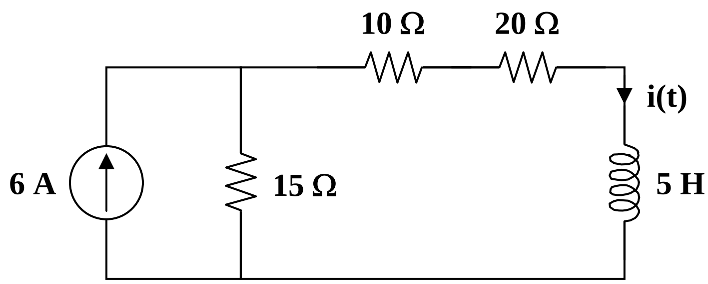
$$R_{th} = 10 + 5 = 15, \qquad \tau = \frac{L}{R_{th}} = \frac{1.5}{15} = 0.1$$

$$i(\infty) = \frac{60}{10+5} = 4\text{A}$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$
$$i(t) = 4 + (6-4)e^{-10t}$$

i(t) = **(4 + 2e^{-10t}) A for all t > 0**

P.P.7.13 For 0 < t < 2, the given circuit is equivalent to that shown below.



Since switch S_1 is open at $t = 0^-$, $i(0^-) = 0$. Also, since i cannot jump, $i(0) = i(0^-) = 0$.

$$i(\infty) = \frac{90}{15+10+20} = 2\text{ A}$$
$$R_{th} = 45\,\Omega, \quad \tau = \frac{L}{R_{th}} = \frac{5}{45} = \frac{1}{9}$$
$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$
$$i(t) = 2 + (0-2)e^{-9t}$$
$$i(t) = 2(1 - e^{-9t})\text{ A}$$

When switch S_2 is closed, the 20 ohm resistor is short-circuited.

$$i(2^+) = i(2^-) = 2(1 - e^{-18}) \cong 2$$

This will be the initial current

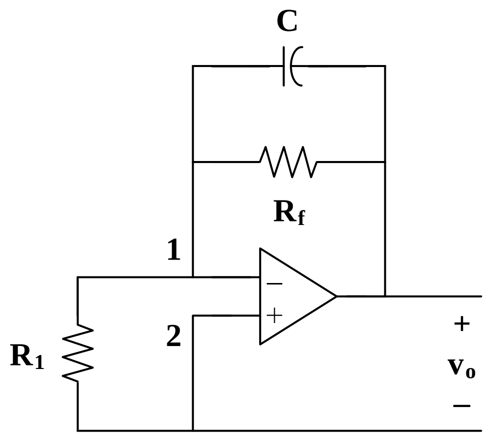
$$i(\infty) = \frac{90}{15+10} = 3.6\text{ A}$$
$$R_{th} = 25\,\Omega, \quad \tau = \frac{5}{25} = \frac{1}{5}$$
$$i(t) = i(\infty) + \left[\, i(2^+) - i(\infty)\right] e^{-(t-2)/\tau}$$
$$i(t) = 3.6 + (2-3.6)e^{-5(t-2)}$$
$$i(t) = 3.6 - 1.6e^{-5(t-2)}$$

Thus, $i(t) = \begin{cases} \mathbf{0} & \mathbf{t < 0} \\ \mathbf{2(1 - e^{-9t})\ A} & \mathbf{0 < t < 2} \\ \mathbf{3.6 - 1.6e^{-5(t-2)}\ A} & \mathbf{t > 2} \end{cases}$

At $t = 1$, $i(1) = 2(1 - e^{-9}) = \mathbf{1.9997\ A}$

At $t = 3$, $i(3) = 3.6 - 1.6e^{-5} = \mathbf{3.589\ A}$

P.P.7.14 The op amp circuit is shown below.



Since nodes 1 and 2 must be at the same potential, there is no potential difference across R_1. Hence, no current flows through R_1. Applying KCL at node 1,

$$\frac{v}{R_f} + C\frac{dv}{dt} = 0 \longrightarrow \frac{dv}{dt} + \frac{v}{CR_f} = 0$$

which is similar to Eq. (7.4).

Hence,

$$v(t) = v_o\, e^{-t/\tau},\ \tau = R_f C$$

$$v(0) = v_o = 4, \qquad \tau = (50\times10^3)(10\times10^{-6}) = 0.5$$

$$v(t) = 4e^{-2t}\ \text{V}, \quad t > 0$$

Alternatively, since no current flows through R_1, the feedback loop forms a first order RC circuit with $v(0) = 4$ and $\tau = R_f C = 0.5$. Hence,

$$v(t) = 4e^{-2t}\ \text{V}, \quad t > 0$$

To get to v_o from v, we notice that v is the potential difference between node 1 and the output terminal, i.e.

$$0 - v_o = v \longrightarrow v_o = -v \text{ or } v_o(t) = \mathbf{-4e^{-2t}\ V, \quad t > 0}$$

P.P.7.15 Let v_1 be the potential at the inverting terminal.

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

where $\tau = RC = 100 \times 10^3 \times 10^{-6} = 0.1$, $v(0) = 0$

$$v_1 = 0 \text{ for all t}$$

$$v_1 - v_o = v \qquad (1)$$

For t > 0, the switch is closed and the op amp circuit is an inverting amplifier with

$$v_o(\infty) = \frac{-100}{10}(4 \text{ mV}) = -40 \text{ mV}$$

From (1),

$$v(\infty) = 0 - v_o(\infty) = 40 \text{ mV}$$

Thus, $v(t) =$ **$40(1 - e^{-10t})u(t)$ mV**

$$v_o = v_1 - v = -v$$

$v_o =$ **$40(e^{-10t} - 1)u(t)$ mV**

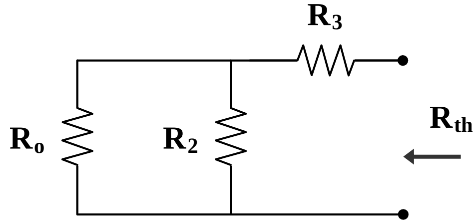
P.P.7.16 This is a noninverting amplifier so that the output of the op amp is

$$v_a = \left(1 + \frac{R_f}{R_1}\right) v_i$$

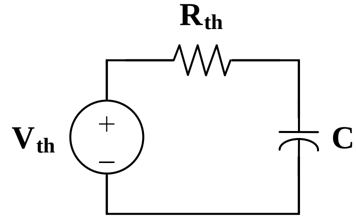
$$v_{th} = v_a = \left(1 + \frac{R_f}{R_1}\right) v_i = \left(1 + \frac{40}{20}\right) 4.5\,u(t) = 13.5\,u(t)$$

To get R_{th}, consider the circuit shown in Fig. (a), where R_o is the output resistance of the op amp. For an ideal op amp, $R_o = 0$ so that

$$R_{th} = R_3 = 10\text{ k}\Omega$$



(a)



(b)

$$\tau = R_{th}C = 10 \times 10^3 \times 2 \times 10^{-6} = \frac{1}{50}$$

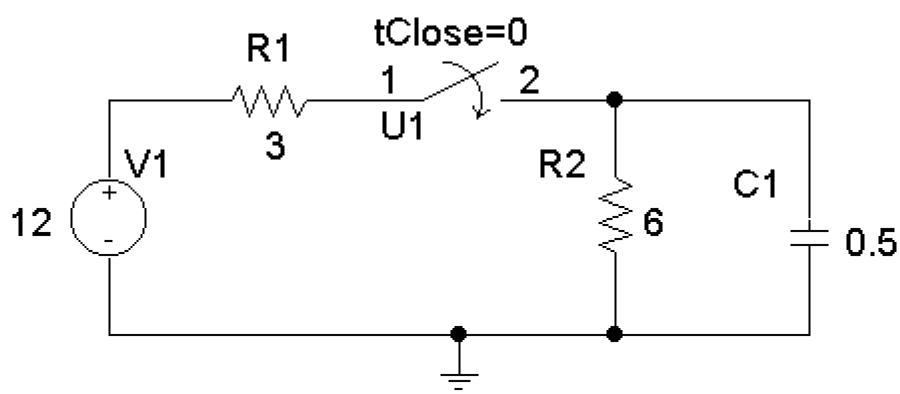
The Thevenin equivalent circuit is shown in Fig. (b), which is a first order circuit.

Hence,

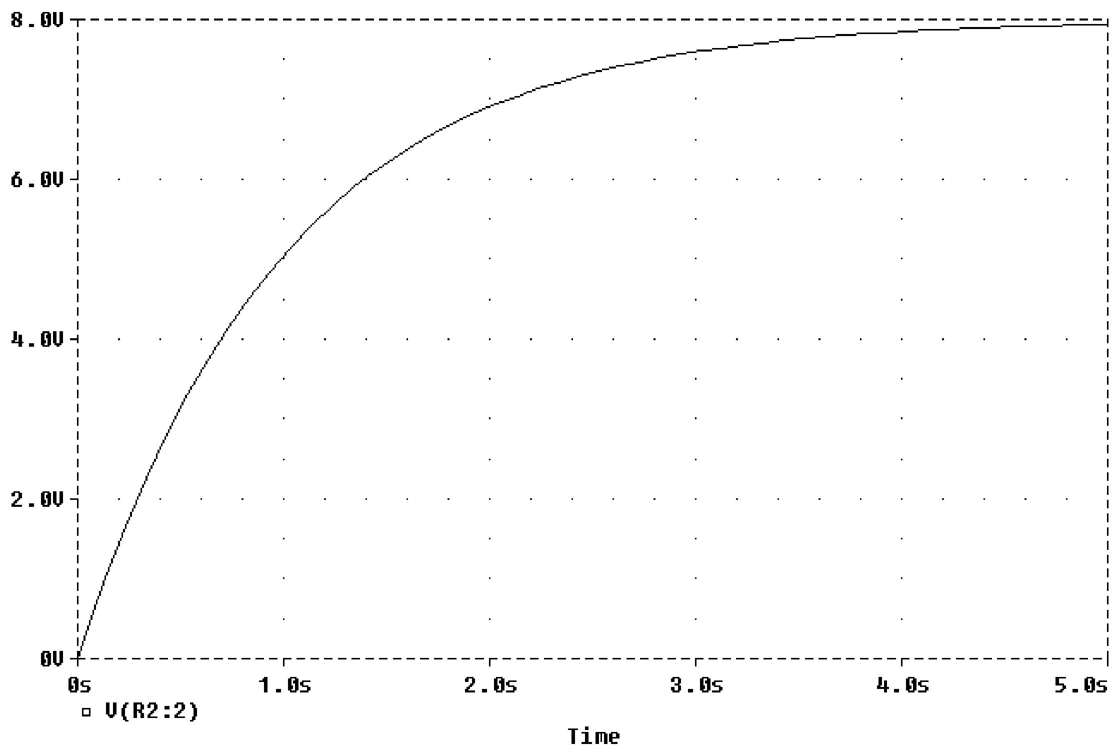
$$v_o(t) = 13.5\left(1 - e^{-t/\tau}\right)u(t)$$

$$v_o(t) = \mathbf{13.5(1 - e^{-50t})u(t)\ V}$$

P.P.7.17 The schematic is shown in Fig. (a). Construct and save the schematic. Select Analysis/Setup/Transient to change the Final Time to 5 s. Set the Print Step slightly greater than 0 (20 ns is default). The circuit is simulated by selecting Analysis/ Simulate. In the Probe menu, select Trace/Add and display V(R2:2) as shown in Fig. (b).

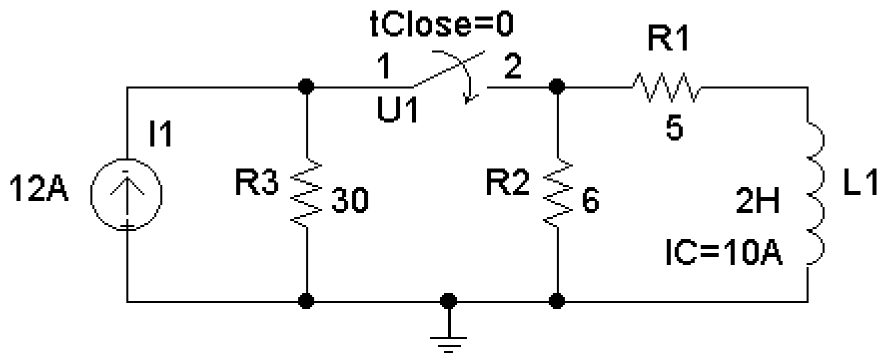


(a)

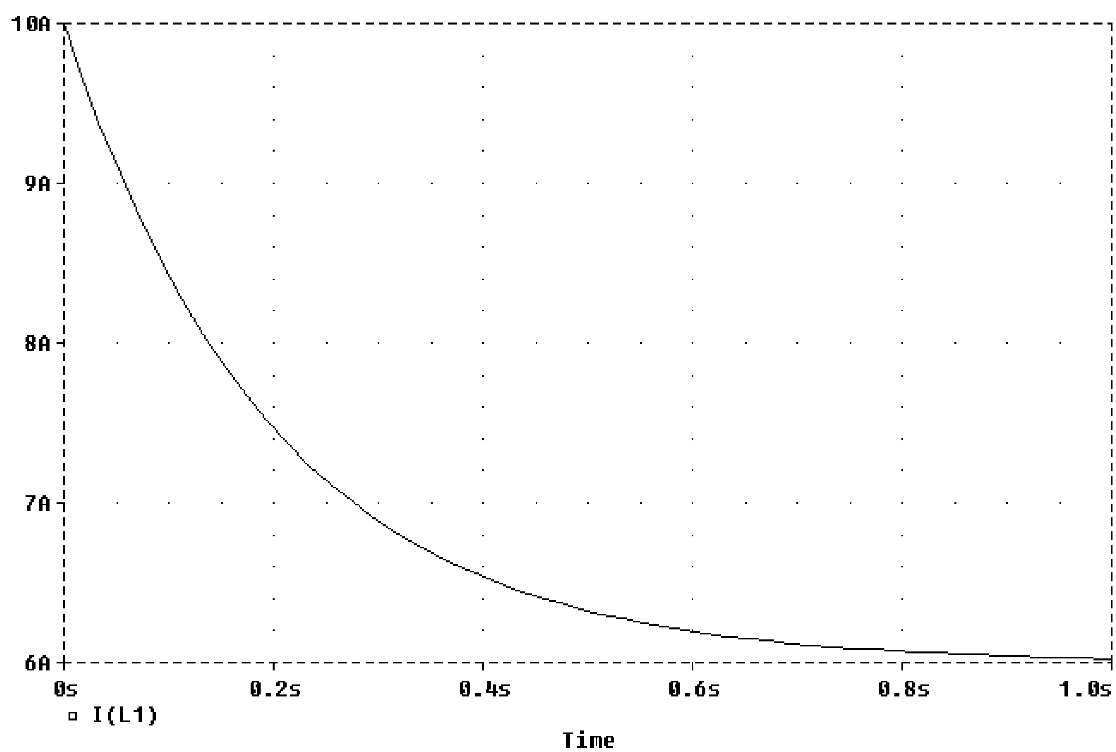


(b)

P.P.7.18 The schematic is shown in Fig. (a). While constructing the circuit, rotate L1 counterclockwise through 270° so that current i(t) enters pin 1 of L1 and set IC = 10 for L1. After saving the schematic, select Analysis/Setup/Transient to change the Final Time to 1 s. Set the Print Step slightly greater than 0 (20 ns is default). The circuit is simulated by selecting Analysis/ Simulate. After simulating the circuit, select Trace/Add in the Probe menu and display I(L1) as shown in Fig. (b).

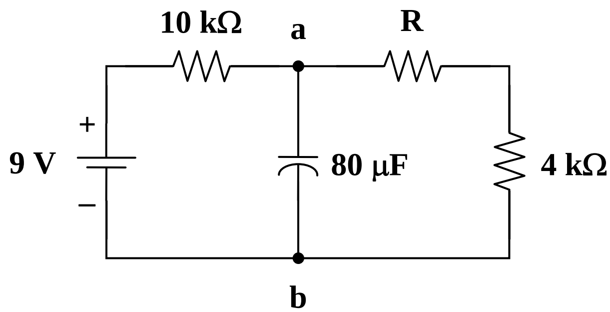


(a)



(b)

P.P.7.19 $v(0) = 0$. When the switch is closed, we have the circuit shown below.



We find the Thevenin equivalent at terminals a-b.

$$R_{th} = (R+4) \parallel 10 = \frac{10(R+4)}{R+14}$$

$$v_{th} = v(\infty) = \frac{R+4}{R+14}(9)$$

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}, \qquad \tau = R_{th}C$$

$$v(t) = v(\infty)\left(1 - e^{-t/\tau}\right)$$

Since $v(0) = 0$,

$$i(t) = \frac{v(t)}{R+4} = \frac{9}{R+4}\left(1 - e^{-t/\tau}\right) \text{ mA}$$

Assuming R is in kΩ,

$$120 \times 10^{-6} = \frac{9}{R+14}\left(1 - e^{-t_0/\tau}\right) \times 10^{-3}$$

$$(0.12)\frac{R+14}{9} = 1 - e^{-t_0/\tau}$$

or $$e^{-t_0/\tau} = 1 - \frac{0.12R + 1.68}{9} = \frac{7.32 - 0.12R}{9}$$

$$t_0 = \tau \ln\left(\frac{9}{7.32 - 0.12R}\right)$$

$$t_0 = \frac{10(R+4)}{R+14} \times 80 \times 10^{-6} \times \ln\left(\frac{9}{7.32 - 0.12R}\right)$$

When R = 0,

$$t_0 = \frac{40 \times 80 \times 10^{-6}}{14} \times \ln\left(\frac{9}{7.32}\right) = 0.04723 \text{ s}$$

When R = 6 kΩ,

$$t_0 = \frac{100}{20} \times 80 \times 10^{-6} \times \ln\left(\frac{9}{6.6}\right) = 0.124 \text{ s}$$

The time delay is **between 47.23 ms and 124 ms**.

P.P.7.20 (a) $q = CV = (2\text{x}10^{-3})(80) = \mathbf{160\ mC}$.

(b) $W = \frac{1}{2}CV^2 = \frac{1}{2}(2 \times 10^{-3})(6400) = \mathbf{6.4\ J}$

(c) $\Delta I = \frac{\Delta q}{\Delta t} = \frac{0.16}{0.8 \times 10^{-3}} = \mathbf{200\ A}$

(d) $p = \frac{\Delta w}{\Delta t} = \frac{6.4}{0.8 \times 10^{-3}} = \mathbf{8\ kW}$

(e) $\Delta t = \frac{\Delta q}{\Delta I} = \frac{0.16}{5 \times 10^{-3}} = \mathbf{32\ s}$

P.P.7.21 $\tau = \frac{L}{R} = \frac{500 \times 10^{-3}}{200} = 2.5 \text{ ms}$

$i(0) = 0, \qquad i(\infty) = \frac{110}{200} = 550 \text{ mA}$

$i(t) = 550\left(1 - e^{-t/\tau}\right) \text{ mA}$

$350 \text{ mA} = i(t_0) = 550\left(1 - e^{-t_0/\tau}\right) \text{ mA}$

$\frac{35}{55} = 1 - e^{-t_0/\tau} \longrightarrow e^{-t_0/\tau} = \frac{20}{55}$

$e^{t_0/\tau} = \frac{55}{20}$

$t_0 = \tau \ln\left(\frac{55}{20}\right) = 2.5 \ln\left(\frac{55}{20}\right) \text{ ms}$

$t_0 = \mathbf{2.529\ ms}$

P.P.7.22 (a) $t = 5\tau = \frac{5L}{R} = \frac{5 \times 20 \times 10^{-3}}{5} = \mathbf{20\ ms}$

(b) $W = \frac{1}{2}LI^2 = \frac{1}{2}\left(20 \times 10^{-3}\right)\left(\frac{12}{5}\right)^2 = \mathbf{57.6\ mJ}$

(c) $V = L\frac{di}{dt} = 20 \times 10^{-3}\left(\frac{12/5}{2 \times 10^{-6}}\right) = \mathbf{24\ kV}$