Differential Equations for Engineers and Scientists

By

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ANSWERS TO SELECTED PROBLEMS IN CHAPTER ONE

(Answers to Section Review problems are in the textbook)

CHAPTER 1

1-33 $z'' = -g - \frac{F_{air}}{m}$ with $z(0) = 0$ and $z'(0) = V(0) = V_i$

1-35 $\frac{dT(t)}{dt} = \frac{hA}{mc}(T - T_0)$ with $T(0) = T_i$

1-37 $\frac{dM}{dt} = -kM, k > 0$

1-41C The slope at the given point is 1.

1-43C This can be possible if $(\frac{\partial f}{\partial x})_y = 0$ or $\frac{dx}{dy} = 0$, or both are zero.

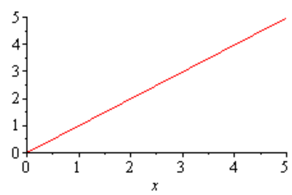
1-45C High pressure lines are steeper than the low pressure lines.

1-47 **(a)** $f(x) = x^2 - 1$ is a continuous function on $(-\infty, +\infty)$

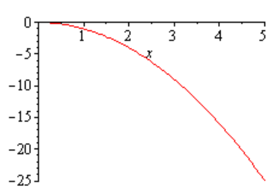
(b) $f(x) = \sqrt{x}$ is defined on $[0, +\infty)$ and continuous in that interval
(c) $f(x) = \frac{x}{\sin 2x}$ is continuous for all x except for $x = 0$
(d) $(x) = \frac{e^{2x}}{x(x-1)}$ is continuous on $\Re - \{0, 1\}$, where $\Re$ denotes set of reel numbers

1-49 $\frac{dT}{dP} = \frac{1}{R}[(1 + a/v^2)(v - b)]$ =constant

1-51 **(a)** $f(x) = x$ satisfies the given condition



(b) $f(x) = -x^2$ satisfies the given condition



(c) No elementary function can satisfy the given condition.

1-53 **(a) Given:** $f_1 = 7x^4 - \sin 3x^3 + 2e^{-3x}$
Solution: $\frac{\partial f_1}{\partial x} = \frac{df_1}{dx} = 28x^3 - 9x^2\cos 3x^3 - 6e^{-3x}$

(b) Given: $f_2 = 7x^4 - \sin 3x^3 t + t^2 e^{-3x}$

Solution: $\frac{\partial f_2}{\partial x} = 28x^3 - 9x^2 t\cos 3x^3 - 3t^2 e^{-3x}$

(c) Given: $f_3 = 7t^4 - \sin 3t^3 x + t^2 e^{-3t}$
Solution: $\frac{\partial f_3}{\partial x} = 28x^3 - \sin 3t^3$

1-55 **(a) Given:** $f = \ln(x^2 + 1)$
Solution: $\frac{df}{dx} = \frac{2x}{x^2+1}$

(b) Given: $f = x^4\cos(2x)$
Solution: $\frac{df}{dx} = 4x^3\cos(2x) - 2x^4\sin(2x)$

c) Given: $f = \frac{5x}{2x^3\sin(x)} = \frac{5}{2x^2\sin(x)}$
Solution: $\frac{df}{dx} = -\frac{5}{x^3\sin(x)} - \frac{5\cos(x)}{2x^2\sin^2(x)} == -\frac{5}{2}\,\frac{2\sin(x)+x\cos(x)}{x^3\sin^2(x)}$

d) Given: $f = \ln(e^{2x})$
Solution: $\frac{df}{dx} = 2$

1-57 **(a) Given:** $f(x) = x^{2t} + \sin(2\omega t) + 3t^2 x$
Solution: $I = \frac{1}{2t+1}\left[-1 + 2\sin(2\omega t) + 24t^3 + 12t^2 + 4t\sin(2\omega t) + 3^{2t+1}\right]$

(b) Given: $f = y''(x) + 3e^{-2tx} + \cosh(2\omega x)$
Solution: $I = y' - \frac{3e^{-2tx}}{2t} + \frac{\sinh(2\omega x)}{\omega} + C$

1-59 **(a) Given:** $f(x) = 3x^4 + xe^{2x} + \cosh(3x)$
Solution: $I = \frac{3}{5}x^5 + \frac{e^{2x}}{4}(2x-1) + \frac{\sinh(3x)}{3} + C$

(b) Given: $f(x) = \frac{a}{x} + 4\sin(3x)\cos(3x) - \sinh(2x)$
Solution: $I = a\ln(2) + \frac{1}{3}[\cos(12) - \cos(24)] - \frac{1}{2}[\cos h(4) - \cosh(8)]$

(c) Given: $f = y''(x) + t^3\sin(2\omega x) + e^{-2tx}$
Solution: $I = \left[y'(8) - y'(x)\right] + \frac{t^3}{2\omega}[\cos(16\omega) - \cos(2\omega x)] + \frac{1}{2t}\left[e^{-16t} - e^{-2tx}\right]$

(d) Given: $f = 4y(x)y'(x) + xy''(x) + \frac{be^{-3t}}{x^2}$
Solution: $I = 2y'(x)^2 + xy'(x) - y(x) - \frac{be^{-3t}}{x} + C$

1-63 **(a)** $y'' + 2y' = \sin(x) + 1$ (Linear, constant coefficient)
(b) $y''' + \sin(x)\,e^{-2x}y' = 0$ (Linear, variable coefficient)
(c) $y''' + \sin(2x)\,y'' + x^4 y = 0$ (Linear, variable coefficient)
(d) $y'' + 3y' - y = \frac{\sin(3x)}{x}$ (Linear, constant coefficient)

(e) $y'' + e^{2x}e^{-y} = 0$ (Nonlinear, variable coefficient)

1-69 y_1 and y_2 are the solutions of the differential equation.

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1-73 y_1, y_2 and y_3 are the solutions of the differential equation.

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1-77 $z'' = 0$ with $z'(0) = -V_0$ and $z(t_0) = 0$ (Upward direction is positive)

1-79 **(a)** $y = C_1x + C_2$, where C_1 and C_2 are arbitrary constants
(b) $y'' + 4ye^{-3x} = 0$ cannot be solved by direct integration
(c) $y = \frac{e^{-4x}}{8}(2x + 1) + C_1x + C_2$, where C_1 and C_2 are arbitrary constants.
(d) $y'' - xy = 0$ cannot be solved by direct integration
(e) The unknown function $y(x)$ cannot be found in terms of elementary functions.

1-81 **(a)** $y = \frac{ax^2}{2} + C$, where C is an arbitrary constant.
(b) $y''' + 4y\sinh(2x) = 0$ cannot be solved by direct
(c) $y = \frac{bx^2}{2}\ln(ax) - \frac{3bx^2}{4} + C_1x + C_2$, where C_1 and C_2 are arbitrary constants
(d) $y' - e^y\cos(x) = 0$ cannot be solved by direct integration
(e) The unknown function $y(x)$ cannot be found in terms of elementary functions.

1-83C The ginput function gives the result $x = 0.5379$ rad.

1-87 The result is 0.4304077247.

1-89

(a) The answer is $\frac{1}{2t+1}[-1 + 4\sin\omega t\cos\omega t + 24t^3 + 12t^2 + 8t\sin\omega t\cos\omega t + 3(9^t)]$ if $t \neq -1/2$, and $\ln 3 - 2\sin\omega + 3$ If $t = -1/2$.

(b) The answer is $y' - \frac{3e^{-2tx}}{2t} + \frac{\sinh 2\omega x}{\omega} + C$

1-91

(a) The answer is $\frac{3}{5}x^5 + \frac{e^{2x}}{4}(2x-1) + \frac{e^{3x}}{6} - \frac{1}{6e^{3x}}$ or $\frac{3}{5}x^5 + \frac{e^{2x}}{4}(2x-1) + \frac{\sinh(3x)}{3}$

(b) The answer is $a\ln(2) + \frac{1}{3}[\cos(12) - \cos(24)] - \frac{1}{2}[\cosh 4 - \cosh 8]$

Another form returned is $a\ln(2) + \frac{1}{3}[\cos(12) - \cos(24)] + (\cosh 2)^2 - (\cosh 4)^2$

(c) The answer is $y'(8) - y'(x) - \frac{t^3}{2\omega}[\cos(16\omega) - \cos(2\omega x)] - \frac{1}{2t}[e^{-16t} - e^{-2tx}]$

(d) The answer is $2y'(x)^2 + xy'(x) - y(x) - \frac{be^{-3t}}{x}$ or $\frac{[4y(x)-1]^2}{8} + xy'(x) - y(x) - \frac{be^{-3t}}{x}$

1-93

(a) $y = C_1x^2 + C_2x + C_3$

(b) $C_1e^{\sqrt[3]{5}x} + C_2e^{-\sqrt[3]{5}x/2}\cos(\sqrt[3]{5}x/2) + C_3e^{-\sqrt[3]{5}x/2}\sin(\sqrt[3]{5}x/2)$

(c) $y = \frac{5}{24}x^4 + C_1x^2 + C_2x + C_3$

(d) $y'' = \pm\sqrt{C_1 - 4e^{-2x}}$,

1-95 **(a)** $m_1 = i$ and $m_1 = -i$.
(b) $m_{1,2} = -1$.
(c) $m_1 = 1$ and $m_1 = -1$.

1-97 **(a)** $m_1 = -1$ and $m_1 = -4$.
(b) $m_{1,2} = -3$.
(c) $m_{1,2} = -\frac{1}{2}(1 \pm i\sqrt{11})$.

1-99 **(a)** $m_{1,2} = -5$.
(b) $m_{1,2} = -\frac{5}{2}(1 \pm i\sqrt{3})$.
(c) $m_{1,2} = -5(1 \pm i\sqrt{2})$

1-101 **(a)** $r_{1,2} = \frac{1}{2}(1 \pm i\sqrt{3})$.
(b) $r_{1,2} = \pm\sqrt{2}$.

1-103 **(a)** $r_{1,2} = 2 \pm i\sqrt{2}$.
(b) $r_{1,2} = -2$.

1-105 $V_0 = 2.5$ m/s.

1-107 $T(r) = T(R) + \frac{g_0}{4k}R^2 - \frac{g_0}{4k}r^2 \rightarrow T(r) = T(R) + \frac{g_0}{4k}(R^2 - r^2)$ and $T(0) = 210$ °C

1-109 $T(x) = T(L) + \frac{g_0}{pk}(x - L) + \frac{g_0}{p^2k}(e^{pL} - e^{px})$ and $T(0) = 323$ °C.

1-111 **(a)** $C(t) = \frac{1}{\sqrt{y(t)}} = \frac{C(0)}{\sqrt{1+2kC(0)t}}$ and **(b)** $C(t) = \frac{C(0)}{\sqrt{1+2kC(0)t}}$