Wednesday, June 29, 2011

CHAPTER 19

P.P.19.1 Comparing the network with that in Fig. 19.5, we observe that

$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{3\ \Omega}$
$\mathbf{z}_{22} - \mathbf{z}_{12} = 0$ or $\mathbf{z}_{22} = \mathbf{z}_{12} = \mathbf{3\ \Omega}$
$z_{11} - z_{12} = 4$ or $z_{11} = 4 + 3 = \mathbf{7\ \Omega}$

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{7} & \mathbf{3} \\ \mathbf{3} & \mathbf{3} \end{bmatrix} \Omega$$

P.P.19.2 $\mathbf{V}_1 = 6\mathbf{I}_1 - j4\mathbf{I}_2$ (1)
$\mathbf{V}_2 = -j4\mathbf{I}_1 + 8\mathbf{I}_2$ (2)

But $\mathbf{V}_2 = 0$
and $2\angle 30° = \mathbf{V}_1 + 2\mathbf{I}_1 \longrightarrow \mathbf{V}_1 = 2\angle 30° - 2\mathbf{I}_1$

Substituting these into (1) and (2),
$2\angle 30° - 2\mathbf{I}_1 = 6\mathbf{I}_1 - j4\mathbf{I}_2$
$2\angle 30° = 8\mathbf{I}_1 - j4\mathbf{I}_2$ (3)

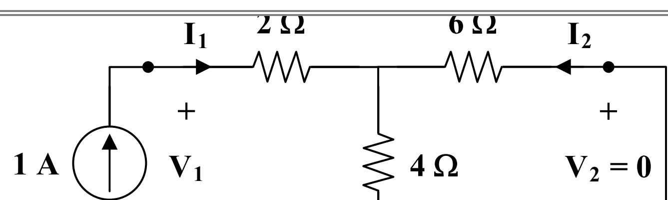
$0 = -j4\mathbf{I}_1 + 8\mathbf{I}_2$
$\mathbf{I}_1 = -j2\mathbf{I}_2$ (4)

Substituting (4) into (3),
$2\angle 30° = -j16\mathbf{I}_2 - j4\mathbf{I}_2 = -j20\mathbf{I}_2$

$$\mathbf{I}_2 = \frac{2\angle 30°}{20\angle -90°} = \mathbf{100\angle 120^\circ\ mA}$$

$$\mathbf{I}_1 = -j2\mathbf{I}_2 = \mathbf{200\angle 30^\circ\ mA}$$

P.P.19.3 Consider the circuit in Fig. (a) for $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$.



$\mathbf{V_1}$ = [2+(4x6/(4+6))]$\mathbf{I_1}$ = 4.4 $\mathbf{I_1}$

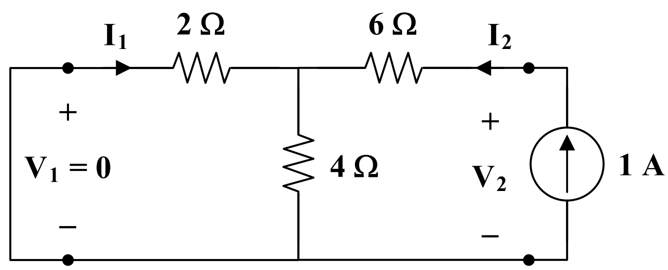
$\mathbf{y_{11}}$ = ($\mathbf{I_1}$/$\mathbf{V_1}$) = 1/4.4 = **0.2273 S**

By current division,

$$\mathbf{I}_2 = \frac{-4}{10}\mathbf{I}_1 = -0.4\mathbf{I}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -0.4\mathbf{I_1}/(4.4\mathbf{I_1}) = \mathbf{-0.09091\ S}$$

For $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$, consider the circuit in Fig. (b).



(b)

$\mathbf{V_2}$ = [6 +(4x2/(4+2))]$\mathbf{I_2}$ = 7.333$\mathbf{I_2}$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = 1/7.333 = \mathbf{0.13636\ S}$$

By current division,

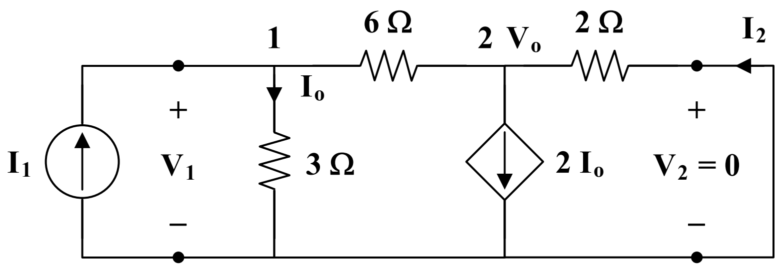
$\mathbf{I_1}$ = [–4/(4+2)]$\mathbf{I_2}$ = –(2/3)$\mathbf{I_2}$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = -(2/3)\mathbf{I_2}/7.333\mathbf{I_2} = \mathbf{-0.09091\ S}$$

Therefore,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{227.3} & \mathbf{-90.91} \\ \mathbf{-90.91} & \mathbf{136.36} \end{bmatrix} \boldsymbol{mS}$$

P.P.19.4 Consider the circuit in Fig (a).



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{3} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{6}$$

$$6\mathbf{I}_1 = 3\mathbf{V}_1 - \mathbf{V}_o \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{6} + \mathbf{I}_2 = 2\mathbf{I}_o = \frac{2}{3}\mathbf{V}_1$$

But $\mathbf{I}_2 = \frac{0 - \mathbf{V}_o}{2} = \frac{-\mathbf{V}_o}{2}$

Hence, $\frac{\mathbf{V}_1 - \mathbf{V}_o}{6} - \frac{\mathbf{V}_o}{2} = \frac{2}{3}\mathbf{V}_1 \longrightarrow \mathbf{V}_1 = \frac{-4}{3}\mathbf{V}_o \qquad (2)$

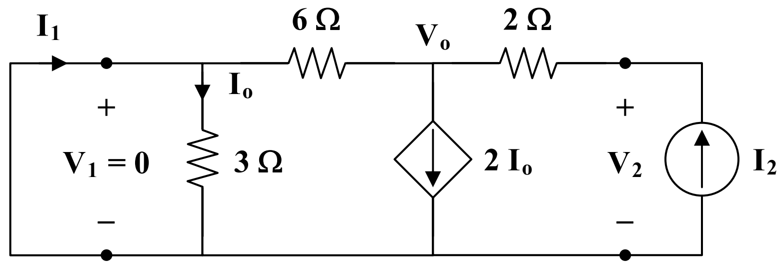
From (1) and (2),

$$6\mathbf{I}_1 = -4\mathbf{V}_o - \mathbf{V}_o = -5\mathbf{V}_o \longrightarrow \mathbf{I}_1 = \frac{-5}{6}\mathbf{V}_o$$

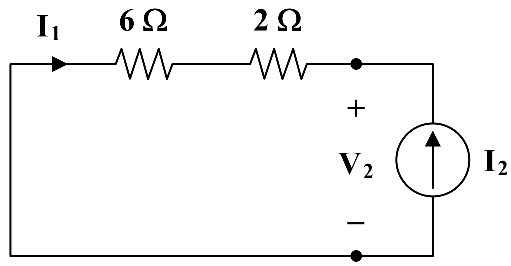
Thus, $\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{(-5/6)\mathbf{V}_o}{(-4/3)\mathbf{V}_o} = \frac{5}{8} = 0.625\text{ S}$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{(-1/2)\mathbf{V}_o}{(-4/3)\mathbf{V}_o} = \frac{3}{8} = 0.375\text{ S}$$

Consider the circuit in Fig. (b). The 3-Ω resistor is short-circuited so that $\mathbf{I}_o = 0$. Consequently, the circuit is equivalent to that shown in Fig.(c).



(b)



(c)

$$\mathbf{V}_2 = 8\mathbf{I}_2, \qquad \mathbf{I}_1 = -\mathbf{I}_2$$

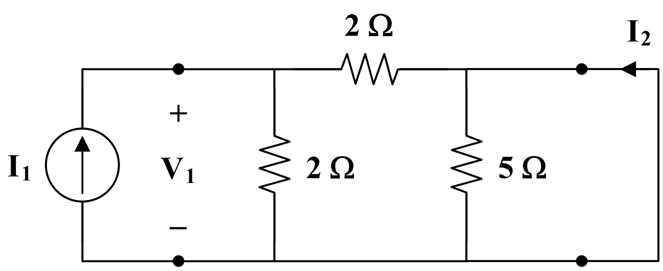
$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{8} = 0.125\text{ S}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\mathbf{I}_2}{8\mathbf{I}_2} = -0.125\text{ S}$$

Therefore,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{0.625} & \mathbf{-0.125} \\ \mathbf{0.375} & \mathbf{0.125} \end{bmatrix}\mathbf{S} \ \ \mathbf{or} \ \begin{bmatrix} \mathbf{625} & \mathbf{-125} \\ \mathbf{375} & \mathbf{125} \end{bmatrix}\mathbf{mS}$$

P.P.19.5 To find $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, we use the circuit in Fig. (a). Note that the 5-Ω resistor is short-circuited.



(a)

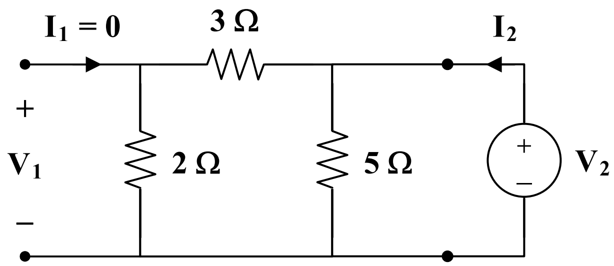
$$\mathbf{V_1} = [2x3/(2+3)]\mathbf{I_1} = 1.2\mathbf{I_1}$$

$$\mathbf{h_{11}} = \frac{\mathbf{V_1}}{\mathbf{I_1}} = \mathbf{1.2\ \Omega}$$

$$-\mathbf{I}_2 = \frac{2}{2+3}\mathbf{I}_1 = 0.4\mathbf{I}_1 = (4/(4+6))\mathbf{I_1} = 0.4\mathbf{I_1}$$

$$\mathbf{h_{21}} = \frac{\mathbf{I_2}}{\mathbf{I_1}} = \mathbf{-0.4}$$

To get $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$, we use the circuit in Fig. (b).



(b)

$$\mathbf{V_1} = [2/(2+3)]\mathbf{V_1} = 0.4\mathbf{V_1}$$

$$\mathbf{h_{12}} = \frac{\mathbf{V_1}}{\mathbf{V_2}} = \mathbf{0.4}$$

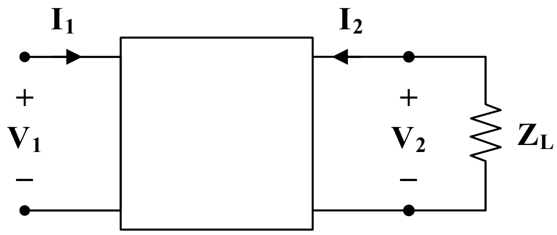
$$\mathbf{V_2} = [5x5/(5+5)]\mathbf{I_2} = 2.5\mathbf{I_2}$$

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{2.5} = 0.4\ S$$

Therefore,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{1.2\ \Omega} & \mathbf{0.4} \\ \mathbf{-0.4} & \mathbf{400}\ \boldsymbol{mS} \end{bmatrix}$$

P.P.19.6 Our goal is to get $\mathbf{Z}_{in} = \mathbf{V}_1/\mathbf{I}_1$.



$$\mathbf{V}_1 = \mathbf{h}_{11}\,\mathbf{I}_1 + \mathbf{h}_{12}\,\mathbf{V}_2 \qquad (1)$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\,\mathbf{I}_1 + \mathbf{h}_{22}\,\mathbf{V}_2 \qquad (2)$$

But $\mathbf{V}_2 = -\mathbf{Z}_L\,\mathbf{I}_2$

Substituting this equation into (1) and (2),

$$\mathbf{I}_2 = \mathbf{h}_{21}\,\mathbf{I}_1 - \mathbf{Z}_L\,\mathbf{h}_{22}\,\mathbf{I}_2 \longrightarrow \mathbf{I}_2 = \frac{\mathbf{h}_{21}\,\mathbf{I}_1}{1+\mathbf{Z}_L\,\mathbf{h}_{22}}$$

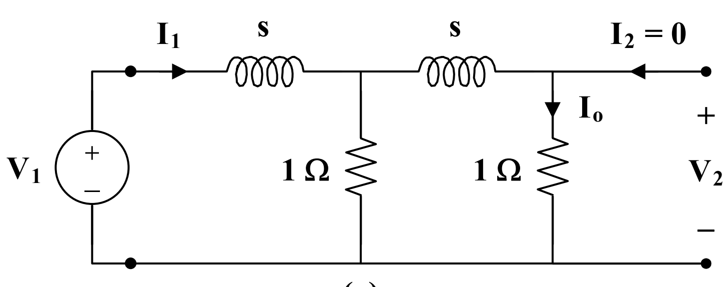
$$\mathbf{V}_2 = -\mathbf{Z}_L\,\mathbf{I}_2 = \frac{-\mathbf{Z}_L\,\mathbf{h}_{21}\,\mathbf{I}_1}{1+\mathbf{Z}_L\,\mathbf{h}_{22}}$$

$$\mathbf{V}_1 = \mathbf{h}_{11}\,\mathbf{I}_1 - \frac{\mathbf{Z}_L\,\mathbf{h}_{12}\,\mathbf{h}_{21}\,\mathbf{I}_1}{1+\mathbf{Z}_L\,\mathbf{h}_{22}}$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \mathbf{h}_{11} - \frac{\mathbf{Z}_L\,\mathbf{h}_{12}\,\mathbf{h}_{21}}{1+\mathbf{Z}_L\,\mathbf{h}_{22}}$$

$$\mathbf{Z}_{in} = 2000 - \frac{(50\times10^3)(10^{-4})(100)}{1+(50\times10^3)(10^{-5})} = \mathbf{1.6667\ k\Omega}$$

P.P.19.7 We get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$ using the circuit in Fig. (a).



(a)

$$\mathbf{V}_1 = [\,s + 1 \,\|\, (s+1)\,]\ \mathbf{I}_1$$

$$\frac{\mathbf{V}_1}{\mathbf{I}_1} = s + \frac{s+1}{s+2} = \frac{s^2+3s+1}{s+2}$$

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{s+2}{s^2+3s+1}$$

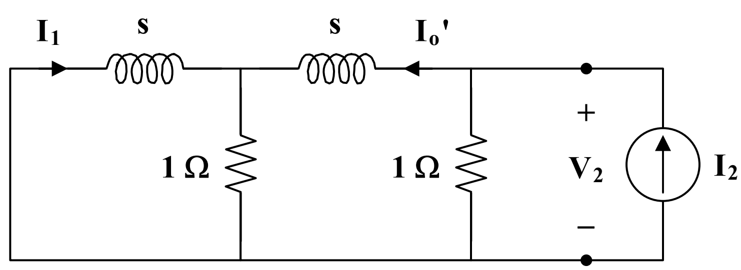
By current division,

$$\mathbf{I}_o = \frac{1}{s+2}\mathbf{I}_1 = \frac{\mathbf{V}_1}{s^2+3s+1}$$

$$\mathbf{V}_2 = \mathbf{I}_o = \frac{\mathbf{V}_1}{s^2+3s+1}$$

$$\mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{s^2+3s+1}$$

We obtain $\mathbf{g}_{12}$ and $\mathbf{g}_{22}$ using the circuit in Fig. (b).



(b)

$$\mathbf{V}_2 = [1 \,\|\, (s+1 \,\|\, s)]\,\mathbf{I}_2 = \left[1 \,\|\left(s + \frac{s}{s+1}\right)\right]\mathbf{I}_2$$

$$\frac{\mathbf{V}_2}{\mathbf{I}_2} = 1 \,\|\, \frac{s^2+2s}{s+1} = \frac{\frac{s^2+2s}{s+1}}{1+\frac{s^2+2s}{s+1}} = \frac{s(s+2)}{s^2+3s+1}$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{s(s+2)}{s^2+3s+1}$$

By current division,

$$\mathbf{I}_1 = \frac{-1}{s+1}\mathbf{I}_o'$$

and $$\mathbf{I}_o' = \frac{1}{1+\frac{s^2+2s}{s+1}}\mathbf{I}_2 = \frac{s+1}{s^2+3s+1}\mathbf{I}_2$$

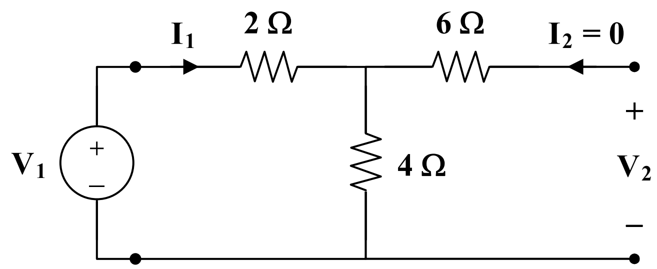
$$\mathbf{I}_1 = \frac{-1}{s^2+3s+1}\mathbf{I}_2$$

$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-1}{s^2 + 3s + 1}$$

Therefore,

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{s+2}{s^2+3s+1} & \dfrac{-1}{s^2+3s+1} \\ \dfrac{1}{s^2+3s+1} & \dfrac{s(s+2)}{s^2+3s+1} \end{bmatrix}$$

P.P.19.8 To get **A** and **C**, we use the circuit in Fig. (a).

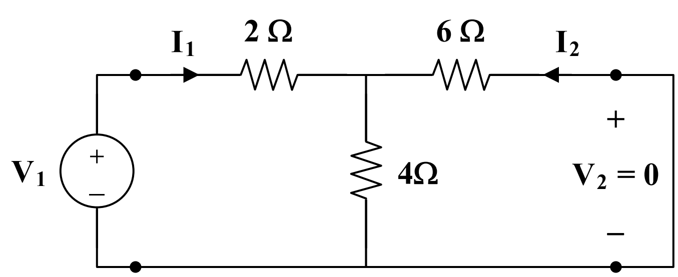


(a)

$\mathbf{A} = \mathbf{V}_1/\mathbf{V}_2 = (2+4)\mathbf{I}_1/(4\mathbf{I}_1) = \mathbf{1.5}$

$\mathbf{C} = \mathbf{I}_1/\mathbf{V}_2 = \mathbf{I}_1/(4\mathbf{I}_1) = \mathbf{0.25\ S}$

To get **B** and **D**, we use the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = \frac{-4}{10}\mathbf{I}_1 \longrightarrow \mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{10}{4} = 2.5$$

Also, $\mathbf{V}_1 = [2+(4\text{x}6/(4+6))]\mathbf{I}_1 = 4.4\mathbf{I}_1$
$\mathbf{B} = -\mathbf{V}_1/\mathbf{I}_2 = -4.4\mathbf{I}_1/(-0.4\mathbf{I}_1) = \mathbf{11\ \Omega}$

Therefore,

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{1.5} & \mathbf{11\,\Omega} \\ \mathbf{250\ \mathit{mS}} & \mathbf{2.5} \end{bmatrix}$$

P.P.19.9 From Eq. (19.22),

$$\mathbf{V}_1 = 5\mathbf{V}_2 - 10\mathbf{I}_2 \qquad (1)$$

$$\mathbf{I}_1 = 0.4\mathbf{V}_2 - \mathbf{I}_2 \qquad (2)$$

At the output port, $\mathbf{V}_2 = \text{-}10\mathbf{I}_2$. At the input port, $\mathbf{V}_1 = 14 - 2\mathbf{I}_1$.
Substituting these into (1) and (2),

$$14 - 2\mathbf{I}_1 = \text{-}50\mathbf{I}_2 - 10\mathbf{I}_2$$

$$14 = 2\mathbf{I}_1 - 60\mathbf{I}_2 \qquad (3)$$

$$\mathbf{I}_1 = \text{-}4\mathbf{I}_2 - \mathbf{I}_2$$

$$\mathbf{I}_1 = \text{-}5\mathbf{I}_2 \qquad (4)$$

Substituting (4) into (3),

$$14 = \text{-}10\mathbf{I}_2 - 60\mathbf{I}_2$$

$$\mathbf{I}_2 = \frac{\text{-}14}{70} = \mathbf{-\,0.2\ A}$$

$$\mathbf{I}_1 = (\text{-}5)(\text{-}0.2) = \mathbf{1\ A}$$

P.P.19.10

$$[\mathbf{z}] = \begin{bmatrix} 6 & 4 \\ 4 & 6 \end{bmatrix} \qquad \Delta_z = 36 - 16 = 20$$

$$\mathbf{z}_{11} = 6 = \mathbf{z}_{22} \qquad \mathbf{z}_{12} = \mathbf{z}_{21} = 4$$

From Table 19.1,

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{6}{20} = 0.3\ \text{S} \qquad \mathbf{y}_{12} = \frac{\text{-}\,\mathbf{z}_{12}}{\Delta_z} = \frac{\text{-}\,4}{20} = \text{-}0.2\ \text{S}$$

$$\mathbf{y}_{21} = \frac{\text{-}\,\mathbf{z}_{21}}{\Delta_z} = \text{-}0.2\ \text{S} \qquad \mathbf{y}_{22} = \frac{\mathbf{z}_{11}}{\Delta_z} = 0.3\ \text{S}$$

$$\mathbf{A} = \frac{\mathbf{z}_{11}}{\mathbf{z}_{21}} = \frac{6}{4} = 1.5 \qquad \mathbf{B} = \frac{\Delta_z}{\mathbf{z}_{21}} = \frac{20}{4} = 5\,\Omega$$

$$\mathbf{C} = \frac{1}{\mathbf{z}_{21}} = \frac{1}{4} = 0.25\ \text{S} \qquad \mathbf{D} = \frac{\mathbf{z}_{22}}{\mathbf{z}_{21}} = \frac{6}{4} = 1.5$$

Therefore,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{0.3} & \mathbf{-0.2} \\ \mathbf{-0.2} & \mathbf{0.3} \end{bmatrix} \boldsymbol{S} \qquad [\mathbf{T}] = \begin{bmatrix} \mathbf{1.5} & \mathbf{5\,\Omega} \\ \mathbf{0.25\ S} & \mathbf{1.5} \end{bmatrix}$$