

**P.P.19.11**

$$\mathbf{I}_1 = \frac{\mathbf{V}_1 - 0}{\mathbf{R}_1} \longrightarrow \mathbf{V}_1 = \mathbf{I}_1 \mathbf{R}_1$$

Also,  $\mathbf{I}_1 = \frac{0 - \mathbf{V}_2}{\mathbf{R}_2} \longrightarrow \mathbf{V}_2 = -\mathbf{I}_1 \mathbf{R}_2$

Comparing these with

$$\mathbf{V}_1 = \mathbf{z}_{11} \mathbf{I}_1 + \mathbf{z}_{12} \mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{21} \mathbf{I}_1 + \mathbf{z}_{22} \mathbf{I}_2$$

shows that

$$\mathbf{z}_{11} = \mathbf{R}_1, \quad \mathbf{z}_{21} = -\mathbf{R}_2, \quad \mathbf{z}_{12} = \mathbf{z}_{21} = 0$$

Hence,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{R}_1 & \mathbf{0} \\ -\mathbf{R}_2 & \mathbf{0} \end{bmatrix}$$

Since  $\Delta_z = \mathbf{z}_{11} \mathbf{z}_{22} - \mathbf{z}_{12} \mathbf{z}_{21} = 0$ , **[z]<sup>-1</sup> does not exist**. Consequently, **[y] does not exist**.

**P.P.19.12** This is a series connection of two two-ports.

$$\text{For } N_a, \quad \mathbf{z}_{12a} = \mathbf{z}_{21a} = 20, \quad \mathbf{z}_{11a} = 20 - j15, \quad \mathbf{z}_{22a} = 20 + j10$$

$$\text{For } N_b, \quad \mathbf{z}_{12b} = \mathbf{z}_{21b} = 50, \quad \mathbf{z}_{11b} = 50 + j40, \quad \mathbf{z}_{22b} = 50 - j20$$

Thus,  $[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b]$

$$[\mathbf{z}] = \begin{bmatrix} 20 - j15 & 20 \\ 20 & 20 + j10 \end{bmatrix} + \begin{bmatrix} 50 + j40 & 50 \\ 50 & 50 - j20 \end{bmatrix}$$

$$[\mathbf{z}] = \begin{bmatrix} 70 + j25 & 70 \\ 70 & 70 - j10 \end{bmatrix}$$

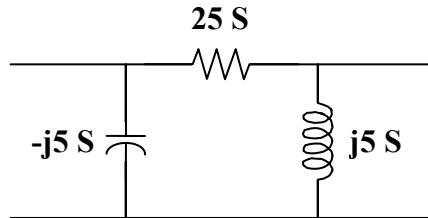
$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{z}_{12} \mathbf{Z}_L}{(\mathbf{z}_{11} + \mathbf{Z}_s)(\mathbf{z}_{22} + \mathbf{Z}_L) - \mathbf{z}_{12} \mathbf{z}_{21}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{(70)(40)}{(70 + j25 + 5)(70 - j10 + 40) - 4900}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{2800}{8250 - j750 + j2750 + 250 - 4900}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{2800}{3600 + j2000} = 0.6799 \angle -29.05^\circ$$

**P.P.19.13** We convert the upper T network  $N_a$  to a  $\Pi$  network, as shown below.



$$y_a = \frac{y_1 y_2 + y_2 y_3 + y_3 y_1}{y_2} = \frac{(-j5)(j5) + (j5)(1) + (1)(-j5)}{j5} = -j5$$

$$y_b = 5, \quad y_c = 25$$

For  $N_a$ ,

$$y_{12a} = -25 = y_{21a}, \quad y_{11a} = 25 - j5, \quad y_{22a} = 25 + j5$$

$$[y_a] = \begin{bmatrix} 25 - j5 & -25 \\ -25 & 25 + j5 \end{bmatrix}$$

For  $N_b$ ,

$$y_{12b} = j10 = y_{21b}, \quad y_{11b} = 2 - j10 = y_{22b}$$

$$[y_b] = \begin{bmatrix} 2 - j10 & j10 \\ j10 & 2 - j10 \end{bmatrix}$$

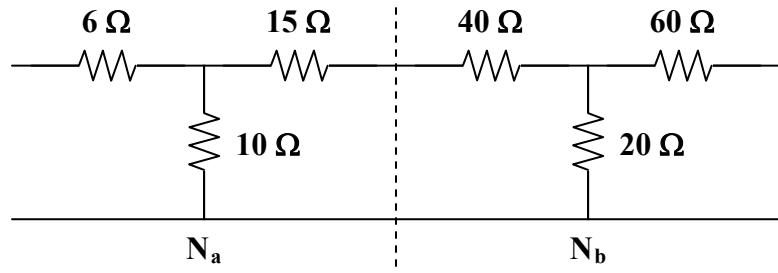
Since  $N_a$  and  $N_b$  are in parallel,  $[y] = [y_a] + [y_b]$

$$[y] = \begin{bmatrix} 27 - j15 & -25 + j10 \\ -25 + j10 & 27 - j5 \end{bmatrix} S$$

**P.P.19.14** Convert the left  $\Pi$  network to a T network.

$$R_1 = \frac{(20)(30)}{20 + 30 + 50} = 6, \quad R_2 = \frac{(20)(50)}{100} = 10, \quad R_3 = \frac{(30)(50)}{100} = 15$$

Putting this network into the given network produces the network shown below. This may be regarded as a cascaded connection of T two-port networks.



For  $N_a$ ,

$$A_a = 1 + \frac{6}{10} = 1.6, \quad B_a = 15 + \left(\frac{6}{10}\right)(25) = 30$$

$$C_a = \frac{1}{10} = 0.1, \quad D_a = 1 + \frac{15}{10} = 2.5$$

$$[T_a] = \begin{bmatrix} 1.6 & 30 \\ 0.1 & 2.5 \end{bmatrix}$$

For  $N_b$ ,

$$A_b = 1 + \frac{40}{20} = 3, \quad B_b = 60 + \left(\frac{40}{20}\right)(80) = 220$$

$$C_b = \frac{1}{20} = 0.05, \quad D_b = 1 + \frac{60}{20} = 4$$

$$[T_b] = \begin{bmatrix} 3 & 220 \\ 0.05 & 4 \end{bmatrix}$$

Hence,

$$[T] = [T_a][T_b] = \begin{bmatrix} 1.6 & 30 \\ 0.1 & 2.5 \end{bmatrix} \begin{bmatrix} 3 & 220 \\ 0.05 & 4 \end{bmatrix}$$

We can now use MATLAB to obtain  $T$ .

```
>> Ta=[1.6,30;0.1,2.5]
```

```
Ta =
```

```
1.6000 30.0000
0.1000 2.5000
```

```
>> Tb=[3,220;0.05,4]
```

```
Tb =
```

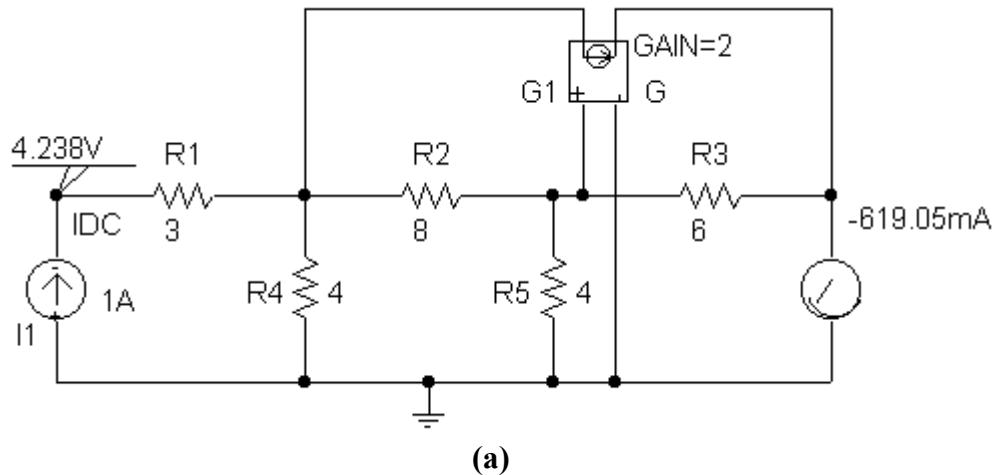
```
3.0000 220.0000
```

```

0.0500 4.0000
>> T=Ta*Tb
T =
6.3000 472.0000
0.4250 32.0000
[T]=
[ 6.3      472 Ω
 0.425 S    32 ]

```

**P.P.19.15** To obtain  $\mathbf{h}_{11}$  and  $\mathbf{h}_{21}$ , simulate the schematic in Fig. (a) using PSpice.

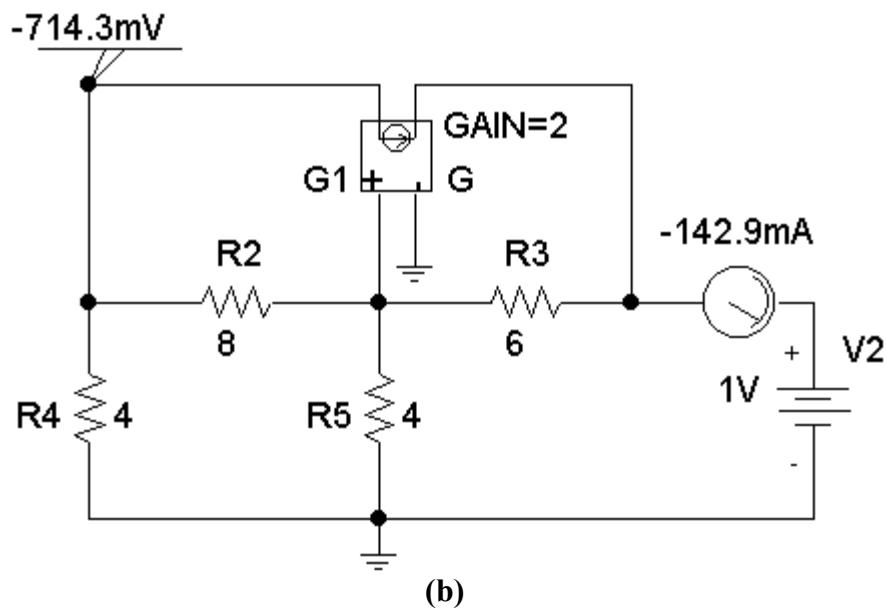


Insert a 1-A dc current source to account for  $\mathbf{I}_1 = 1 \text{ A}$ . Also, include pseudocomponents VIEWPOINT and IPROBE to display  $\mathbf{V}_1$  and  $\mathbf{I}_2$  respectively. When the circuit is saved and run, the values of  $\mathbf{V}_1$  and  $\mathbf{I}_2$  are displayed on the pseudocomponents as shown in Fig. (a). Thus,

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{1} = 4.238 \Omega, \quad \mathbf{h}_{21} = \frac{\mathbf{I}_2}{1} = -0.6190$$

To obtain  $\mathbf{h}_{12}$  and  $\mathbf{h}_{22}$ , insert a 1-V dc voltage source at the output port to account for  $\mathbf{V}_2 = 1 \text{ V}$ . The pseudocomponents VIEWPOINT and IPROBE are included to display  $\mathbf{V}_1$  and  $\mathbf{I}_2$  respectively. After simulation, the schematic displays the results as shown in Fig. (b).

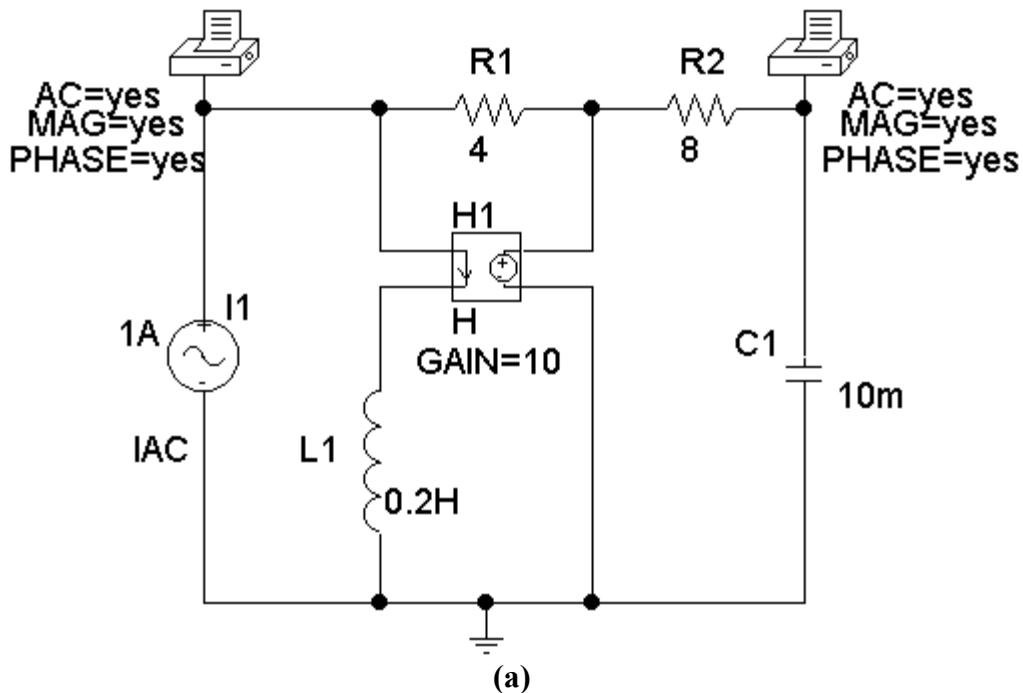
$$\mathbf{h}_{12} = \frac{\mathbf{V}_1}{1} = -0.7143, \quad \mathbf{h}_{22} = \frac{\mathbf{I}_2}{1} = -0.1429 \text{ S}$$



Thus,

$$[h] = \begin{bmatrix} 4.238 \Omega & -0.7143 \\ -0.6190 & -0.1429 S \end{bmatrix}$$

**P.P.19.16** Insert a 1-A ac current source at the output terminals to account for  $I_1 = 1$  A. Include two VPRINT1 pseudocomponents to output  $V_1$  and  $V_2$ . For each VPRINT1, set the attributes to AC = yes, PHASE = yes, and MAG = yes. In the AC Sweep and Noise Analysis dialog box, set Total pt : 1, Start Freq : 60, and End Freq : 60. The schematic is shown in Fig. (a).



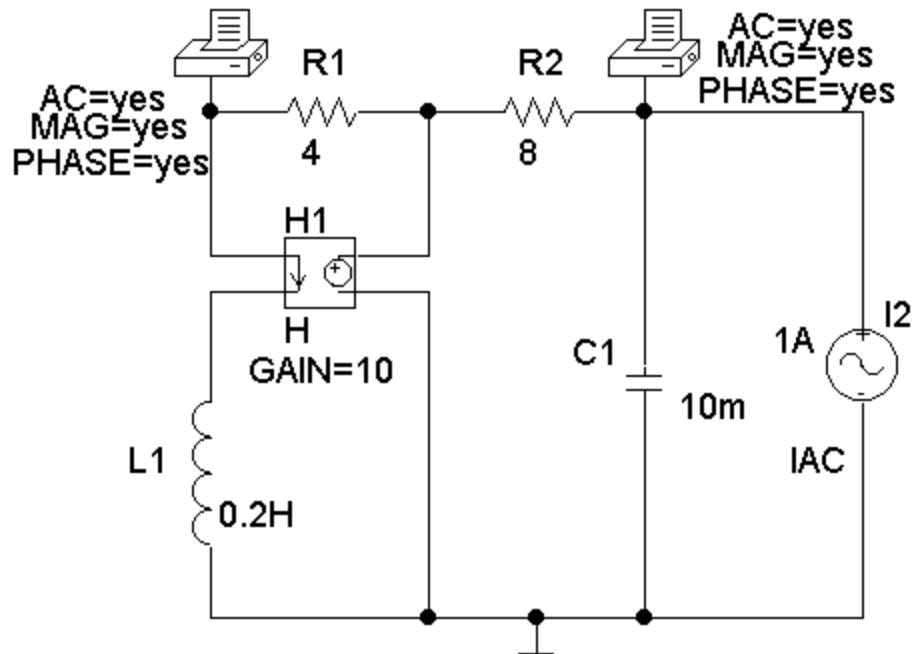
Once the schematic is saved and run, the output results include :

FREQ	VM(\$N_0002)	VP(\$N_0002)
6.000E+01	3.987E+00	1.755E+02
FREQ	VM(\$N_0003)	VP(\$N_0003)
6.000E+01	1.752E-02	-2.651E+00

From this table,

$$Z_{11} = \frac{V_1}{I_1} = 3.987 \angle 175.5^\circ, \quad Z_{21} = 0.0175 \angle -2.65^\circ$$

Similarly, insert a 1-A ac source at the output port with the two pseudocomponents in place as in Fig. (a). The result is the schematic in Fig. (b).



(b)

When the schematic is saved and run, the output results include :

FREQ	VM(\$N_0002)	VP(\$N_0002)
6.000E+01	1.000E-30	0.000E+00
FREQ	VM(\$N_0003)	VP(\$N_0003)
6.000E+01	2.651E-01	9.190E+01

From this table,

$$\mathbf{Z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_{12}} \approx 0 \quad \mathbf{Z}_{22} = 0.265 \angle 91.9^\circ$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} 3.987 \angle 175.5^\circ & 0 \\ 0.0175 \angle -2.65^\circ & 0.2651 \angle 91.9^\circ \end{bmatrix} \Omega$$

**P.P.19.17** In this case,  $R_s = 150 \text{ k}\Omega$ ,  $R_L = 3.75 \text{ k}\Omega$ .

$$h_{ie} h_{oe} - h_{re} h_{fe} = (6 \times 10^3)(8 \times 10^{-6}) - (1.5 \times 10^{-4})(200) = 18 \times 10^{-3}$$

The gain for the transistor is given as,

$$A_v = \frac{-(200)(3750)}{6000 + (18 \times 10^{-3})(3.75 \times 10^3)} = V_o / V_b = -123.61$$

To calculate the gain of the circuit we need to use,

$$-V_s + 150kI_b + V_b = 0 \text{ or } 0.002 = 150k(0.002/156k) - V_c/123.61$$

$$V_c = -9.506 \text{ mV} \text{ which leads to the gain} = -9.506/2 = -4.753$$

$$A_i = \frac{200}{1 + (8 \times 10^{-6})(3.75 \times 10^3)} = 194.17$$

The input resistance for the transistor is equal to  $h_{ie} = 6 \text{ k}\Omega$ .

The input resistance for the circuit is equal to,

$$Z_{in} = 150,000 + 6000 - (1.5 \times 10^{-4})(194.17) \approx 156 \text{ k}\Omega$$

$$\begin{aligned} Z_{out} &= \frac{150 \times 10^3 + 6 \times 10^3}{(150 \times 10^3)(8 \times 10^{-6}) - (1.5 \times 10^{-4})(200)} \\ &= \frac{156}{1.248 - 0.03} \text{ k}\Omega = 128.08 \text{ k}\Omega \end{aligned}$$

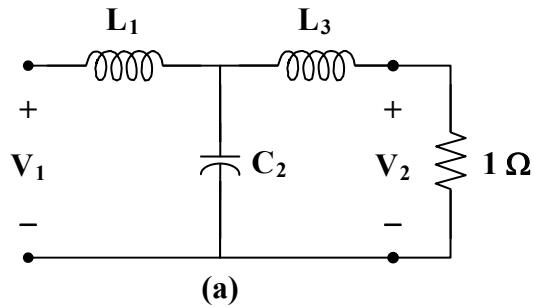
**P.P.19.18** Let  $D(s) = (s^3 + 4s) + (s^2 + 2)$

Dividing both numerator and denominator by  $s^3 + 4s$  gives

$$H(s) = \frac{\frac{2}{s^3 + 4s}}{1 + \frac{s^2 + 2}{s^3 + 4s}}$$

i.e.  $y_{21} = \frac{-2}{s^3 + 4s}$        $y_{22} = \frac{s^2 + 2}{s^3 + 4s}$

As a third order function, we can realize  $H(s)$  by the **LC network shown in Fig. (a)**.

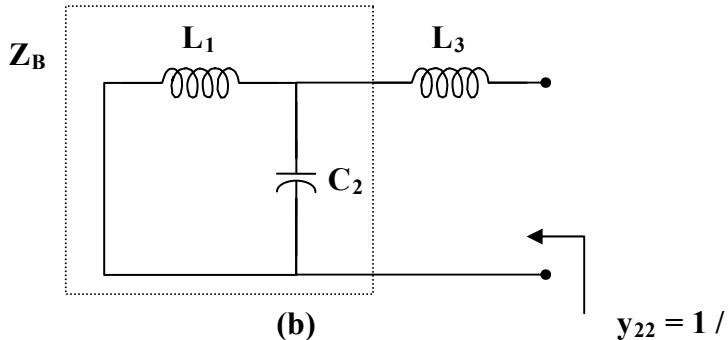


(a)

$$Z_A = \frac{1}{Y_{22}} = \frac{s^3 + 4s}{s^2 + 2} = s + \frac{2s}{s^2 + 2} = sL_3 + Z_B$$

$$L_3 = 1 \text{ H}$$

$$Z_B = \frac{2s}{s^2 + 2}$$



(b)

$$y_{22} = 1 /$$

$$Y_B = \frac{1}{Z_B} = \frac{s^2 + 2}{2s} = 0.5s + \frac{1}{s} = sC_2 + \frac{1}{Y_C}$$

$$C_2 = 0.5 \text{ F}$$

$$Y_C = \frac{1}{sL_1} = \frac{1}{s} \longrightarrow L_1 = 1 \text{ H}$$

Hence,

$$L_1 = 1 \text{ H}, C_2 = 500 \text{ mF}, L_3 = 1 \text{ H}.$$