CHAPTER 12

P.P.12.1 For the abc sequence, $\mathbf{V}_{an}$ leads $\mathbf{V}_{bn}$ by 120° and $\mathbf{V}_{bn}$ leads $\mathbf{V}_{cn}$ by 120°.

Hence,

$$\mathbf{V}_{an} = 110\angle(30° + 120°) = \mathbf{110\angle 150°\ V}$$
$$\mathbf{V}_{cn} = 110\angle(30° - 120°) = \mathbf{110\angle{-90}°\ V}$$

P.P.12.2

(a)

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = 120\angle 30° - 120\angle{-90}°$$
$$\mathbf{V}_{ab} = (103.92 + j60) + j120$$
$$\mathbf{V}_{ab} = \mathbf{207.8\angle 60°\ V}$$

Alternatively, using the fact that $\mathbf{V}_{ab}$ leads $\mathbf{V}_{an}$ by 30° and has a magnitude of $\sqrt{3}$ times that of $\mathbf{V}_{an}$,

$$\mathbf{V}_{ab} = \sqrt{3}(120)\angle(30° + 30°) = \mathbf{207.8\angle 60°\ V}$$

Following the abc sequence,

$$\mathbf{V}_{bc} = \mathbf{207.8\angle{-60}°\ V}$$
$$\mathbf{V}_{ca} = \mathbf{207.8\angle \pm 180°\ V}$$

(b)

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}}$$

$$\mathbf{Z} = (0.4 + j0.3) + (24 + j19) + (0.6 + j0.7)$$
$$\mathbf{Z} = 25 + j20 = 32\angle 38.66°$$

$$\mathbf{I}_a = \frac{120\angle 30°}{32\angle 38.66°} = \mathbf{3.75\angle{-8.66}°\ A}$$

Following the abc sequence,

$$\mathbf{I}_b = \mathbf{I}_a\angle{-120}° = \mathbf{3.75\angle{-128.66}°\ A}$$
$$\mathbf{I}_c = \mathbf{I}_a\angle{-240}° = \mathbf{3.75\angle 111.34°\ A}$$

P.P.12.3
The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{120\angle -20°}{20\angle 40°} = \mathbf{6\angle{-60}° \ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \mathbf{6\angle 180° \ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \mathbf{6\angle 60° \ A}$$

The line currents are

$$\mathbf{I}_{a} = \mathbf{I}_{AB}\sqrt{3}\angle -30° = 6\sqrt{3}\angle -90° = \mathbf{10.392\angle{-90}° \ A}$$

$$\mathbf{I}_{b} = \mathbf{I}_{a}\angle -120° = \mathbf{10.392\angle 150° \ A}$$

$$\mathbf{I}_{c} = \mathbf{I}_{a}\angle 120° = \mathbf{10.392\angle 30° \ A}$$

P.P.12.4 In a delta load, the phase current leads the line current by 30° and has a magnitude $\frac{1}{\sqrt{3}}$ times that of the line current. Hence,

$$\mathbf{I}_{AB} = \frac{\mathbf{I}_{a}}{\sqrt{3}}\angle 30° = \frac{9.609}{\sqrt{3}}\angle 65° = \mathbf{5.548\angle 65° \ A}$$

$$\mathbf{Z}_{\Delta} = 18 + j12 = 21.63\angle 33.69° \ \Omega$$

$$\mathbf{V}_{AB} = \mathbf{I}_{AB}\,\mathbf{Z}_{\Delta} = (5.548\angle 65°)(21.63\angle 33.69°)$$
$$\mathbf{V}_{AB} = \mathbf{120\angle 98.69° \ V}$$

P.P.12.5 $\mathbf{Z}_{Y} = 12 + j15 = 19.21\angle 51.34°$

After converting the Δ-connected source to a Y-connected source,

$$\mathbf{V}_{an} = \frac{240}{\sqrt{3}}\angle(150° - 30°) = 138.56\angle -15°$$

$$\mathbf{I}_{a} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{Y}} = \frac{138.56\angle -15°}{19.21\angle 51.34°} = \mathbf{7.21\angle -66.34° \ A}$$

$$\mathbf{I}_{b} = \mathbf{I}_{a}\angle -120° = \mathbf{7.21\angle 173.66° \ A}$$
$$\mathbf{I}_{c} = \mathbf{I}_{a}\angle 120° = \mathbf{7.21\angle 53.66° \ A}$$

P.P.12.6
For the source,

$$\mathbf{S} = 3\,\mathbf{V}_p\,\mathbf{I}_p^* = (3)(120\angle 30°)(3.75\angle 8.66°)$$
$$\mathbf{S} = -1350\angle 38.66° = \mathbf{[-1.054.2 - j0.8433]\ kVA}$$

For the load,

$$\mathbf{S} = 3\left|\,\mathbf{I}_p\,\right|^2 \mathbf{Z}$$

where $\mathbf{Z} = 24 + j19 = 30.61\angle 38.37°$
$\mathbf{I}_p = 3.75\angle -8.66°$

$$\mathbf{S} = (3)(3.75)^2\,(30.61\angle 38.37°)$$
$$\mathbf{S} = 1291.36\angle 38.37° = \mathbf{[1.012{+}j0.8016]\ kVA}$$

P.P.12.7 $\quad P = S\cos\theta \longrightarrow S = \dfrac{P}{\cos\theta} = \dfrac{30\times 10^3}{0.85} = 35.29\ \text{kVA}$

$$S = \sqrt{3}\,V_L I_L \longrightarrow I_L = \frac{S}{\sqrt{3}\,V_L} = \frac{35.29\times 10^3}{\sqrt{3}\,(440)} = \mathbf{46.31\ A}$$

Alternatively,

$$P_p = \frac{30\times 10^3}{3} = 10\ \text{kW}, \qquad V_p = \frac{440}{\sqrt{3}}\ \text{V}$$

$$P_p = V_p\,I_p\cos\theta$$

$$I_p = \frac{P_p}{V_p\cos\theta} = \frac{(10\times 10^3)\sqrt{3}}{(440)(0.85)} = \mathbf{46.31\ A}$$

P.P.12.8

(a) For load 1,

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{840}{\sqrt{3}}$$

$$\mathbf{I}_{a1} = \frac{\mathbf{V}_a}{\mathbf{Z}_p} = \frac{840\angle 0°}{\sqrt{3}}\cdot\frac{1}{30 + j40} = 9.7\angle -53.13°$$

$$\mathbf{S}_1 = \frac{V_{rms}^2}{\mathbf{Z}^*} = \frac{(840)^2}{50\angle -53.15°} = 14.112\angle 53.13°\ \text{kVA}$$

For load 2,

$$S_2 = \frac{P_2}{\cos\theta_2} = \frac{48}{0.8} = 60 \text{ kVA}$$

$$Q_2 = S_2 \sin\theta_2 = (60)(0.6) = 36 \text{ kVAR}$$

$$\mathbf{S}_2 = 48 + j36 \text{ kVA}$$

$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}$ = **[56.47 + j47.29] kVA**

$\mathbf{S} = 73.65\angle 39.94° \text{ kVA}$

with $\text{pf} = \cos(39.94°) = 0.7667$

(b) $Q_c = P(\tan\theta_{old} - \tan\theta_{new})$

$Q_c = (56.47)(\tan 39.94° - \tan 0°) = 47.29 \text{ kVAR}$

For each capacitor, the rating is **15.76 kVAR**

(c) At unity pf, $\mathbf{S} = P = 56.47 \text{ kVA}$

$$I_L = \frac{\mathbf{S}}{\sqrt{3}V_L} = \frac{56470}{\sqrt{3}(840)} = \mathbf{38.81\ A}$$

P.P.12.9

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{AB}} = \frac{440\angle 0°}{10 - j5} = 39.35\angle 26.56° = 35.2 + j17.595$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{BC}} = \frac{440\angle -120°}{16} = 27.5\angle -120° = -13.75 - j23.82$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{CA}} = \frac{440\angle 120°}{8 + j6} = 44\angle 83.13° = 5.263 + j43.68$$

The line currents are

$\mathbf{I_a} = \mathbf{I_{AB}} - \mathbf{I_{CA}}$ = (35.2+j17.595) – (5.263+j43.68)
= 29.94–j26.08 = $\mathbf{39.71\angle{-41.06}°\ A}$.
$\mathbf{I_b} = \mathbf{I_{BC}} - \mathbf{I_{AB}}$ = –48.95–j41.42 = $\mathbf{64.12\angle{-139.8}°\ A}$.
$\mathbf{I_c} = \mathbf{I_{CA}} - \mathbf{I_{BC}}$ = 19.013+j67.5 = $\mathbf{70.13\angle 74.27°\ A}$.

P.P.12.10
The phase currents are

$$\mathbf{I}_{AB} = \frac{220\angle 0°}{-j5} = j44$$

$$\mathbf{I}_{BC} = \frac{220\angle 0°}{j10} = 22\angle 30°$$

$$\mathbf{I}_{CA} = \frac{220\angle 120°}{10} = 22\angle -120°$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = (j44) - (-11 - j19.05)$$
$$\mathbf{I}_a = 11 + j63.05 = \mathbf{64\angle 80.1° \ A}$$

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = (19.05 + j11) - (j44)$$
$$\mathbf{I}_b = 19.05 - j33 = \mathbf{38.1\angle -60° \ A}$$

$$\mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} = (-11 - j19.05) - (19.05 + j11)$$
$$\mathbf{I}_c = -30.05 - j30.05 = \mathbf{42.5\angle 225° \ A}$$

The real power is absorbed by the resistive load

$$P = |\mathbf{I}_{CA}|^2 (10) = (22)^2 (10) = \mathbf{4.84\ kW}$$

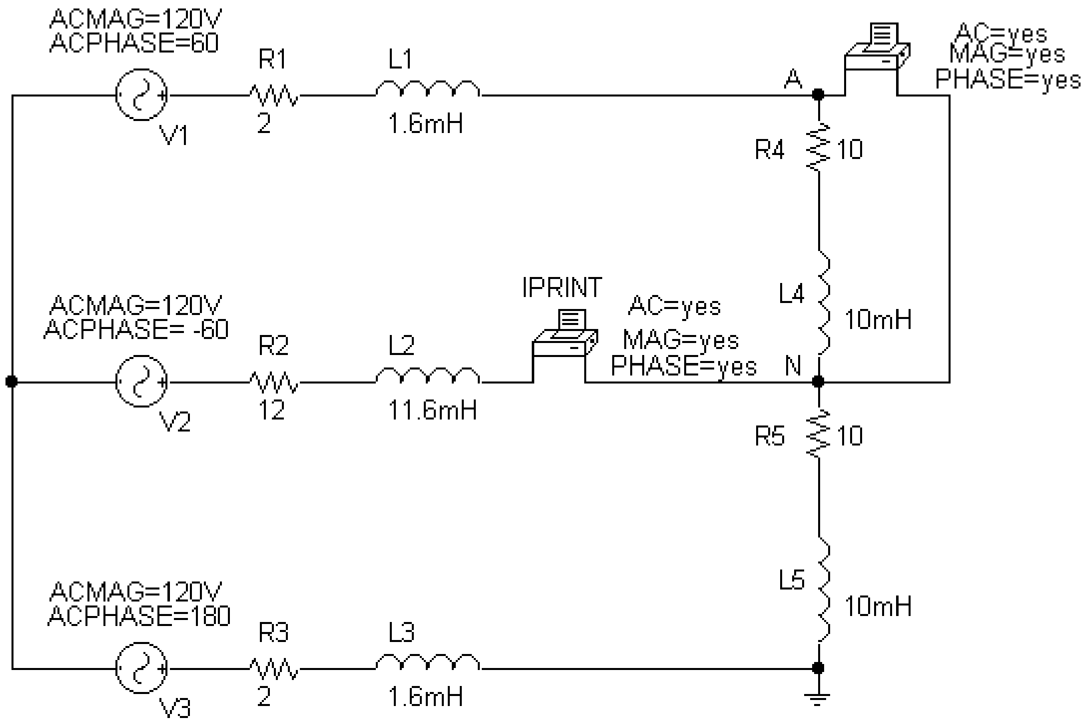
P.P.12.11 The schematic is shown below. First, use the **AC Sweep** option of the **Analysis Setup**. Choose a *Linear* sweep type with the following *Sweep Parameters* : *Total Pts* = 1, *Start Freq* = 100, and *End Freq* = 100. Once the circuit is saved and simulated, we obtain an output file whose contents include the following results.

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.000E+02	8.547E+00	-9.127E+01

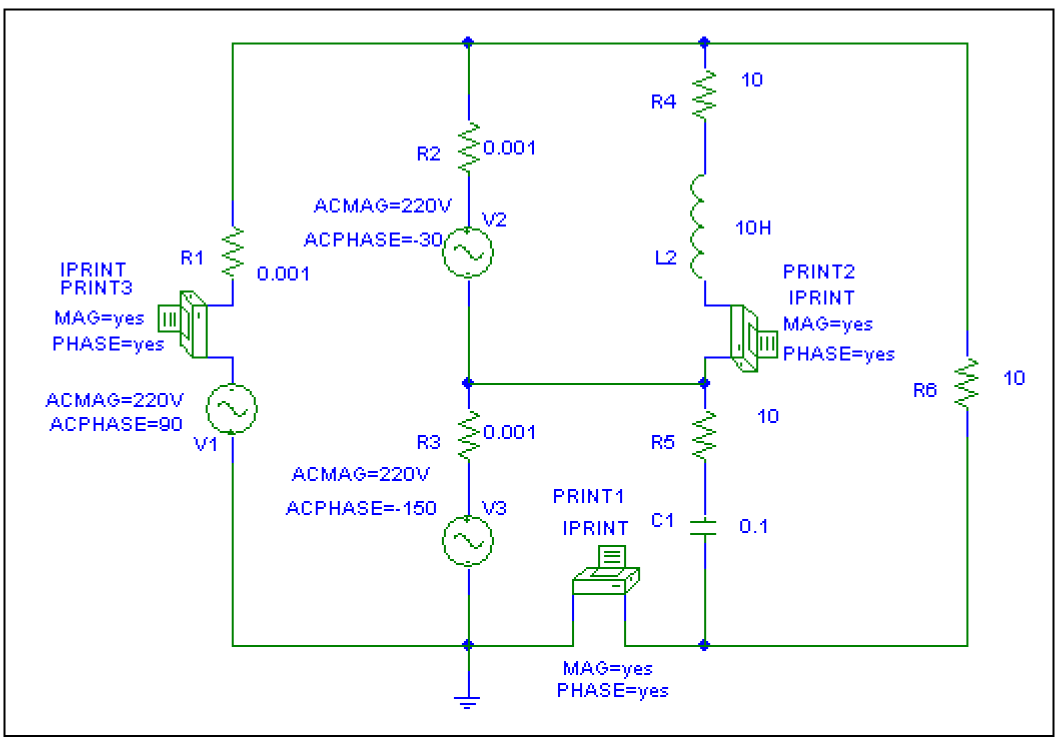
FREQ	VM(A,N)	VP(A,N)
1.000E+02	1.009E+02	6.087E+01

From this we obtain,

$$\mathbf{V}_{an} = \mathbf{100.9\angle 60.87° \ V}, \quad \mathbf{I}_{bB} = \mathbf{8.547\angle -91.27° \ A}$$



P.P.12.12 The schematic is shown below.



In this case, we may assume that $\omega = 1\text{ rad/s}$, so that $f = 1/2\pi = 0.1592\text{ Hz}$. Hence, $L = X_L/\omega = 10$ and $C = 1/\omega X_c = 0.1$.

Use the **AC Sweep** option of the **Analysis Setup**. Choose a *Linear* sweep type with the following *Sweep Parameters* : *Total Pts* = 1, *Start Freq* = 0.1592, and *End Freq* = 0.1592. Once the circuit is saved and simulated, we obtain an output file whose contents include the following results.

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592E-01	3.724E+01	8.379E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592E-01	1.555E+01	-7.501E+01

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592E-01	2.468E+01	-9.000E+01

From this we obtain,

$$\mathbf{I}_{ca} = \mathbf{24.68\angle -90° \ A} \qquad \mathbf{I}_{cC} = \mathbf{37.25\angle 83.79° \ A} \qquad \mathbf{I}_{AB} = \mathbf{15.55\angle -75.01° A}$$

P.P.12.13

(a) If point o is connected to point B, $P_2 = \mathbf{0\ W}$

$P_1 = \text{Re}(\mathbf{V}_{AB}\,\mathbf{I}_a^*)$

$P_1 = (440)(39.71)\cos(0° + 41.06°) = \mathbf{13.175kW}$

$P_3 = \text{Re}(\mathbf{V}_{CB}\,\mathbf{I}_c^*)$

where $\mathbf{V}_{CB} = -\mathbf{V}_{BC} = 240\angle(-120° + 180°) = 240\angle 60°$

$P_3 = (440)(70.13)\cos(60° - 74.27°) = \mathbf{29.91kW}$

(b) Total power is = (13.175+29.91) kW = **43.08 kW**.

P.P.12.14 $V_L = 208\ V$, $P_1 = -560\ W$, $P_2 = 800\ W$

(a) $P_T = P_1 + P_2 = -560 + 800 = \mathbf{240\ W}$

(b) $Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(800 + 560) = \mathbf{2.356\ kVAR}$

(c) $\tan\theta = \dfrac{Q_T}{P_T} = \dfrac{2355.6}{240} = 9.815 \longrightarrow \theta = 84.18°$

$\text{pf} = \cos\theta = \mathbf{0.1014}$ (**lagging / inductive**)

It is inductive because $P_2 > P_1$

(d) For a Y-connected load,

$$I_p = I_L, \qquad V_p = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120 \text{ V}$$

$$P_p = V_p I_p \cos\theta \longrightarrow I_p = \frac{80}{(120)(0.1014)} = 6.575 \text{ A}$$

$$Z_p = \frac{V_p}{I_p} = \frac{120}{6.575} = 18.25$$

$$\mathbf{Z}_p = Z_p \angle\theta = \mathbf{18.25\angle 84.18^\circ\ \Omega}$$

The impedance is **inductive**.

P.P.12.15 $\mathbf{Z}_\Delta = 30 - j40 = 50\angle -53.13^\circ$

The equivalent Y-connected load is

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 16.67\angle -53.13^\circ$$

$$V_p = \frac{440}{\sqrt{3}} = 254 \text{ V}$$

$$I_L = \frac{V_p}{|\mathbf{Z}_Y|} = \frac{254}{16.67} = 15.24$$

$$P_1 = V_L I_L \cos(\theta + 30^\circ)$$

$$P_1 = (440)(15.24)\cos(-53.13^\circ + 30^\circ) = \mathbf{6.167 \text{ kW}}$$

$$P_2 = V_L I_L \cos(\theta - 30^\circ)$$

$$P_2 = (440)(15.24)\cos(-53.13^\circ - 30^\circ) = \mathbf{802.1 \text{W}}$$

$$P_T = P_1 + P_2 = \mathbf{6.969 \text{ kW}}$$

$$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(802.1 - 6167)$$

$$Q_T = \mathbf{-9.292 \text{ kVAR}}$$