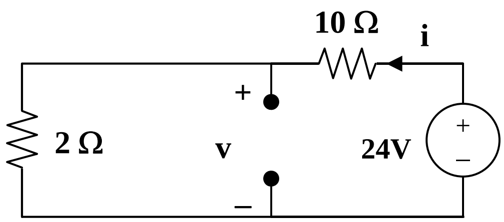
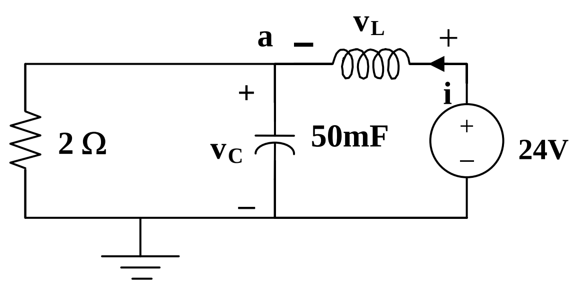
CHAPTER 8

P.P.8.1

(a) At $t = 0^-$, we have the equivalent circuit shown in Figure (a).



(a)



(b)

$$i(0^-) = 24/(2 + 10) = \mathbf{2\ A},\ \ v(0^-) = 2i(0^-) = 4\ \text{V}$$

$$\text{hence,}\ \ v(0^+) = v(0^-) = \mathbf{4\ V}.$$

(b) At $t = 0^+$, the switch is closed.

$$L(di/dt) = v_L,\ \ \text{leads to } (di/dt) = v_L/L$$

But, $v_C(0^+) + v_L(0^+) = 24 = 4 + v_L(0+)$, or $v_L(0+) = 20$ V

$$(di(0^+)/dt) = 20/0.4 = \mathbf{50\ A/s}$$

$$C(dv/dt) = i_C \ \ \text{leading to } (dv/dt) = i_C/C$$

But at node a, KCL gives $i(0^+) = i_C(0^+) + v(0^+)/2 = 2 = i_C(0^+) + 4/2$

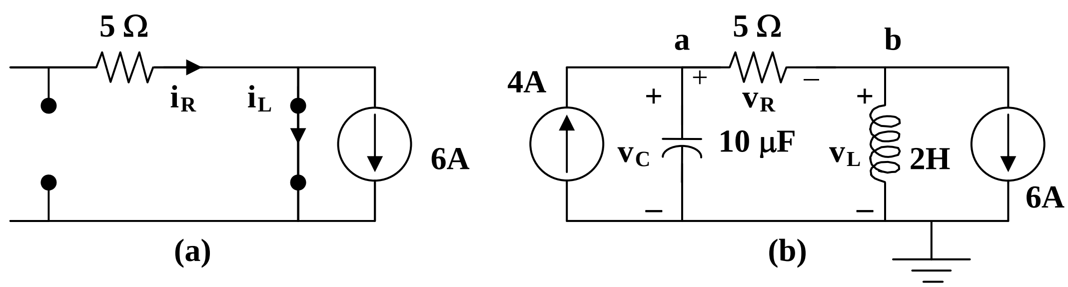
$$\text{or}\ \ i_C(0^+) = 0,\ \ \text{hence } (dv(0^+)/dt) = \mathbf{0\ V/s}$$

(c) As t approaches infinity, the capacitor is replaced by an open circuit and the inductor is replaced by a short circuit.

$$v(\infty) = \mathbf{24\ V},\ \ \text{and}\ \ i(\infty) = \mathbf{12\ A}.$$

P.P.8.2

(a) At $t = 0-$, we have the equivalent circuit shown in (a).



(a) (b)

$$i_L(0-) = -6A, \quad v_L(0^-) = 0, \quad v_R(0^-) = 0$$

At $t = 0^+$, we have the equivalent circuit in Figure (b). At node b,

$$i_R(0^+) = i_L(0^+) + 6, \text{ since } i_L(0^+) = i_L(0^-) = -6A, \; i_R(0^+) = 0,$$

and $v_R(0+) = 5i_R(0+) = 0$. Thus, $i_L(0) = \mathbf{-6\ A}$, $v_C(0) = \mathbf{0}$, and $v_R(0^+) = \mathbf{0}$.

(b) $dv_C(0^+)/dt = i_C(0^+)/C = 4/0.2 = 20$ V/s.

To get (dv_R/dt), we apply KCL to node b, $i_R = i_L + 6$, thus $di_R/dt = di_L/dt$.

Since $v_R = 5i_R$, $dv_R/dt = 5di_R/dt = 5di_L/dt$. But $Ldi_L/dt = v_L$, $di_L/dt = v_L/L$.

$$\text{Hence, } dv_R(0^+)/dt = 5v_L(0^+)/L.$$

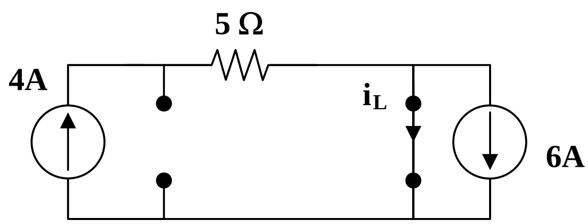
Applying KVL to the middle mesh in Figure (b),

$$-v_C(0^+) + v_R(0^+) + v_L(0^+) = 0 = 0 + 0 + v_R(0^+), \text{ or } v_R(0^+) = 0$$

$$\text{Hence, } dv_R(0^+)/dt = 0 = di_L(0^+)/dt;$$

$$di_L(0^+)/dt = \mathbf{0}, \quad dv_C(0^+)/dt = \mathbf{20\ V/s}, \quad dv_R(0^+)/dt = \mathbf{0}.$$

(c) As t approaches infinity, we have the equivalent circuit shown below.



$$4 = 6 + i_L(\infty) \text{ leads to } i_L(\infty) = -2\text{A}$$

$$v_C(\infty) = v_R(\infty) = 4\text{x}5 = 20\text{V}$$

Thus, $i_L(\infty)$ = **–2 A**, $v_C(\infty) = v_R(\infty)$ = **20 V**

P.P.8.3

(a) $\alpha = R/(2L) = 10/(2\text{x}5) = \mathbf{1}$, $\omega_o = 1/\sqrt{LC} = 1/\sqrt{5\text{x}2\text{x}10^{-2}} = \mathbf{10}$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1 \pm \sqrt{1-100} = \mathbf{-1 \pm j9.95}.$$

(b) Since $\alpha < \omega_o$, we clearly have an **underdamped** response.

P.P.8.4 For t < 0, the inductor is connected to the voltage source and when the circuit reaches steady state, the inductor acts like a short circuit.

$$i(0\text{-}) = 50/10 = 5 = i(0+) = i(0)$$

The voltage across the capacitor is $0 = v(0\text{-}) = v(0+) = v(0)$.

For t > 0, we have a source-free RLC circuit.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1\text{x}\frac{1}{9}} = 3$$

$$\alpha = R/(2L) = 5/(2\text{x}1) = 2.5$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.5 \pm \sqrt{6.25 - 9} = -2.5 \pm j1.6583$$

$$i(t) = e^{-2.5t}[A_1\cos 1.6583t + A_2\sin 1.6583t]$$

We now determine A_1 and A_2.

$$i(0) = 10 = A_1$$

$$di/dt = -2.5\{e^{-2.5t}[A_1\cos(1.6583t) + A_2\sin(1.6583t)]\} + 1.6583e^{-2.5t}[-A_1\sin(1.6583t) + A_2\cos(1.6583t)]$$

$$di(0)/dt = -(1/L)[Ri(0) + v(0)] = -2.5A_1 + 1.6583A_2$$

$$= -1[5x10+0] = -1[50] = -2.5(10) + 1.6583A_2$$

$$A_2 = -15.076$$

Thus, $i(t) = \mathbf{e^{-2.5t}[10\cos(1.6583t) - 15.076\sin(1.6583t)]\ A}$

P.P.8.5 $\alpha = 1/(2RC) = 1/(2x2x25x10^{-3}) = 10$

$$\omega_0 = 1/\sqrt{LC} = 1/\sqrt{0.4x25x10^{-3}} = 10$$

since $\alpha = \omega_o$, we have a critically damped response. Therefore,

$$v(t) = [(A_1 + A_2t)e^{-10t}]$$

$v(0) = 0 = A_1 + A_2x0 = A_1$, which leads to $v(t) = [A_2te^{-10t}]$.

$$dv(0)/dt = -(v(0) + Ri(0))/(RC) = -2x0.05/(2x25x10^{-3}) = -2$$

$$dv/dt = [(A_2 - 10A_2t)e^{-10t}]$$

At $t = 0$, $-2 = A_2$ therefore, $v(t) = \mathbf{(-2t)e^{-10t}u(t)\ V}$

P.P.8.6 For t < 0, the switch is closed. The inductor acts like a short circuit while the capacitor acts like an open circuit. Hence,

$$i(0) = 3A \text{ and } v(0) = 0.$$

$$\alpha = 1/(2RC) = 1/(2x20x4x10^{-3}) = 6.25$$

$$\omega_o = 1\Big/\sqrt{LC} = 1\Big/\sqrt{10x4x10^{-3}} = 5$$

Since $\alpha > \omega_o$, this is an overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o} = -6.25 \pm \sqrt{(6.25)^2 - 25} = -2.5 \text{ and } -10$$

$$\text{Thus,} \quad v(t) = A_1 e^{-2.5t} + A_2 e^{-10t}$$

$$v(0) = 0 = A_1 + A_2, \text{ which leads to } A_2 = -A_1$$

$$dv(0)/dt = -(v(0) + Ri(0))/(RC) = -(20x4.5)12.5 = -1125$$

$$\text{But, } dv/dt = -2.5A_1 e^{-2.5t} - 10A_2 e^{-10t}$$

At $t = 0$, $-1125 = -2.5A_1 - 10A_2 = 7.5A_1$ since $A_1 = -A_2$

$$A_1 = -150, \quad A_2 = 150$$

$$\text{Thus,} \quad v(t) = \mathbf{150(e^{-10t} - e^{-2.5t})\ V}$$

P.P.8.7 The initial capacitor voltage is obtained when the switch is in position a.

$$v(0) = [2/(2 + 1)]18 = 12\ V$$

The initial inductor current is $i(0) = 0$.

When the switch is in position b, we have the RLC circuit with the voltage source.

$$\alpha = R/(2L) = 10/(2x2.5) = 2$$

$$\omega_o = 1\Big/\sqrt{LC} = 1\Big/\sqrt{(5/2)x(1/40)} = 4$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o} = -2 \pm \sqrt{(2)^2 - 16} = -2 \pm j\,3.464$$

$$\text{Thus,}\quad v(t) = v_f + [(A_1\cos 3.464t + A_2\sin 3.464t)e^{-2t}]$$

where $v_f = v(\infty) = 15$, the final capacitor voltage. We now impose the initial conditions to get A_1 and A_2.

$$v(0) = 12 = 15 + A_1 \text{ leads to } A_1 = -3$$

The initial capacitor current is the same as the initial inductor current.

$$i(0) = C(dv(0)/dt) = 0 \text{ therefore, } dv(0)/dt = 0$$

$$\text{But,}\quad dv/dt = 3.464[\{-A_1\sin(3.464t) + A_2\cos(3.464t)\}e^{-2t}] \\ -2[\{A_1\cos(3.464t) + A_2\sin(3.464t)\}e^{-2t}]$$

$$dv(0)/dt = 0 - 2A_1 + 3.464A_2, \text{ which leads to } A_2 = -6/3.464 = -1.7321$$

$$\text{Thus,}\quad v(t) = \mathbf{\{15 + [(-3\cos(3.464t) - 1.7321\sin(3.464t)]e^{-2t}\}\ V}$$

$$i = C(dv/dt),\ v_R = Ri = RC(dv/dt) = (1/4)dv/dt$$

$$= (1/4)[(6 - 6)\cos(3.464t) + (2x1.7321 + 3x3.464)\sin(3.464t)]e^{-2t}$$

$$v_R(t) = \mathbf{(3.464\sin(3.464t)e^{-2t})\ V}$$

P.P.8.8 When $t < 0$, $v(0) = 0$, $i(0) = 0$; for $t > 0$,

$$\alpha = 0, \omega_o = 1/\sqrt{LC} = 1/\sqrt{0.2x20} = 0.25$$

$$i(t) = i_s + A_1\cos t + A_2\sin t = 10 + A_1\cos(0.25t) + A_2\sin(0.25t)$$

$$i(0) = 0 = 10 + A_1, \text{ therefore } A_1 = -10$$

$$Ldi(0)/dt = v(0) = 0$$

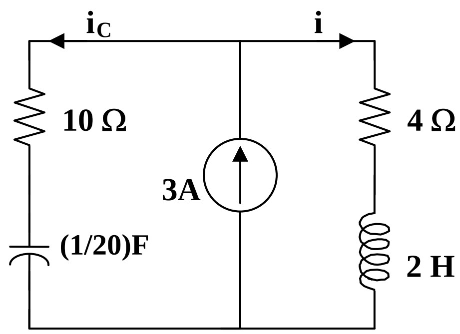
But $di/dt = -A_1\,0.25\sin(0.25t) + A_2\,0.25\cos(0.25t)$

At $t = 0$, $di(0)/dt = 0 = 0 + 0.25A_2$ leading to $i(t) = \mathbf{10(1 - \cos(0.25t))\ A}$

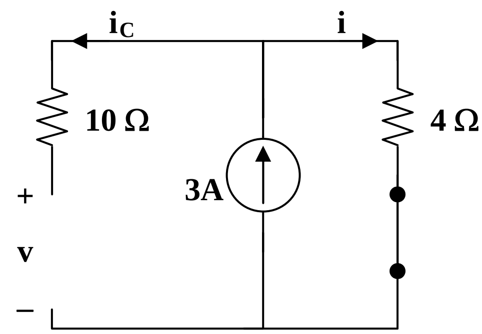
$v(t) = Ldi/dt = 20\text{x}10\text{x}0.25\sin t = \mathbf{50\sin(0.25t)\ V}$

P.P.8.9 At $t = 0$, the switch is open so that $v(0) = 0$, $i(0^-) = 0$ (1)

For $t > 0$, the switch is closed. We have the equivalent circuit as in Figure (a).



(a)



(b)

$$v(0^+) = 0,\ i(0^+) = 0 \qquad (2)$$

$$-3 + i_C + i = 0 \qquad (3)$$

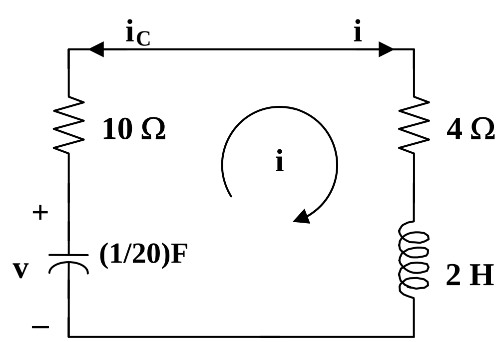
From (3), $i(0^+) = 0$ means that $i_C(0^+) = 3$, but $i_C(0^+) = Cdv(0^+)/dt$

which leads to $dv(0^+)/dt = i_C(0^+)/C = 3/(1/20) = 60$ V/s

As t approaches infinity, we have the equivalent circuit in (b).

$$i(\infty) = 3\text{ A},\ v(\infty) = 4i(\infty) = 12\text{V} \qquad (5)$$

Next we find the network response by turning off the current source as shown in Figure (c).



(c)

Applying KVL gives $-v - 10i_C + 4i + 2di/dt = 0$ (6)

Applying KCL to the top node, $i - i_C = 0$

Namely, $i = i_C = -Cdv/dt = -(1/20)dv/dt$ (7)

Combining (6) and (7), $-v - (10/20)dv/dt - (4/20)dv/dt - (2/20)d^2v/dt^2 = 0.$

or $(d^2v/dt^2) + 7(dv/dt) + 10 = 0$

The characteristic equation is $s^2 + 7s + 10 = 0 = (s + 2)(s + 5)$

This means that $v_n = (Ae^{-2t} + Be^{-5t})$ and $v_f = v(\infty) = 12$ V.

Thus, $v = v_f + v_n$ $v = 12 + (Ae^{-2t} + Be^{-5t})$ (8)

$v(0) = 0 = 12 + A + B$, or $A + B = -12$ (9)

$dv/dt = (-2Ae^{-2t} - 5Be^{-5t})$

$dv(0)/dt = 60 = -2A - 5B$

$2A + 5B = -60$ (10)

From (9) and (10), $A = 0$ and $B = -12$.

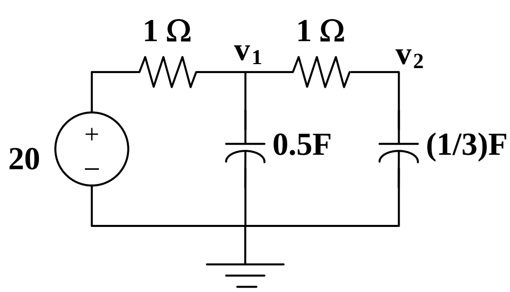
Thus, $v(t) = \mathbf{12(1 - e^{-5t})}$ **V for all t > 0**.

But, from (3), $i = 3 - i_C = 3 - (1/20)dv/dt = 3 - (1/20)(60)e^{-5t}$

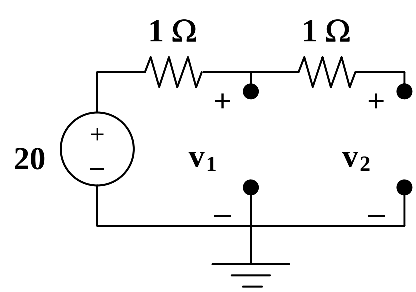
$i(t) = \mathbf{3(1 - e^{-5t})}$ **A for all t > 0**.

P.P.8.10 For $t < 0$, $5u(t) = 0$ so that $v_1(0\text{-}) = v_2(0\text{-}) = 0$ (1)

For $t > 0$, the circuit is as shown in Figure (a).



(a)



(b)

$$i_1 = C_1 dv_1/dt, \text{ or } dv_1/dt = i_1/C_1; \text{ likewise } dv_2/dt = i_2/C_2$$

$$i_2(0^+) = (v_1(0^+) - v_2(0^+))/1 = (0 - 0)/1 = 0$$

$$(20 - v_1(0^+))/1 = i_1(0^+) + i_2(0^+), \text{ or } 20 = i_1(0^+)$$

Hence,

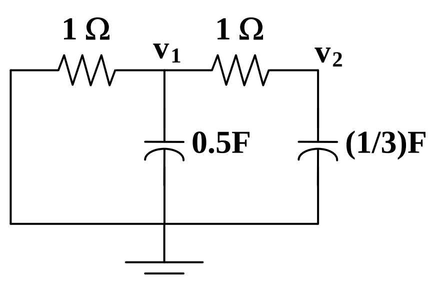
$$dv_1(0^+)/dt = 20/(1/2) = 40 \text{ V/s} \quad (2a)$$

$$dv_2(0^+)/dt = 0 \quad (2b)$$

As t approaches infinity, the capacitors can be replaced by open circuits as shown in Figure (b). Thus,

$$v_1(\infty) = v_2(\infty) = 20\text{V} \quad (3)$$

Next we obtain the network response by considering the circuit in Figure (c).



(c)

Applying KCL at node 1 gives $(v_1/1) + (1/2)(dv_1/dt) + (v_1 - v_2)/1 = 0$

or

$$v_2 = 2v_1 + (1/2)dv_1/dt \quad (4)$$

Applying KCL at node 2 gives $(v_1 - v_2)/1 = (1/3)dv_2/dt$

$$\text{or } v_1 = v_2 + (1/3)dv_2/dt \quad (5)$$

Substituting (5) into (4) yields,

$$v_2 = 2v_2 + (2/3)(dv_2/dt) + (1/2)(dv_2/dt) + (1/6)d^2v_2/dt^2$$

$$\text{or,} \quad (d^2v_2/dt^2) + (7dv_2/dt) + 6v_2 = 0$$

Now we have, $s^2 + 7s + 6 = 0 = (s + 1)(s + 6)$

$$\text{Thus,}\quad v_{2n} = (Ae^{-t} + Be^{-6t}) \text{ and } v_{2f} = v_2(\infty) = 20\text{V}.$$

$$v_2 = v_{2n} + v_{2f} = 20 + (Ae^{-t} + Be^{-6t})$$

$$v_2(0) = 0 \text{ which implies that } A + B = -20 \qquad (6)$$

$$dv_2/dt = (-Ae^{-t} - 6Be^{-6t})$$

$$dv_2(0) = 0 = -A - 6B \qquad (7)$$

From (6) and (7), $A = -24$ and $B = 4$.

$$\text{Thus,}\quad v_2(t) = (20 - 24e^{-t} + 4e^{-6t})\text{ V}$$

From (5), $v_1 = v_2 + (1/3)dv_2/dt$

$$\text{Thus,}\quad v_1(t) = (20 - 16e^{-t} - 4e^{-6t})\text{ V}$$

Now we can find, $v_o = v_1 - v_2 =$ $\mathbf{8(e^{-t} - e^{-6t})\ V,\ t > 0}$

P.P.8.11 Let v_1 equal the voltage at non-inverting terminal of the op amp. Then v_o is equal to the output of the op amp.

At the non-inverting terminal, $(v_s - v_o)/R_1 = C_1dv_1/dt$ (1)

At the output terminal of the op amp, $(v_1 - v_o)/R_2 = C_2dv_o/dt$ (2)

We now eliminate v_1 from (2), $v_1 = v_o + R_2C_2dv_o/dt$ (3)

From (1) $v_s = v_1 + R_1C_1dv_1/dt$ (4)

Substituting (3) into (4) gives

$$v_s = v_o + R_2C_2dv_o/dt + R_1C_1dv_o/dt + R_1C_1R_2C_2d^2v_o/dt^2$$

$$\text{or}\quad d^2v_o/dt^2 + [(1/(R_1C_1)) + (1/(R_2C_2))]dv_o/dt + v_o/(R_1R_2C_1C_2) = v_s/(R_1R_2C_1C_2)$$

With the given parameters,

$$(R_1R_2C_1C_2) = 10^4\text{x}10^4\text{x}20\text{x}10^{-6}\text{x}100\text{x}10^{-6} = 2\text{x}10^{-2}$$

$$1/(R_1R_2C_1C_2) = 5$$

$$[(1/(R_1C_1)) + (1/(R_2C_2))] = 10^{-4}[(1/20\text{x}10^{-6}) + (1/200\text{x}10^{-6})] = 6$$

$$\text{Hence, we now have } s^2 + 6s + 5 = 0 = (s+1)(s+5)$$

$$\text{Therefore } v_{on} = Ae^{-t} + Be^{-5t}, \text{ and } v_{of} = 10\text{V}$$

Thus,

$$v_o = 10 + Ae^{-t} + Be^{-5t} \tag{5}$$

$$\text{For } t < 0,\ v_s = 0,\ v_1(0^-) = 0 = v_o(0^-)$$

$$\text{For } t > 0,\ v_s = 10, \text{ but}$$

$$v_1(0^+) - v_o(0^+) = 0 \tag{6}$$

From (2),

$$dv_o(0^+)/dt = [v_1(0^+) - v_o(0^+)]/R_2C_2 = 0 \tag{7}$$

Imposing these conditions on $v_o(t)$,

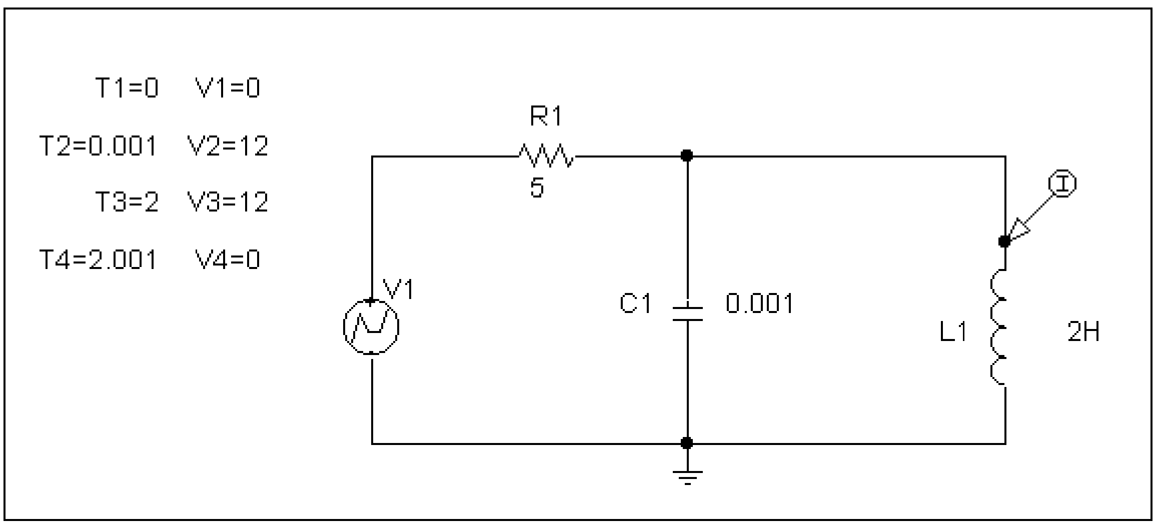
$$0 = 10 + A + B \tag{8}$$

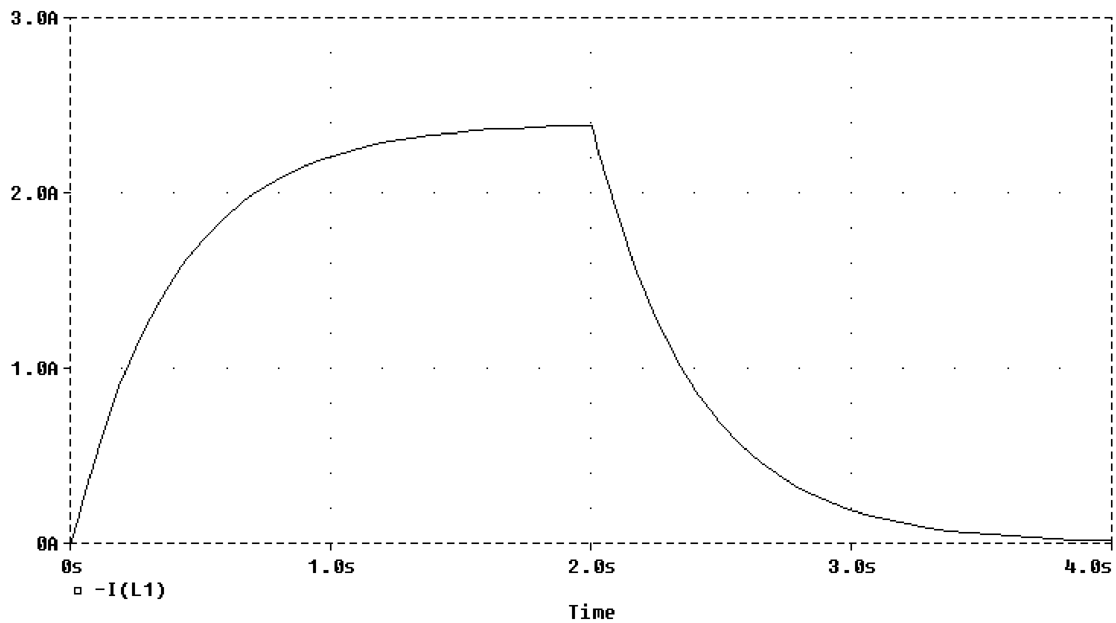
$$0 = -A - 5B \text{ or } A = -5B \tag{9}$$

From (8) and (9), $A = -12.5$ and $B = 2.5$

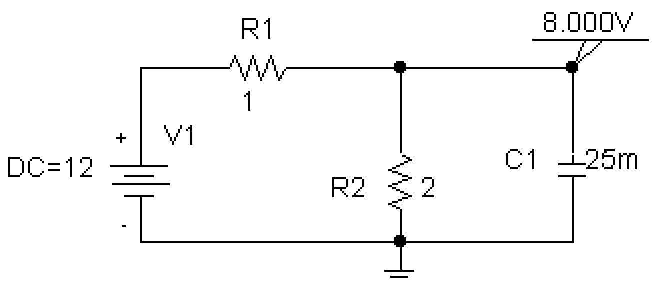
$$v_o(t) = \mathbf{(10 - 12.5e^{-t} + 2.5e^{-5t})\ V,\ \ t > 0}$$

P.P.8.12 We follow the same procedure as in Example 8.12. The schematic is shown in Figure (a). The current marker is inserted to display the inductor current. After simulating the circuit, the required inductor current is plotted in Figure (b).



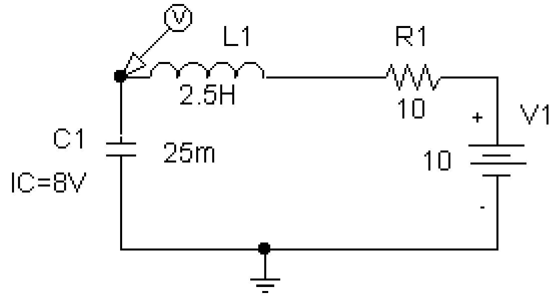


P.P.8.13 When the switch is at position a, the schematic is as shown in Figure (a). We carry out dc analysis on this to obtain initial conditions. It is evident that $v_C(0) = 8$ volts.

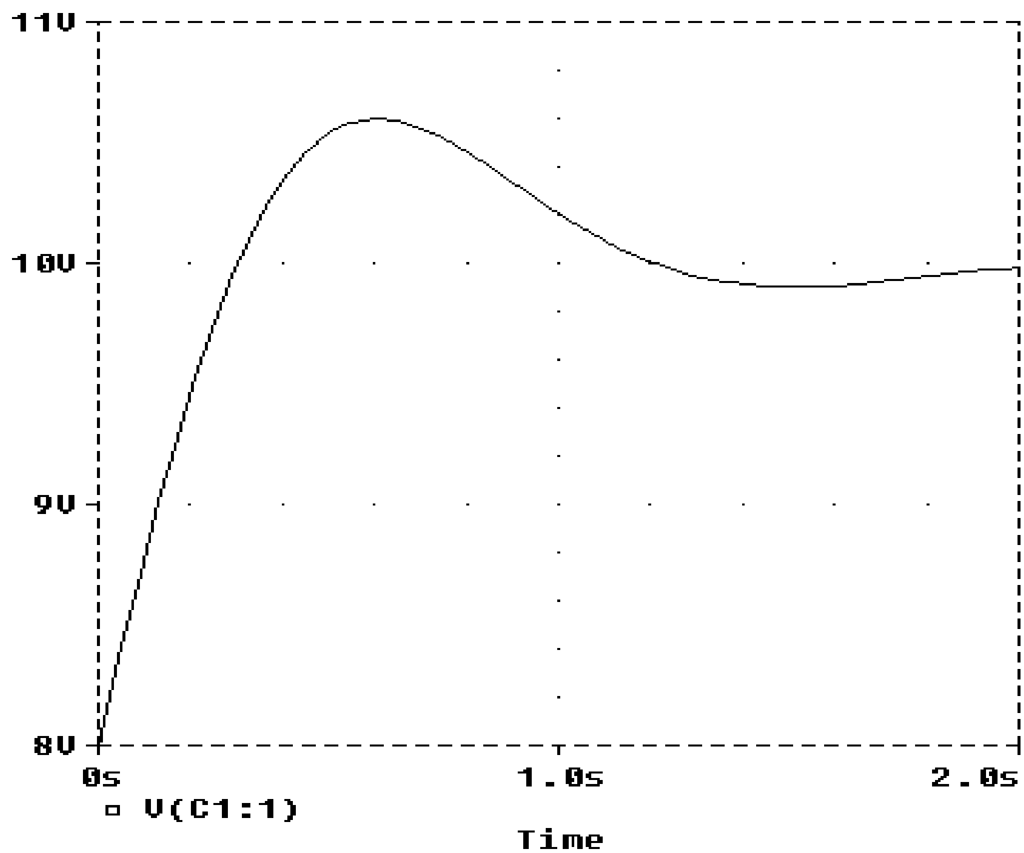


(a)

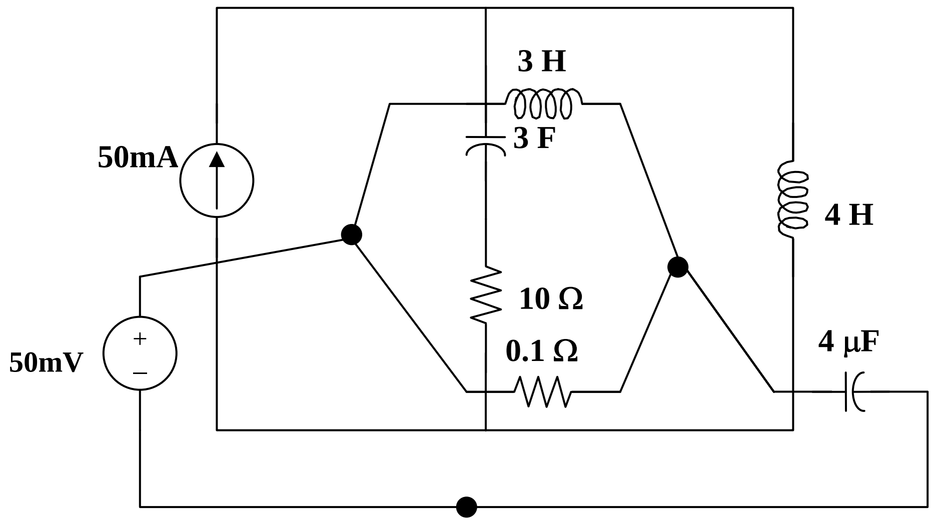
With the switch in position b, the schematic is as shown in Figure (b). A voltage marker is inserted to display the capacitor voltage. When the schematic is saved and run, the output is as shown in Figure (c).



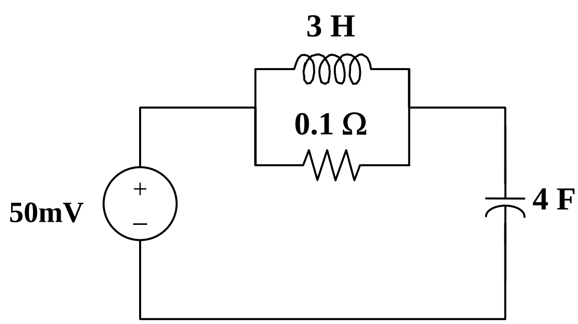
(b)



P.P.8.14 The dual circuit is obtained from the original circuit as shown in Figure (a). It is redrawn as shown in Figure (b).

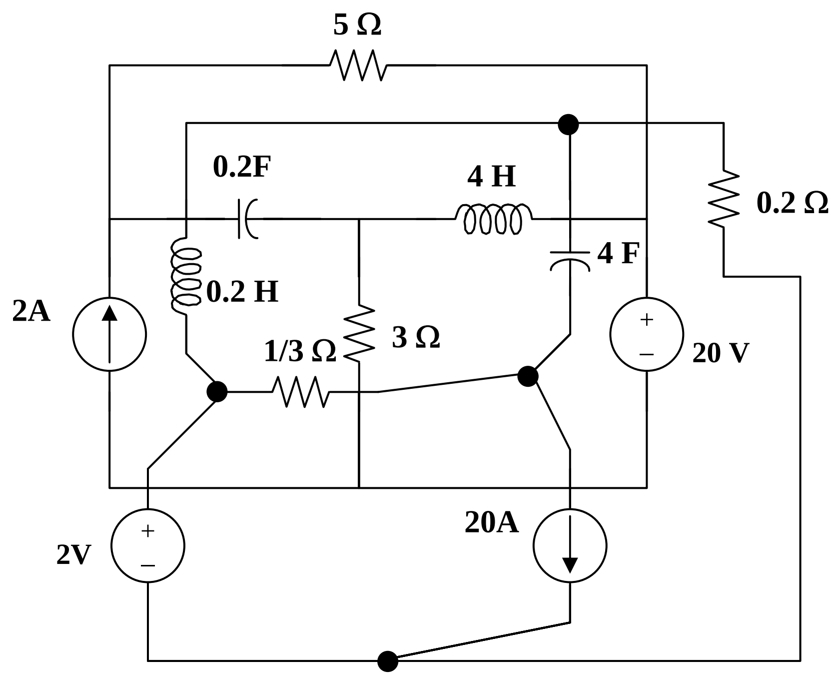


(a)

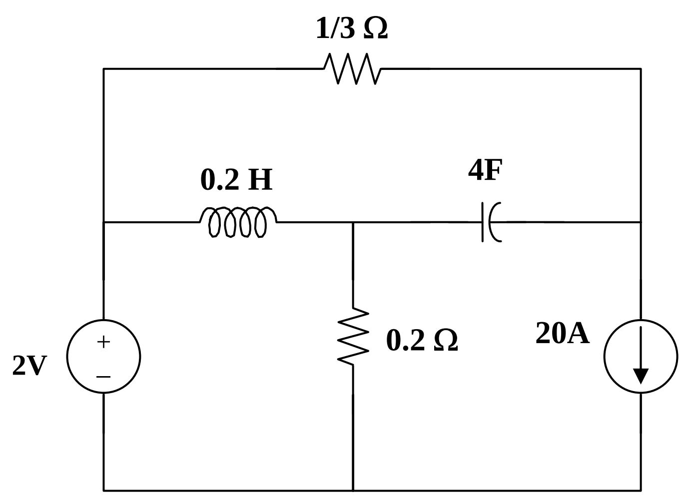


(b)

P.P.8.15 The dual circuit is obtained in Figure (a) and redrawn in Figure (b).



(a)



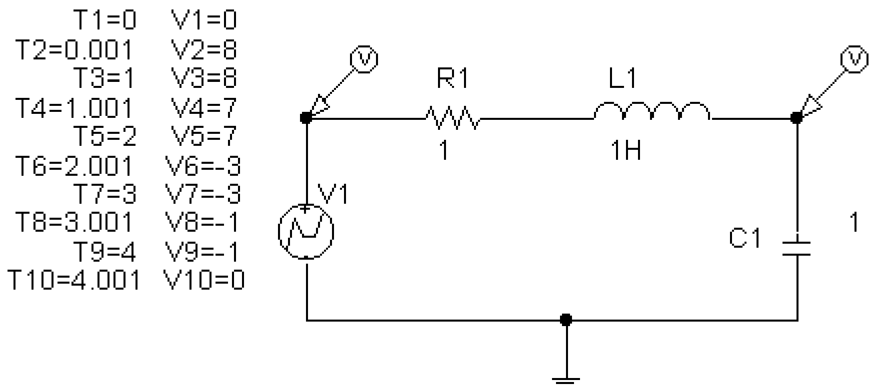
(b)

P.P.8.16 Since $12 = 4i + v_L + v_C$ or $v_C = 12 - 4i - v_L$

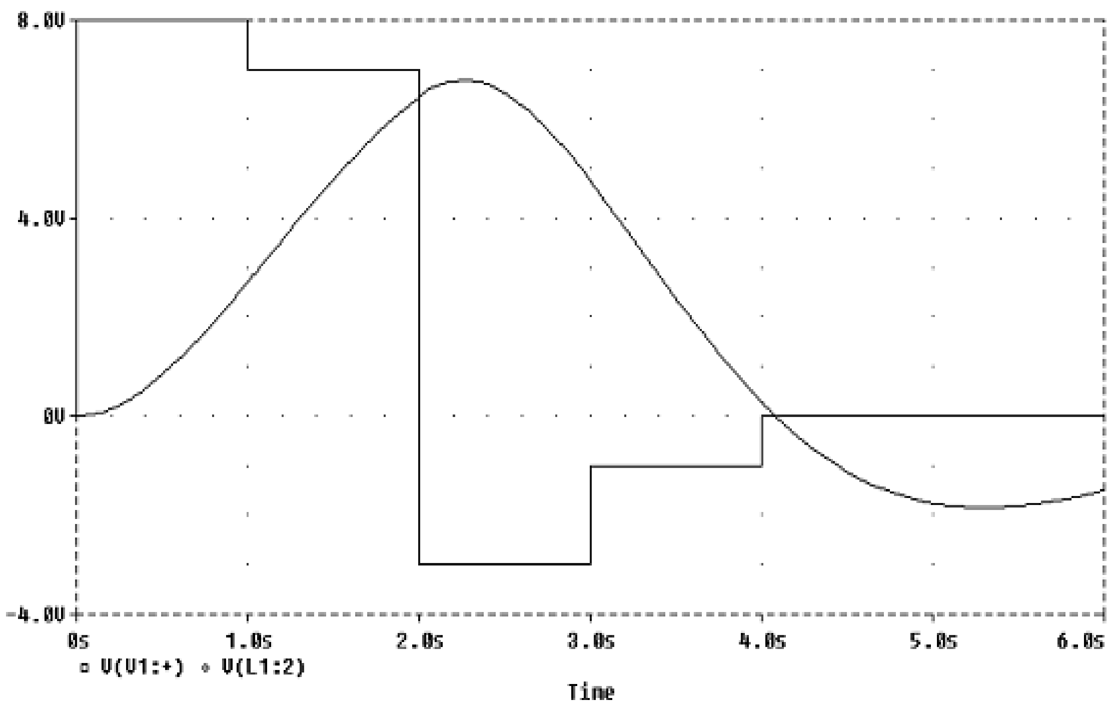
$$-(v_C - 12) = 4i + v_L = e^{-250t}(12\cos\omega_d t + 0.2684\sin\omega_d t - 268\sin\omega_d t)$$

$$v_C(t) = \mathbf{[12 - 12e^{-250t}\cos(11{,}180t) + 267.7e^{-250t}\sin(11{,}180t)]\ V}$$

P.P.8.17 We follow the same procedure as in Example 8.17. The schematic is as shown in Figure (a) with two voltage markers to display both input and output voltages. When the schematic is saved and run, the result is as displayed in Figure (b).



(a)



(b)