CHAPTER 18

P.P.18.1 (a) $g(t) = 4u(t+1) - 4u(t-2) = \begin{cases} 4, & 1 < t < 2 \\ 0, & \text{otherwise} \end{cases}$

$$G(\omega) = \int_1^2 4 \cdot e^{-j\omega t} dt = -\frac{4}{j\omega} e^{-j\omega t}\Big|_1^2$$

$$= \mathbf{\frac{4(e^{-j\omega} - e^{-j2\omega})}{j\omega}}$$

(b) $F(t) = 4\delta(t+2)$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} 4\delta(t+2)e^{-j\omega t}dt$$

$$= 4e^{j\omega t}\Big|_{t=2} = \mathbf{4e^{j2\omega}}$$

(c) $F(t) = 10\sin(\omega_o t)$

$$F(\omega) = F\left[\frac{-10e^{j\omega_o t}}{2j}\right] = \frac{10}{j2}\left[F\left(e^{j\omega_o t}\right) - F\left(e^{-j\omega_o}\right)\right]$$

$$= \mathbf{-j10\pi\left[\delta(\omega - \omega_o) - \delta(\omega + \omega_o)\right]} \text{ or } \mathbf{j10\pi[\delta(\omega+\omega_o) - \delta(\omega - \omega_o)]}$$

P.P.18.2 $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} = \int_{-1}^{0} 10e^{-j\omega t}dt + \int_{0}^{1}(-10)e^{-j\omega t}dt$

$$= \frac{10e^{-j\omega t}}{-j\omega}\Bigg|_{-1}^{0} - \frac{10e^{-j\omega t}}{-j\omega}\Bigg|_{0}^{1} = \frac{j10}{\omega}\left[1 - e^{j\omega} - e^{-j\omega} + 1\right]$$

$$= \mathbf{\frac{20(\cos\omega - 1)}{j\omega}}$$

P.P.18.3 $f(t) = \begin{vmatrix} 10e^{at}, & t < 0 \\ 0, & t > 0 \end{vmatrix}$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{j\omega t}dt = \int_{-\infty}^{\infty} 10e^{at}e^{-j\omega t}dt$$

Let x = -t, then dt = -dx

$$F(\omega) = \int_{\infty}^{0} 10e^{-ax}e^{j\omega x}(-dx) = -10\int_{\infty}^{0} e^{-(a-j\omega)x}dx$$

$$= \frac{10}{a - j\omega}e^{a-j\omega)x} = \mathbf{\frac{10}{a - j\omega}}$$

P.P.18.4 (a) $g(t) = u(t) - u(t - 1)$

$F(\omega) = u(\omega) - e^{-j\omega}u(\omega) = (1 - e^{-j\omega})u(\omega)$

$= \mathbf{(1 - e^{-j\omega})(\pi\delta(\omega) + 1/(j\omega))}$

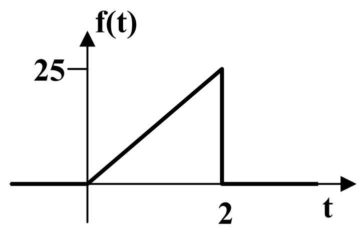
(b) $f(t) = te^{-2t}u(t)$

Let $g(t) = e^{-2t}u(t) \longrightarrow G(\omega) = 1/(2 + j\omega)$

$f(t) = tg(t) \longrightarrow j\dfrac{dG}{d\omega} = j(-1)\,(2 + j\omega)^{-2}(j)$

$$F(\omega) = \mathbf{\frac{1}{(2 + j\omega)^2}}$$

(c) f(t) is sketched below.



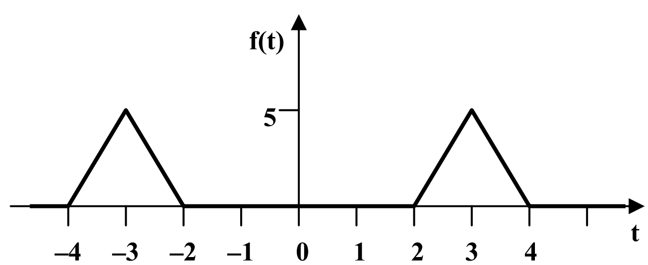
$f'(t) = -50\delta(t-2) - 100\delta(t-2)$

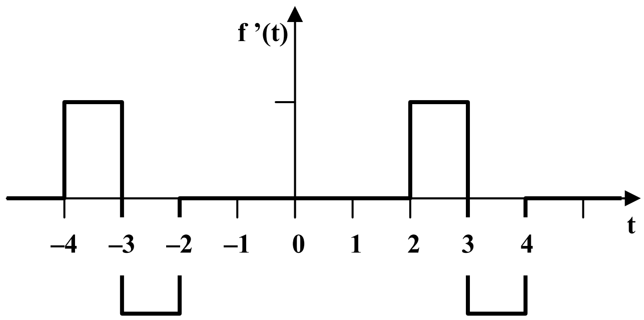
$f''(t) = 50\delta(t) - 50\delta(t - 2) - 100\delta'(t - 2)$

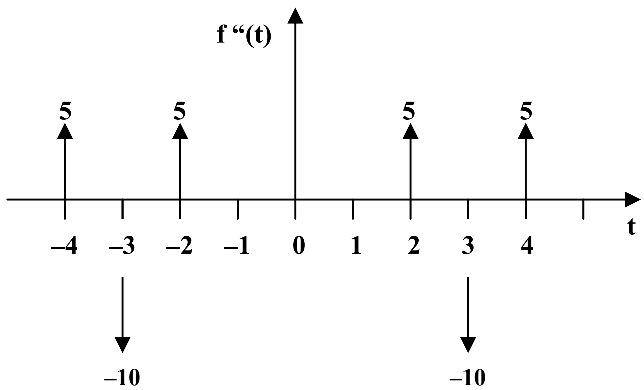
$(j\omega)^2F(\omega) = 50(1 - e^{-j\omega 2}) - 100j\omega e^{-j\omega 2}$

$$F(\omega) = \frac{50\left(e^{-j\omega 2} - 1\right)}{\omega^2} + \frac{100je^{-j\omega 2}}{\omega}$$

P.P.18.5 Given f(t), f '(t) and f "(t) are sketched below:







$f''(t) = 5\delta(t + 4) - 10\delta(t + 3) + 5\delta(t + 2) + 5\delta(t - 2) - 10\delta(t + 3) + 5\delta(t - 4)$

We take the Fourier transform of each term.

$(j\omega)^2F(\omega) = 5(e^{j4\omega} + e^{-j4\omega}) - 10(e^{j3\omega} + e^{-j3\omega}) + 5(e^{j2\omega} + e^{-j2\omega})$
$= 4\cos 4\omega - 8\cos 3\omega + 4\cos 2\omega$
$F(\omega) =$ $\mathbf{[1/(\omega^2)](20\cos 3\omega - 10\cos 4\omega - 10\cos 2\omega)}$

P.P.18.6 (a) $H(\omega) = \dfrac{6(2j\omega+3)}{(j\omega+1)(j\omega+4)(j\omega+2)}$

$$= \frac{2}{j\omega+1} + \frac{3}{j\omega+2} - \frac{5}{j\omega+4}$$

$h(t) =$ $\mathbf{(2e^{-t} + 3e^{-2t} - 5e^{-4t})u(t)}$

(b) $y(t) = u(t) + 2e^{-t}\cos 4t\, u(t)$
$=$ $\mathbf{(1 + 2e^{-t}\cos 4t)\, u(t)}$

P.P.18.7 $v_i = 5\,\text{sgn}(t) \longrightarrow V_i(\omega) = 10/(j\omega)$
$H(\omega) = 4/(4 + j\omega)$

$$V_o(\omega) = H(\omega)V_i(\omega) = \frac{40}{j\omega(4+j\omega)} = \frac{A}{j\omega} + \frac{B}{4+j\omega}$$

$$= \frac{10}{j\omega} - \frac{10}{4 + j\omega}$$

$v_o(t) = 5 \text{ sgn }(t) – 10e^{-4t}u(t) = 5[–1 + u(t)] – 10e^{-4t}u(t)$

$= \mathbf{–5 + 10\ [1 – e^{-4t}]u(t)\ V}$

P.P.18.8 $I_s(\omega) = 20\pi[\delta(\omega + 4) + \delta(\omega - 4)]$

$$H(\omega) = \frac{6 + j\omega 2}{10 + 6 + j2\omega} = \frac{3 + j\omega}{8 + j\omega}$$

$$I_0(\omega) = H(\omega)I_s(\omega) = \left(\frac{3 + j\omega}{8 + j\omega}\right)(20\pi)[\delta(\omega + 4) + \delta(\omega - 4)]$$

$$i_o(t) = \mathcal{F}^{-1}I_o(\omega) = \frac{20\pi}{2\pi}\int_{-\infty}^{\infty}\left(\frac{3 + j\omega}{8 + j\omega}\right)[\delta(\omega + 4) + \delta(\omega - 4)e^{j\omega t}d\omega]$$

$$= 10\left[\frac{3 - j4}{8 - j\omega}e^{-j4t} + \frac{3 + j4}{8 + j4}e^{j4t}\right]$$

But

$$\frac{3 + j4}{8 + j4} = \frac{5\angle 53.13^\circ}{\sqrt{80}\angle 26.56^\circ} = 0.559\angle 26.57^\circ$$

$$i_o(t) = 5.59\left(e^{-j(4t + 26.57^\circ)} + e^{j(4t + 26.57^\circ)}\right)$$

$i_o(t) = \mathbf{11.18\ cos\ (4t + 26.57°)\ A}$

P.P.18.9 (a) $W_{1\Omega} = \int_{-\infty}^{\infty} 100e^{-4|t|}\,dt = 200\int_0^{\infty} e^{-4t}\,dt$

since $|t|$ is even.

$$W_{1\Omega} = \frac{200e^{-4t}}{-4}\Big|_0^{\infty} = \mathbf{50\ J}$$

(b) $H(\omega) = \frac{40}{4 + \omega^2}$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{\infty}\frac{1600}{(4 + \omega^2)^2}d\omega = \frac{1600}{\pi}\cdot\frac{1}{8}\left(\frac{\omega}{\omega^2 + 4} + \frac{1}{2}\tan^{-1}\frac{\omega}{2}\right)\Big|_0^{\infty}$$

$$W_{1\Omega} = \frac{200}{\pi}\left(0 + \frac{\pi}{4} - 0 - 0\right) = \mathbf{50\ J}$$

P.P.18.10 $F(\omega) = \frac{2}{1 + j\omega} \longrightarrow |F(\omega)|^2 = \frac{4}{1 + \omega^2}$

$$W_{2\Omega} = \frac{8}{\pi}\int_0^{\infty}\frac{d\omega}{1 + \omega^2} = \frac{8}{\pi}\tan^{-1}\omega\Big|_0^{\infty} = \frac{8}{\pi}\cdot\frac{\pi}{2} = 4\,J$$

for -4 < ω < 4,

$$W = \frac{8}{\pi}\int_0^4 \frac{d\omega}{1+\omega^2} = \frac{8}{\pi}\tan^{-1}\omega\Big|_0^4 = \frac{8}{\pi}\cdot\frac{76}{180}\pi = 3.378\,\text{J}$$

Percentage energy = (3.378/4)100 = **84.4%** of the total energy.

P.P.18.11 If f_c = 2 MHz, f_m = 4 kHz

upper sideband = 2,000,000 + 4,000 = **2,004,000 Hz**
Carrier = **2,000,000 Hz**
Lower sideband = 2,000,000 -4,000 = **1,996,000 Hz**

P.P.18.12 W = 12.5 kHz, f_s = 2W = 25 kHz

$$T_s = \frac{1}{f_s} = \frac{1}{25x10^3} = \mathbf{40\ \mu s}$$