CHAPTER 13

P.P. 13.1 For mesh 1,

$$141.42+j141.42 = 4(1 + j2)\mathbf{I_1} + j\mathbf{I_2} \qquad (1)$$

For mesh 2, $0 = j\mathbf{I_1} + (10 + j5)\mathbf{I_2}$ (2)

For the matrix form $\begin{bmatrix} 141.42 + j141.42 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j8 & j \\ j & 10 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = j100, \ \ \Delta_2 = 141.42–j141.42$$

$$\mathbf{I_2} = \Delta_2/\Delta = (141.42–j141.42)/j100$$

$$\mathbf{V_o} = 10I_2 = 10(–1.4142–j1.4142) = \mathbf{20\angle{–135°}\ V}$$

P.P. 13.2 Since $\mathbf{I_1}$ enters the coil with reactance 2Ω and $\mathbf{I_2}$ enters the coil with reactance 6Ω, the mutual voltage is positive. Hence, for mesh 1,

$$100\angle 60^o = (5 + j2 + j6 – j\ 3x2)\mathbf{I_1} – j6\mathbf{I_2} + \mathbf{j3I_2}$$

or $100\angle 60^o = (5 + j2)\mathbf{I_1} – j3\mathbf{I_2}$ (1)

For mesh 2, $0 = (j6 – j4)\mathbf{I_2} – j6\mathbf{I_1} + j3\mathbf{I_1}$

or $\mathbf{I_2} = 1.5\mathbf{I_1}$ (2)

Substituting this into (1), $100\angle 60^o = (5 – j2.5)\mathbf{I_1}$

$$\mathbf{I_1} = (100\angle 60^o)/(5.59\angle{–26.57^o}) = \mathbf{17.889\angle 86.57^o\ A}$$

$$\mathbf{I_2} = 1.5\mathbf{I_1} = \mathbf{26.83\angle 86.57^o\ A}$$

P.P. 13.3 The coupling coefficient is, $k = m/\sqrt{L_1L_2} = 1/\sqrt{2x1} = \mathbf{0.7071}$

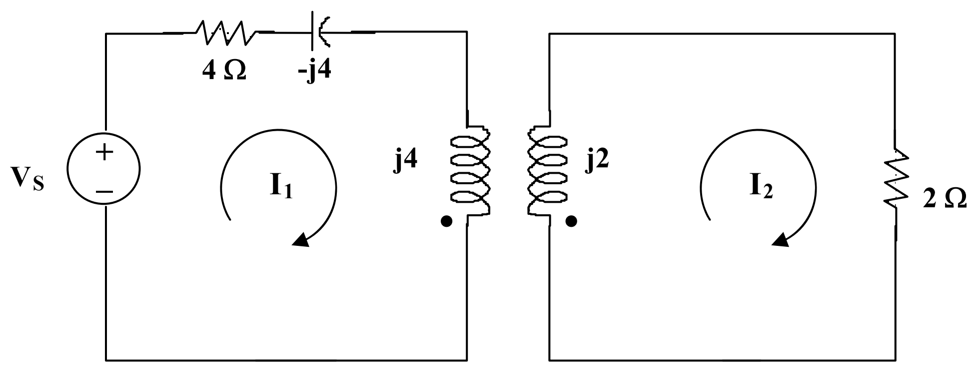
To obtain the energy stored, we first obtain the frequency-domain circuit shown below.

$$100\cos(\omega t) \ \text{becomes} \ 100\angle 0^o,\ \omega = 2$$

1H becomes $j\omega 1 = j2$

2H becomes $j\omega 2 = j4$

(1/8) F becomes $1/j\omega C = -j4$



For mesh 1, $100 = (4 - j4 + j4)\mathbf{I_1} - j2\mathbf{I_2}$

or $50 = 2\mathbf{I_1} - j\mathbf{I_2}$ (1)

For mesh 2, $-j2\mathbf{I_1} + (2 + j2)\mathbf{I_2} = 0$

or $\mathbf{I_1} = (1 - j)\mathbf{I_2}$ (2)

Substituting (2) into (1), $(2 - j3)\mathbf{I_2} = 50$

$$\mathbf{I_2} = 50/(2 - j3) = 13.87\angle 56.31^o$$

$$\mathbf{I_1} = 19.658\angle 11.31^o$$

In the time domain, $i_1 = 19.658\cos(2t + 11.31^o)$
$i_2 = 13.87\cos(2t + 56.31^o)$

At $t = 1.5$, $2t = 3 \text{ rad} = 171.9^o$

$i_1 = 19.658\cos(171.9^o + 11.31^o) = -19.62$ A
$i_2 = 13.87\cos(171.9^o + 56.31^o) = -9.25$ A

The total energy stored in the coupled inductors is given by,

$$W = 0.5L_1(i_1)^2 + 0.5L_2(i_2)^2 - 0.5M(i_1i_2)$$
$$= 0.5(2)(-19.62)^2 + 0.5(1)(-9.25)^2 - (1)(-19.62)(-9.25)$$
$$= \mathbf{246.2\ J}$$

P.P. 13.4 $\mathbf{Z_{in}} = 4 + j8 + [3^2/(j10 - j6 + 6 + j4)]$

$$= 4 + j8 + 9/(6 + j8)$$

$$= \mathbf{8.58\angle 58.05^o\ \Omega}$$

The current from the voltage is,

$$\mathbf{I} = \mathbf{V/Z} = 40\angle 0^o/8.58\angle 58.05^o = \mathbf{4.662\angle{-58.05^o}\ A}$$

P.P. 13.5 $L_1 = 10,\ L_2 = 4,\ M = 2$

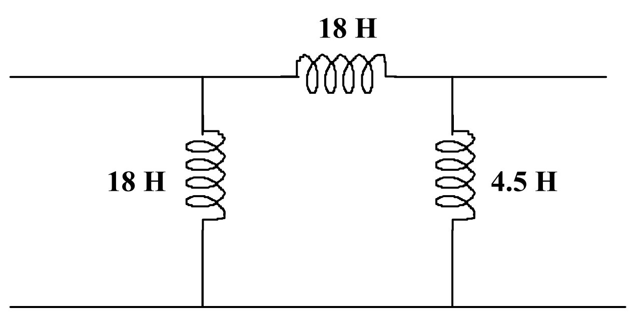
$$L_1L_2 - M^2 = 40 - 4 = 36$$

$$L_A = (L_1L_2 - M^2)/(L_2 - M) = 36/(4 - 2) = \mathbf{18\ H}$$

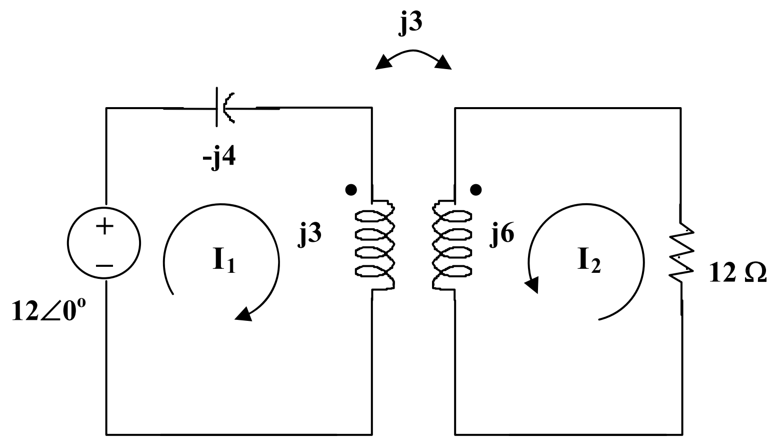
$$L_B = (L_1L_2 - M^2)/(L_1 - M) = 36/(10 - 2) = \mathbf{4.5\ H}$$

$$L_C = (L_1L_2 - M^2)/M = 36/2 = \mathbf{18\ H}$$

Hence, we get the π equivalent circuit as shown below.



P.P. 13.6 If we reverse the direction of I_2 so that we replace I_2 by $-I_2$, we have the circuit shown in Figure (a).



(a)

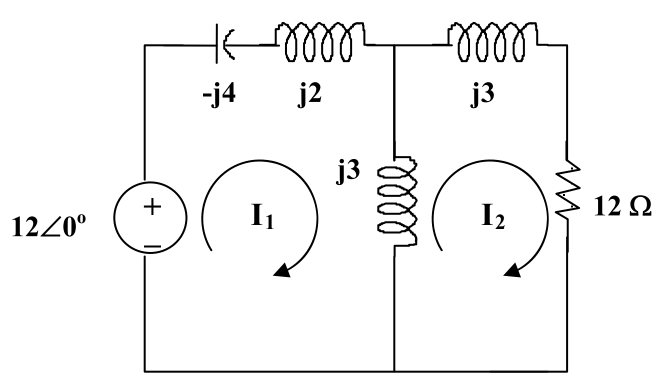
We now replace the coupled coil by the T-equivalent circuit and assume $\omega = 1$.

$$L_a = 5 - 3 = 2\text{ H}$$

$$L_b = 6 - 3 = 3\text{ H}$$

$$L_c = 3\text{ H}$$

Hence the equivalent circuit is shown in Figure (b). We apply mesh analysis.



(b)

$$12 = i_1(-j4 + j2 + j3) + j3i_2$$

or $12 = ji_1 + j3i_2$ (1)

Loop 2 produces, $0 = j3i_1 + (j3 + j3 + 12)i_2$

or $i_1 = (-2 + j4)i_2$ (2)

Substituting (2) into (1), $12 = (-4 + j)i_2$, which leads to $i_2 = 12/(-4 + j)$

$$I_2 = -i_2 = 12/(4 - j) = \mathbf{2.91\angle 14.04^o\ A}$$

$$I_1 = i_1 = (-2 + j4)i_2 = 12(2 - j4)/(4 - j) = \mathbf{13\angle -49.4^o\ A}$$

P.P. 13.7

(a) $n = V_2/V_1 = 110/2200 = \mathbf{1/20}$ (a step-down transformer)

(b) $S = V_1I_1 = 2200x5 = \mathbf{11\ kVA}$

(c) $I_2 = I_1/n = 5/(1/20) = \mathbf{100\ A}$

P.P. 13.8 The 16 – j24-ohm impedance can be reflected to the primary resulting in

$$Z_{in} = 2 + (16 - j24)/16 = 3 - j1.5$$

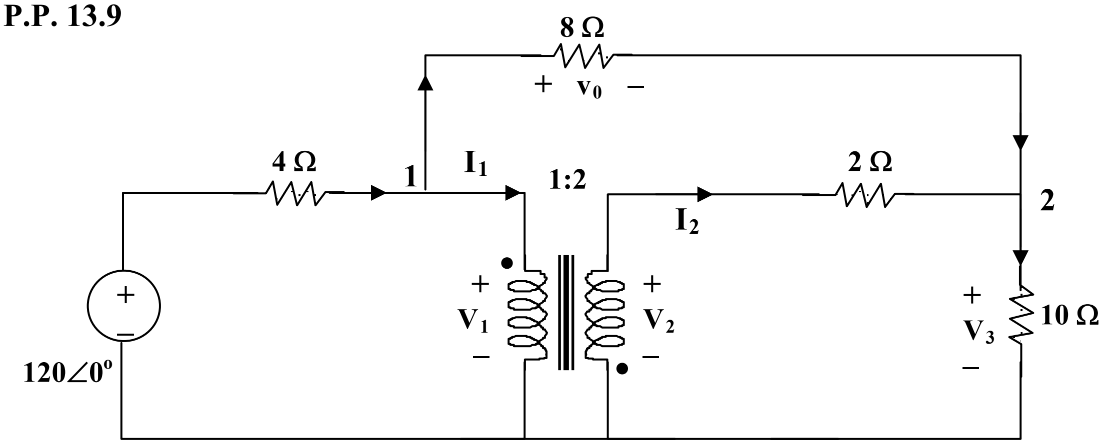
$$I_1 = 240/(3 - j1.5) = 240/(3.354\angle -26.57°) = 71.56\angle 26.57^o$$

$$I_2 = -I_1/n = -17.89\angle 26.57^o$$

$$V_o = -j24i_2 = (24\angle -90^o)(-17.89\angle 26.57^o) = \mathbf{429.4\angle 116.57^o V}$$

$$S_1 = V_1I_1 = (240)(71.56\angle 26.57^o) = \mathbf{17.174\angle -26.57^o\ kVA}.$$

P.P. 13.9



Consider the circuit shown above.

At node 1, $(120 - \mathbf{V_1})/4 = \mathbf{I_1} + (\mathbf{V_1} - \mathbf{V_3})/8$ (1)

At node 2, $[(\mathbf{V_1} - \mathbf{V_3})/8] + [(\mathbf{V_2} - \mathbf{V_3})/2] = (\mathbf{V_3})/8$ (2)

At the transformer terminals, $\mathbf{V_2} = -2\mathbf{V_1}$ and $\mathbf{I_2} = -\mathbf{I_1}/2$ (3)

But $I_2 = (\mathbf{V_2} - \mathbf{V_3})/2 = -\mathbf{I_1}/2$ which leads to $\mathbf{I_1} = (\mathbf{V_3} - \mathbf{V_2})/1 = \mathbf{V_3} + 2\mathbf{V_1}$.

Substituting all of this into (1) and (2) leads to,

$(120 - \mathbf{V_1})/4 = \mathbf{V_3} + 2\mathbf{V_1} + (\mathbf{V_1} - \mathbf{V_3})/8$ which leads $240 = 19\mathbf{V_1} + 7\mathbf{V_3}$ (4)

$[(\mathbf{V_1} - \mathbf{V_3})/8] + [(-2\mathbf{V_1} - \mathbf{V_3})/2] = \mathbf{V_3}/8$ which leads to
$\mathbf{V_3} = -7\mathbf{V_1}/6$ (5)

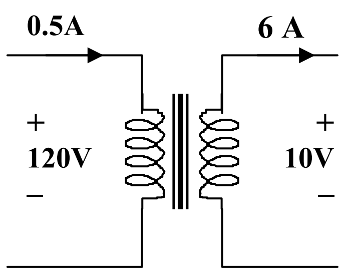
From (4) and (5),

$$240 = 10.833\mathbf{V_1} \text{ or } \mathbf{V_1} = 22.155 \text{ volts}$$

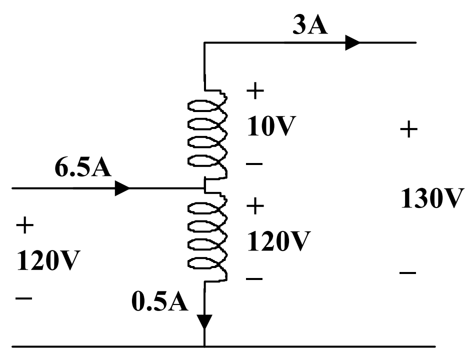
$$\mathbf{V_3} = -7\mathbf{V_1}/6 = -25.85 \text{ volts}$$

$$\mathbf{V_o} = \mathbf{V_1} - \mathbf{V_3} = \mathbf{48\ volts}$$

P.P. 13.10 We should note that the current and voltage of each winding of the autotransformer in Figure (b) are the same for the two-winding transformer in Figure (a).



(a)



(b)

For the two-winding transformer,

$$\mathbf{S_1} = 120\text{x}0.5 = 60\text{ VA}$$

$$\mathbf{S_2} = 6(10) = 60\text{ VA}$$

For the autotransformer,

$$\mathbf{S_1} = 120(6.5) = 780\text{ VA}$$

$$\mathbf{S_2} = 130(6) = \mathbf{780\text{ VA}}$$

P.P. 13.11 $(\mathbf{I_2})^* = \mathbf{S_2/V_2} = 16{,}000/1000 = \mathbf{16\text{ A}}$

Since $\mathbf{S_1} = \mathbf{V_1(I_1)}^* = \mathbf{V_2(I_2)}^* = \mathbf{S_2}$, $\mathbf{V_2/V_1} = \mathbf{I_1/I_2}$, $1000/2500 = \mathbf{I_1}/32$,

$$\text{or } \mathbf{I_1} = 1000\text{x}16/2500 = \mathbf{6.4\text{ A}}.$$

At the top, KCL produces $\mathbf{I_1 + I_o} = \mathbf{I_2}$, or $\mathbf{I_o} = \mathbf{I_2 - I_1} = 16 - 6.4 = \mathbf{9.6\text{ A}}$.

P.P. 13.12

(a) $S_T = (\sqrt{3})V_L I_L$, but $S_T = P_T/\cos\theta = 40\text{x}10^6/0.85 = 47.0588$ MVA

$$I_{LS} = S_T/(\sqrt{3})V_{LS} = 47.0588\text{x}10^6/[(\sqrt{3})12.5\text{x}10^3] = \mathbf{2.174\ kA}$$

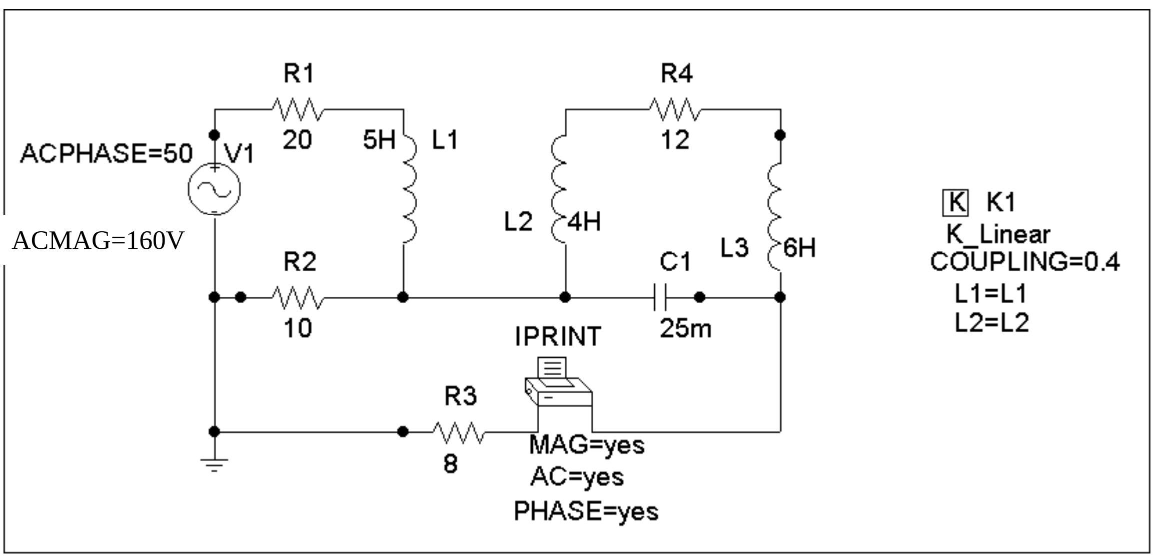
(b) $V_{LS} = 12.5$ kV, $V_{LP} = 625$ kV, $n = V_{LS}/V_{LP} = 12.5/625 =$ **0.02**

(c) $I_{LP} = nI_{LS} = 0.02\text{x}2173.6 =$ **43.47 A**

or $I_{LP} = S_T/[(\sqrt{3})v_{LP}] = 47.0588\text{x}10^6/[(\sqrt{3})625\text{x}10^3] =$ **43.47 A**

(d) The load carried by each transformer is $(1/3)S_T =$ **15.69 MVA**

P.P. 13.13 The process is essentially the same as in Example 13.13. We are given the coupling coefficient, $k = 0.4$, and can determine the operating frequency from the value of $\omega = 4$ which implies that $f = 4/(2\pi) = 0.6366$ Hz.



Saving and then simulating produces,

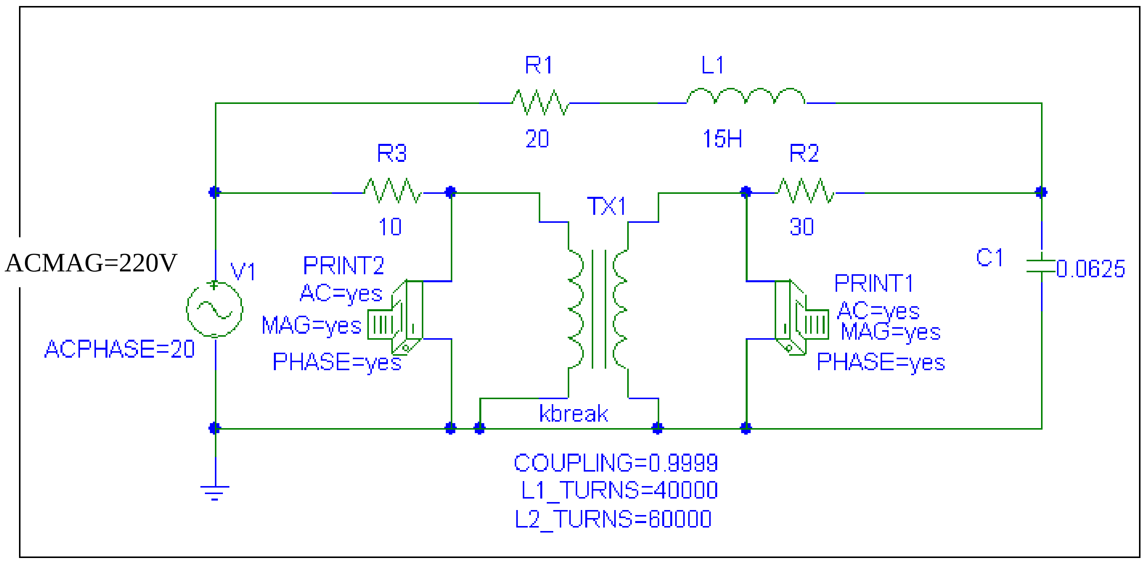
$$i_o = \mathbf{2.012cos(4t + 68.52^o)\ A}$$

P.P. 13.14 Following the same basic steps in Example 13.14, we first assume $\omega = 1$. This then leads to following determination of values for the inductor and the capacitor.

$$j15 = j\omega L \text{ leads to } L = 15\ H$$

$$-j16 = 1/(\omega C) \text{ leads to } C = 62.5\ mF$$

The schematic is shown below.



FREQ	VM($N_0005,0)	VP($N_0005,0)
1.592E-01	1.530E+02	2.185E+00

FREQ	VM($N_0001,0)	VP($N_0001,0)
1.592E-01	2.302E+02	2.091E+00

Thus,

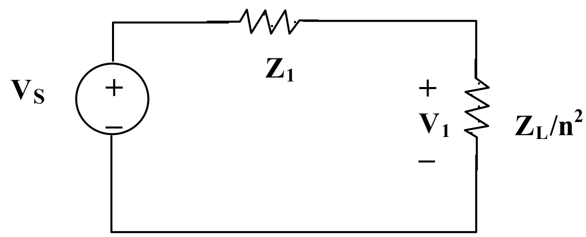
$$\mathbf{V_1 = 153\angle 2.18^\circ\ V}$$

$$\mathbf{V_2 = 230.2\angle 2.09^\circ\ V}$$

Note, if we divide V_2 by V_1 we get $1.5046\angle -.09^\circ$ which is in good agreement that the transformer is ideal with a voltage ratio of 1:1.5 (or 2:3)!

P.P. 13.15 $V_2/V_1 = 120/13{,}200 = \mathbf{1/110} = 1/n$

P.P. 13.16



As in Example 13.16, $n^2 = \mathbf{Z_L}/\mathbf{Z_1} = 400/(2.5\text{x}10^3) = 4/25$, $n = \mathbf{0.4}$

By voltage division, $\mathbf{V_1} = \mathbf{V_s}/2$ (since $\mathbf{Z_1} = \mathbf{Z_L}/n^2$), therefore $V_1 = 60/2 = 30$ volts, and

$$V_2 = nV_1 = (0.4)(30) = \mathbf{12\ volts}$$

P.P. 13.17

(a) $S = 12\text{x}60 + 350 + 4{,}500 = \mathbf{5.57\ kW}$

(b) $I_P = S/V_P\ \ 5570/2400 = \mathbf{2.321\ A}$