CHAPTER 17

P.P.17.1 $T = 2, \ \omega_o = 2\pi/T = \pi$

$$f(t) = \begin{array}{ll} 1, & 0 < t < 1 \\ -1, & 1 < t < 2 \end{array}$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\left[\int_0^1 (1)dt + \int_1^2 (-1)dt\right] = 0.5(1-1) = 0$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o dt = \frac{2}{2}\left[\int_0^1 1\cos n\pi tdt + \int_1^2 (-1)\cos n\pi tdt\right]$$

$$= \frac{1}{n\pi}\left[\sin n\pi t\right]_0^1 - \frac{1}{n\pi}\left[\sin n\pi t\right]_1^2 = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o dt = \frac{2}{2}\left[\int_0^1 1\sin n\pi tdt + \int_1^2 (-1)\sin n\pi tdt\right]$$

$$= \frac{-1}{n\pi}\left[\cos n\pi t\right]_0^1 + \frac{1}{n\pi}\left[\cos n\pi t\right]_1^2 = \frac{2}{n\pi}\left[1 - \cos n\pi\right]$$

$$\begin{array}{ll} b_n = & 4/(n\pi), \text{ for } n = \text{ odd} \\ = & 0, \quad \text{ for } n = \text{ even} \end{array}$$

$$f(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin n\pi t, \qquad n = 2k - 1$$

See Fig. 17.6 for the spectra.

P.P.17.2 $T = 1, \ \omega_o = 2\pi/T = 2\pi, \ f(t) = 6t, \ 0 < t < 1.$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \left[\int_0^1 (6t)dt\right] = \frac{3t^2}{1}\Big|_0^1 = 3$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o dt = \frac{2}{1}\left[\int_0^1 6t\cos n\pi tdt\right]$$

$$= 2\left[\frac{6}{(2n\pi)^2}\left[\cos 2n\pi t\right]+\frac{6t}{2n\pi}\left[\sin 2n\pi t\right]\right]_0^1$$

$$= \frac{12}{4n^2\pi^2}\left[\left[\cos 2n\pi 1\right]-1\right] = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o dt = \frac{2}{1}\left[\int_0^1 6t\sin 2n\pi t\, dt\right]$$

$$= 2\left[\frac{6}{4n^2\pi^2}\left[\sin 2n\pi t\right]-\frac{6t}{2n\pi}\left[\cos 2n\pi t\right]\right]_0^1 = \frac{-12}{2n\pi}\left[\cos 2n\pi\right] = -6/(n\pi)$$

$$f(t) = 3 - \frac{6}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin 2n\pi t$$

P.P.17.3

$$f(t) = \begin{array}{ll} 8, & -\pi < t < 0 \\ -8, & 0 < t < \pi \end{array}$$

f(t) is an odd function, $\qquad a_o = 0 = a_n$

$$T = 2\pi, \quad \omega_o = 2\pi/T = 1$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_o dt = \frac{4}{2\pi}\left[\int_0^{\pi}(-8)\sin nt\, dt\right]$$

$$= \left[\frac{16}{n\pi}\left[\cos nt\right]\right]_0^{\pi} = \frac{16}{n\pi}\left[\cos n\pi -1\right]$$

$$= \begin{array}{ll} -32/(n\pi), & n = odd \\ 0, & n = even \end{array}$$

$$f(t) = \frac{-32}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin nt, \qquad n = 2k-1$$

P.P.17.4 $\qquad\qquad f(t) = 8t/\pi, \ 0 < t < \pi, \ T = 2\pi, \ \omega_o = 1$

This is an even function, $b_n = 0$.

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{2\pi}\left[\int_0^\pi (8t/\pi)dt\right] = \left(\frac{1}{\pi^2}\right)\left(\frac{8t^2}{2}\right)\Big|_0^\pi = \mathbf{4}$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_o dt = \frac{4}{2\pi}\left[\int_0^\pi \frac{8t}{\pi}\cos nt\, dt\right]$$

$$= \frac{2}{\pi^2}\left[\left[(8t/n)\sin nt\right]_0^\pi - \frac{8}{n}\int_0^\pi \sin nt\, dt\right]$$

$$= \frac{(-2)}{n\pi^2}\frac{(-8)}{n}\cos nt\,\Big|_0^\pi = \frac{16}{n^2\pi^2}(\cos n\pi - 1)$$

$$= \ \begin{array}{ll} \mathbf{-32/(n^2\pi^2),} & \mathbf{n = odd}\\ \mathbf{0,} & \mathbf{n = even}\end{array}$$

$$f(t) = \mathbf{4} - \frac{\mathbf{32}}{\pi^2}\sum_{k=1}^\infty \frac{\mathbf{1}}{n^2}\cos nt, \qquad\qquad \mathbf{n = 2k - 1}$$

P.P.17.5 $\qquad\qquad f(t) = 5t/\pi, \ 0 < t < \pi, \ \omega_o = 2\pi/T = 1$

This is half-wave symmetric. For odd n,

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_o dt = \frac{4}{2\pi}\left[\int_0^\pi \frac{5t}{\pi}\cos nt\, dt\right]$$

$$= \frac{2}{\pi^2}\left[\left[(5t/n)\sin nt\right]_0^\pi - \frac{1}{n}\int_0^\pi 5\sin nt\, dt\right]$$

$$= \frac{(-2)}{n\pi^2}\frac{(-5)}{n}\cos nt\,\Big|_0^\pi = \frac{10}{n^2\pi^2}(\cos n\pi - 1)$$

$$= \ \begin{array}{ll} \mathbf{-20/(n^2\pi^2),} & \mathbf{n = odd}\\ \mathbf{0,} & \mathbf{n = even}\end{array}$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_o dt = \frac{4}{2\pi}\left[\int_0^{\pi}\frac{5t}{\pi}\sin nt\, dt\right]$$

$$= \left[\frac{10}{n^2\pi^2}[\sin nt - nt\cos nt]\right]_0^{\pi} = \frac{10}{n\pi}, \qquad n = \text{odd}$$

Thus, $\displaystyle f(t) = \frac{10}{\pi}\sum_{k=1}^{\infty}\left(\frac{-2}{n^2\pi}\cos nt + \frac{1}{n}\sin nt\right),$ **n = 2k – 1**

P.P.17.6

$$v_s(t) = 1.5 - (3/\pi)\sum_{n=1}^{\infty}\frac{1}{n}\sin 2\pi nt, \quad \omega = 2\pi n$$

$$v_o(\omega) = (1/(j\omega C))v_s/(R + (1/j\omega C)) = v_s/(1 + j\omega RC) = v_s/(1 + j2\omega), \quad RC = 2$$

For the DC component $(\omega = 0,\ \text{or}\ n = 0)$, $\quad v_s = 1.5$ and $v_o = 1.5$

For the nth harmonic, $v_s = -(3/(n\pi))$ or

$$v_o = -(3/(n\pi))/\sqrt{1 + 4\omega^2}\ \angle\tan^{-1}2\omega$$

or $\quad v_o = -3\angle(-\tan^{-1}2\omega)/(n\pi\sqrt{1 + 4\omega^2})$

Hence, $\quad \displaystyle v_o(t) = 1.5 - (3/\pi)\sum_{n=1}^{\infty}\frac{1}{n\sqrt{1 + 4\omega^2}}\sin(2\pi nt - \tan^{-}1(2\omega))$

$$= \frac{3}{2} - \frac{3}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2\pi nt - \tan^{-1} 4\pi n)}{n\sqrt{1 + 16\pi^2 n^2}}\, V$$

P.P.17.7

$$v(t) = (1/3) + (1/\pi^2)\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\cos nt - \frac{\pi}{n}\sin nt\right)$$

$$= (1/3) + (1/\pi^2)\sum_{n=1}^{\infty} A_n\cos(nt - \phi_n)$$

where
$$A_n = \frac{1}{n}\sqrt{\frac{1}{n^2} + \pi^2} = \frac{1}{n^2}\sqrt{1 + n^2\pi^2}$$

$$\phi_n = \tan^{-1}(b_n/a_n) = \tan^{-1}(-n\pi)$$

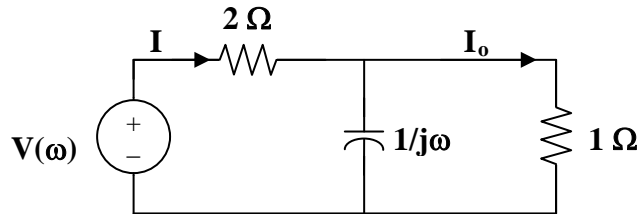$$v(t) = (1/3) + \frac{1}{\pi^2}\sum \frac{1}{n^2}\sqrt{1 + n^2\pi^2}\, \cos[nt - \tan^{-1}(-n\pi)]$$

$$Z = 2 + 1\|(1/(j\omega)), \quad \omega = n$$

$$= 2 + (1/(j\omega))/(1 + (1/(j\omega))) = 2 + (1/(1 + (j\omega)))$$

$$= (3 + 2j\omega)/(1 + j\omega), \quad \omega = n$$

$$= (3 + j2n)/(1 + jn)$$

$$I = V/Z = [(1 + jn)/(3 + j2n)]\, V$$

By current division, $\quad I_o = (1/j\omega)I/[1 + (1/j\omega)] = I/(1 + j\omega) = V/(3 + j2n)$

For the DC component ($n = 0$), $\quad V = 1/3$ and $I_o = V/3 = 1/9$

For the nth harmonic, $\quad V = [1/(n^2\pi^2)]\sqrt{1 + n^2\pi^2}\, \angle -\tan^{-1}(-n\pi)$

$$I_o = V/[\sqrt{9 + 4n^2}\, \angle -\tan^{-1}(2n/3)]$$

$$= \sqrt{1 + n^2\pi^2}\, \angle[-\tan^{-1}(-n\pi) - \tan^{-1}(2n/3)]/[n^2\pi^2\sqrt{9 + 4n^2}\,]$$

But, $\qquad \tan^{-1}(-n\pi) = -\tan^{-1}(n\pi)$

In the time domain,
$$i_o(t) = \{(1/9) + \sum_{n=1}^{\infty} \frac{\sqrt{1 + n^2\pi^2}}{n^2\pi^2\sqrt{9 + 4n^2}}\, \cos[nt - \tan^{-1}(2n/3) + \tan^{-1}(n\pi)]\}A$$

P.P.17.8
$$P = V_{DC} I_{DC} + 0.5 \sum_{n=0}^{\infty} V_n I_n \cos(\phi_n - \theta_n)$$

$$= 128(0) + 0.5(192)(4) \cos(10°) + 0.5(96)(1.6) \cos(30°)$$

$$= 378.2 + 66.51$$

$$= \mathbf{444.7 \ watts}$$

P.P.17.9
$$I^2_{rms} = 8^2 + 0.5[30^2 + 20^2 + 15^2 + 10^2]$$

$$= 64 + 0.5 \times 1625 = 876.5$$

$$= \mathbf{29.61 \ A}$$

P.P.17.10
$$f(t) = 1, \qquad 0 < t < 1$$

$$= 0, \qquad 1 < t < 2$$

$$T = 2, \quad \omega_o = 2\pi/T = \pi$$

$$C_n = (1/T) \int_0^T f(t) e^{-jn\omega_o t} dt = 0.5[\int_0^1 1 e^{-jn\pi t} dt + 0]$$

$$= 0.5[1/(-jn\pi)] e^{-jn\pi t} \Big|_0^1 = [j/(2n\pi)](e^{-jn\pi} - 1)$$

But $e^{-jn\pi} = \cos(n\pi) - j\sin(n\pi) = \cos(n\pi) = (-1)^n$

$$C_n = [j/(2\pi n)][(-1)^n - 1] \qquad = 0, \qquad\qquad n = even$$

$$= [-j/(n\pi)], \quad n = odd \neq 0$$

For $n = 0$, $\qquad C_o = 0.5 \int_0^1 1 dt = 0.5$

Hence, $\qquad f(t) = \dfrac{1}{2} - \sum_{\substack{n=-\infty \\ n \neq 0 \\ n=odd}}^{\infty} \dfrac{j}{n\pi} e^{jn\pi t}$

P.P.17.11 $f(t) = t, \ -1 < t < 1, \ T = 2, \ \omega_o = 2\pi/T = \pi$

$$C_n = (1/T)\int_{-T/2}^{T/2} f(t)e^{-jn\omega_o t}dt = 0.5\int_{-1}^{1} te^{-jn\pi t}dt$$

$$= 0.5[e^{-jn\pi t}/(-jn\pi)^2](-jn\pi t - 1)|_{-1}^{1}$$

$$= [-1/(2n^2\pi^2)][e^{-jn\pi}(-jn\pi - 1) - e^{jn\pi}(jn\pi - 1)]$$

$$= [-1/(2n^2\pi^2)][(\cos n\pi - j\sin n\pi)(-jn\pi - 1) \\ - (\cos n\pi + j\sin n\pi)(jn\pi - 1)]$$

$$= [j\cos n\pi]/(n\pi)$$

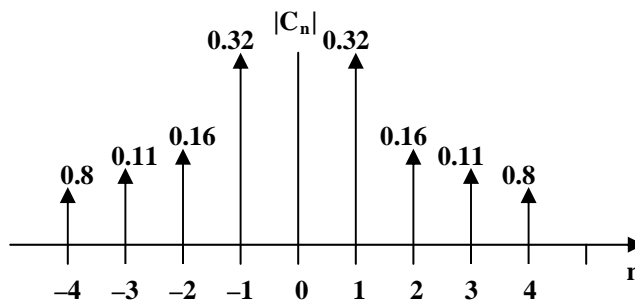$$C_n = j(-1)^n/n\pi, \ n \neq 0$$

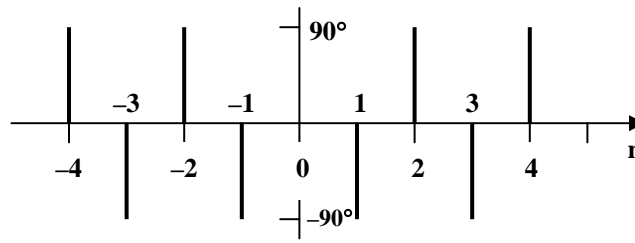For $n = 0$, $\qquad C_o = (1/T)\int_{-T/2}^{T/2} f(t)dt = 0$

Thus, $\qquad f(t) = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{j(-1)^n}{n\pi} e^{jn\pi t}$

$$|C_n| = 1/(n\pi), \ n \neq 0, \ \theta_n = (-1)^n 90°, n \neq 0$$
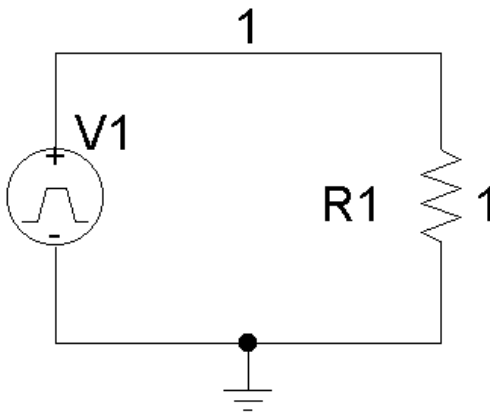
The amplitude and phase spectra are shown below.

P.P.17.12   The schematic is shown below. The attributes of the voltage source is entered as shown. After entering the final time (5 or 6T), the Print Step, the Step Ceiling, and the Center Frequency in the transient dialog box, the circuit is saved. Once the PSpice is run, the output contains the following Fourier coefficients.

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FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)
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DC COMPONENT = **4.950000E-01**

HARMONIC FREQUENCY FOURIER NORMALIZED PHASE NORMALIZED
NO	(HZ)	COMPONENT	COMPONENT	(DEG)	PHASE (DEG)
1	1.000E+00	3.184E-01	1.000E+00	-1.782E+02	0.000E+00
2	2.000E+00	1.593E-01	5.002E-01	-1.764E+02	1.800E+00
3	3.000E+00	1.063E-01	3.338E-01	-1.746E+02	3.600E+00
4	4.000E+00	7.979E-02	2.506E-01	-1.728E+02	5.400E+00
5	5.000E+00	6.392E-02	2.008E-01	-1.710E+02	7.200E+00
6	6.000E+00	5.337E-02	1.676E-01	-1.692E+02	9.000E+00
7	7.000E+00	4.584E-02	1.440E-01	-1.674E+02	1.080E+01
8	8.000E+00	4.021E-02	1.263E-01	-1.656E+02	1.260E+01
9	9.000E+00	3.584E-02	1.126E-01	-1.638E+02	1.440E+01

TOTAL HARMONIC DISTORTION = 7.363360E+01 PERCENT

P.P.17.13   The schematic is shown below. Since T = 1/f = 0.55 ms, in the transient dialog box, we set Print Step = 0.01 ms, Final Time = 4 ms, Center Frequency = 2,000 Hz, Number of Harmonics = 5, and Output Vars = V(R1:1). Once the circuit is saved, we simulate it and obtain the following results.
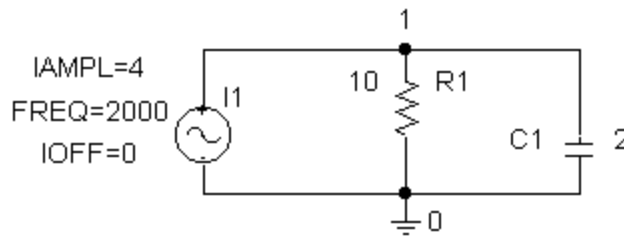
DC COMPONENT = -1.507149E-04

HARMONIC FREQUENCY FOURIER NORMALIZED PHASE NORMALIZED

NO	(HZ)	COMPONENT	COMPONENT	PHASE (DEG)	PHASE (DEG)
1	2.000E+03	1.455E-04	1.000E+00	9.006E+01	0.000E+00
2	4.000E+03	1.851E-06	1.273E-02	9.597E+01	5.910E+00
3	6.000E+03	1.406E-06	9.662E-03	9.323E+01	3.167E+00
4	8.000E+03	1.010E-06	6.946E-03	8.077E+01	-9.292E+00

TOTAL HARMONIC DISTORTION = 1.830344E+00 PERCENT

From this, we use the amplitude and phase of the Fourier components to get

$$v(t) = \{-150.72 + 145.5\sin(4\pi10^3t + 90°) + 1.845\sin(8\pi10^3t + 96.24°) + - - -\}\mu V$$

P.P.17.14   From Example 16.14,

$$2\omega_o = 4\pi = 12.566 \text{ rad/s}$$
$$3\omega_o = 6\pi = 18.84 \text{ rad/s}$$
$$4\omega_o = 8\pi = 25.13 \text{ rad/s}$$
$$5\omega_o = 10\pi = 31.41 \text{ rad/s}$$
$$6\omega_o = 12\pi = 37.7 \text{ rad/s}$$

Since the ideal bandpass filter passes only $15 < \omega < 25$, it means that only the 3rd, 4th, and 5th harmonics will be passed. Hence,

$$y(t) = (-1/3\pi)\sin(3\omega_o t) - (1/(4\pi))\sin(4\omega_o t) - (/(5\pi))\sin(5\omega_o t), \quad \omega_o = 2\pi$$