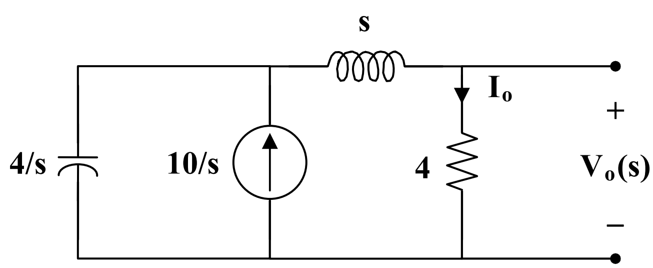
CHAPTER 16

P.P.16.1 Consider the circuit shown below.



Using current division,

$$I_o = \frac{\frac{4}{s}}{\frac{4}{s}+s+4} \cdot \frac{10}{s} = \frac{40}{s(s^2+4s+4)}$$

$$V_o(s) = 4I_o = \frac{160}{s(s+2)^2}$$

$$\frac{160}{s(s+2)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$160 = A(s^2+4s+4) + B(s^2+2s) + Cs$$

Equating coefficients :

s^0: $80 = 4A \longrightarrow A = 40$

s^1: $0 = 4A + 2B + C$

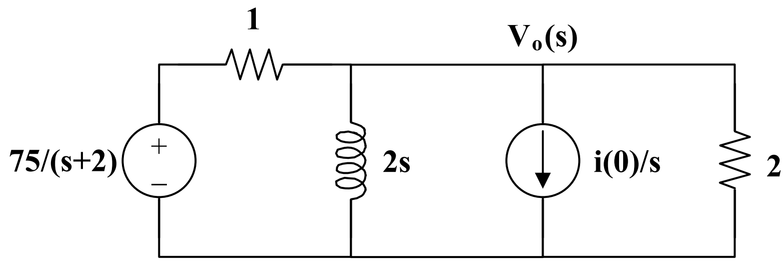
s^2: $0 = A + B \longrightarrow B = -A = -40$

Hence, $0 = 4A + 2B + C \longrightarrow C = -80$

$$V_o(s) = \frac{40}{s} - \frac{40}{s+2} - \frac{80}{(s+2)^2}$$

$$v_o(t) = \mathbf{40(1 - e^{-2t} - 2t\,e^{-2t})\,u(t)\,V}$$

P.P.16.2 The circuit in the s-domain is shown below.



At node o,

$$\frac{V_o - \frac{75}{s+2}}{1} + \frac{V_o}{2s} + \frac{V_o}{2} + \frac{i(0)}{s} = 0 \qquad \text{where} \quad i(0) = 0\text{A}$$

$$\left(1 + \frac{1}{2} + \frac{1}{2s}\right)V_o = \frac{75}{s+2}$$

$$V_o = \frac{150s}{(s+2)(3s+1)} = \frac{50s}{(s+2)(s+1/3)} = \frac{A}{s+2} + \frac{B}{s+1/3}$$

Solving for A and B we get,

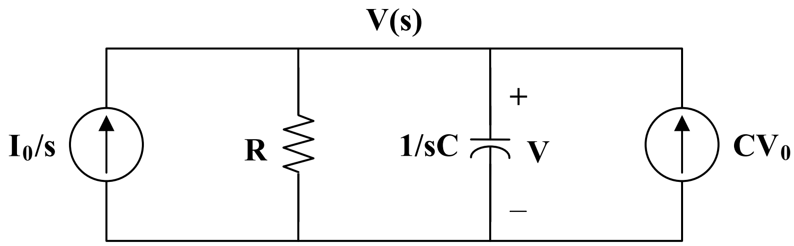
A = [50(–2)]/(–2+1/3) = 300/5 = 60, B = [50(–1/3)]/[(–1/3)+2] = –150/15 = –10

$$V_o = \frac{60}{s+2} - \frac{10}{s+1/3}$$

Hence,

$$v_o(t) = \left(60\,e^{-2t} - 10e^{-t/3}\right)u(t)V$$

P.P.16.3 $v(0) = V_0$ is incorporated as shown below.



We apply KCL to the top node.

$$\frac{I_0}{s} + CV_0 = \frac{V}{R} + sCV = \left(sC + \frac{1}{R}\right)V$$

$$V = \frac{I_0}{s(sC + 1/R)} + \frac{CV_0}{sC + 1/R}$$

$$V = \frac{V_0}{s + 1/RC} + \frac{I_0/C}{s(s + 1/RC)}$$

$$V = \frac{V_0}{s + 1/RC} + \frac{A}{s} + \frac{B}{s + 1/RC}$$

where $A = \dfrac{I_0/C}{1/RC} = I_0R$, $\quad B = \dfrac{I_0/C}{-1/RC} = -I_0R$

$$V(s) = \frac{V_0}{s + 1/RC} + \frac{I_0R}{s} - \frac{I_0R}{s + 1/RC}$$

$$v(t) = \boldsymbol{((V_0 - I_0R)\,e^{-t/\tau} + I_0R), \quad t > 0, \quad where\ \tau = RC}$$

P.P.16.4 We solve this problem the same as we did in Example 16.4 up to the point where we find V_1. Once we have V_1, all we need to do is to divide V_1 by 5s to and add in the contribution from i(0)/s to find I_L.

$I_L = V_1/5s – i(0)/s = 7/(s(s+1)) – 6/(s(s+2)) – 1/s$
$= 7/s – 7/(s+1) – 3/s + 3/(s+2) – 1/s = 3/s – 7/(s+1) + 3/(s+2)$

Which leads to $i_L(t) =$ $\mathbf{(3 – 7e^{–t} + 3e^{–2t})u(t)A}$

P.P.16.5 We can use the same solution as found in Example 16.5 to find i_L.

All we need to do is divide each voltage by 5s and then add in the contribution from i(0). Start by letting $i_L = i_1 + i_2 + i_3$.

$I_1 = V_1/5s - 0/s = 6/(s(s+1)) - 6/(s(s+2)) = 6/s - 6/(s+1) - 3/s + 3/(s+2)$

or $i_1 = (3 - 6e^{-t} + 3e^{-2t})u(t)A$

$I_2 = V_2/5s - 1/s = 2/(s(s+1)) - 2/(s(s+2)) - 1/s = 2/s - 2/(s+1) - 1/s + 1/(s+2) - 1/s$

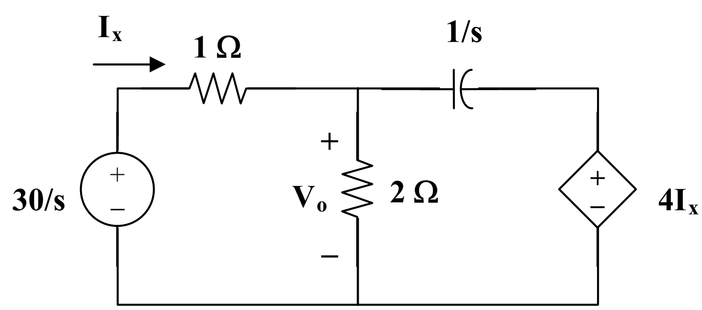
or $i_2 = (-2e^{-t} + e^{-2t})u(t)A$

$I_3 = V_3/5s - 0/s = -1/(s(s+1)) + 2/(s(s+2)) = -1/s + 1/(s+1) + 1/s - 1/(s+2)$

or $i_3 = (e^{-t} - e^{-2t})u(t)A$

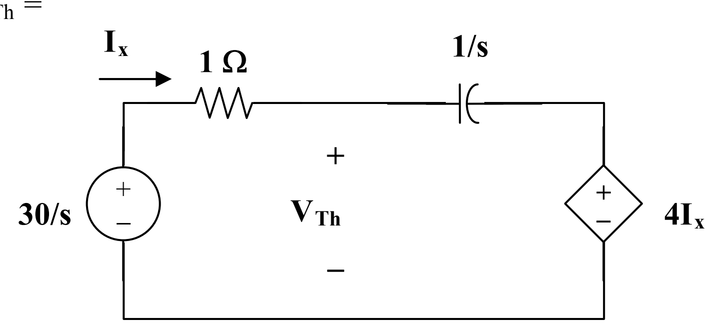
This leads to $i_L(t) = i_1 + i_2 + i_3 =$ $\mathbf{(3 - 7e^{-t} + 3e^{-2t})u(t)\ A}$

P.P.16.6



(a) Take out the 2 Ω and find the Thevenin equivalent circuit.

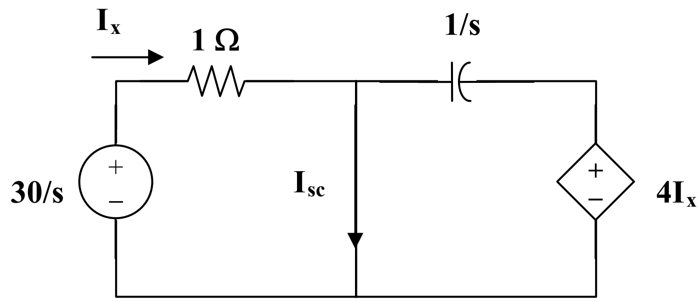
$V_{Th} =$



Using mesh analysis we get,

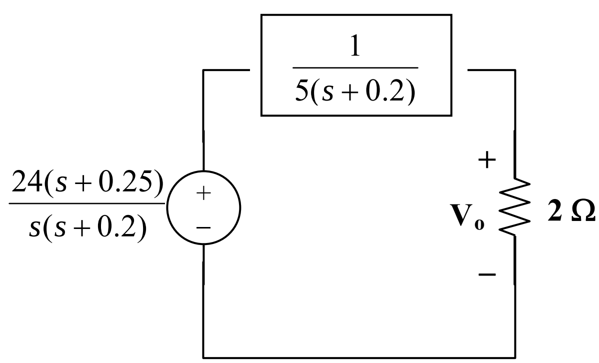
$-30/s + 1I_x + I_x/s + 4I_x = 0$ or $(1 + 1/s + 4)I_x = 30/s$ or $I_x = 30/(5s+1)$

$V_{Th} = 30/s - 30/(5s+1) = (150s+30-30s)/(s(5s+1))$
$= 30(4s+1)/(s(5s+1)) = \mathbf{24(s+0.25)/(s(s+0.2))}$



$I_x = (30/s)/1 = 30/s$ $I_{sc} = 30/s + 4(30/s)/(1/s) = 30/s + 120 = (120s+30)/s = 120(s+0.25)/s$

$Z_{Th} = V_{Th}/I_{sc} = \{24(s+0.25)/(s(s+0.2))\}/\{120(s+0.25)/s\} = \mathbf{1/(5(s+0.2))}$



$$V_o = \frac{\dfrac{24(s+0.25)}{s(s+0.2)}}{\dfrac{1}{5(s+0.2)}+2}2 = \frac{24(s+0.25)}{s(0.2+2s+0.4)}2 = \mathbf{\frac{24(s+0.25)}{s(s+0.3)}} \ \textbf{\textit{or}} \ \mathbf{\frac{60(4s+1)}{s(10s+3)}}$$

(b) Initial value: $v_o(0^+) = \lim_{s\to\infty} sV_o = \mathbf{24\ V}$

Final value: $v_o(\infty) = \lim_{s\to 0} sV_o = 24(0+0.25)/(0+0.3) = \mathbf{20\ V}$

(c) Partial fraction expansion leads to $V_o = 20/s + 4/(s+0.3)$

Taking the inverse Laplace transform we get,

$$v_o(t) = \mathbf{(20 + 4e^{-0.3t})u(t)V}$$

P.P.16.7 If $x(t) = 10e^{-3t}\,u(t)$, then $X(s) = \dfrac{10}{s+3}$.

$$Y(s) = H(s)\,X(s) = \frac{20s}{(s+3)(s+6)} = \frac{A}{s+3} + \frac{B}{s+6}$$

$$A = Y(s)\,(s+3)\big|_{s=-3} = -20$$
$$B = Y(s)\,(s+6)\big|_{s=-6} = 40$$

$$Y(s) = \frac{-20}{s+3} + \frac{40}{s+6}$$

$$y(t) = \mathbf{(-20e^{-3t} + 40\,e^{-6t})u(t)}$$

$$H(s) = \frac{2s}{(s+6)} = \frac{2(s+6-6)}{s+6} = 2 - \frac{12}{s+6}$$

$$h(t) = \mathbf{2\delta(t) - 12\,e^{-6t}\,u(t)}$$

P.P.16.8 By current division,

$$I_1 = \frac{2+1/2s}{s+4+2+1/2s} I_0$$

$$H(s) = \frac{I_1}{I_0} = \frac{2+1/2s}{s+4+2+1/2s} = \mathbf{\frac{4s+1}{2s^2+12s+1}}$$

P.P.16.9

(a) $$\frac{V_o}{V_i} = \frac{1\,||\,2/s}{1+1\,||\,2/s} = \frac{\dfrac{2/s}{1+2/s}}{1+\dfrac{2/s}{1+2/s}} = \frac{2}{s+4}$$

$$H(s) = \frac{V_o}{V_i} = \frac{\mathbf{2}}{\mathbf{s+4}}$$

(b) $h(t) = \mathbf{2e^{-4t}\,u(t)}$

(c) $$V_o(s) = H(s)\,V_i(s) = \frac{2}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4}$$

$$A = s\,V_o(s)\Big|_{s=0} = \frac{1}{2}, \qquad B = (s+4)\,V_o(s)\Big|_{s=-4} = \frac{-1}{2}$$

$$V_o(s) = \frac{1}{2}\left(\frac{1}{s} - \frac{1}{s+4}\right)$$

$$v_o(t) = \mathbf{\frac{1}{2}(1 - e^{-4t})\,u(t)\ V}$$

(d) $$v_i(t) = 8\cos(2t) \longrightarrow V_i(s) = \frac{8s}{s^2+4}$$

$$V_o(s) = H(s)\,V_i(s) = \frac{16s}{(s+4)(s^2+4)} = \frac{A}{(s+4)} + \frac{Bs+C}{(s^2+4)}$$

$$A = (s+4)\,V_o(s)\Big|_{s=-4} = \frac{-16}{5}$$

Multiplying both sides by $(s+4)(s^2+4)$ gives

$$16s = A(s+4) + B(s^2+4s) + C(s+4)$$

Equating coefficients :

$$s^2: \quad 0 = A + B \longrightarrow B = -A = \frac{16}{5} \qquad (1)$$

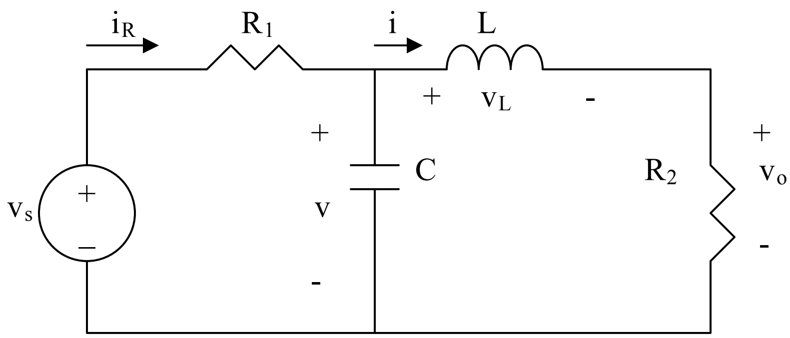
$$s^1: \quad 16 = 4B + C \longrightarrow C = \frac{16}{5} \qquad (2)$$

$$s^0: \quad 0 = 4A + 4C \longrightarrow C = -A \qquad (3)$$

$$V_o(s) = \frac{16}{5}\left(\frac{-1}{s+4} + \frac{s+1}{s^2+4}\right) = \frac{16}{5}\left(\frac{-1}{s+4} + \frac{s}{s^2+4} + \frac{1}{2}\cdot\frac{2}{s^2+4}\right)$$

$$v_o(t) = \mathbf{3.2\left[-e^{-4t} + \cos(2t) + 0.5\sin(2t)\right]u(t)\,V}$$

P.P. 16.10 Consider the circuit below.



$$i_R = i + C\frac{dv}{dt}$$

$$v_o = R_2 i \qquad (1)$$

But $\quad i_R = \dfrac{v_s - v}{R_1}$

Hence,

$$\frac{v_s - v}{R_1} = i + C\frac{dv}{dt}$$

or

$$\dot{v} = -\frac{v}{R_1 C} - \frac{i}{C} + \frac{v_s}{R_1 C} \qquad (2)$$

Also,

$$- \quad v + v_L + v_o = 0$$

$$v_L = L\frac{di}{dt} = v - v_o$$

But $v_o = iR_2$. Hence

$$\dot{i} = v/L - v_o/L = \frac{v}{L} - \frac{iR_2}{L} \qquad (3)$$

Putting (1) to (3) into the standard form

$$\begin{bmatrix} \dot{v} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1 C} & -\frac{1}{C} \\ \frac{I}{L} & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1 C} \\ 0 \end{bmatrix} v_s$$

$$v_o = \begin{bmatrix} 0 & R_2 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix}$$

If we let $R_1 = 1$, $R_2 = 2$, $C = ½$, $L = 1/5$, then

$$A = \begin{bmatrix} -2 & -2 \\ 5 & -10 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 2 \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s+2 & 2 \\ -5 & s+10 \end{bmatrix}$$

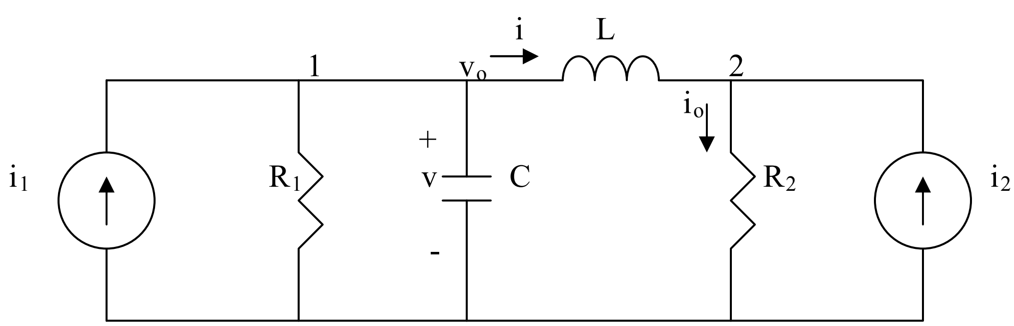
$$(sI - A)^{-1} = \frac{\begin{bmatrix} s+10 & -2 \\ 5 & s+2 \end{bmatrix}}{s^2 + 12s + 30}$$

$$\text{H(s)} = \text{C(sI} - \text{A)}^{-1}\text{B} = \frac{\begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} s+10 & -2 \\ 5 & s+2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}}{s^2 + 12s + 30}$$

$$= \frac{20}{s^2 12s + 30}$$

$$= \mathbf{\frac{20}{s^2 + 12s + 30}}$$

P.P. 16.11 Consider the circuit below.



At node 1,

$$i_1 = \frac{v}{R_1} + C\dot{v} + i$$

or
$$\dot{v} = -\frac{1}{R_1 C}v - \frac{1}{C}i + \frac{i_1}{C} \qquad (1)$$

This is one state equation.
At node 2,

$$i_o = i + i_2 \qquad (2)$$

Applying KVL around the loop containing C, L, and R_2, we get

$$-v + L\dot{i} + i_o R_2 = 0$$

or
$$\dot{i} = \frac{v}{L} - \frac{R_2}{L}i_o \qquad (3)$$

Substituting (2) into (3) gives

$$\dot{i} = \frac{v}{L} - \frac{R_2}{L}i - \frac{R_2}{L}i_2 \qquad (4)$$

$$v_o = v \qquad (5)$$

From (1), (3), (4), and (5), we obtain the state model as

$$\begin{bmatrix} \dot{v} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1 C} & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} \frac{1}{C} & 0 \\ 0 & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} v_o \\ i_o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Substituting $R_1 = 1$, $R_2 = 2$, $C = ½$, $L = ¼$ yields

$$\begin{bmatrix} \dot{v} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 4 & -8 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & -8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} v_o \\ i_o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

P.P. 16.12

Let $x_1 = y$ (1)
so that

$$\dot{x}_1 = \dot{y} \qquad (2)$$

Let $$x_2 = \dot{x}_1 = \dot{y} \qquad (3)$$

Finally, let

$$x_3 = \dot{x}_2 = \ddot{y} \qquad (4)$$

then

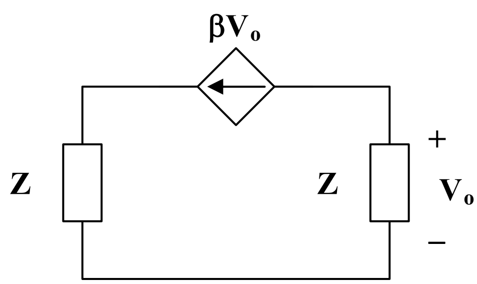
$$\dot{x}_3 = \dddot{y} = \quad 10\ddot{y} \quad 20\dot{y} \quad 5y + z \qquad (5)$$

From (1) to (5), we obtain,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -20 & -18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} z(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

P.P.16.13 The circuit in the s-domain is equivalent to the one shown below.



$$-V_o = (\beta V_o)Z \longrightarrow -1 = \beta Z, \qquad \text{where}$$

$$Z = R \parallel 1/sC = \frac{R}{1 + sRC}$$

Thus, $$-1 = \frac{\beta R}{1 + sRC} \quad \text{or} \quad -(1 + sRC) = \beta R$$

For stability,

$$\beta R > -1 \qquad \text{or} \qquad \beta > \frac{-1}{R}$$

From another viewpoint,

$$V_o = -(\beta V_o)Z \longrightarrow (1 + \beta Z)V_o = 0$$

$$\left(1+\frac{\beta R}{1+sRC}\right)V_o = 0$$

$$(sRC+\beta R+1)\,V_o = 0$$

$$\left(s+\frac{\beta R+1}{RC}\right)V_o = 0$$

For stability $\frac{\beta R+1}{RC}$ must be positive, i.e.

$$\beta R+1>0 \qquad \text{or} \qquad \boldsymbol{\beta > \frac{-1}{R}}$$

P.P.16.14

(a) Following Example 15.24, the circuit is stable when

$$25+\alpha>0 \qquad \text{or} \qquad \boldsymbol{\alpha > -25}$$

(b) For oscillation,

$$25+\alpha=0 \qquad \text{or} \qquad \boldsymbol{\alpha = -25}$$

P.P.16.15

$$\frac{V_o}{V_i} = \frac{R}{R+sL+\frac{1}{sC}} = \frac{s\cdot\frac{R}{L}}{s^2+s\cdot\frac{R}{L}+\frac{1}{LC}}$$

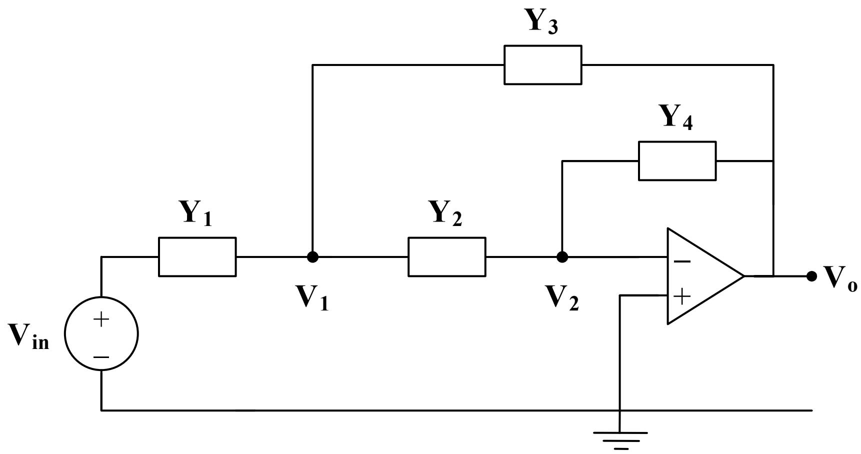
Comparing this with the given transfer function,

$$\frac{R}{L}=4 \qquad \text{and} \qquad \frac{1}{LC}=20$$

If we select $R=2$, then

$$L=\frac{2}{4}=\mathbf{500}\ \boldsymbol{mH} \qquad \text{and} \qquad C=\frac{1}{20L}=\frac{1}{10}=\mathbf{100}\ \boldsymbol{mF}$$

P.P.16.16 Consider the circuit shown below.



Clearly, $V_2 = 0$

At node 1,

$$(V_{in} - V_1)\,Y_1 = (V_1 - V_o)\,Y_3 + (V_1 - 0)\,Y_2$$

or $$V_{in}\,Y_1 = V_1\,(Y_1 + Y_2 + Y_3) - V_o\,Y_3 \qquad (1)$$

At node 2,

$$(V_1 - 0)\,Y_2 = (0 - V_o)\,Y_4$$

or $$V_1 = \frac{-Y_4}{Y_2}V_o \qquad (2)$$

Substituting (2) into (1),

$$V_{in}\,Y_1 = \frac{-Y_4}{Y_2}V_o\,(Y_1 + Y_2 + Y_3) - V_o\,Y_3$$

or $$\frac{V_o}{V_{in}} = \frac{-Y_1 Y_2}{Y_4\,(Y_1 + Y_2 + Y_3) + Y_2 Y_3}$$

If we select $Y_1 = \frac{1}{R_1}$, $Y_2 = sC_1$, $Y_3 = sC_2$, and $Y_4 = \frac{1}{R_2}$, then

$$\frac{V_o}{V_{in}} = \frac{-s \cdot \frac{C_1}{R_1}}{\frac{1}{R_2}\left(\frac{1}{R_1} + sC_1 + sC_2\right) + s^2 C_1 C_2}$$

$$\frac{V_o}{V_{in}} = \frac{-s \cdot \frac{1}{R_1 C_2}}{s^2 + s \cdot \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

Comparing this with the given transfer function shows that

$$\frac{1}{R_1 C_2} = 2, \qquad \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 6, \qquad \frac{1}{R_1 R_2 C_1 C_2} = 10$$

If $R_1 = 10\ \text{k}\Omega$, then

$$C_2 = \frac{1}{2 \times 10^3} = 0.5\ \text{mF}$$

$$\frac{1}{R_2 C_1} = 5 \longrightarrow \frac{1}{R_2} = 5C_1$$

$$\frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 6 \longrightarrow 5\left(1 + \frac{C_1}{C_2}\right) = 6 \longrightarrow C_1 = \frac{C_2}{5} = 0.1\ \text{mF}$$

$$R_2 = \frac{1}{5C_1} = \frac{1}{(5)(0.1 \times 10^{-3})} = 2\ \text{k}\Omega$$

Therefore,

C_1 = **100 μF**, C_2 = **500 μF**, R_2 = **2 kΩ**.