Differential Equations for Engineers and Scientists

by

Y. Cengel and W. Palm III

ANSWERS TO SELECTED PROBLEMS

(*Answers to Section Review problems are in the textbook*)

CHAPTER 1

1-33 $z'' = -g - \frac{F_{air}}{m}$ with $z(0) = 0$ and $z'(0) = V(0) = V_i$

1-35 $\frac{dT(t)}{dt} = \frac{hA}{mc}(T - T_0)$ with $T(0) = T_i$

1-37 $\frac{dM}{dt} = -kM, k > 0$

1-41C The slope at the given point is 1.

1-43C This can be possible if $(\frac{\partial f}{\partial x})_y = 0$ or $\frac{dx}{dy} = 0$, or both are zero.

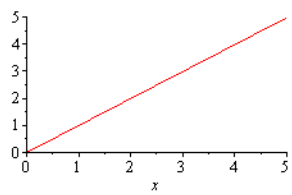
1-45C High pressure lines are steeper than the low pressure lines.

1-47 **(a)** $f(x) = x^2 - 1$ is a continuous function on $(-\infty, +\infty)$

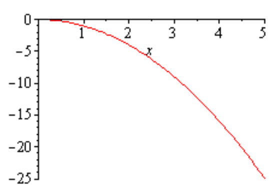
(b) $f(x) = \sqrt{x}$ is defined on $[0, +\infty)$ and continuous in that interval
(c) $f(x) = \frac{x}{\sin 2x}$ is continuous for all x except for $x = 0$
(d) $(x) = \frac{e^{2x}}{x(x-1)}$ is continuous on $\Re - \{0, 1\}$, where $\Re$ denotes set of reel numbers

1-49 $\frac{dT}{dP} = \frac{1}{R}[(1 + a/v^2)(v - b)]$ =constant

1-51 **(a)** $f(x) = x$ satisfies the given condition



(b) $f(x) = -x^2$ satisfies the given condition



(c) No elementary function can satisfy the given condition.

1-53 **(a) Given:** $f_1 = 7x^4 - \sin 3x^3 + 2e^{-3x}$
Solution: $\frac{\partial f_1}{\partial x} = \frac{df_1}{dx} = 28x^3 - 9x^2\cos 3x^3 - 6e^{-3x}$

(b) Given: $f_2 = 7x^4 - \sin 3x^3 t + t^2 e^{-3x}$

Solution: $\frac{\partial f_2}{\partial x} = 28x^3 - 9x^2 t\cos 3x^3 - 3t^2 e^{-3x}$

(c) Given: $f_3 = 7t^4 - \sin 3t^3 x + t^2 e^{-3t}$
Solution: $\frac{\partial f_3}{\partial x} = 28x^3 - \sin 3t^3$

1-55 **(a) Given:** $f = \ln(x^2 + 1)$
Solution: $\frac{df}{dx} = \frac{2x}{x^2+1}$

(b) Given: $f = x^4 \cos(2x)$
Solution: $\frac{df}{dx} = 4x^3 \cos(2x) - 2x^4 \sin(2x)$

c) Given: $f = \frac{5x}{2x^3 \sin(x)} = \frac{5}{2x^2 \sin(x)}$
Solution: $\frac{df}{dx} = -\frac{5}{x^3 \sin(x)} - \frac{5\cos(x)}{2x^2 \sin^2(x)} == -\frac{5}{2}\,\frac{2\sin(x) + x\cos(x)}{x^3 \sin^2(x)}$

d) Given: $f = \ln(e^{2x})$
Solution: $\frac{df}{dx} = 2$

1-57 **(a) Given:** $f(x) = x^{2t} + \sin(2\omega t) + 3t^2 x$
Solution: $I = \frac{1}{2t+1}\left[-1 + 2\sin(2\omega t) + 24t^3 + 12t^2 + 4t\sin(2\omega t) + 3^{2t+1}\right]$

(b) Given: $f = y''(x) + 3e^{-2tx} + \cosh(2\omega x)$
Solution: $I = y' - \frac{3e^{-2tx}}{2t} + \frac{\sinh(2\omega x)}{\omega} + C$

1-59 **(a) Given:** $f(x) = 3x^4 + xe^{2x} + \cosh(3x)$
Solution: $I = \frac{3}{5}x^5 + \frac{e^{2x}}{4}(2x - 1) + \frac{\sinh(3x)}{3} + C$

(b) Given: $f(x) = \frac{a}{x} + 4\sin(3x)\cos(3x) - \sinh(2x)$
Solution: $I = a\ln(2) + \frac{1}{3}[\cos(12) - \cos(24)] - \frac{1}{2}[\cos h(4) - \cosh(8)]$

(c) Given: $f = y''(x) + t^3 \sin(2\omega x) + e^{-2tx}$
Solution: $I = \left[y'(8) - y'(x)\right] + \frac{t^3}{2\omega}[\cos(16\omega) - \cos(2\omega x)] + \frac{1}{2t}\left[e^{-16t} - e^{-2tx}\right]$

(d) Given: $f = 4y(x)y'(x) + xy''(x) + \frac{be^{-3t}}{x^2}$
Solution: $I = 2y'(x)^2 + xy'(x) - y(x) - \frac{be^{-3t}}{x} + C$

1-63 **(a)** $y'' + 2y' = \sin(x) + 1$ (Linear, constant coefficient)
(b) $y''' + \sin(x)\, e^{-2x} y' = 0$ (Linear, variable coefficient)
(c) $y''' + \sin(2x)\, y'' + x^4 y = 0$ (Linear, variable coefficient)
(d) $y'' + 3y' - y = \frac{\sin(3x)}{x}$ (Linear, constant coefficient)

(e) $y'' + e^{2x}e^{-y} = 0$ (Nonlinear, variable coefficient)

1-69 y_1 and y_2 are the solutions of the differential equation.

1-71 y_1 and y_2 are the solutions of the differential equation.

1-73 y_1, y_2 and y_3 are the solutions of the differential equation.

1-75 y_1, y_2 and y_3 are the solutions of the differential equation.

1-77 $z'' = 0$ with $z'(0) = -V_0$ and $z(t_0) = 0$ (Upward direction is positive)

1-79 **(a)** $y = C_1x + C_2$, where C_1 and C_2 are arbitrary constants
(b) $y'' + 4ye^{-3x} = 0$ cannot be solved by direct integration
(c) $y = \frac{e^{-4x}}{8}(2x + 1) + C_1x + C_2$, where C_1 and C_2 are arbitrary constants.
(d) $y'' - xy = 0$ cannot be solved by direct integration
(e) The unknown function $y(x)$ cannot be found in terms of elementary functions.

1-81 **(a)** $y = \frac{ax^2}{2} + C$, where C is an arbitrary constant.
(b) $y''' + 4y\sinh(2x) = 0$ cannot be solved by direct
(c) $y = \frac{bx^2}{2}\ln(ax) - \frac{3bx^2}{4} + C_1x + C_2$, where C_1 and C_2 are arbitrary constants
(d) $y' - e^y\cos(x) = 0$ cannot be solved by direct integration
(e) The unknown function $y(x)$ cannot be found in terms of elementary functions.

1-83C The ginput function gives the result $x = 0.5379$ rad.

1-87 The result is 0.4304077247.

1-89

(a) The answer is $\frac{1}{2t+1}[-1 + 4\sin\omega t\cos\omega t + 24t^3 + 12t^2 + 8t\sin\omega t\cos\omega t + 3(9^t)]$ if $t \neq -1/2$, and $\ln 3 - 2\sin\omega + 3$ If $t = -1/2$.

(b) The answer is $y' - \frac{3e^{-2tx}}{2t} + \frac{\sinh 2\omega x}{\omega} + C$

1-91

(a) The answer is $\frac{3}{5}x^5 + \frac{e^{2x}}{4}(2x-1) + \frac{e^{3x}}{6} - \frac{1}{6e^{3x}}$ or $\frac{3}{5}x^5 + \frac{e^{2x}}{4}(2x-1) + \frac{\sinh(3x)}{3}$

(b) The answer is $a\ln(2) + \frac{1}{3}[\cos(12) - \cos(24)] - \frac{1}{2}[\cosh 4 - \cosh 8]$

Another form returned is $a\ln(2) + \frac{1}{3}[\cos(12) - \cos(24)] + (\cosh 2)^2 - (\cosh 4)^2$

(c) The answer is $y'(8) - y'(x) - \frac{t^3}{2\omega}[\cos(16\omega) - \cos(2\omega x)] - \frac{1}{2t}[e^{-16t} - e^{-2tx}]$

(d) The answer is $2y'(x)^2 + xy'(x) - y(x) - \frac{be^{-3t}}{x}$ or $\frac{[4y(x)-1]^2}{8} + xy'(x) - y(x) - \frac{be^{-3t}}{x}$

1-93

(a) $y = C_1x^2 + C_2x + C_3$

(b) $C_1e^{\sqrt[3]{5}x} + C_2e^{-\sqrt[3]{5}x/2}\cos(\sqrt[3]{5}x/2) + C_3e^{-\sqrt[3]{5}x/2}\sin(\sqrt[3]{5}x/2)$

(c) $y = \frac{5}{24}x^4 + C_1x^2 + C_2x + C_3$

(d) $y'' = \pm\sqrt{C_1 - 4e^{-2x}}$,

1-95 **(a)** $m_1 = i$ and $m_1 = -i$.
(b) $m_{1,2} = -1$.
(c) $m_1 = 1$ and $m_1 = -1$.

1-97 **(a)** $m_1 = -1$ and $m_1 = -4$.
(b) $m_{1,2} = -3$.
(c) $m_{1,2} = -\frac{1}{2}(1 \pm i\sqrt{11})$.

1-99 **(a)** $m_{1,2} = -5$.
(b) $m_{1,2} = -\frac{5}{2}(1 \pm i\sqrt{3})$.
(c) $m_{1,2} = -5(1 \pm i\sqrt{2})$

1-101 **(a)** $r_{1,2} = \frac{1}{2}(1 \pm i\sqrt{3})$.
(b) $r_{1,2} = \pm\sqrt{2}$.

1-103 **(a)** $r_{1,2} = 2 \pm i\sqrt{2}$.
(b) $r_{1,2} = -2$.

1-105 $V_0 = 2.5$ m/s.

1-107 $T(r) = T(R) + \frac{g_0}{4k}R^2 - \frac{g_0}{4k}r^2 \rightarrow T(r) = T(R) + \frac{g_0}{4k}(R^2 - r^2)$ and $T(0) = 210$ °C

1-109 $T(x) = T(L) + \frac{g_0}{pk}(x - L) + \frac{g_0}{p^2k}(e^{pL} - e^{px})$ and $T(0) = 323$ °C.

1-111 **(a)** $C(t) = \frac{1}{\sqrt{y(t)}} = \frac{C(0)}{\sqrt{1+2kC(0)t}}$ and **(b)** $C(t) = \frac{C(0)}{\sqrt{1+2kC(0)t}}$

CHAPTER 2

2-37 **(a)** $y' + e^x y = 2\sqrt{x} \rightarrow$ linear **(b)** $y'y^2 + \cos(y) = x \rightarrow$ nonlinear

2-39 **(a)** $yy' + xy = x \rightarrow$ nonlinear **(b)** $y'^2 - y^2 = x^2 \rightarrow$ nonlinear

2-41 **(a)** $y = \frac{1}{3}x^2 + C\sqrt{x}$, **(b)** $y = \left(1 - \frac{1}{x}\right)e^x + \frac{C}{x}$

2-43 **(a)** $y = \frac{x}{9}(3x - 4 + C_1 e^{-3x})$ where $C_1 = 9C$, **(b)** $y = 2(x\tanh(x) - 1 + C\mathrm{sech}(x))$

2-45
(a) $y = \frac{1}{20x^2}[-10x^2\cos(2x) + 5\cos(2x) + 10x\cos(2x) - 4x^5 + C_1]$ where $C_1 = 20C$.
(b) $y = \frac{e^{2x}}{4x^2}(2x - 1) + \frac{C}{x^2}$

2-47 **(a)** $y = -e^{-x} + 2e^{2x-2} + e^{2x-1}$, **(b)** $y = -x + 5\sqrt{\frac{x^2-1}{3}}$

2-49 **(a)** $y = \frac{1}{x}(4\ln(x) + 3)$, **(b)** $y = \frac{1}{30}\frac{5x^6-24x^5-32}{x-4}$

2-51 $x = 1 - e^{\frac{1-y}{y}}$

2-53 $Cy_1(x)$ is also a solution of $y' + P(x)y = 0$, no matter what value of C is.

2-55 $Cy_1(x)$ cannot be a solution of the given DE.

2-59 $t = 6$ hours

2-63 **(a)** At steady state, y = 24/3 = 8. **(b)** 5.04 is 63% of 8, so it will take one time constant, or 8 , to reach 5.04. **(c)** 7.84 is 98% of 8, so it will take four time constants, or 32 , to reach 7.84.

2-65 $T(2) = 13.3$ °C.

2-67 $t \approx 4.8$ min

2-69 $E/E_0 = 0.243$ or 24.3%. Therefore we conclude that the fraction of the light that will reach the bottom of the pond is $1 - 0.243 = 0.757 = 75.7\%$.

2-71 The amount of salt after 30 minutes will be 10 kg, and it will never drop to 1 kg.

2-73 $\frac{dV}{dt} + \frac{k}{m}V = g\left(1 - \frac{\rho_w}{\rho_b}\right)$, The solution is $V(t) = \frac{mg}{k}\left(1 - \frac{\rho_w}{\rho_b}\right)\left(1 - e^{-\frac{k}{m}t}\right)$, The terminal velocity is $V_t = \frac{mg}{k}\left(1 - \frac{\rho_w}{\rho_b}\right)$

2-75 $A(8 \text{ years}) = \$25{,}260.22$

2-77 $i = 13.86\%$ per year.

2-79

a) Theorem 2-2 guarantees both existence and uniqueness of a solution in a neighborhood of any x.

b) The given differential equation $y' = xy/(x^2 - 1)$ must have a unique solution near any point in the xy-plane where $x \neq -1$ or $x \neq 1$.

2-81

a) Theorem 2-2 guarantees nothing in some neighborhood of $x = 0$.

b) The Theorem 2-2 guarantees both existence and uniqueness in some neighborhood of $x = 1$.

2-83

a) Theorem 2-2 guarantees both existence and uniqueness in some neighborhood of $x = 0$.

b) Theorem 2-2 guarantees both existence and uniqueness in some neighborhood of $x = 1$.

2-87 **(a)** $y(x) = \pm\sin(2\sqrt{x} + C)$, **(b)** $y(x) = e^{ax - \frac{bx^2}{2} + C}$

2-89 **(a)** $\ln\left(\frac{1+\sin(y)}{\cos(y)}\right) = \sin(x) - x\cos(x) + C$, **(b)** $-(1+y)e^{-y} = e^{x+1}$

2-91 **(a)** $e^{3(y-x)}(3y - 3x - 1) = 18x + C$, where $C = 9C_1$.
(b) $\sqrt{x + 2y - 3} - \frac{1}{2}\ln\left(2\sqrt{x + 2y - 3} + 1\right) = x + C$

2-93 **(a)** $y(x) = \frac{\tan(x^2)}{2}$, **(b)** $y(x) = \tanh\left(\frac{x-1}{x}\right)$

2-95 **(a)** $y(x) = e^{x^3 - 8}$, **(b)** $\cos(y) + y\sin(y) = \sin(x) - x\cos(x)$

2-97 **(a)** $y(x) = e^{\frac{3x^4}{2} + C}$, **(b)** $y(x) = -\frac{e^{K - bx} - c}{b}$

2-99 $y(t) = \left(H^{5/2} - \frac{5}{2}\frac{r^2\sqrt{2g}}{(R/H)^2}t\right)^{2/5}$, The time required for the tank to be empty can be evaluated by setting $y(T) = 0$ which yields $T = \frac{1}{5}\sqrt{2H/g}\left(\frac{R}{r}\right)^2$

2-101 $\left(\frac{N}{N_0}\right)^A \left(\frac{a - bN}{a - bN_0}\right)^B \left(\frac{1 - cN_0}{1 - cN}\right)^C = e^{-t}$, where $A = \frac{1}{a}$, $B = \frac{b}{a^2c - ab}$, and $C = \frac{c}{ac - b}$
Equilibrium points are $N = \frac{a}{b}$, $N = \frac{1}{c}$ and $N = 0$.

2-103 **(a)** $v' = \frac{T - mg}{m} - \frac{0.027v^2}{m} = A - Bv^2$, $v = C\tanh BCt$, where $C = \sqrt{A/B}$
(b) $v(4) = 200.9 \quad \text{m/s}$

2-105 **(a)** $xy = K$, **(b)** $y(x - k) = C$

2-109 **(a)** $\frac{dy}{dx} = \frac{x^2 - y^2}{xy}$ is homogeneous, **(b)** $\frac{dy}{dx} = \frac{x^3 - 2xy^2}{x^2 + y}$ is **not** homogeneous .

2-111 **(a)** $\frac{dy}{dx} = \frac{x^4 - 3x^2y^2 + y^4}{xy^3 - 4y^4}$ is homogeneous, **(b)** $\frac{dy}{dx} = x^3 - y^3$ is **not** homogeneous.

2-113 **(a)** $y(x) = \frac{x^2}{x-C}$, **(b)** $\ln\left(K\sqrt{x^2+y^2}\right) = 2\arctan\left(\frac{y}{x}\right)$, where $K = 1/C$.

2-115 **(a)** $y^6 + 2x^3y^3 + x^6 = Ky^3$, where $K = C^3$

(b) $\frac{4\sqrt{5}}{5}\text{arctanh}\left(\frac{\sqrt{5}}{5}\left(2\frac{y}{x}+1\right)\right) = \ln\left(\frac{C}{y^2+xy-x^2}\right)$

2-117 **(a)** $y(x) = 2x\sin\left(\ln(C/x^{10})\right)$, **(b)** $y(x) = \frac{1}{2x}(x^2+K)$, where $K = C^2$.

2-119 **(a)** $y = x/\ln(y)$, **(b)** $y(x) = \frac{2x}{2-9x^{1/3}}$

2-121 **(a)** $y(x) = -\frac{2}{3} + x + C(x-1)^3$,

(b) $\ln\left(\sqrt{K(x^2+y^2-3(x+y-1)+xy)}\right) = \sqrt{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{y-1}{x-1}+1\right)\right)$

2-123 **(a)** $y = (x-4)\ln\left(C(x-4)\right) - 4$, **(b)** $y = x + Ce^{\left(\frac{x-1}{-x+y+1}\right)} - 1$

2-127 **(a)** $3(x^2+y^2) + 2(x-y) = K$, where $K = 2C$

(b) The differential equation is inexact

2-129 **(a)** $-e^y\cos(x) + 2x = C$, **(b)** The differential equation is inexact.

2-131 **(a)** $x^3 - y^3 + 3\left(\sin(y) - \cos(x)\right) = K$, **(b)** The differential equation is inexact.

2-133 **(a)** $x^2e^y + y = C$, **(b)** The differential equation is inexact.

2-135 $\frac{2x^3}{3} + y^4 - y^2 + x + y = 1$

2-137 $x^2 - y^2 + 3y(x+1) - x = 0$

2-139 $e^x(x^2-2x+2) - e^y(y^2-2y+2) + x + y = 6 - 10e^4$

2-147 $y = C + \frac{\sin 2x}{8} + \frac{x[2(\sin x)^2 - 1]}{4}$

2-149 Maple gives the answer: $y(x) = \mathrm{e}^{\frac{1}{3}\text{LambertW}\left(\frac{1}{3}\left(\mathrm{e}^{\_C1}\right)^3 x^3\right) - \_C1}$

2-151 The equation in the first printing is incorrect. It should be $y' = 2(x-y)^2$.The solution is $y(x) = x - \frac{\sqrt{2}}{2}\tanh\left(\sqrt{2}x\right)$

2-159 $y(x) = \begin{cases} Ce^{4x}, & x < 0 \\ -\frac{5}{2} + Ce^{4x}, & x \geq 0 \end{cases}$

2-161 $y(x) = \begin{cases} -x + 1 + Ce^{-x}, & x < 0 \\ x - 1 + Ce^{-x}, & x \geq 0 \end{cases}$

2-163 $t = 2.56$ secs.

2-165 **(a)** $z_{max} = 101.20$ m, **(b)** $t = 5.85$ secs. **(c)**$V(5.85) = -32.06$ m/s (downwards).

2-167 $P(t) = \frac{-b+eCe^{-Kt}}{-a+fCe^{-Kt}} = \frac{-b+eCe^{-k(ae-bf)t}}{-a+fCe^{-k(ae-bf)t}}$

2-169 $x(t) = \frac{ab\left(e^{k(a-b)t}-1\right)}{ae^{k(a-b)t}-b}$

Taking $a = 2b$ we end up with the $x(t) = \frac{2b\left(e^{kbt}-1\right)}{2e^{kbt}-1}$ whose limit is b as $t \to \infty$.

2-171 $x^2 + 3xy - 2x - 2y^2 - 3y = C$

2-173 $-\frac{1}{2}e^x\big(\cos(x) - \sin(x)\big) + y^2x - 3x - \frac{y^3}{3} = C$

2-175 $C_1(t) = K_2 - \frac{K_1}{2k}e^{-2kt}$

2-177 $y(x) = x + \ln\left(-\frac{e+1}{e^x - e^{x+1} - 2e}\right)$

2-179 $y(x) = \frac{1}{8}\left(1 - x\ln^2\left(\frac{K}{x^4}\right)\right)$

2-181 $y(x) = x\tan\big(\ln(x)\big)$

2-183 $\ln\left(\frac{Ky^2}{y^2 - x^2}\right) = \frac{x^2}{y^2 - x^2}$

2-185 $y(x) = \frac{2}{x + 2C}$

2-187 $y(x) = \frac{1}{2}e^x + C_1e^{-x} + C_2x + C_3$

2-189 $y(x) = \frac{1}{2}x^2 + 1$

2-191 $y(x) = \frac{1}{12}(x^4 + 6x^2 - 32x)$

2-193 $y(x) = -\frac{x^3}{6} - \frac{x^2}{2} + (x + 1)\ln(x + 1) - 1$

2-195 $y(x) = \ln\left(\frac{2/e}{2-e}e^{x + \ln(2-e)} - 1\right) - x - \ln(2 - e)$

2-197 $y(x) = 1.$

2-199 $y(x) = \sqrt[3]{\frac{1}{-1 + 2e^{-3x}}}$

2-201 $\frac{1}{y} = (1 + x)\left(\frac{C - \arcsin(x)}{\sqrt{1 - x^2}} - 1\right)$

2-203 $y(x) = \pm\sqrt{\frac{a}{-b + aCe^{-2ax}}}$

2-207 $y(x) = \frac{2}{2Ce^{2x} - 1}$

2-209 $y(x) = \frac{e^{2x}(5 + 7Ce^{7x})}{-2 + 7Ce^{7x}}$

CHAPTER 3

3-57 **(a)** $y'' - 5y' + \cos y = x + 1$; Nonlinear, nonhomogeneous, constant coefficients
(b) $y'' = 0$; Linear, homogeneous, constant coefficients
(c) $y'' + 2x^2y' + 5y = 0$; Linear, homogeneous, variable coefficients
(d) $y'' + e^x y = \frac{1}{x}$; Linear, nonhomogeneous, variable coefficients

3-59 **(a)** $y'' + \frac{1}{y} = 1$; Nonlinear, nonhomogeneous, constant coefficients
(b) $y'' + 8y' - e^{\ln y} = 0$; Noninear, homogeneous, constant coefficients
(c) $y'' - \sin 2xy' + y = 0$; Linear, homogeneous, variable coefficients
(d) $y'' + y = 7$; Linear, nonhomogeneous, constant coefficients

3-61 **(a)** The initial-value problem has a unique solution in the interval $-\infty < x < +\infty$.

(b) The initial-value problem has a unique solution in the interval $-\infty < x < 2$.

3-63 **(a)**The initial-value problem has a unique solution in the interval $-\infty < x < +\infty$.

(b) The initial-value problem has a unique solution in the interval $-2 < x < 2$.

3-65 **(a)** $y'' + 2y' + 10y = 0$, **(b)** $4x^2y'' + 4xy' + (4x^2 - 1)y = 0$

3-67 **(a)** $x^2y'' - 2xy' - 4y = 0$, **(b)** $y'' + 4y = 0$

3-73 **(a)** y_1 and y_2 are linearly dependent, **(b)** y_1 and y_2 are linearly dependent.

3-75 **(a)** y_1 and y_2 are linearly independent, **(b)** y_1 and y_2 are linearly independent.

3-77 **(a)** y_1 and y_2 are linearly independent, **(b)** y_1 and y_2 are linearly independent.

3-79 **(a)** y_1 and y_2 are linearly dependent, **(b)** y_1 and y_2 are linearly dependent.

3-81 y_1, y_2 and y_3 are linearly dependent.

3-83 y_1, y_2 and y_3 are linearly independent.

3-85 y_1, y_2 and y_3 are linearly independent.

3-87 y_1, y_2 and y_3 are linearly dependent.

3-89 y_1, y_2 and y_3 are linearly independent.

3-93 **(a)** ky_1 ***is*** also a solution, **(b)** ky_1 is ***not*** a solution

(c) ky_1 is ***not*** a solution, **(d)** ky_1 is ***not*** a solution

3-95 **(a)** ky_1 is ***not*** a solution, **(b)** ky_1 is ***not*** a solution

(c) ky_1 ***is*** also a solution, **(d)** ky_1 ***is*** also a solution

3-97 (a) $y_1 + y_2$ ***is*** also a solution, **(b)** $y_1 + y_2$ is ***not*** a solution

(c) $y_1 + y_2$ is ***not*** a solution, **(d)** $y_1 + y_2$ is ***not*** a solution

3-99 (a) $y_1 + y_2$ is ***not*** a solution, **(b)** $y_1 + y_2$ ***is*** also a solution

(c) $y_1 + y_2$ ***is*** also a solution, **(d)** $y_1 + y_2$ ***is*** also a solution

3-101 (a) The Wronskian of y_1 and y_2 is never zero for $x > 0$

(b) The Wronskian of y_1 and y_2 is zero

(c) The Wronskian of y_1 and y_2 is zero

3-103 (a) The Wronskian of y_1 and y_2 is never zero for $x > 0$

(b) The Wronskian of y_1 and y_2 is zero

(c) The Wronskian of y_1 and y_2 is zero

3-105 (a) $y(x) = \frac{C_1}{x} + C_2 \frac{\ln x}{x}$

(b) $y(x) = \frac{2C_1}{x} + C_2 \frac{\ln x}{x}$

(c) y_1 and y_2 does ***not*** form a fundamental set of solutions.

3-107 (a) $y(x) = e^x\left(C_1 \sin\sqrt{2}x + C_2 \cos\sqrt{2}x\right)$

(b) $y(= e^x\left(C_1 \sin\sqrt{2}x + C_2 \cos\sqrt{2}x\right)$

(c) $y(x) = e^x\left(C_1 \cos\sqrt{2}x + C_2 \sin\sqrt{2}x\right)$

3-111 $y = (C_1 + C_2x)e^{-x}$

3-113 $y = C_1e^{2x} + C_2e^{-2x}$

3-115 $y = C_1 \sin 3x + C_2 \cos 3x$

3-117 $y = \frac{C_1}{x} + \frac{C_2 \ln x}{x}$

3-119 $y = C_1x + C_2e^{2x}\left(\frac{x}{2} - 1\right)$

3-121 $y = C_1x^{1/3} - \frac{C_2}{x^{13/3}}$

3-129 (a) $y = C_1 \cos\lambda x + C_2 \sin\lambda x$, **(b)** $y = (C_1 + C_2x)e^{2x}$, **(c)** $y = C_1e^{\lambda x} + C_2e^{-\lambda x}$

3-131 (a) $y = (C_1 + C_2x)e^{3x}$, **(b)** $y = e^{-3x/2}\left(C_1 \cos\frac{\sqrt{7}}{2}x + C_2 \sin\frac{\sqrt{7}}{2}x\right)$,

(c) $y = C_1e^{(3+\sqrt{13})x} + C_2e^{(3-\sqrt{13})x}$

3-133 $y = \frac{1}{4}(e^{2x} - e^{-2x})$

3-134 $y = e^{x-1} + e^{-4(x-1)}$

3-135 $y = -\frac{17}{3}e^{-x} + \frac{2}{3}e^{x/2}$

3-137 $y = \frac{1}{2}e^{\pi - 2x}\sin 4x$

3-139 $y(x) = \frac{e^{4x-4} + 4e^{16-x}}{4e^{15}+1}$

3-141 $T(x) = -29.6e^{3.75x} + 129.6e^{-3.75x}$

3-143 $T(x) = 0.0083e^{12.613x} + 199.991e^{-12.613x}$

3-147 **(a)** $y = C_1\cos x + C_2\sin x + e^x$, **(b)** $y = C_1\cos x + C_2\sin x + e^x$

3-149 **(a)** $y = C_1e^{-2x} + C_2xe^{-2x} + x^2e^{-2x}$, **(b)** $y = C_1e^{-2x} + (C_2+1)xe^{-2x} + C_2xe^{-2x}$

3-151 **(a)** $= C_1 + C_2x + \frac{1}{12}x^4 - \frac{1}{2}x^2$, **(b)** $y = C_1 + C_2x + \frac{1}{12}x^4 - \frac{1}{2}x^2$

3-153 $y = C_1\cos 2x + C_2\sin 2x + 2x^2 + x + 2e^x$

3-155 $y = C_1\cos x + C_2\sin x - 2\sin 2x + 2$

3-161 **(a)** $y = C_1e^{2x} + C_2xe^{2x} - 2e^{3x}$, **(b)** $y = C_1e^{2x} + C_2xe^{2x} + x^2e^{2x+3}$

(c) $y = C_1e^{2x} + C_2xe^{2x} + \frac{5}{6}x^3e^{2x}$, **(d)** $y = C_1e^{2x} + C_2xe^{2x} - \frac{1}{25}(3\cos 2x + 4\sin 2x)e^x$

3-163 **(a)** $y = C_1 + C_2e^{3x} - \frac{1}{6}x^2 + \frac{5}{9}x$, **(b)** $y = C_1 + C_2e^{3x} + \left(-\frac{1}{2}x + \frac{3}{4}\right)e^x$

(c) $y = C_1 + C_2e^{3x} + \frac{1}{27}(7x - 3x^2 - 3x^3)$

(d) $y = C_1 + C_2e^{3x} + \frac{e^x}{200}(10x\cos 2x - 19\cos 2x - 8\sin 2x - 30x\sin 2x)$

3-165 **(a)** $y = C_1\cos x + C_2\sin x - \frac{x}{2}(2\cos x + 3\sin x)$

(b) $y = C_1\cos x + C_2\sin x + x^2 + 3 - \frac{1}{2}e^x$

(c) $y = C_1\cos x + C_2\sin x + \frac{1}{2}(x^2 - 2x)e^x$

(d) $y = C_1\cos x + C_2\sin x - \frac{1}{40}(3\cos 3x + \sin 3x)e^{2x}$

3-167 $y = \left(-10\sin x + \frac{17}{2}\cos x\right)e^x + \frac{1}{2}x^3 + \frac{3}{2}x^2 + \frac{3}{2}x - \frac{5}{2}$

3-169 $y = -\frac{3}{2}e^x + \frac{5}{2}e^{-x} + 2xe^x + \frac{1}{2}\sin x - \frac{x}{2}\cos x$

3-171 **(a)** $y = C_1e^{-2x} + C_2e^{2x} + \frac{1}{64}(8x^2 - 4x + 1)e^{2x}$

(b) $y = C_1e^{-2x} + C_2e^{2x} + \frac{1}{8}(2x^2 - 2x + 1)$

3-173 **(a)** $y = C_1e^{2x}\sin x + C_2e^{2x}\cos x - \frac{1}{8}e^{2x}(2\cos 2x - 1)\cos x$

(b) $y = C_1e^{2x}\sin x + C_2e^{2x}\cos x - e^{2x}\cos x\ln(\sec x + \tan x)$

3-175 **(a)** $y = C_1 + C_2e^{4x} - \frac{1}{8}x^2 - \frac{21}{16}x$
(b) $y = C_1 + C_2e^{4x} + \frac{1}{2(x-2)}$

3-177 **(a)** $y = C_1e^x + C_2xe^x + e^{2x} + 8$, **(b)** $y = C_1e^x + C_2xe^x + x^{-2}e^x$

3-181 **(a)** $y = C_1x^{-1+\sqrt{3}} + C_2x^{-1-\sqrt{3}}$, **(b)** $y = C_1(x-1)^{-1+\sqrt{3}} + C_2(x-1)^{-1-\sqrt{3}} - 3$

3-183 **(a)** $y = C_1\frac{1}{x} + C_2\frac{\ln x}{x}$, **(b)** $y = C_1\frac{1}{x} + C_2\frac{\ln x}{x} + \frac{2}{9}x^2$

3-185 **(a)** $y = x^2\left[C_1\cos\left(\sqrt{2}\ln x\right) + C_2\sin\left(\sqrt{2}\ln x\right)\right]$

(b) $y = x^2\left[C_1\cos\left(\sqrt{2}\ln x\right) + C_2\sin\left(\sqrt{2}\ln x\right)\right] - \frac{1}{6}x + \frac{1}{12}$

3-193 $\omega_0 = 31.62\ \text{s}^{-1}, T \approx 0.2\ \text{s}, A = 0.316\ \text{m}$

3-195 $x(t) = \frac{200}{981-\omega^2}(\cos\omega t - \cos 31.32t)$, $\omega = \omega_0 = 31.32\ \text{s}^{-1}$ will cause the resonance.

3-197 $v(t) = 4\cos 20t - 4\cos 30t$, $v_{\max} = 8\ \text{m/s}$, $\Delta t = \frac{\pi}{5}\ \text{s}$

3-199 The mass will pass through its static equilibrium position at the time $t = 0.182$ s, with a velocity of $V(0.182) = -0.0245\text{m/s}$ (upward)

3-201 $x(t) = e^{-20t}\left(C_1\sin\frac{\sqrt{362}}{2}t + C_2\cos\frac{\sqrt{362}}{2}t\right) + \frac{F_0\cos\left(\omega t - \arctan\left(\frac{c\omega}{k-m\omega^2}\right)\right)}{\sqrt{(mk-m^2\omega^2)+c^2\omega^2}}, \quad \omega = 0$

3-203 $mx'' + (k_1 + k_2)x = k_1y$

3-205 $x(0.4643) = 0.2080\ \text{m}$

3-207 $mL^2\varphi'' = mgL\sin\varphi - kL_1^2\varphi - cL_1^2\varphi'$

3-209 $x = \frac{b\omega^2}{k-m\omega^2}\left(\sin\omega t - \frac{\omega}{\omega_o}\sin\omega_o t\right)$

3-211 $x(t) = e^{-\zeta\omega_o t}\left(\frac{x'(0)+\zeta\omega_o x(0)}{\omega_d}\sin\omega_d t + x(0)\cos\omega_d t\right)$

3-215 $Q(t) = C_1\sin\omega_0 t + C_2\cos\omega_0 t + \frac{E_0/L}{\omega_0^2-\omega^2}\cos\omega t$
The charge of capacitor would be, *at least mathematically*, unbounded as $t \to \infty$

3-217

If $R^2 - 4\frac{L}{C} > 0$ then there are two real and distinct roots, m_1 and m_2. Thus the general solution of the differential equation is

$$I(t) = C_1e^{m_1t} + C_2e^{m_2t}$$

If $R^2 - 4\frac{L}{C} = 0$ then there are two real and equal roots, $m_1 = m_2 = m = -\frac{R}{2L}$. Thus the general solution of the differential equation is

$$I(t) = (C_1 + C_2 t)e^{mt}$$

If $R^2 - 4\frac{L}{C} < 0$ then there are two complex and conjugate roots, $m_{1,2} = \alpha \mp i\beta$. Thus the general solution of the differential equation is

$$I(t) = e^{\alpha t}(C_1 \cos\beta t + C_2 \sin\beta t)$$

where $\alpha = -\frac{R}{2L}$ and $\beta = \frac{\sqrt{4L/C - R^2}}{2L}$

3-219 $x(t) = Ae^{-3t} + Bte^{-3t}$

3-221 $y(t) = A\sin 2t + B\cos 2t$

3-223 $y = -\frac{17}{3}e^{-x} + \frac{2}{3}e^{x/2}$

3-225 $y = e^{-2x}\left(\frac{e^{\pi}}{2}\sin 4x\right) = \frac{1}{2}e^{\pi-2x}\sin 4x$

3-227 $y = 6xe^{2-x}$

3-229 $y = e^{-2x} + C_2e^{2x} + \left(-\frac{3}{5}x^2 + \frac{36}{25}x - \frac{186}{125}\right)e^{3x}$

3-230 $y = C_1\cos 3x + C_2\sin 3x + \frac{1}{4}\sin x$

3-231 $y = C_1\cos 3x + C_2\sin 3x + \frac{1}{2}x\cos x + \frac{1}{16}\sin x$

3-233 $y = C_1 + C_2e^{3x} + \frac{e^x}{200}(10x\cos 2x - 19\cos 2x - 8\sin 2x - 30x\sin 2x)$

3-235 $y = \frac{3}{4}\cos 4x + \frac{1}{24}\sin 4x - \frac{1}{4}\cos 2x + \frac{1}{12}\sin 2x$

3-237 Note: the equation is incorrect in the first printing of the textbook. It should be $x^2y'' + y = 0$. The solution is $y = x^{1/2}\left[C_1\cos\left(\frac{\sqrt{3}}{2}\ln x\right) + C_1\sin\left(\frac{\sqrt{3}}{2}\ln x\right)\right]$

3-241 $y = C_1e^{4x} + C_2e^{-4x}$

3-243 $y = C_1 + C_2e^{-x}$

3-245 $y = c_1 + C_2x^{-3/2} + \frac{1}{14}x^2 - \frac{1}{3}\ln x$

3-247 $y = C_1 + C_2e^{-2x}$

3-249 $y = (C_1 + C_2x)e^{3x}$

3-251 $y = 1 - e^{-x}$

3-253 $y = C_1 + C_2 e^{4x} - \frac{1}{8}x^2 - \frac{5}{16}x$

3-255 $y = C_1 + C_2 e^x - \frac{1}{3}x^3 - x^2 - 3x + \frac{e^x}{2}(\sin x + \cos x)$

3-257 $y = C_1 e^x + C_2 e^{8x} - \frac{1}{500}(50x^2 - 30x + 19)e^{3x}$

3-259 $y = \cosh x$

3-261 $y = C_1 x^{2+\sqrt{3}} + C_2 x^{2-\sqrt{3}}$

3-263 $y = x^{1/2}\left[C_1 \cos\left(\frac{\sqrt{15}}{2}\ln x\right) + C_2 \sin\left(\frac{\sqrt{15}}{2}\ln x\right)\right]$

3-265 $y(x) = -\frac{47}{128}e^{-4x} + \frac{1}{12}x^3 - \frac{1}{16}x^2 - \frac{15}{32}x + \frac{47}{128}$

3-267 $v(t) = 0.39047 \sin 10t + 0.060992 \cos 10t,\ T \approx 0.628$ s.

3-269 $I_{st} \cong 0.002435 \cos(60t) - 0.0003985 \sin(60t)$, $C = 1.67$ farad

CHAPTER 4

4-25 **(a)** $y^{(iv)} - 5y' + \cos y = x + 1$; Nonlinear, nonhomogeneous, constant coefficients

(b) $y^{(iv)} = 0$; Linear, homogeneous, constant coefficients

(c) $y^{(iv)} + 2x^2y' + 5y = 0$; Linear, homogeneous, variable coefficients

(d) $y^{(iv)} + e^x y = \frac{1}{x}$; Linear, nonhomogeneous, variable coefficients

4-27 **(a)** $y^{(v)} + \frac{1}{y} = 1$; Nonlinear, nonhomogeneous, constant coefficients

(b) $y^{(v)} - 8y' - e^{\ln y} = 0$; Nonlinear, homogeneous, constant coefficients

(c) $y^{(v)} - \sin 2x\, y' + y = 0$; Linear, homogeneous, variable coefficients

(d) $y^{(v)} + y = 7$; Linear, nonhomogeneous, constant coefficients

4-29 **(a)** The initial-value problem has a unique solution in the interval $-\infty < x < +\infty$.

(b) The initial-value problem has a unique solution in the interval $-\infty < x < 2$.

4-31 **(a)** The initial-value problem has a unique solution in the interval $-\infty < x < +\infty$.

(b) The initial-value problem has a unique solution in the interval $-2 < x < 2$.

4-35 **(a)** The Wronskian of these three solution functions is never zero for $x > 0$.

(b) The solutions e^x, $2e^{2+x}$ and -5 are linearly dependent.

4-37 **(a)** $y_1 = 1, y_2 = e^x$ and $y_3 = e^{-x}$ do not form a set of fundamental solutions.

(b) $y_1 = 1, y_2 = \sinh x$ and $y_3 = \cosh x$ do not form a set of fundamental solutions.

4-39 **(a)** y_1, y_2 and y_3 are linearly independent.

(b) y_1, y_2 and y_3 are linearly independent.

4-43 $y_3(x) = \frac{1}{3}\sin 3x$

4-45 $y_3(x) = -\frac{1}{3x}$

4-53 **(a)** $y = (C_1 + C_2x + C_3x^2)e^{-x}$, **(b) Given:** $y = C_1 + C_2e^{-3x} + C_3xe^{-3x}$

4-55 **(a)** $y = C_1e^{-x} + e^{3x/2}\left(C_2 \cos\frac{\sqrt{7}}{2}x + C_3 \sin\frac{\sqrt{7}}{2}x\right)$,

(b) $y = e^{-x}\left(C_2 \cos\sqrt{3}x + C_3 \sin\sqrt{3}x\right) + xe^{-x}\left(C_3 \cos\sqrt{3}x + C_4 \sin\sqrt{3}x\right)$

4-57 $y(x) = \frac{1}{3}e^x + \frac{2}{3}e^{-x/2}\cos\frac{\sqrt{3}}{2}x$

4-59 $y(x) = \left(1 - x + \frac{x^2}{2}\right)e^x$

4-61 $x(t) = Ae^{-6037t} + e^{-1982t}(B\sin 16{,}500t + C\cos 16{,}500t)$

4-63 $\beta = \pm i\sqrt{2\alpha}, \pm i\sqrt{\alpha} = \pm i\sqrt{\frac{2k}{m}}, \pm i\sqrt{\frac{k}{m}}$

4-67 (a) $y(x) = (C_1 + C_2x)e^x + C_3e^{-2x} - \frac{1}{10}e^{3x}$

(b) $y(x) = (C_1 + C_2x)e^x + C_3e^{-2x} + \frac{2}{9}xe^{3-2x}$

(c) $y(x) = (C_1 + C_2x)e^x + C_3e^{-2x} + \left(\frac{5}{18}x^2 + \frac{10}{27}x\right)e^{-2x}$

(d) $y(x) = (C_1 + C_2x)e^x + C_3e^{-2x} - \left(\frac{3}{52}\cos 2x + \frac{1}{26}\sin 2x\right)e^x$

4-69 (a) $y(x) = C_1e^{-x} + C_2e^x + C_3\cos x + C_4\sin x - x + 2$

(b) $y(x) = C_1e^{-x} + C_2e^x + C_3\cos x + C_4\sin x + \frac{1}{8}(x^2 - 5x)e^x$

(c) $y(x) = C_1e^{-x} + C_2e^x + C_3\cos x + C_4\sin x - x^2 + 1$

(d) $y = C_1e^{-x} + C_2e^x + C_3\cos x + C_4\sin x + \left(\frac{3}{80}x - \frac{41}{800}\right)e^x\cos 2x - \left(\frac{1}{80}x + \frac{19}{400}\right)e^x\sin 2x$

4-71 (a) $y(x) = C_1e^{-2x} + e^x\left(C_2\cos\sqrt{3}x + C_3\sin\sqrt{3}x\right) - \frac{1}{65}(22\cos x - 19\sin x)$

(b) $y(x) = C_1e^{-2x} + e^x\left(C_2\cos\sqrt{3}x + C_3\sin\sqrt{3}x\right) + \frac{1}{8}x^2 - \frac{1}{9}e^x$

(c) $y(x) = C_1e^{-2x} + e^x\left(C_2\cos\sqrt{3}x + C_3\sin\sqrt{3}x\right) + \frac{1}{81}(9x^2 - 6x - 13)e^x$

(d) $y(x) = C_1e^{-2x} + e^x\left(C_2\cos\sqrt{3}x + C_3\sin\sqrt{3}x\right) - \frac{1}{1525}(9\cos 3x + 38\sin 3x)$

4-73 $x_{p1} = \alpha^2, x_{p2}(t) = C_1\sin 0.618\alpha t + C_2\cos 0.618\alpha t + C_3\sin 1.618\alpha t + C_4\cos 1.618\alpha t + \alpha^2$
$(\alpha = \sqrt{k/m})$

4-77 (a) $y_p = \frac{1}{10}\sin 2x$

(b) $y_p = \frac{1}{9}\ln(\sec 2x + \tan 2x) + \sin 3x\left(\frac{2}{9}\cos x - \frac{\sqrt{2}}{18}\operatorname{arctanh}\left(\sqrt{2}\cos x\right)\right)$
$+(\cos 3x)\left(-\frac{2}{9}\sin x + \frac{\sqrt{2}}{18}\operatorname{arctanh}\left(\sqrt{2}\sin x\right)\right)$

4-79 (a) $y_p = \frac{1}{4}x^4 - 3x^2 - x + 6$

(b) $y_p = \ln|x| - \sin x\int\frac{\sin x}{x}dx - \cos x\int\frac{\cos x}{x}dx$

4-81 (a) $y_p = (x^2 - 6x + 12)e^x$

(b) $y_p = -x\ln|x|$

4-87 $y(x) = \frac{C_1}{x} + \frac{C_2}{x^2} + C_3x^3$

4-89 $y(x) = C_1x + x\left[C_2\cos\left(\sqrt{2}\ln x\right) + C_3\sin\left(\sqrt{2}\ln x\right)\right]$

4-91 $y(x) = C_1x^3 + \frac{C_2}{x^2} + \frac{C_3 \ln x}{x^2}$

4-93

(a) $-2, -6, -8$
(b) $-5, -2 \pm 5.6465i$
(c) $-2, -2 \pm 5.3 \times 10^{-5}i$
(d) $-3, -5 \pm 8i$
(e) $-5 \pm 8i, -3 \pm 6i$
(f) $-3, -3, -5 \pm 8i$
(g) $-3 \pm 5i, -3 \pm 5i$

4-95

(a) $y(x) = -\frac{13}{18} - \frac{1}{6}x + \_C1\, e^{3x} + \_C2\, e^{-x}\cos(x) + \_C3\, e^{-x}\sin(x)$

(b) $y(x) = \frac{1}{26}e^{3x} + \_C1\, e^{x} + \_C2\, e^{-\frac{1}{2}x}\cos\left(\frac{1}{2}\sqrt{3}\,x\right) + \_C3\, e^{-\frac{1}{2}x}\sin\left(\frac{1}{2}\sqrt{3}\,x\right)$

4-97 $y(x) = -1 + e^{2x} - e^{2x}x$

4-99 (a) $2e^{3x}$, (b) 0

4-103 $y(x) = y_h + y_p = C_1 + C_2e^x + C_3e^{-x} + e^{2x}\left(-\frac{1}{10}x + \frac{3}{25}\right)\cos x + \frac{2}{25}e^{2x}\sin x$

4-105 $y_p = \frac{x^2}{2}\ln|x| - \frac{3}{4}x^2 - 3\sin x + x\cos x$

4-107 $y(x) = C_1e^{-x} + e^{-x}(C_2\cos\sqrt{3}x + C_3\sin\sqrt{3}x) + \frac{1}{4}x^3 - \frac{9}{8}x^2 + \frac{9}{4}x - \frac{41}{16}$

4-109 $y = C_1 + C_2x + C_3x\ln x - \frac{1}{4x} - 2\ln x$

4-111 $y = C_1e^x + C_2xe^x + C_3x^2e^x$

4-113 $y(x) = C_1e^{-x} + e^{x/2}\left(C_2\cos\frac{\sqrt{3}}{2}x + C_3\sin\frac{\sqrt{3}}{2}x\right) + \frac{1}{7}e^{3x} - x$

4-115 $y(x) = C_1e^x + e^{3x/2}\left(C_2\cos\frac{\sqrt{21}}{2}x + C_3\sin\frac{\sqrt{21}}{2}x\right) + \frac{1}{5}e^{2x} - \frac{1}{3}$

4-117 $y = e^{x/\sqrt{2}}\left(C_1\sin\frac{x}{\sqrt{2}} + C_2\cos\frac{x}{\sqrt{2}}\right) + e^{-x/\sqrt{2}}\left(C_3\sin\frac{x}{\sqrt{2}} + C_4\cos\frac{x}{\sqrt{2}}\right) + \cos x + \frac{x}{2}\sin x$

4-119 $y = e^{\sqrt{2}x}(C_1\sin\sqrt{2}x + C_2\cos\sqrt{2}x) + e^{-\sqrt{2}x}(C_3\sin\sqrt{2}x + C_4\cos\sqrt{2}x) + \frac{1}{32}(x-1)e^{2x} - \frac{1}{16}$

4-121 $y(x) = C_1 + C_2\sin 3x + C_3\cos 3x - \frac{3}{50}x\cos 2x + \left(\frac{1}{10}x^2 - \frac{69}{500}\right)\sin 2x$

4-123 $y(x) = \frac{1}{64}(1 - \cosh 2x\cos 2x)$

4-125 $y(x) = \frac{C_1}{x} + x^2[C_1\cos(\sqrt{2}\ln x) + C_2\sin(\sqrt{2}\ln x)]$

CHAPTER 5

5 – 41 (a) $5x\ \sum_{n=1}^{\infty}(n+1)^2\ x^{n+3} = 20x^5 + 45x^6 + 80x^7 + 5x\ \sum_{n=4}^{\infty}(n+1)^2\ x^{n+3}$

(b) $\sum_{n=2}^{\infty}\frac{n+5}{n+3}C_{2n+1}x^{2n+1} = \frac{7}{5}C_5x^5 + \frac{4}{3}C_7x^7 + \frac{9}{7}C_9x^9 + \sum_{n=5}^{\infty}\frac{n+5}{n+3}C_{2n+1}x^{2n+1}$

5 – 43 (a) $\sum_{n=4}^{\infty}(n-4)^2\ 2^{n-3}x^n$, **(b)** $\sum_{n=4}^{\infty}C_{n-1}x^n$

5 – 45 (a) $\sum_{n=1}^{\infty}(n+2)\ x^{n-1}$, **(b)** $\sum_{n=3}^{\infty}C_{n+1}x^{n-1}$

5 – 47 (a) $\sum_{n=5}^{\infty}C_{n-1}x^{n+1}$, **(b)** $\sum_{n=1}^{\infty}(n+3)^2 2^{n+1}x^{n+1}$

5 – 49 $15C_0\frac{1}{x} - \frac{1}{x^2} + \sum_{n=0}^{\infty}\left[(n+1)^2 - 5(n-2)C_{n+1} + \frac{n+1}{n+3}\right]x^n = 0$

5 – 51 The equality holds for any x value.

5 – 53 The equality holds for any x value.

5 – 55 Not correct

5 – 57 (a) $\rho = 1/3,\ -1/3 \le x < 1/3$, **(b)** $\rho = 1/2,\ 1/2 \le x < 3/2$

5 – 59 (a) $C_{n+2} = \frac{3C_n}{(n+2)(n+1)}$, **(b)** $C_{n+2} = -\frac{1}{2}\frac{n(n+1)C_{n+1}+(2-n)C_n}{(n+2)(n+1)}$

5 – 61 (a) $C_{n+2} = \frac{-(n+1)C_{n+1}+2C_n}{(n+2)(n+1)}$, **(b)** $C_{n+2} = \frac{nC_n}{(n+1)}$

5 – 63 (a) $y(x) = C_0\left(1 + \frac{x^2}{2} + \frac{x^4}{24} + \cdots\right) + C_1\left(x + \frac{x^3}{6} + \frac{x^5}{120} + \cdots\right)$

(b) $y(x) = C_0\left(1 + 2x^2 + \frac{2}{3}x^4 + \frac{4}{45}x^6 + \cdots\right) + C_1\left(x + \frac{2}{3}x^3 + \frac{2}{15}x^5 + \cdots\right)$

5 – 65 $$y(x) = C_0 + C_1x + (-2C_1 + 6C_0)x^2 + \left(\frac{14}{3}C_1 - 8C_0\right)x^3 + \left(-\frac{20}{3}C_1 + 14C_0\right)x^4 + \left(\frac{122}{15}C_1 + 16C_0\right)x^5 + \left(-\frac{364}{45}C_1 + \frac{244}{14}C_0\right)x^6 + \cdots$$

5 – 69 (a) All points are ordinary points.

(b) Both $x = -2$ and $x = 2$ are the regular singular points of the differential equation.

5 – 71 (a) Both $x = -1$ and $x = 1$ are the regular singular points of the differential equation.

(b) All points are ordinary points of the differential equation.

5 – 73 $\rho = 1$.

5 – 75 $\rho = 4$.

5 – 77 $\rho = 1$.

5 – 79 $y(x) = C_0\left(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4 - \frac{11}{720}x^6 - \cdots\right) + C_1\left(x - \frac{1}{6}x^3 - \frac{1}{24}x^5 - \frac{19}{1008}x^7 - \cdots\right)$

Interval of convergence: $-1 < x < 1$.

5 – 81 $y(x) = C_0\left(\frac{33}{10} - 3x - 3x^2 + 7x^3 - \frac{9}{2}x^4 + \frac{6}{5}x^5 + \cdots\right)$

$$+C_1\left(-\frac{9}{5} + \frac{7}{2}x - 3x^2 + 2x^3 - x^4 + \frac{3}{10}x^5 + \cdots\right)$$

5 – 83 $y(x) = C_0\left(1 + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{7}{24}x^4 + \frac{4}{15}x^5 + \cdots\right) + C_1\left(x + \frac{1}{6}x^3 + \frac{1}{6}x^4 + \frac{19}{120}x^5 + \cdots\right)$

Interval of the convergence: $-1 < x < 1$.

5 – 85 $y(x) = C_0\left(\frac{7}{60} - \frac{8}{3}x^2 + \frac{59}{24}x^3 - \frac{7}{12}x^4 + \frac{11}{120}x^5 + \cdots\right) + C_1(x - 1)$

5 – 87 $y(x) = C_0\left(1 - \frac{1}{2}x^2 - \frac{1}{2}x^3 - \frac{5}{12}x^4 - \frac{1}{3}x^5 - \cdots\right) + C_1\left(x + \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^5 - \frac{1}{8}x^6 \ldots\right)$
Interval of the convergence: $-1 < x < 1$.

5 – 89 $y(x) = C_0\left(1 + \frac{1}{3}x^3 - \frac{1}{12}x^4 + \frac{1}{60}x^5 + \cdots\right) + C_1\left(x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{8}x^4 - \frac{3}{40}x^5 + \cdots\right)$
Interval of convergence is $(-\infty, \infty)$.

5 – 91 $y(x) = 1 + \frac{2}{3}x^3 + \frac{4}{45}x^6 + \frac{2}{405}x^9 + \frac{2}{13365}x^{12} + \cdots$

5 – 97 $y(x) = C_0(1 - 3x^2) + C_1\left(x - \frac{2}{3}x^3 - \frac{1}{5}x^5 - \frac{4}{35}x^7 - \frac{5}{63}x^9 - \cdots\right)$, $-1 < x < 1$.

5 – 99 $P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$

5 – 105 **(a)** $r_1 = 1 + \sqrt{3}i$ and $r_2 = 1 - \sqrt{3}i$

(b) $r_1 = 1 + \sqrt{3}i$ and $r_2 = 1 - \sqrt{3}i$

5 – 107 **(a)** $r_1 = \frac{1}{2} + \frac{\sqrt{15}}{2}i$ and $r_2 = \frac{1}{2} - \frac{\sqrt{15}}{2}i$

(b) $r_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $r_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

5 – 109
(a) $y_1(x) = x^{r_1}\sum_{n=0}^{\infty} a_n x^n$ and $y_2(x) = x^{r_2}\sum_{n=0}^{\infty} b_n x^n$, where $a_0 \neq 0$ and $b_0 \neq 0$. Since $x = 0$ is the only singular point for the given differential equation, the series solution converges for all $x > 0$.
(b) $y_1(x) = x^r\sum_{n=0}^{\infty} a_n x^n$ and $y_2(x) = y_1(x)\ln x + x^r\sum_{n=0}^{\infty} b_n x^n$, where $a_0 \neq 0$. It is clear from either $P(x)$ or $Q(x)$ that $x = 1$ is another singular point of the given differential equation. Therefore the series will converge for all x such that $0 < x < 1$.

5 – 111
(a) $y_1(x) = x^{r_1}\sum_{n=0}^{\infty} a_n x^n$ and $y_2(x) = Cy_1(x)\ln x + x^{r_2}\sum_{n=0}^{\infty} b_n x^n$, where $a_0 \neq 0$ and $b_0 \neq 0$, whereas the constant C may be zero. The series solution will converge for any $x > 0$.
(b) $y_1(x) = x^{r_1}\sum_{n=0}^{\infty} a_n x^n$ and $y_2(x) = Cy_1(x)\ln x + x^{r_2}\sum_{n=0}^{\infty} b_n x^n$, where $a_0 \neq 0$ and $b_0 \neq 0$, whereas the constant C may be zero. The series solution will converge for any $x > 0$.

5 – 113

(a) $y_1(x) = x^{r_1}\sum_{n=0}^{\infty} a_n\, x^n$ and $y_2(x) = x^{r_2}\sum_{n=0}^{\infty} b_n\, x^n$, where $a_0 \neq 0$ and $b_0 \neq 0$. Since $x = 0$ is the only singular point for the given differential equation, the series solution converges for all $x > 0$.

(b) $y_1(x) = x^{r_1}\sum_{n=0}^{\infty} a_n\, x^n$ and $y_2(x) = Cy_1(x)\ln x + x^{r_2}\sum_{n=0}^{\infty} b_n\, x^n$, where $a_0 \neq 0$ and $b_0 \neq 0$, whereas the constant C may be zero. It is clear from either $P(x)$ or $Q(x)$ that $x = \mp 2$ are two other singular points of the given differential equation. Therefore the series solution will converge for all x such that $0 < x < 2$.

5 – 115 **(a)** $y(x) = C_1 x^{1+\sqrt{3}}\left(1 - \frac{1}{4}x^2 + \frac{1}{32}\frac{3+\sqrt{3}}{2+\sqrt{3}}x^4 + \cdots\right) + C_2 x^{1-\sqrt{3}}\left(1 - \frac{1}{4}x^2 + \frac{1}{32}\frac{-3+\sqrt{3}}{-2+\sqrt{3}}x^4 + \cdots\right)$

(b) $y(x) = C_1 x + C_2\left(x\ln x + \frac{1}{4}x^3 + \frac{3}{32}x^5 + \frac{5}{96}x^7 + \frac{35}{1024}x^9 + \cdots\right)$

5 – 117

(a) $y(x) = C_1 x^{\frac{5}{4}+\frac{\sqrt{41}}{4}} + C_2 x^{\frac{5}{4}-\frac{\sqrt{41}}{4}}$

(b) $y(x) = C_1 x^{\frac{3}{2}}\left[1 - \frac{1}{2}x + \frac{23}{128}x^2 - \frac{281}{3840}x^3 + \frac{7397}{245760}x^4 - \frac{222991}{17203200}x^5 + \cdots\right]$

$$+\frac{7C_2}{32}x^{\frac{3}{2}}\left[1 - \frac{1}{2}x + \frac{23}{128}x^2 - \frac{281}{3840}x^3 + \frac{7397}{245760}x^4 - \frac{222991}{17203200}x^5 + \cdots\right]\ln x$$

$$+C_2 x^{\frac{1}{2}}\left[1 - \frac{1}{2}x + \frac{1}{12}x^3 - \frac{1385}{49152}x^4 + \frac{76739}{7372800}x^5 + \cdots\right]$$

5 – 119 **(a)** $y(x) = \frac{C_1}{x^2} + \frac{C_2 \ln x}{x^2}$, **(b)** $y(x) = C_1 x^{\frac{4}{3}} + C_2 x^{-1}$

5-125 $J_2(x) = \sum_{n=0}^{\infty}\frac{(-1)^n}{2^{2n+2}n!(n+2)!}x^{2n+2} = \frac{1}{8}x^2 - \frac{1}{96}x^4 + \frac{1}{3072}x^6 - \frac{1}{184320}x^8 + \cdots$

5-129

(a) $I = -xJ_0(x) + \int J_0(x)\,dx + C$, The integral in the result cannot be evaluated in finite form in terms of any of the known Bessel's functions.

(b) $I = -x^{-3}J_3(x) + C = -\frac{J_3(x)}{x^3} + C$

5-131

(a) $y(x) = \_C1\ e^{x} + \_C2\ e^{-x}$

The series solution found in Problem 5-63a is

$$y(x) = C_0\left(1 + \frac{x^2}{2} + \frac{x^4}{24} + \cdots\right) + C_1\left(x + \frac{x^3}{6} + \frac{x^5}{120} + \cdots\right)$$

(b) $y(x) = \_C1\ e^{4x}$

The series solution found in Problem 5-63b is

$$y(x) = C_0\left(1 + 2x^2 + \frac{2}{3}x^4 + \frac{4}{45}x^6 + \cdots\right) + C_1\left(x + \frac{2}{3}x^3 + \frac{2}{15}x^5 + \cdots\right)$$

5-133 $y(x) = \_C1\, e^{2x} + \_C2\, e^{-6x}$

The series solution found in Problem 5-65 is

$$y(x) = C_0 + C_1 x + (-2C_1 + 6C_0)x^2 + \left(\frac{14}{3}C_1 - 8C_0\right)x^3 + \left(-\frac{20}{3}C_1 + 14C_0\right)x^4$$

$$+\left(\frac{122}{15}C_1 + 16C_0\right)x^5 + \left(-\frac{364}{45}C_1 + \frac{244}{14}C_0\right)x^6 + \cdots$$

5-135

$$y(x) = \frac{\_C1\, \mathrm{HeunC}\left(0, 0, 0, -2\,\mathrm{I}, \mathrm{I}, \frac{1}{2} - \frac{1}{2}\,\mathrm{I}\,x\right)\sqrt{\mathrm{I}\,x + 1}}{\sqrt{x - \mathrm{I}}}$$

$$+ \frac{1}{\sqrt{x - \mathrm{I}}}\left(\_C2\, \mathrm{HeunC}\left(0, 0, 0, -2\,\mathrm{I}, \mathrm{I}, \frac{1}{2}\right.\right.$$

$$\left.- \frac{1}{2}\,\mathrm{I}\,x\right)\sqrt{\mathrm{I}\,x + 1}\left(\right.$$

$$\left.\left.\int \frac{x - \mathrm{I}}{(x^2 + 1)\,\mathrm{HeunC}\left(0, 0, 0, -2\,\mathrm{I}, \mathrm{I}, \frac{1}{2} - \frac{1}{2}\,\mathrm{I}\,x\right)^2 (\mathrm{I}\,x + 1)}\,\mathrm{d}x\right)\right)$$

The series solution found in Problem 5-80 is

$$y(x) = C_0\left(1 + \frac{1}{6}x^3 - \frac{1}{10}x^5 + \frac{1}{180}x^6 + \cdots\right) + C_1\left(x - \frac{1}{3}x^3 + \frac{1}{12}x^4 + \frac{1}{5}x^5 + \cdots\right)$$

5-137

$$y(x) = \_C1\,(x^2 - 1)\,\mathrm{hypergeom}\left(\left[\frac{3}{4} + \frac{1}{4}\sqrt{17}, \frac{3}{4} - \frac{1}{4}\sqrt{17}\right],\right.$$

$$\left.\left[\frac{1}{2}\right], x^2\right) + \_C2\,(x^3 - x)\,\mathrm{hypergeom}\left(\left[\frac{5}{4} - \frac{1}{4}\sqrt{17}, \frac{5}{4}\right.\right.$$

$$\left.\left. + \frac{1}{4}\sqrt{17}\right], \left[\frac{3}{2}\right], x^2\right)$$

The series solution found in Problem 5-82 is

$$y(x) = C_0\left(1 - 2x^2 + \frac{1}{3}x^4 + \frac{4}{45}x^6 + \cdots\right) + C_1\left(x - \frac{2}{3}x^3 - \frac{1}{15}x^5 - \frac{8}{315}x^7 + \cdots\right)$$

5-139 $y(x) = \_C1\,\sqrt{x}\,\mathrm{BesselJ}\left(1, 2\sqrt{2}\sqrt{x}\right) + \_C2\,\sqrt{x}\,\mathrm{BesselY}\left(1,\right.$
$\left.2\sqrt{2}\sqrt{x}\right)$

The series solution found in Problem 5-84 is

$$y(x) = C_0\left(\frac{2}{15} - \frac{8}{3}x + \frac{7}{3}x^2 - \frac{2}{3}x^3 + \frac{1}{12}x^4 - \frac{1}{240}x^5 + \cdots\right)$$

$$+C_1\left(-1 + \frac{3}{2}x + \frac{1}{3}x^2 - \frac{1}{2}x^3 + \frac{1}{8}x^4 - \frac{1}{96}x^5 + \cdots\right)$$

5-141 $y(x) = \_C1\ \mathrm{AiryAi}(2^{2/3}\,x) + \_C2\ \mathrm{AiryBi}(2^{2/3}\,x)$

The series solution found in Problem 5-86 is

$$y(x) = C_0\left(1 + \frac{2}{3}x^3 + \frac{4}{45}x^6 + \frac{2}{405}x^9 + \cdots\right) + C_1\left(x + \frac{1}{3}x^4 + \frac{2}{63}x^7 + \frac{4}{2835}x^{10} + \cdots\right)$$

5-143

$$y(x) = \_C1\ \mathrm{WhittakerM}\left(2\,\mathrm{I}, \frac{1}{2}, 4\,\mathrm{I}\,(x+2)\right) + \_C2\ \mathrm{WhittakerW}\left(2\,\mathrm{I}, \frac{1}{2}, 4\,\mathrm{I}\,(x+2)\right)$$

The series solution found in Problem 5-88 is

$$y(x) = C_0\left(1 - \frac{1}{3}x^2 + \frac{1}{12}x^4 - \frac{1}{40}x^5 + \cdots\right) + C_1\left(x - \frac{1}{6}x^4 + \frac{1}{20}x^5 - \frac{1}{60}x^6 + \cdots\right)$$

5-145

$$y(x) = \_C1\ (x^2-1)\ \mathrm{hypergeom}\left(\left[\frac{1}{4}\sqrt{5} + \frac{3}{4}, \frac{3}{4} - \frac{1}{4}\sqrt{5}\right], \left[\frac{1}{2}\right], x^2\right) + \_C2\ (x^3 - x)\ \mathrm{hypergeom}\left(\left[\frac{5}{4} - \frac{1}{4}\sqrt{5}, \frac{5}{4} + \frac{1}{4}\sqrt{5}\right], \left[\frac{3}{2}\right], x^2\right)$$

The general series solution found in Problem 5-90 is

$$y(x) = C_0\left(1 - \frac{1}{2}x^2 - \frac{1}{24}x^4 - \frac{11}{720}x^6 - \cdots\right) + C_1\left(x - \frac{1}{6}x^3 - \frac{1}{25}x^5 - \frac{19}{1008}x^7 - \cdots\right)$$

The assumed power series solution suggests that $y(0) = y_0 = C_0$ and $y'(0) = y'_0 = C_1$. Therefore the solution of the given initial-value problem can be acquired by simply plugging in $C_0 = 0$ and $C_1 = 0$ in the general solution. Then the solution of the initial-value problem is $y(x) = 0$.

5-147 $y(x) = \sqrt{2}\,\sin(\sqrt{2}\,\arcsin(x)) + 2\cos(\sqrt{2}\,\arcsin(x))$

The general series solution found in Problem 5-92 is

$$y(x) = C_0\left(1 - x^2 - \frac{1}{6}x^4 - \frac{7}{90}x^6 - \cdots\right) + C_1\left(x - \frac{1}{6}x^3 - \frac{7}{120}x^5 - \frac{23}{720}x^7 - \cdots\right)$$

The assumed power series solution suggests that $y(0) = y_0 = C_0$ and $y'(0) = y_0' = C_1$. Therefore the solution of the given initial-value problem can be acquired by simply plugging in $C_0 = 2$ and $C_1 = 2$ in the general solution. Then the solution of the initial-value problem is obtained to be $y(x) = 2 + 2x - 2x^2 - \frac{1}{3}x^3 - \frac{1}{3}x^4 - \frac{14}{120}x^5 - \frac{7}{45}x^6 - \frac{23}{360}x^7 - \cdots$

5-149

(a) $$y(x) = \_C1\, e^{-\frac{1}{4}x^2}\, x\, \mathrm{BesselI}\left(\frac{1}{2}\sqrt{3}, \frac{1}{4}x^2\right) + \_C2\, e^{-\frac{1}{4}x^2}\, x\, \mathrm{BesselK}\left(\frac{1}{2}\sqrt{3}, \frac{1}{4}x^2\right)$$

The solution found in Problem 5-115(a) is

$$y(x) = C_1 x^{1+\sqrt{3}}\left(1 - \frac{1}{4}x^2 + \frac{1}{32}\frac{3+\sqrt{3}}{2+\sqrt{3}}x^4 + \cdots\right) + C_2 x^{1-\sqrt{3}}\left(1 - \frac{1}{4}x^2 + \frac{1}{32}\frac{-3+\sqrt{3}}{-2+\sqrt{3}}x^4 + \cdots\right)$$

(b) $y(x) = \_C1\, x + \_C2\, x \arctan\left(\frac{1}{\sqrt{x^2-1}}\right)$ or using MuPAD $\left\{C66\, x + C67\, x \arctan\left(\sqrt{x^2-1}\right)\right\}$

The solutions found from Maple and MuPAD differ, but they are both correct solutions. The solution found in Problem 5-115(b) is

$$y(x) = C_1 x + C_2\left(x \ln x + \frac{1}{4}x^3 + \frac{3}{32}x^5 + \frac{5}{96}x^7 + \frac{35}{1024}x^9 + \cdots\right)$$

5-151

(a) $y(x) = \_C1\, x^{\frac{5}{4}+\frac{1}{4}\sqrt{33}} + \_C2\, x^{\frac{5}{4}-\frac{1}{4}\sqrt{33}}$

(b) $$y(x) = DESol\left(\left\{\frac{d^2}{dx^2}\_Y(x) - \frac{4\left(\frac{d}{dx}\_Y(x)\right)}{x^2-4} + \frac{3\_Y(x)}{x^2(x^2-4)}\right\}, \{\_Y(x)\}\right)$$

Maple is unable to solve this problem. The solution found in Problem 5-117(b) is

$$y(x) = C_1 x^{\frac{3}{2}}\left[1 - \frac{1}{2}x + \frac{23}{128}x^2 - \frac{281}{3840}x^3 + \frac{7397}{245760}x^4 - \frac{222991}{17203200}x^5 + \cdots\right]$$
$$+\frac{7C_2}{32}x^{\frac{3}{2}}\left[1 - \frac{1}{2}x + \frac{23}{128}x^2 - \frac{281}{3840}x^3 + \frac{7397}{245760}x^4 - \frac{222991}{17203200}x^5 + \cdots\right]\ln x$$
$$+C_2 x^{\frac{1}{2}}\left[1 - \frac{1}{2}x + \frac{1}{12}x^3 - \frac{1385}{49152}x^4 + \frac{76739}{7372800}x^5 + \cdots\right]$$

5-153

(a) $y(x) = \frac{\_C1}{x^2} + \frac{\_C2\ \ln(x)}{x^2}$, The solution found in Problem 5-119(a) is $y(x) = \frac{C_1}{x^2} + \frac{C_2 \ln x}{x^2}$

(b) $y(x) = \_C1\ x^{4/3} + \frac{\_C2}{x}$,The solution found in Problem 5-119(b) is $y(x) = C_1 x^{4/3} + C_2 x^{-1}$

CHAPTER 6

6-21
(a)

$x_1' = x_2$

$x_2' = x_3$

$x_3' = -2x_1^2\, x_2 - 2x_1 + te^{-3t}$

(b)

$x_1' = x_2$

$x_2' = x_3$

$x_3' = -5x_2 + kx_1$

6-23
(a)

$x_1' = x_2$

$x_2' = x_3$

$x_3' = x_4$

$x_4' = 5x_2 - \cos x_1 + t + 1$

(b)

$x_1' = x_2$

$x_2' = x_3$

$x_3' = x_4$

$x_4' = 0$

6-25
(a)

$x_1' = x_2$

$x_2' = -e^x x_2 + 2x_1 + 6$

(b)

$x_1' = x_2$

$x_2' = x_3$

$x_3' = 2x_2 - x_1 + t^3 \cos 2t$

6-27

$x_1' = x_2$

$x_2' = x_3$

$x_3' = x_1 x_4$

$x_4' = x_5$

$x_5' = \frac{2t}{(t-1)^3} x_1 x_5 - \frac{1}{(t-1)^3} x_5 + \frac{e^{-t}}{(t-1)^3}$

6-29

$x_1' = x_2$

$x_2' = x_3$

$x_3' = x_1 + x_5 + x_1 x_4 - x_4 x_6 - 1 - 3t$

$x_4' = x_5$

$x_5' = t^2 x_4 - x_1 x_6$

$x_6' = x_7$

$x_7' = x_1 x_4 - x_4 x_6 - 1$

6-31 The system is nonlinear due to term $2txy'$, nonhomogeneous due to $e^{-t} - 1$, and has variable coefficients due to term $2txy'$.

6-33 The system is nonlinear due to terms xz, xy and yz, nonhomogeneous due to -1, and has variable coefficients due to term $t^2 y$.

6-35 The system is linear, nonhomogeneous due to terms e^t and 3 , and has constant coefficients.

6-37 The system is linear, nonhomogeneous due to 1, and has variable coefficients due to terms tx and $t^2(x - z)$.

6-39

$$m_1 \frac{d^2 x_1}{dt^2} + k_1 x_1 + k_2 (x_1 - x_2) = 0$$

$$m_2 \frac{d^2 x_2}{dt^2} + k_3 (x_2 - x_3) - k_2 (x_1 - x_2) = 0$$

$$m_3 \frac{d^2 x_3}{dt^2} - k_3 (x_2 - x_3) = F(t)$$

6-41

$$R\frac{dI_1}{dt}+\frac{1}{C}(I_1-I_2)=0$$
$$L\frac{d^2I_2}{dt^2}-\frac{1}{C}(I_1-I_2)=0$$

6-43

$$L\frac{d^2I_1}{dt^2}-L\frac{d^2I_2}{dt^2}+R_1\frac{dI_1}{dt}=\frac{dE(t)}{dt}$$
$$-L\frac{d^2I_1}{dt^2}+L\frac{d^2I_2}{dt^2}+R_2\frac{dI_2}{dt}+R_4\frac{dI_2}{dt}-R_4\frac{dI_3}{dt}=0$$
$$-R_4\frac{dI_2}{dt}+\ R_3\frac{dI_3}{dt}+R_4\frac{dI_3}{dt}+\ \frac{1}{C}I_3=0$$

6-45

$$\frac{dx_1}{dt}=\frac{1}{20}(x_2-2x_1)+2.5$$
$$\frac{dx_2}{dt}=\frac{1}{10}(x_1-x_2)$$

6-47
(a)

$$x(t)=C_1e^{\left(\frac{5}{2}+\frac{\sqrt{13}}{2}\right)t}+C_2e^{\left(\frac{5}{2}-\frac{\sqrt{13}}{2}\right)t}$$
$$y(t)=\frac{C_1}{2}\left(-3-\sqrt{13}\right)e^{\left(\frac{5}{2}+\frac{\sqrt{13}}{2}\right)t}+\frac{C_2}{2}\left(-3+\sqrt{13}\right)e^{\left(\frac{5}{2}-\frac{\sqrt{13}}{2}\right)t}$$

(b)

$$x(t)=C_1e^{\left(\frac{5}{2}+\frac{\sqrt{13}}{2}\right)t}+C_2e^{\left(\frac{5}{2}-\frac{\sqrt{13}}{2}\right)t}+\frac{4}{3}t+\frac{5}{9}+(t-3)e^t$$
$$y(t)=\frac{C_1}{2}\left(-3-\sqrt{13}\right)e^{\left(\frac{5}{2}+\frac{\sqrt{13}}{2}\right)t}+\frac{C_2}{2}\left(-3+\sqrt{13}\right)e^{\left(\frac{5}{2}-\frac{\sqrt{13}}{2}\right)t}+\frac{1}{3}t+\frac{2}{9}-e^t$$

6-49
(a)

$$x(t)=C_1e^{2t}\sin 2t+C_2e^{2t}\cos 2t$$
$$y(t)=\frac{1}{2}C_1e^{2t}\cos 2t-\frac{1}{2}C_2e^{2t}\sin 2t$$

(b)

$$x(t) = C_1 e^{2t} \sin 2t + C_2 e^{2t} \cos 2t - \left(t^2 + \frac{6}{5}t - \frac{22}{25}\right) e^{3t}$$

$$y(t) = \frac{1}{2} C_1 e^{2t} \cos 2t - \frac{1}{2} C_2 e^{2t} \sin 2t + \left(t^2 - \frac{4}{5}t - \frac{2}{25}\right) e^{3t}$$

6-51

(a)

$$x(t) = C_1 e^{\sqrt{5}t} + C_2 e^{-\sqrt{5}t}$$

$$y(t) = C_1\left(-2 + \sqrt{5}\right) e^{\sqrt{5}t} + C_2\left(-2 - \sqrt{5}\right) e^{-\sqrt{5}t}$$

(b)

$$x(t) = C_1 e^{\sqrt{5}t} + C_2 e^{-\sqrt{5}t} - \frac{3}{5}t^2 - \frac{16}{25}$$

$$y(t) = C_1\left(-2 + \sqrt{5}\right) e^{\sqrt{5}t} + C_2\left(-2 - \sqrt{5}\right) e^{-\sqrt{5}t} + \frac{6}{5}t^2 - \frac{6}{5}t + \frac{7}{25}$$

6-53

(a)

$$x(t) = C_1 e^{-2\sqrt{3}t} + C_2 e^{2\sqrt{3}t}$$

$$y(t) = C_1\left(2 + \sqrt{3}\right) e^{-2\sqrt{3}t} + C_2\left(2 - \sqrt{3}\right) e^{2\sqrt{3}t}$$

(b)

$$x(t) = C_1 e^{-2\sqrt{3}t} + C_2 e^{2\sqrt{3}t} - \frac{1}{3}t^2 - t + \frac{17}{18}$$

$$y(t) = C_1\left(2 + \sqrt{3}\right) e^{-2\sqrt{3}t} + C_2\left(2 - \sqrt{3}\right) e^{2\sqrt{3}t} - \frac{1}{6}t^2 - \frac{5}{3}t + \frac{8}{9}$$

6-55

(a)

$$x(t) = C_1 e^{t} \sin \sqrt{5}\, t + C_2 e^{t} \cos \sqrt{5}\, t$$

$$y(t) = -\frac{C_1}{\sqrt{5}} e^{t} \cos \sqrt{5}\, t + \frac{C_2}{\sqrt{5}} e^{t} \sin \sqrt{5}\, t$$

(b)

$$x(t) = C_1 e^{t} \sin \sqrt{5}\, t + C_2 e^{t} \cos \sqrt{5}\, t + 2$$

$$y(t) = -\frac{C_1}{\sqrt{5}} e^t \cos\sqrt{5}\,t + \frac{C_2}{\sqrt{5}} e^t \sin\sqrt{5}\,t + 1$$

6-57

(a)

$$x(t) = C_1 e^{2t} + C_2 e^{-2t}$$

$$y(t) = \frac{1}{2}(C_1 e^{2t} - C_2 e^{-2t})$$

(b)

$$x(t) = C_1 e^{2t} + C_2 e^{-2t} - \frac{4}{3} e^t + 3$$

$$y(t) = \frac{1}{2}(C_1 e^{2t} - C_2 e^{-2t}) - \frac{1}{3} e^t - \frac{1}{4}$$

6-59

(a)

$$x(t) \cong 33.74695 C_1 e^{-3.2443t} + e^{2.62215t}[(0.260655C_2 - 0.37348C_3)\cos 1.067t$$
$$+(-0.37348C_2 - 0.260655C_3)\sin 0.8297t]$$

$$y(t) \cong -6.2443 C_1 e^{-3.2443t} + e^{2.62215t}[(1.067C_2 - 0.37785C_3)\cos 1.067t$$
$$+(-0.37785C_2 - 1.067C_3)\sin 1.067t]$$

$$z(t) \cong C_1 e^{-3.2443t} + e^{2.62215t}(C_2 \sin 1.067t + C_3 \cos 1.067t)$$

(b)

$$x(t) \cong 33.74695 C_1 e^{-3.2443t} + e^{2.62215t}[(0.260655C_2 - 0.37348C_3)\cos 1.067t$$
$$+(-0.37348C_2 - 0.260655C_3)\sin 0.8297t] + 0.26923077t^2 + 0.3787t + 0.36481$$

$$y(t) \cong -6.2443 C_1 e^{-3.2443t} + e^{2.62215t}[(1.067C_2 - 0.37785C_3)\cos 1.067t$$
$$+(-0.37785C_2 - 1.067C_3)\sin 1.067t] - 0.11538t^2 - 0.043923t + 1.0355$$

$$z(t) \cong C_1 e^{-3.2443t} + e^{2.62215t}(C_2 \sin 1.067t + C_3 \cos 1.067t) + \frac{1}{26}t^2 - \frac{54}{169}t - \frac{3333}{4394}$$

6-61

(a)

$$x(t) = C_1 \frac{\sin\left(\frac{\sqrt{71}}{2}\ln t\right)}{\sqrt{t}} + C_2 \frac{\cos\left(\frac{\sqrt{71}}{2}\ln t\right)}{\sqrt{t}} + \frac{5}{3}$$

$$y(t) = \frac{C_1}{12}t^2\left[\frac{\sqrt{71}\cos\left(\frac{\sqrt{71}}{2}\ln t\right)}{\sqrt{t^3}} - \frac{\sin\left(\frac{\sqrt{71}}{2}\ln t\right)}{\sqrt{t^3}}\right] - \frac{C_2}{12}t^2\left[\frac{\cos\left(\frac{\sqrt{71}}{2}\ln t\right)}{\sqrt{t^3}} - \frac{\sqrt{71}\sin\left(\frac{\sqrt{71}}{2}\ln t\right)}{\sqrt{t^3}}\right]$$

(b)

$$x(t) = C_1\frac{\sin\left(\frac{\sqrt{7}}{2}\ln t\right)}{\sqrt{t}} + C_2\frac{\cos\left(\frac{\sqrt{7}}{2}\ln t\right)}{\sqrt{t}} + 2t - 1$$

$$y(t) = -\frac{C_1}{4}t^2\left[\frac{\sqrt{7}\cos\left(\frac{\sqrt{7}}{2}\ln t\right)}{\sqrt{t^3}} - \frac{\sin\left(\frac{\sqrt{7}}{2}\ln t\right)}{\sqrt{t^3}}\right] + \frac{C_2}{4}t^2\left[\frac{\cos\left(\frac{\sqrt{7}}{2}\ln t\right)}{\sqrt{t^3}} + \frac{\sqrt{71}\sin\left(\frac{\sqrt{7}}{2}\ln t\right)}{\sqrt{t^3}}\right] + 2t^2$$

6-63

$$x(t) = \left(\frac{33}{49} - \frac{121\sqrt{2}}{196}\right)e^{(3+\sqrt{2})t} + \left(\frac{33}{49} + \frac{121\sqrt{2}}{196}\right)e^{(3-\sqrt{2})t} - \frac{4}{7}t - \frac{17}{49}$$

$$y(t) = -\left(\frac{33}{49} - \frac{121\sqrt{2}}{196}\right)(\sqrt{2}+1)e^{(3+\sqrt{2})t} + \left(\frac{33}{49} + \frac{121\sqrt{2}}{196}\right)(\sqrt{2}-1)e^{(3-\sqrt{2})t} - \frac{1}{7}t - \frac{6}{49}$$

6-65

$$x(t) = \frac{287\sqrt{71}}{639}e^{-\frac{1}{2}t}\sin\frac{\sqrt{71}}{2}t + \frac{17}{9}e^{-\frac{1}{2}t}\cos\frac{\sqrt{71}}{2}t + \frac{1}{9}$$

$$y(t) = \frac{517\sqrt{71}}{1278}e^{-\frac{1}{2}t}\sin\frac{\sqrt{71}}{2}t - \frac{59}{18}e^{-\frac{1}{2}t}\cos\frac{\sqrt{71}}{2}t + \frac{5}{18}$$

6-67 $x(t) = 0$ and $y(t) = 0$

6-69

$$h_2'' + \frac{5B}{3}h_2' + \frac{1}{3}B^2h_2 = \frac{q_{mi}}{3\rho A}(q_{mi}' + Bq_{mi})$$

6-73
(a)

$$x(t) = C_1e^{2t} + C_2e^{-2t}$$

$$y(t) = -C_1e^{2t} + 3C_2e^{-2t}$$

(b)

$$x(t) = C_1e^{2t} + C_2e^{-2t} - \frac{1}{4}t^2 + \frac{1}{4}t + \frac{1}{8}$$

$$y(t) = -C_1e^{2t} + 3C_2e^{-2t} + \frac{3}{4}t^2 + \frac{3}{4}t - \frac{9}{8}$$

6-75
(a)

$$x(t) = C_1 e^{(2+2\sqrt{6})t} + C_2 e^{(2-2\sqrt{6})t}$$

$$y(t) = C_1(-5+2\sqrt{6})e^{(2+2\sqrt{6})t} + C_2(-5-2\sqrt{6})e^{(2-2\sqrt{6})t}$$

(b)

$$x(t) = C_1 e^{(2+2\sqrt{6})t} + C_2 e^{(2-2\sqrt{6})t} + \frac{1}{10}$$

$$y(t) = C_1(-5+2\sqrt{6})e^{(2+2\sqrt{6})t} + C_2(-5-2\sqrt{6})e^{(2-2\sqrt{6})t} + \frac{3}{10}$$

6-77
(a)

$$x(t) = C_1 e^{\sqrt{7}t} + C_2 e^{-\sqrt{7}t}$$

$$y(t) = \frac{C_1}{2}(1+\sqrt{7})e^{\sqrt{7}t} + \frac{C_2}{2}(1-\sqrt{7})e^{-\sqrt{7}t}$$

(b)

$$x(t) = C_1 e^{\sqrt{7}t} + C_2 e^{-\sqrt{7}t} + \frac{3}{11}\sin 2t - \frac{6}{11}\cos 2t + \frac{4}{7}$$

$$y(t) = \frac{C_1}{2}(1+\sqrt{7})e^{\sqrt{7}t} + \frac{C_2}{2}(1-\sqrt{7})e^{-\sqrt{7}t} - \frac{9}{11}\sin 2t + \frac{2}{7}$$

6-79
(a)

$$x(t) = C_1 e^{(4+\sqrt{3})t} + C_2 e^{(4-\sqrt{3})t}$$

$$y(t) = C_1(2-\sqrt{3})e^{(4+\sqrt{3})t} + C_2(2+\sqrt{3})e^{(4-\sqrt{3})t}$$

(b)

$$x(t) = C_1 e^{(4+\sqrt{3})t} + C_2 e^{(4-\sqrt{3})t} + e^{2t} + \frac{1}{13}$$

$$y(t) = C_1(2-\sqrt{3})e^{(4+\sqrt{3})t} + C_2(2+\sqrt{3})e^{(4-\sqrt{3})t} + (t+4)e^{2t} + \frac{6}{13}$$

6-81
(a)

$$x(t) = C_1 e^{(1+4\sqrt{3})t} + C_2 e^{1-4\sqrt{3}t}$$

$$y(t) = \left(1 + \frac{2\sqrt{3}}{3}\right) C_1 e^{(1+4\sqrt{3})t} + \left(1 - \frac{2\sqrt{3}}{3}\right) C_2 e^{1-4\sqrt{3}t}$$

(b)

$$x(t) = C_1 e^{(1+4\sqrt{3})t} + C_2 e^{(1-4\sqrt{3})t} - \frac{6}{47}t^2 + \frac{24}{2209}t + \frac{14851}{103823}$$

$$y(t) = \left(1 + \frac{2\sqrt{3}}{3}\right) C_1 e^{(1+4\sqrt{3})t} + \left(1 - \frac{2\sqrt{3}}{3}\right) C_2 e^{(1-4\sqrt{3})t} - \frac{5}{47}t^2 - \frac{74}{2209}t - \frac{4740}{103823}$$

6-83

$$x(t) = \left(\frac{16}{49} - \frac{75\sqrt{2}}{1916}\right) e^{(3+\sqrt{2})t} + \left(\frac{16}{49} + \frac{75\sqrt{2}}{1916}\right) e^{(3-\sqrt{2})t} + \frac{4}{7}t + \frac{17}{49}$$

$$y(t) = \left(\frac{43}{98} + \frac{11\sqrt{2}}{196}\right) e^{(3+\sqrt{2})t} + \left(\frac{43}{98} - \frac{11\sqrt{2}}{196}\right) e^{(3-\sqrt{2})t} + \frac{1}{7}t + \frac{6}{49}$$

6-85

$$x(t) \cong -0.00244e^{3t} - 2.30616e^{-2t} - \frac{1}{6}t + \frac{19}{36}$$

$$y(t) \cong -0.00061e^{3t} + 2.30616e^{-2t} - \frac{1}{6}t - \frac{11}{36}$$

6-87

$$LI\omega'' + RL\omega' + K_T K_b \omega = -LT_L' - RT_L + K_T V_a$$

6-91

$$\omega = \sqrt{\frac{4 + 2\sqrt{7}}{3}\alpha}, \sqrt{\frac{-4 + 2\sqrt{7}}{3}\alpha}$$

$$\frac{A_1}{A_2} = \frac{3}{2 + 2\sqrt{7}}$$

6-93

$$\left\{ x1(t) = \frac{55}{3} + \frac{1}{20} e^{-\frac{21}{400}t} \left(-\sin\left(\frac{3}{400}\sqrt{31}\, t\right) \_C2 - 3\cos\left(\frac{3}{400}\sqrt{31}\, t\right)\sqrt{31}\, \_C2 - \cos\left(\frac{3}{400}\sqrt{31}\, t\right) \_C1 + 3\sin\left(\frac{3}{400}\sqrt{31}\, t\right)\sqrt{31}\, \_C1 \right), x2(t) = -\frac{50}{3} + e^{-\frac{21}{400}t} \left(\sin\left(\frac{3}{400}\sqrt{31}\, t\right) \_C2 + \cos\left(\frac{3}{400}\sqrt{31}\, t\right) \_C1 \right) \right\}$$

6-95

(a)

$$\left\{x(t) = \_C1\, e^{\frac{1}{2}(5+\sqrt{13})t} + \_C2\, e^{-\frac{1}{2}(-5+\sqrt{13})t}, y(t) = -\frac{3}{2}\_C1\, e^{\frac{1}{2}(5+\sqrt{13})t} - \frac{1}{2}\_C1\, e^{\frac{1}{2}(5+\sqrt{13})t}\sqrt{13} - \frac{3}{2}\_C2\, e^{-\frac{1}{2}(-5+\sqrt{13})t} + \frac{1}{2}\_C2\, e^{-\frac{1}{2}(-5+\sqrt{13})t}\sqrt{13}\right\}$$

(b)

$$\left\{x(t) = e^{\frac{1}{2}(5+\sqrt{13})t}\_C2 + e^{-\frac{1}{2}(-5+\sqrt{13})t}\_C1 + \frac{4}{3}t + t\,e^{t} - 3\,e^{t} + \frac{5}{9}, y(t) = -\frac{3}{2}e^{\frac{1}{2}(5+\sqrt{13})t}\_C2 - \frac{1}{2}e^{\frac{1}{2}(5+\sqrt{13})t}\_C2\sqrt{13} - \frac{3}{2}e^{-\frac{1}{2}(-5+\sqrt{13})t}\_C1 + \frac{1}{2}e^{-\frac{1}{2}(-5+\sqrt{13})t}\_C1\sqrt{13} + \frac{2}{9} - e^{t} + \frac{1}{3}t\right\}$$

6-97

$$x(t) = \frac{389\sqrt{95}}{32110}e^{3/2t}\sin\left(\frac{1}{2}\sqrt{95}t\right) + \frac{373}{338}e^{3/2t}\cos\left(\frac{1}{2}\sqrt{95}t\right) - \frac{35}{338} - \frac{3}{13}t$$

$$y(t) = \frac{5841\sqrt{95}}{64220}e^{3/2t}\sin\left(\frac{1}{2}\sqrt{95}t\right) - \frac{127}{676}e^{3/2t}\cos\left(\frac{1}{2}\sqrt{95}t\right) + \frac{127}{676} - \frac{1}{26}t$$

6-99

$$x(t) = \frac{1}{9} + e^{-1/2t}\left[\frac{287\sqrt{71}}{639}\sin\left(\frac{1}{2}\sqrt{71}t\right) + \frac{17}{9}\cos\left(\frac{1}{2}\sqrt{71}t\right)\right]$$

$$y(t) = \frac{5}{18} + e^{-1/2t}\left[\frac{517\sqrt{71}}{1278}\sin\left(\frac{1}{2}\sqrt{71}t\right)\sqrt{71} - \frac{59}{18}\cos\left(\frac{1}{2}\sqrt{71}t\right)\right]$$

6-101

$$x(t) = C_1 e^{2t} + C_2 e^{\frac{1}{2}(5+\sqrt{29})t} + C_3 e^{\frac{1}{2}(5-\sqrt{29})t}$$

$$y(t) = C_1 e^{2t} + \frac{C_2}{6}\left(5-\sqrt{29}\right)e^{\frac{1}{2}(5+\sqrt{29})t} + \frac{C_3}{6}\left(5+\sqrt{29}\right)e^{\frac{1}{2}(5-\sqrt{29})t}$$

$$z(t) = \frac{C_2}{3}\left(6-\sqrt{29}\right)e^{\frac{1}{2}(5+\sqrt{29})t} + \frac{C_3}{6}\left(6+\sqrt{29}\right)e^{\frac{1}{2}(5-\sqrt{29})t}$$

6-103

$$x(t) = -19C_1 e^{-3t} - \frac{1}{3}C_2 e^{\frac{5}{2}t}\sin\frac{\sqrt{103}}{2}t - \frac{1}{3}C_3 e^{\frac{5}{2}t}\cos\frac{\sqrt{103}}{2}t$$

$$y(t) = 2C_1 e^{-3t} + \frac{1}{6}\left(C_2 + \sqrt{103}C_3\right)e^{\frac{5}{2}t}\sin\frac{\sqrt{103}}{2}t + \frac{1}{6}\left(C_3 - \sqrt{103}C_2\right)e^{\frac{5}{2}t}\cos\frac{\sqrt{103}}{2}t$$

$$z(t) = C_1 e^{-3t} + C_2 e^{\frac{5}{2}t}\sin\frac{\sqrt{103}}{2}t + C_3 e^{\frac{5}{2}t}\cos\frac{\sqrt{103}}{2}t$$

CHAPTER 7

7-39

(a) $\mathbf{A}+\mathbf{B}=\begin{pmatrix}3 & -3\\ 1 & 8\end{pmatrix}$, **(b)** $2\mathbf{A}=\begin{pmatrix}4 & 0\\ -14 & 10\end{pmatrix}$

(c) $3\mathbf{A}-\mathbf{B}=\begin{pmatrix}5 & 3\\ -29 & 12\end{pmatrix}$, **(d)** $-3\mathbf{AB}=\begin{pmatrix}-6 & 18\\ -99 & -108\end{pmatrix}$

7-41

(a) $5\mathbf{A}=\begin{pmatrix}35 & -15\\ 30 & 60\end{pmatrix}$, **(b)** $2\mathbf{A}+3\mathbf{B}=\begin{pmatrix}47 & -33\\ 24 & 27\end{pmatrix}$,

(c) $2\mathbf{AB}=\begin{pmatrix}130 & -132\\ 228 & -84\end{pmatrix}$, **(d)** $det\mathbf{A}=102$

7-43

(a) $\mathbf{A}-4\mathbf{B}=\begin{pmatrix}16 & -18 & 23\\ -5 & -12 & 3\\ 9 & -36 & -2\end{pmatrix}$, **(b)** $\mathbf{AB}=\begin{pmatrix}-23 & 37 & -17\\ -6 & 23 & 8\\ 3 & 6 & 17\end{pmatrix}$

c) $\mathbf{BA}=\begin{pmatrix}-1 & 6 & -7\\ 1 & -2 & 12\\ -24 & 6 & 20\end{pmatrix}$, **(d)** $det\mathbf{B}=-103$

7-45

(a) $\mathbf{A}+\mathbf{B}=\begin{pmatrix}18 & -12\\ 10 & 13\end{pmatrix}$, $\mathbf{B}+\mathbf{A}=\begin{pmatrix}18 & -12\\ 10 & 13\end{pmatrix}$

b) $2(\mathbf{A}+\mathbf{B})=\begin{pmatrix}36 & -24\\ 20 & 26\end{pmatrix}$, $2\mathbf{A}+2\mathbf{B}=\begin{pmatrix}36 & -24\\ 20 & 26\end{pmatrix}$

c) $\mathbf{AB}=\begin{pmatrix}65 & -66\\ 114 & -42\end{pmatrix}$, $\mathbf{BA}=\begin{pmatrix}23 & -141\\ 34 & 0\end{pmatrix}$

7-47

(a) $(\mathbf{A}+\mathbf{B})+\mathbf{C}=\begin{pmatrix}0 & 8\\ 11 & 8\end{pmatrix}$, $\mathbf{A}+(\mathbf{B}+\mathbf{C})=\begin{pmatrix}0 & 8\\ 11 & 8\end{pmatrix}$

b) $\mathbf{A}(\mathbf{BC}) = \begin{pmatrix} 168 & -12 \\ -7 & -71 \end{pmatrix}, \qquad (\mathbf{AB})\mathbf{C} = \begin{pmatrix} 168 & -12 \\ -7 & -71 \end{pmatrix}$

c) $\mathbf{A}(\mathbf{B}+\mathbf{C}) = \begin{pmatrix} 56 & 31 \\ 38 & 12 \end{pmatrix}, \qquad \mathbf{AB}+\mathbf{AC} = \begin{pmatrix} 56 & 31 \\ 38 & 12 \end{pmatrix}$

7-49

(a)

$$\int_0^t \mathbf{A}dt = \begin{pmatrix} \ln\frac{1}{1-t} & 1-3\cos 3t \\ \frac{1}{2}-\frac{1}{2}e^{-2t} & \frac{1}{2}t^2+t \end{pmatrix}$$

(b)

$$\frac{d\mathbf{A}}{dt} = \begin{pmatrix} \frac{1}{(1-t)^2} & 9\cos 3t \\ -2e^{-2t} & 1 \end{pmatrix}$$

7-51

(a)

$$\mathbf{B}\int_0^1 \mathbf{A}dt = \begin{pmatrix} \frac{11}{3}-e & \frac{7}{2}-\frac{4}{e}+\frac{1}{2}\cos 2 \\ -\frac{17}{3}+7e & \frac{11}{2}-\frac{2}{e}-\frac{7}{2}\cos 2 \end{pmatrix}$$

(b)

$$\int_0^1 (\mathbf{BA})dt = \begin{pmatrix} \frac{11}{3}-e & \frac{7}{2}-\frac{4}{e}+\frac{1}{2}\cos 2 \\ -\frac{17}{3}+7e & \frac{11}{2}-\frac{2}{e}-\frac{7}{2}\cos 2 \end{pmatrix}$$

(c)

$$\mathbf{B}\frac{d\mathbf{A}}{dt} = \begin{pmatrix} \frac{2}{\sqrt{t}}-e^t & -4e^{-t}-2\cos 2t \\ \frac{1}{\sqrt{t}}+7e^t & -2e^{-t}+14\cos 2t \end{pmatrix}$$

(d)

$$\frac{d}{dt}(\mathbf{BA}) = \begin{pmatrix} \frac{2}{\sqrt{t}}-e^t & -4e^{-t}-2\cos 2t \\ \frac{1}{\sqrt{t}}+7e^t & -2e^{-t}+14\cos 2t \end{pmatrix}$$

7-53 $m_1 x_1'' = -k_1 x_1 + k_2(x_2 - x_1) - c_1 x_1' + c_2(x_2' - x_1')$

$m_2 x_2'' = f - k_2(x_2 - x_1) - c_2(x_2' - x_1')$

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_2 + k_1}{m_1} & \frac{k_2}{m_1} & -\frac{c_1 + c_2}{m_1} & \frac{c_2}{m_1} \\ \frac{k_2}{m_2} & \frac{-k_2}{m_2} & \frac{c_2}{m_2} & -\frac{c_2}{m_2} \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

7-65
(a)

$$\mathbf{A^{-1}} = \begin{pmatrix} \frac{2}{17} & \frac{1}{34} \\ -\frac{1}{17} & \frac{7}{102} \end{pmatrix}$$

(b)

$$\mathbf{B^{-1}} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{3}{2} \\ -\frac{1}{3} & \frac{1}{3} & -\frac{5}{3} \end{pmatrix}$$

7-67
(a)

$$\mathbf{A^{-1}} = \begin{pmatrix} 1 & 0 \\ \frac{2}{5} & \frac{1}{5} \end{pmatrix}$$

(b)

$$\mathbf{B^{-1}} = \begin{pmatrix} \frac{8}{1727} & -\frac{62}{1727} & -\frac{166}{1727} & \frac{89}{1727} \\ \frac{298}{1727} & \frac{281}{1727} & -\frac{139}{1727} & \frac{293}{1727} \\ \frac{122}{1727} & -\frac{82}{1727} & -\frac{59}{1727} & \frac{62}{1727} \\ -\frac{273}{3554} & -\frac{43}{3454} & \frac{26}{1727} & \frac{201}{3454} \end{pmatrix}$$

7-69
(a)

$$\mathbf{A}^{-1} = \begin{pmatrix} \frac{1}{47} & \frac{9}{47} \\ -\frac{4}{47} & \frac{11}{47} \end{pmatrix}$$

(b)

$$\mathbf{B}^{-1} = \begin{pmatrix} \frac{1}{17} & 0 & \frac{3}{17} \\ -\frac{1}{2} & 1 & 0 \\ -\frac{5}{34} & 0 & \frac{1}{17} \end{pmatrix}$$

7-71
(a)

$$\mathbf{A}^{-1} = \begin{pmatrix} \frac{1}{11} & \frac{1}{11} \\ -\frac{8}{33} & \frac{1}{11} \end{pmatrix}$$

(b)

$$\mathbf{B}^{-1} = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} & -\frac{1}{8} \\ \frac{1}{2} & 1 & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & \frac{1}{16} \end{pmatrix}$$

7-73
(a)

$$\mathbf{A}^{-1} = \begin{pmatrix} -\frac{2}{39} & \frac{7}{39} \\ \frac{5}{39} & \frac{2}{39} \end{pmatrix}$$

(b) The inverse of the square matrix **B** does not exist. This is a singular matrix.

7-75 (*This problem is identical with 7-14, and will be removed in the second press run*)

(a)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

(b)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

(c)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} -\frac{3}{4} \\ -\frac{1}{8} \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{5}{2} \\ \frac{5}{4} \\ 0 \end{pmatrix}$$

(d)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{9}{7} \\ \frac{5}{7} \\ \frac{16}{7} \end{pmatrix}$$

7-77

(a)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} \frac{2}{5} \\ -\frac{7}{5} \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{8}{5} \\ \frac{2}{5} \\ 0 \end{pmatrix}$$

(b)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} \frac{2}{5} \\ -\frac{7}{5} \\ 1 \end{pmatrix}$$

(c) The system has no solution.

(d)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$$

7-79

(a)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} -\frac{3}{7} \\ \frac{5}{7} \\ 1 \end{pmatrix} + \begin{pmatrix} -\frac{13}{7} \\ \frac{31}{7} \\ 0 \end{pmatrix}$$

(b)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} -\frac{3}{7} \\ \frac{5}{7} \\ 1 \end{pmatrix}$$

(c)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -\frac{9}{5} \\ 3 \\ -\frac{14}{5} \end{pmatrix}$$

(d)

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -\frac{48}{25} \\ \frac{16}{5} \\ -\frac{63}{25} \end{pmatrix}$$

7-81 The vectors are linearly independent.

7-83 The vectors are linearly independent.

7-85 The vectors are linearly independent.

7-87 The vectors are linearly independent in the given interval.

7-89 The vectors are linearly dependent in $-\infty < t < \infty$.

7-91

(a) $\mathbf{v}_1 = \begin{pmatrix} \frac{1}{2} + \frac{1}{2}i \\ 1 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} \frac{1}{2} - \frac{1}{2}i \\ 1 \end{pmatrix}$

(b) $\mathbf{v}_1 = \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} \frac{3}{2} \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{v}_3 = \begin{pmatrix} -\frac{1}{6} \\ -\frac{2}{3} \\ 1 \end{pmatrix}$

7-93

(a) $\mathbf{v}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

(b) $\mathbf{v}_1 = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ $\mathbf{v}_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ and $\mathbf{v}_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

7-95

(a) $\mathbf{v}_1 = \begin{pmatrix} \frac{3}{2}i \\ 1 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} -\frac{3}{2}i \\ 1 \end{pmatrix}$

(b) $\mathbf{v}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 1 \\ \frac{2}{5} \\ 1 \end{pmatrix}$ and $\mathbf{v}_3 = \begin{pmatrix} -1 \\ -\frac{4}{7} \\ 1 \end{pmatrix}$

7-97

(a) $\mathbf{v}_1 = \begin{pmatrix} \frac{\sqrt{6}}{4}i \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} -\frac{\sqrt{6}}{4}i \\ 1 \end{pmatrix}$

(b) $\mathbf{v}_1 = \begin{pmatrix} -4 \\ -2 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} -\frac{1}{6} - \frac{\sqrt{11}}{3}i \\ -\frac{1}{3} - \frac{\sqrt{11}}{3}i \\ 1 \end{pmatrix}$ and $\mathbf{v}_3 = \begin{pmatrix} -\frac{1}{6} + \frac{\sqrt{11}}{3}i \\ -\frac{1}{3} + \frac{\sqrt{11}}{3}i \\ 1 \end{pmatrix}$

7-99

(a) $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} -\frac{7}{3} \\ 1 \end{pmatrix}$

(b) $\mathbf{v}_1 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} \frac{\sqrt{2}i-2}{-2+3\sqrt{2}i} \\ -\frac{\sqrt{2}i+6}{-2+3\sqrt{2}i} \\ 1 \end{pmatrix}$ and $\mathbf{v}_3 = \begin{pmatrix} \frac{\sqrt{2}i+2}{2+3\sqrt{2}i} \\ \frac{-\sqrt{2}i+6}{2+3\sqrt{2}i} \\ 1 \end{pmatrix}$

7-105 $\mathbf{x}_1$ and $\mathbf{x}_2$ are not solutions to the given system, and they are linearly independent.

7-107 $\mathbf{x}_1$ and $\mathbf{x}_2$ are the solutions to the given system, and they are linearly dependent.

7-109 $\mathbf{x}_1$ and $\mathbf{x}_2$ are the solutions to the given system, and they are linearly independent. Thus, the general solution of the given system is

$$\mathbf{x} = C_1\mathbf{x}_1 + C_2\mathbf{x}_2 = C_1\begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2\begin{pmatrix} e^{-5t} \\ -2e^{-5t} \end{pmatrix}$$

7-111 $\mathbf{x}_1, \mathbf{x}_2$ and $\mathbf{x}_3$ are not solutions to the given system, and they are linearly dependent.

7-113 $\mathbf{x}_1, \mathbf{x}_2$ and $\mathbf{x}_3$ are the solutions to the given system, and they are linearly dependent

7-115 The vector $\mathbf{x}_p$ satisfies the given system, and it is a solution.

7-117 The vector $\mathbf{x}_p$ does **not** satisfy the given system, and it is not a particular solution.

7-119 The vector $\mathbf{x}_p$ does **not** satisfy the given system.

7-125 $\mathbf{x} = C_1\mathbf{x}_1 + C_2\mathbf{x}_2 = C_1\begin{pmatrix} \frac{1}{4} \\ 1 \end{pmatrix} e^{-3t} + C_2\begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t}$

7-127 $\mathbf{x} = C_1\mathbf{x}_1 + C_2\mathbf{x}_2 = C_1\begin{pmatrix} \frac{1}{-5+2\sqrt{6}} \\ 1 \end{pmatrix} e^{(2+2\sqrt{6})t} + C_2\begin{pmatrix} -\frac{1}{5+2\sqrt{6}} \\ 1 \end{pmatrix} e^{(2-2\sqrt{6})t}$

7-129 $\mathbf{x} = C_1\mathbf{x}_1 + C_2\mathbf{v}_2 = C_1\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} e^{3t} + C_2\begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-3t}$

7-131 $\mathbf{x} = C_1\mathbf{x}_1 + C_2\mathbf{v}_2 = C_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3t} + C_2\begin{pmatrix} 1+t \\ t \end{pmatrix} e^{3t}$

7-133 $\mathbf{x} = C_1\mathbf{x}_1 + C_2\mathbf{x}_2 = C_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{9t} + C_2\begin{pmatrix} -3 \\ 1 \end{pmatrix} e^{t}$

7-135 $\mathbf{x} = C_1\mathbf{x}_1 + C_2\mathbf{x}_2 + C_3\mathbf{x}_3 = C_1\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} e^{3t} + C_2\begin{pmatrix} \frac{2}{5}\cos t - \frac{6}{5}\sin t \\ \frac{2}{5}\cos t - \frac{1}{5}\sin t \\ \cos t \end{pmatrix} + C_3\begin{pmatrix} \frac{6}{5}\cos t + \frac{2}{5}\sin t \\ \frac{1}{5}\cos t + \frac{2}{5}\sin t \\ \sin t \end{pmatrix}$

7-137 $\mathbf{x} = \begin{pmatrix} e^t \\ e^t \end{pmatrix}$

7-139 $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{2}{7e^3}\begin{pmatrix} 6 \\ 1 \end{pmatrix} e^{3t} + \frac{12}{7e^{-4}}\begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-4t}$

7-147 $\mathbf{x} = C_1\begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t} + C_2\begin{pmatrix} \frac{1}{4} \\ 1 \end{pmatrix} e^{-3t} + \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix} \cos 3t + \begin{pmatrix} \frac{1}{6} \\ -\frac{1}{3} \end{pmatrix} \sin 3t$

7-149 $\mathbf{x} = C_1\begin{pmatrix} \frac{1}{-3+2\sqrt{2}} \\ 1 \end{pmatrix} e^{2\sqrt{2}t} + C_2\begin{pmatrix} -\frac{1}{3+2\sqrt{2}} \\ 1 \end{pmatrix} e^{-2\sqrt{2}t} + \begin{pmatrix} -\frac{8}{7} \\ \frac{2}{7} \end{pmatrix} te^{t} + \begin{pmatrix} -\frac{23}{49} \\ -\frac{10}{49} \end{pmatrix} e^{t}$

7-151 $\mathbf{x} = C_1\begin{pmatrix}\frac{1}{3}\\1\end{pmatrix}e^{5t} + C_2\begin{pmatrix}-2\\1\end{pmatrix}e^{-2t} + \begin{pmatrix}-\frac{1}{5}\\-\frac{3}{5}\end{pmatrix}t + \begin{pmatrix}\frac{24}{25}\\-\frac{31}{50}\end{pmatrix}$

7-153 $\mathbf{x} = C_1\begin{pmatrix}1\\1\end{pmatrix}e^{4t} + C_2\begin{pmatrix}\frac{1}{2}\\1\end{pmatrix}e^{3t} + \begin{pmatrix}\frac{1}{75}\\\frac{26}{75}\end{pmatrix}\cos 3t + \begin{pmatrix}-\frac{7}{75}\\-\frac{32}{75}\end{pmatrix}\sin 3t$

7-155 $\mathbf{x} = C_1\begin{pmatrix}\frac{1}{2}\\1\end{pmatrix}e^{8t} + C_2\begin{pmatrix}-6\\1\end{pmatrix}e^{-5t} + \begin{pmatrix}\frac{27}{20}\\-\frac{1}{10}\end{pmatrix}$

7-157

$$\mathbf{x} = C_1\begin{pmatrix}1\\0\\1\end{pmatrix}e^{t} + C_2\begin{pmatrix}\frac{6}{5}\cos t - \frac{3}{5}\sin t\\ \frac{1}{5}\cos t + \frac{2}{5}\sin t\\ \cos t\end{pmatrix}e^{t} + C_3\begin{pmatrix}\frac{3}{5}\cos t + \frac{6}{5}\sin t\\ -\frac{2}{5}\cos t + \frac{1}{5}\sin t\\ \sin t\end{pmatrix}e^{t} + \frac{1}{2}\begin{pmatrix}1\\1\\9\end{pmatrix}t + \begin{pmatrix}11\\\frac{9}{2}\\\frac{29}{2}\end{pmatrix}$$

7-159

$$x_1(t) = -\frac{2}{5}e^{5t} - e^{t} + te^{t} + \frac{12}{5}$$
$$x_2(t) = \frac{6}{5}e^{5t} - 2e^{t} + te^{t} + \frac{9}{5}$$

7-161

$$x_1(t) = \frac{16}{7}e^{3(t-1)} - \frac{235}{112}e^{-4(t-1)} - \frac{5}{4}t + \frac{17}{16}$$
$$x_2(t) = \frac{8}{21}e^{3(t-1)} + \frac{235}{112}e^{-4(t-1)} + \frac{1}{4}t + \frac{35}{48}$$

7-163

$$x_1(t) = 2e^{-t} + 2\sin t + 2\cos t - 4$$
$$x_2(t) = -4e^{-t} - 2\sin t + 4$$

7-165

(a) $\mathbf{A}^3 = \begin{bmatrix}63 & 62\\62 & 63\end{bmatrix}$ **(b)** $\mathbf{A}^{-1}\mathbf{I} = \mathbf{A}^{-1} = (6\mathbf{I} - \mathbf{A})/5$ $\mathbf{A}^{-1} = \frac{1}{5}\begin{bmatrix}3 & -2\\-2 & 3\end{bmatrix}$

7-167 Only the second mode is controllable.

7-169 The truncated series solution gives $\boldsymbol{\varphi}(0.1) = \begin{pmatrix}0.9 & 0.107\\-0.3 & 0.51\end{pmatrix}$

whereas from the example $\boldsymbol{\varphi}(0.1) = \begin{pmatrix}0.9002 & 0.0707\\-0.0707 & 0.4651\end{pmatrix}$

7-171

(a) $\boldsymbol{\varphi}(\mathbf{t}) = \begin{pmatrix} \frac{e^{2t}+1}{2e^{4t}} & 3\frac{e^{2t}-1}{2e^{4t}} \\ \frac{e^{2t}-1}{6e^{4t}} & \frac{e^{2t}+1}{2e^{4t}} \end{pmatrix}$, **(b)** $\boldsymbol{\varphi}(\mathbf{t}) = \begin{pmatrix} e^t & te^t \\ 0 & e^t \end{pmatrix}$,

(c) $\boldsymbol{\varphi}(\mathbf{t}) = \begin{pmatrix} e^{3t} & 0 \\ 0 & e^{3t} \end{pmatrix}$, **(d)** $\boldsymbol{\varphi}(\mathbf{t}) = \begin{pmatrix} e^{-t}\left(\cos 3t + \frac{1}{3}\sin 3t\right) & \frac{1}{3}e^{-t}\sin 3t \\ -\frac{10}{3}e^{-t}\sin 3t & e^{-t}\left(\cos 3t - \frac{1}{3}\sin 3t\right) \end{pmatrix}$

7-173

(a) $\mathbf{A} = \begin{bmatrix} -5 & 3 \\ 0 & -4 \end{bmatrix}$ $\quad \mathbf{B} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$ $\quad \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad \mathbf{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} -5 & 3 \\ 1 & -4 \end{bmatrix}$ $\quad \mathbf{B} = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$ $\quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$ $\quad \mathbf{D} = \begin{bmatrix} 0 & 0 \end{bmatrix}$

7-181

$$x_1(t) = \frac{20\sqrt{2}}{21}\sin\frac{\sqrt{2}}{2}t - \frac{5\sqrt{2}}{6}\sin\sqrt{2}t + \frac{5}{14}\sin 2t$$

$$x_2(t) = \frac{40\sqrt{2}}{21}\sin\frac{\sqrt{2}}{2}t + \frac{5\sqrt{2}}{6}\sin\sqrt{2}t - \frac{25}{14}\sin 2t$$

7-183

$$\mathbf{x} = C_1\begin{pmatrix} -e^{3t}\cos 3t - 3e^{3t}\sin 3t \\ e^{3t}\cos 3\,t \end{pmatrix} + C_2\begin{pmatrix} 3e^{3t}\cos 3t - e^{3t}\sin 3t \\ e^{3t}\sin 3t \end{pmatrix}$$

$$+\begin{pmatrix} -\frac{2}{9} \\ \frac{1}{18} \end{pmatrix}t^3 + \begin{pmatrix} -\frac{11}{18} \\ -\frac{1}{18} \end{pmatrix}t^2 + \begin{pmatrix} -\frac{1}{3} \\ \frac{1}{18} \end{pmatrix}t + \begin{pmatrix} -\frac{457}{162} \\ -\frac{43}{81} \end{pmatrix}$$

7-185

$$\mathbf{x} = C_1\begin{pmatrix} \frac{1}{5}e^t\cos 2t - \frac{2}{5}e^t\sin 2t \\ e^t\cos 2\,t \end{pmatrix} + C_2\begin{pmatrix} \frac{2}{5}e^t\cos 2t + \frac{1}{5}e^t\sin 2t \\ e^t\sin 2t \end{pmatrix}$$

$$+\begin{pmatrix} \frac{11}{10} \\ -\frac{5}{2} \end{pmatrix}\sin t + \begin{pmatrix} -\frac{7}{10} \\ -\frac{5}{2} \end{pmatrix}\cos t$$

7-187

$$x_1(t) = -\frac{2}{5}e^{-5t} + \frac{9}{20}e^{-8t} + \frac{4}{5}e^{-3t} + \frac{3}{20}$$

$$x_2(t) = -\frac{2}{5}e^{-5t} - \frac{9}{40}e^{-8t} + \frac{8}{5}e^{-3t} + \frac{1}{40}$$

7-189

$$x_1(t) = -C_1e^{4t}\sin 3t + C_2e^{4t}\cos 3t$$
$$x_2(t) = C_1e^{4t}\cos 3t + C_2e^{4t}\sin 3t$$

7-191 $\mathbf{x} = C_1\begin{pmatrix} -e^{2t}\sin 3t \\ e^{2t}\cos 3t \end{pmatrix} + C_2\begin{pmatrix} e^{2t}\cos 3t \\ e^{2t}\sin 3\,t \end{pmatrix}$

7-193

$$\mathbf{x} = C_1\begin{pmatrix} -\frac{1}{2}e^{8t}(\cos t + \sin t) \\ e^{8t}\cos t \end{pmatrix} + C_2\begin{pmatrix} \frac{1}{2}e^{8t}(\cos t - \sin t) \\ e^{8t}\sin t \end{pmatrix} + \begin{pmatrix} \left(t - \frac{1}{2}\right)e^{8t}(\sin t + \cos t) \\ (-2t+1)e^{8t}(\sin t + \cos t) \end{pmatrix}$$

7-195 $\mathbf{x} = C_1\mathbf{x}_1 + C_2\mathbf{x}_2 = C_1\begin{pmatrix} -2 \\ 1 \end{pmatrix}e^{t} + C_2\begin{pmatrix} -2t + \frac{1}{2} \\ t \end{pmatrix}e^{t}$

Note that the initial conditions specified for the given system of two linear homogeneous differential equations with constant coefficients are both equal to zero. Therefore this initial-value problem has only the trivial solutions $x_1(t) = x_2(t) = 0$.

7-197 $\mathbf{x} = C_1\begin{pmatrix} 2 \\ 1 \end{pmatrix}e^{-t} + C_2\begin{pmatrix} -2 \\ 1 \end{pmatrix}e^{-5t} + \begin{pmatrix} \left(-\frac{2}{3}t + \frac{5}{18}\right)e^{t} + \frac{4}{5}t - \frac{84}{25} \\ \left(-\frac{1}{6}t + \frac{1}{9}\right)e^{t} + \frac{3}{5}t - \frac{58}{25} \end{pmatrix}$

7-199 $\mathbf{x} = C_1\begin{pmatrix} -\frac{6}{7} \\ -\frac{9}{7} \\ 1 \end{pmatrix}e^{5t} + C_2\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}e^{-3t} + C_3\begin{pmatrix} -2 \\ -\frac{5}{2} \\ 1 \end{pmatrix}e^{4t} + \begin{pmatrix} -\frac{14}{15}t + \frac{1477}{900} \\ -\frac{29}{60} + \frac{637}{3600} \\ \frac{11}{30}t - \frac{1513}{1800} \end{pmatrix}$

7-201

$$x_1(t) = \frac{45}{16}e^{-4t} - \frac{7}{4}t - \frac{13}{16}$$

$$x_2(t) = \frac{75}{16}e^{-4t} - \frac{1}{4}t + \frac{5}{16}$$

CHAPTER 8

8-17 **(a)** $\frac{a}{s^2+a^2}$, **(b)** $\frac{5}{s^2}-\frac{3}{s}$, **(c)** $\frac{1}{(s-1)^2}$

8-19 **(a)** $\frac{1}{e}\left(\frac{1}{s+2}\right)$ **(b)** $\frac{1}{2}\left(\frac{s}{s^2+4}+\frac{1}{s}\right)$ **(c)** $\frac{1}{s^2}\left(1-2e^{-s}+e^{-\frac{3s}{2}}\right)$

8-27 **(a)** exponential order, **(b)** exponential order

(c) not exponential order, **(d)** not exponential order

8-37 **(a)** $\frac{6(s^4-6s^2+1)}{(s^2+1)^4}$ **(b)** $\frac{10}{s^6}-\frac{3e}{s-2}$ **(c)** $\frac{1}{s^2-3s}$

8-39 **(a)** $24\frac{s-3}{[(s-3)^2+4]^2}$ **(b)** $\frac{3(s^2+k^2)}{(s^2-k^2)^2}$ **(c)** $\frac{\sqrt{\pi}}{4}\frac{2s+3}{s^{5/2}}$

8-41 **(a)** $\frac{20(3s^2+4)}{(s^2-4)^3}$ **(b)** $\frac{12}{(s+3)^4}$ **(c)** $\frac{2e^{-2}(s-5)[3k^2-(s-5)^2]}{[(s-5)^2+k^2]^3}$

8-49 **(a)** $\frac{1}{s^2}(1-e^{2s})$ **(b)** $e^{-s}+\frac{2}{s}(e^{-2s}-e^{-4s})+e^{-5s}$

8-51 **(a)** $\frac{2}{s^2+4}\left(1+e^{-\frac{\pi}{2}s}\right)$ **(b)** $-\frac{1}{s^2}(2e^{-2s}+e^{-4s})+e^{-6s}$

8-53 **(a)** $\frac{s}{s^2+4}(1-e^{-\pi s})$ **(b)** $\frac{1}{s}(e^{-2s}+e^{-s}-1)$

8-55 **(a)** $\frac{e^{-3s}}{s^3}(9s^2+6s+2)$ **(b)** $5e^{-s}+\frac{5s+2}{s}e^{-2s}$

8-57 $\frac{1}{s^2+1}\coth \pi s$

8-59 $\frac{2}{s(1+e^{-5s})}$

8-61 $\frac{2}{(s^2+4)(1-e^{-\pi s/2})}$

8-65 **(a)** $Y(s)=\frac{(s^2-2)y(0)+sy'(0)+y''(0)}{s^3-2s+5}$ **(b)** $Y(s)=\frac{(s-2)^2y'(0)+s(s-2)^2y(0)+3}{s^2(s-2)^2}$

8-67 **(a)** $Y(s)=\frac{(s^2-6s+13)^2y'(0)+s(s^2-6s+13)^2y(0)-12}{(s^2+5)(s^2-6s+13)^2}$ **(b)** $Y(s)=\frac{s(s-2)y(0)+(e+3)s-6}{s(s+3)(s-2)}$

8-71 **(a)** $u(t-3)\sin(t-3)$ **(b)** $e^{-3t}-\frac{1}{2}e^{3t}t^2(t+1)$ **(c)** $3(1-e^{-t/9})$

8-73 **(a)** $\cosh t+3\sinh t$ **(b)** $2\cos\frac{\sqrt{3}}{2}t$ **(c)** $\frac{1}{120}u(t-2)(t-2)^5$

8-75 **(a)** $\frac{2}{3}u(t-2)\sinh(3t-6)$ **(b)** $e^{-t}(2\cos t+\sin t)$ **(c)** $\cos t-2\sin t$

8-81 **(a)** $-1-t+3e^t$ **(b)** $-\frac{1}{16}+\frac{t}{8}+\frac{1}{16}e^{-2t}\cos 2t+\frac{1}{2}e^{-2t}\sin 2t$

8-83 **(a)** $-1-5t-\frac{1}{2}t^2+\cosh t+5\sinh t$ **(b)** $t-\frac{4}{\sqrt{7}}e^{-\frac{t}{4}}\sin\frac{\sqrt{7}}{4}t$

8-85 **(a)** $\frac{2}{5}+\frac{11}{10}e^{-5t}-\frac{1}{2}e^{-t}$ **(b)** $-\frac{3}{4}-t+e^t-\frac{1}{4}e^{-4t}$

8-91 (a) $8\cosh t - 4t^2 - 8$ **(b)** $\left[-\frac{1}{4}e^{-4\tau}\right]_{\tau=0}^{\tau=t} = -\frac{1}{4}e^{-4t} + \frac{1}{4} = \frac{1}{4}(\sinh 4t - \cosh 4t + 1)$

8-93 (a) $\left(\frac{3}{4}t - \frac{3}{16}\right)e^{-t} + \frac{3}{16}e^{-5t}$ **(b)** $\frac{1}{18}(3\sinh 3t + \cosh 3t) - \frac{1}{2}(\sinh t + \cosh t) + \frac{4}{9}$

8-95 (a) $-\frac{1}{4} + \frac{1}{8}(\cosh 2t + \cos 2t)$ **(b)** $\frac{2}{5} - \frac{2}{5}e^{-t}\cos 2t - \frac{1}{5}e^{-t}\sin 2t$

8-97 (a) $-\frac{3}{4} - \frac{1}{2}t - \frac{1}{2}t^2 + \frac{2}{3}e^t + \frac{1}{12}e^{-2t}$ **(b)** $\frac{3}{8}(1 - \cos 2t)$

8-101 $y(t) = \frac{1}{10}(3\cos t + \sin t - 13e^{-3t})$

8-103 $y(t) = \frac{1000}{19}\left(-e^{-t/50} + e^{-t/1000}\right) = \frac{2000}{19}e^{-\frac{21}{1000}t}\sinh\frac{19}{2000}t$

8-105 $x(t) = 2\sqrt{5}e^{-3t/2}\sinh\frac{\sqrt{5}}{2}t$

8-107 $x(t) = \frac{1}{2}e^{-t}(\cos t - \sin t - 4) + \frac{3}{2}$

8-109

$$y(t) = \frac{6}{\sqrt{115}}e^{-5t}\sin\sqrt{115}\,t + \frac{1}{140}u(t-1)$$
$$-\frac{1}{140}u(t-1)e^{-5(t-1)}\left(\cos\left[\sqrt{115}(t-1)\right] + \frac{5}{\sqrt{115}}\sin\left[\sqrt{115}(t-1)\right]\right)$$

8-111 $y(t) = 0.37654e^{-3t/2}\cosh\frac{\sqrt{17}}{2}t + 0.273973e^{-3t/2}\sinh\frac{\sqrt{17}}{2}t$

8-113 $y(t) = 5\sinh t$

8-115 $y(t) = \frac{25}{48}e^{t/2}\cos\frac{\sqrt{15}}{2}t + \frac{111}{48\sqrt{15}}\sin\frac{\sqrt{15}}{2}t - \frac{1}{3}e^{-t} - \frac{3}{4}t + \frac{1}{2}t^2 + \frac{29}{16}$

8-117 $y(t) = \frac{1}{3}t^4 + \frac{4}{3}t^3 + \frac{1}{3}t^2 = \frac{1}{3}(t^2 + t + 1)t^2$

8-119 $y(t) = -\frac{1}{4}e^{-8(t-\pi)} + \frac{e^\pi}{656}e^{-8(t-\pi)} + \frac{5}{82}e^t\cos t + \frac{2}{41}e^t\sin t + \frac{5}{4} - \frac{e^\pi}{16}$

8-121 $\frac{Y(s)}{F(s)} = \frac{1}{s^2+3s+1}$

8-123

$$x(t) = -\frac{4}{13}e^{-2t} + \frac{1}{26}e^t(8\cos 2t + 27\sin 2t)$$
$$y(t) = -\frac{10}{13}e^{-2t} + \frac{1}{26}e^t(46\cos 2t - 43\sin 2t)$$

8-125

$$x(t) = \frac{1}{4}e^{-t} + \frac{1}{2}\sin t - \frac{5}{4}e^{-\frac{1}{2}t}\cos\frac{\sqrt{7}}{2}t + \frac{1}{4\sqrt{7}}e^{-\frac{1}{2}t}\sin\frac{\sqrt{7}}{2}t$$

$$y(t) = \frac{1}{2} + \frac{1}{2}e^{-t} + \sin t - e^{-\frac{1}{2}t}\cos\frac{\sqrt{7}}{2}t - \frac{2}{\sqrt{7}}e^{-\frac{1}{2}t}\sin\frac{\sqrt{7}}{2}t$$

8-127

$$x(t) = -\frac{1}{2}\sin t + \frac{85}{44}e^{\frac{1}{2}t}\cos\frac{\sqrt{7}}{2} - \frac{127}{44\sqrt{7}}e^{\frac{1}{2}t}\sin\frac{\sqrt{7}}{2} - \frac{7}{4}e^{t} - \frac{2}{11}e^{-4t}$$

$$y(t) = \frac{1}{2}\cos t + \frac{1}{2}\sin t - \frac{8}{11}e^{\frac{1}{2}t}\cos\frac{\sqrt{7}}{2} + \frac{61}{11\sqrt{7}}e^{\frac{1}{2}t}\sin\frac{\sqrt{7}}{2} + \frac{5}{22}e^{-4t}$$

$$z(t) = -\frac{1}{2}\sin t + \frac{53}{44}e^{\frac{1}{2}t}\cos\frac{\sqrt{7}}{2} + \frac{117}{44\sqrt{7}}e^{\frac{1}{2}t}\sin\frac{\sqrt{7}}{2} + \frac{7}{4}e^{t} + \frac{1}{22}e^{-4t}$$

8-129 $\frac{X_2(s)}{G(s)} = \frac{4s}{s^2-2s-12}$

8-131 $\frac{1}{6}e^{-2t} - \frac{1}{6}e^{-4t}$

8-133 e^{-2t}

8-135 $\mathbf{x}(t) = \boldsymbol{\varphi}(t)\mathbf{x}(0) + \mathbf{A}^{-1}[\boldsymbol{\varphi}(t) - \mathbf{I}]\mathbf{Bp}$

8-137 **(a)** $\frac{1}{45e^{5t}} - \frac{1}{18e^{2t}} + \frac{1}{30}$

(b) $\frac{1}{65} - \frac{\left(\cos(3t) + \frac{2\sin(3t)}{3}\right)}{65e^{2t}}$

(c) $\frac{7}{24e^{3t}} - \frac{13}{40e^{5t}} + \frac{1}{30}$

(d) $\frac{2}{3e^{2t}} - \frac{7}{6e^{3t}} + \frac{13}{30e^{5t}} + \frac{1}{15}$

8-139 **(a)** $\frac{1}{9e^{2t}} - \frac{1}{9e^{5t}}$ **(b)** $-\frac{4}{169} + \frac{1}{13}t + \frac{1}{507}(12\cos(3t) - 5\sin(3t))e^{-2t}$

(c) $\frac{13}{40}e^{-5t} + \frac{1}{6}t + \frac{29}{180} - \frac{35}{72}e^{-3t}$ **(d)** $-\frac{13}{30}e^{-5t} + \frac{7}{45} - \frac{5}{3}e^{-2t} + \frac{35}{18}e^{-3t} + \frac{1}{3}t$

8-147 $F(s) = \frac{C}{Ds^2} - \frac{C}{Ds^2}e^{-Ds} - \frac{C}{s}e^{-Ds}$

8-149 $x(t) = \frac{2}{15}t^5 - \frac{2}{3}t^4 + 3t^3 - 9t^2 + 19t - 19 + 19e^{-t}$

8-151 $x(t) = \frac{1}{6}(e^{-2t} - e^{-5t})$

8-153 $x(t) = \frac{2v_1}{11}\sqrt{\frac{10m}{k}}\sin\sqrt{\frac{k}{10m}}t$

8-155 **(a)** $p_0 = 30\times10^3$ Pa $\quad\quad \tau = -0.2/\ln 0.5 = 0.289$

(b) $x(t) = -0.643\cos 10t + 0.2225\sin 10t + 0.643e^{-3.46t}$

8-157 $\ddot{x} = \frac{a}{b^2+\omega_n^2}\left(-b\omega_n \sin\omega_n t - \omega_n^2\cos\omega_n t + \omega_n^2 e^{bt}\right)$

8-159 $x(t) = -\frac{F_0}{kT}t + \frac{F_0}{k} + \frac{F_0}{kT\omega_n}\sin\omega_n t - \frac{F_0}{k}\cos\omega_n t$

8-161 $y(x) = \frac{AL^2}{24}\left(3x - \frac{L}{2}\right) = \frac{f_0L^2}{24EI}\left(3x - \frac{L}{2}\right)$ where $x \geq \frac{L}{2}$

CHAPTER 9

9-29

Strip method, N=1: $\frac{e}{2}$, 35.19%, **Strip method**, N=2: $\frac{1}{8}(e^{\frac{1}{2}} + 3e^{\frac{3}{2}})$, 10.04%

Trapezoidal rule, N=1: $\frac{e^2}{2}$, 76.16%, **Trapezoidal rule**, N=2: $\frac{1}{4}e(e + 1)$, 20.48%

9-31

Trapezoidal rule, N=1: $-\frac{\pi}{2}(e^{\pi} + 1)$, 276.55%, **Trapezoidal rule, N=2:** $-\frac{\pi}{4}(1 + e^{\pi})$, 88.27%.

Simpson's rule, N=1$-\frac{\pi}{6}(1 + e^{\pi})$, 25.52%, **Simpson's rule, N=2:** -10.2288, 1.57%.

9-33

Strip method, N=1: 6, 18.18%, **Strip method, N=2:** 7, 4.54%.

Simpson's rule, N=1 and 2: $\frac{22}{3}$, 0.00%,

9-35

Strip method, N=1: $8e^{-4}$, 70.69%, **Strip method, N=2:** 0.7365, 47.30%.

Trapezoidal rule, N=1: $8e^{-16}$, $\cong$ 100%, **Trapezoidal rule, N=2:** 0.073263, 85.35%.

Simpson's rule, N=1: 0.097683, 80.46%, **Simpson's rule, N=2:** 0.51542, 3.08%.

9-37 Exact results and all numerical methods result in 0. Relative error 0.00%.

9-39

Strip Method for N=10: $EXACT := 2.09726402$ $NUMERICAL := 2.08846084$ $RELERROR := 0.4197457$

Strip Method for N=100: $EXACT := 2.09726402$ $NUMERICAL := 2.09717583$ $RELERROR := 0.0042052$

Trapezoidal Rule for N=10: $EXACT := 2.09726402$ $NUMERICAL := 2.11488449$ $RELERROR := 0.8401647$

Trapezoidal Rule for N=100: $EXACT := 2.09726402$ $NUMERICAL := 2.09744041$ $RELERROR := 0.0084105$

9-41

Trapezoidal Rule for N=10: $EXACT := 107.298331$ $NUMERICAL := 102.965004$ $RELERROR := 4.0385782$

Trapezoidal Rule for N=100: $EXACT := 107.298331$ $NUMERICAL := 107.254214$ $RELERROR := 0.0411159$

Strip Method for N=10: $EXACT := 107.298331$ $NUMERICAL := 109.435382$ $RELERROR := 1.991691$

Strip Method for N=100: $EXACT := 107.298331$ $NUMERICAL := 107.320386$ $RELERROR := 0.020555$

Simpson's Rule for N=10: $EXACT := 107.298331$ $NUMERICAL := 107.278590$ $RELERROR := 0.0183984$

Simpson's Rule for N=100: $EXACT := 107.298331$ $NUMERICAL := 107.298329$ $RELERROR := 0.0000018$

9-43

Strip Method for N=10: $EXACT := 7.33333333$ $NUMERICAL := 7.32000000$ $RELERROR := 0.181818177$

Strip Method for N=100: $EXACT := 7.33333333$ $NUMERICAL := 7.33320000$ $RELERROR := 0.00181817727$

Simpson's Rule for N=10 and N=100: $EXACT := 7.33333333$ $NUMERICAL := 7.33333333$
$RELERROR := 0.$

9-45

Strip Method for N=10: $EXACT := 0.499999943$ $NUMERICAL := 0.506861656$ $RELERROR := 1.3723427$

Strip Method for N=100: $EXACT := 0.499999943$ $NUMERICAL := 0.500066629$ $RELERROR := 0.0133374$

Trapezoidal Rule for N=10: $EXACT := 0.499999943$ $NUMERICAL := 0.486444617$
$RELERROR := 2.7110655$

Trapezoidal Rule for N=100: $EXACT := 0.499999943$ $NUMERICAL := 0.499866588$
$RELERROR := 0.0266710$

Simpson's Rule for N=10: $EXACT := 0.499999943$ $NUMERICAL := 0.500055976$
$RELERROR := 0.0112066214$

Simpson's Rule for N=100: $EXACT := 0.499999943$ $NUMERICAL := 0.499999949$ $RELERROR := 0.0000011$

9-53

<u>**Trapezoidal Rule**</u>

a) **One step:** 3.066, 0.14%.

b) **Two steps:** 3.295, 0.30%.

<u>**Simpson's Rule**</u>

a) **One step:** 3.062, 0.00%.

b) **Two steps:** 3.285, 0.00%.

9-55

<u>**Strip Method**</u>

a) **One step:** −0.922, 0.00%.

b) **Two steps:** −0.849, 0.00%.

<u>**Simpson's Rule**</u>

a) **One step:** −0.922, 0.00%.

b) **Two steps:** −0.849, 0.00%.

9-57

Strip Method

a) **One step:** 1.019, 0.03%.

b) **Two steps:** 1.0717, 0.04%.

Trapezoidal Rule

a) **One step:** 1.0182, 0.05%.

b) **Two steps:** 1.070, 0.08%.

Simpson's Rule

a) **One step:** 1.0188, 0.00%.

b) **Two steps:** 1.071, 0.00%.

9-59

Strip Method

After 10 steps with h=0.2: $y_{10} = 1$, 0.00%.

After 20 steps with h=0.1: $y_{20} = 1$, 0.00%.

Trapezoidal Rule

After 10 steps with h=0.2: $y_{10} = 1$, 0.00%.

After 20 steps with h=0.1: $y_{20} = 1$, 0.00%.

9-61

Trapezoidal Rule

After 10 steps with h=0.2: $y_{10} = 25.9950$, 0.74%.

After 20 steps with h=0.1: $y_{20} = 25.8540$, 0.17%.

Simpson's Rule

After 10 steps with h=0.2: $y_{10} = 25.8040$, 0.00%.

After 20 steps with h=0.1: $y_{20} = 25.8540$, of 0.00%.

9-63

Strip Method

After 10 steps with h=0.2: $y_{10} = -0.3882$, 0.03%.

After 20 steps with h=0.1: $y_{20} \cong -0.3881$, 0.00%.

Simpson's Rule

After 10 steps with h=0.2: $y_{10} = -0.3881$, 0.02%

After 20 steps with h=0.1: $y_{20} \cong -0.3881$, 0.00%

9-65

Strip Method

After 10 steps with h=0.2: $y_{10} = 2.4634$, 0.04%.

After 20 steps with h=0.1: $y_{20} \cong 2.4639$, 0.01%.

Trapezoidal Rule

After 10 steps with h=0.2: $y_{10} = 2.4655$, 0.06%

After 20 steps with h=0.1: $y_{20} \cong 2.4644$, 0.06%.

Simpson's Rule

After 10 steps with h=0.2: $y_{10} = 2.4641$, 0.00%

After 20 steps with h=0.1: $y_{20} \cong 2.4641$, 0.00%.

9-71 After one step: 2, 26.42%. **After two steps:** 4, 45.87%.

9-73 After one step: 3, 2.43%. **After two steps:** 3.216, 15.86%.

9-75 After one step: 1, 0.00%. **After two steps:** 0, 0.00%.

9-77 After one step: 4.01176, 0.06%. **After two steps:** 4.02861, 0.13%.

9-79

After 10 steps with h=0.2: $y_{10} = 5.37762$, 48.29%.

After 20 steps with h=0.1: $y_{20} \cong 7.26788$, 30.11%.

9-81 *There is a typographical error in this problem. The change is yellowed below and will be corrected in the second printing.* **Given:** $y' = 3e^x y$, $y(0) = -2$

After 10 steps with h=0.2: $y_{10} = -25{,}529.4$, $\approx$ 100%

After 20 steps with h=0.1: $y_{20} \cong -432{,}193.00$, 99.90%.

9-83

After 10 steps with h=0.2: $y_{10} = 6.19174$, 16.20%.

After 20 steps with h=0.1: $y_{20} \cong 6.72749$, 8.95%.

9-85 After 10 and 20 steps with h=0.2 and h=0.1: $y_{10} = 0$, 0.00%.

9-87 *There is a typographical error in this problem. The change is yellowed below and will be corrected in the second printing.* **Given:** $y' = -xe^y,\ y(0) = 1$

After 10 steps with h=0.2: $y_{10} = -0.86427$, 0.26%.

After 20 steps with h=0.1: $y_{20} \cong -0.86246$, 0.05%.

9-89

After 10 steps with h=0.2: $y_{10} = 1.13745$, 3.53%.

After 20 steps with h=0.1: $y_{20} \cong 1.11745$, 1.71%.

9-91 *There is a typographical error in this problem. The change is yellowed below and will be corrected in the second printing*. **Given:** $y' = \frac{4xy}{x^2+y^2}$, $y(1) = 2$

After 10 steps with h=0.2: $y_{10} = 5.34311$, 0.15%.

After 20 steps with h=0.1: $y_{20} \cong 5.34731$, 0.07%.

9-93

After 10 steps with h=0.2: $y_{10} = 15.71430$, 7.56%.

After 20 steps with h=0.1: $y_{20} \cong 16.33660$, 3.90%.

9-105 *There is a typographical error in this problem. The change is yellowed below and will be corrected in the second printing.* **Given:** $y' = x^2 e^y$, $y(2) = -2$

After one step: : $LDE_1 = GDE_1 = -1.01\%$

After two steps: $LDE_2 = -1.70\%$, $GDE_2 = -2.96\%$

9-107

After one step: $LDE_1 = GDE_1 = 100\%$

After two steps: $LDE_2 = 67.70\%$, $GDE_2 = 72.97\%$

9-109

(a) $N = 10, h = 0.2$

x	$y_{numerical}$	YY	y_{exact}	$LDE(\%)$	$GDE(\%)$
2.00000	-0.77784	-0.77878	-0.77915	0.04788	0.16917

(b) $N = 20, h = 0.1$

x	$y_{numerical}$	YY	y_{exact}	$LDE(\%)$	$GDE(\%)$
2.00000	-0.77822	-0.77908	-0.77915	0.00964	0.11976

9-111 *There is a typographical error in this problem. The change is yellowed below and will be corrected in the second printing.* **Given:** $y' = x^2 e^y$, $y(0) = -2$

(a) $N = 10, h = 0.2$

x	$y_{numerical}$	YY	y_{exact}	$LDE(\%)$	$GDE(\%)$
2.00000	-1.64591	-1.57570	-1.55232	-1.50652	-6.02925

(b) $N = 20, h = 0.1$

x	$y_{numerical}$	YY	y_{exact}	$LDE(\%)$	$GDE(\%)$
2.00000	-1.60254	-1.55903	-1.55232	-0.43239	-3.23517

9-113

(a) $N = 10, h = 0.2$

x	$y_{numerical}$	YY	y_{exact}	$LDE(\%)$	$GDE(\%)$
2.00000	2.47969	2.68943	2.81201	4.35908	11.81790

(b) $N = 20, h = 0.1$

x	$y_{numerical}$	YY	y_{exact}	$LDE(\%)$	$GDE(\%)$
2.00000	2.64773	2.77967	2.81201	1.15010	5.84214

9-115 $h = \frac{0.1}{200}$

9-117 $h = \frac{0.1}{200}$

9-119 $h = \frac{0.1}{3000}$

9-125 After one step: 4.81667, 0.00%. **After two steps**: 4.65463, 0.00%.

9-127 After one step: 1.49600, 0.83%.**After two steps:** 2.37930, 3.48%.

9-129 After one step: 22.6400, 6.09%. **After two steps:** 58.3569, 14.44%.

9-131 After one step: −1.93713, 0.01%. **After two steps:** −1.84212, 0.03%.

9-133 After one step: 2.00954, 0.01%. **After two steps:** 2.03741, 0.01%.

9-135

After 10 steps with h=0.2: $y_{10} = 2481.28$, 57.26%.

After 20 steps with h=0.1: $y_{20} \cong 4224.97$, 27.23%.

9-137

After 10 steps with h=0.2: $y_{10} = 118.198$, 7.36%

After 20 steps with h=0.1: $y_{20} \cong 124.804$, 2.19%.

9-139

After 10 steps with h=0.2: $y_{10} \cong 17.60930$, 0.95%

After 10 steps with h=0.2: $y_{10} \cong 17.73240$, 0.26%

9-141

After 10 steps with h=0.2: $y_{10} = 5.44462$, 0.00%.

After 20 steps with h=0.1: $y_{20} \cong 5.44451$, 0.00%.

9-143

After 10 steps with h=0.2: $y_{10} = 1.88171$, 0.91%.

After 20 steps with h=0.1: $y_{20} \cong 1.86873$, 0.22%.

9-145

After 10 steps with h=0.2: $y_{10} = 58.74190$, 1.28%.
After 20 steps with h=0.1: $y_{20} \cong 59.29590$, 0.34%.

9-147

After 10 steps with h=0.2: $y_{10} = 0.33501$, 0.50%.

After 20 steps with h=0.1: $y_{20} \cong 0.33373$, 0.12%.

9-153 After one step: 4.82, 0.07%.**After two steps:** 4.65985, 0.11%.

9-155 After one step: 1.48, 1.89%. **After two steps**: 2.30695, 6.41%.

9-157 After one step: 22, 8.75%. **After two steps:** 55.0176, 19.34%.

9-159

After 10 steps with h=0.2: $y_{10} = 1857.48$, 68.01%.

After 20 steps with h=0.1: $y_{20} \cong 3761.1$, 35.22%.

9-161

After 10 steps with h=0.2: $y_{10} = 118.197$, 7.37%

After 20 steps with h=0.1: $y_{20} \cong 124.804$, 2.19%.

9-163

After 10 steps with h=0.2: $y_{10} \cong 17.60930$, 0.95%

After 20 steps with h=0.1: $y_{20} \cong 17.73260$, 0.26%.

9-171 After one step: 4.81650, 0.00%. **After two steps:** 4.65465, 0.00%.

9-173 After one step: 1.50846, 0.00%. **After two steps:** 2.46400, 0.04%.

9-175 After one step: 24.05560, 0.22%. **After two steps:** 67.75830, 0.66%.

9-177 After one step: -1.93739, 0.00%. **After two steps:** -1.84260, 0.00%.

9-179 After one step: 2.00969, 0.00%. **After two steps:** 2.03767, 0.00%.

9-181

After 10 steps with h=0.2: $y_{10} = 5445.76$, 6.21%.

After 20 steps with h=0.1: $y_{20} \cong 5764.32$, 0.72%

9-183

After 10 steps with h=0.2: $y_{10} = 127.52100$, 0.06%

After 20 steps with h=0.1: $y_{20} \cong 124.804$, 2.19%

9-185

After 10 steps with h=0.2: $y_{10} \cong 17.60930$, 0.95%.

After 20 steps with h=0.1: $y_{20} \cong 17.73240$, 0.26%.

9-187

After 10 steps with h=0.2: $y_{10} = 5.44432$, 0.00%.

After 20 steps with h=0.1: $y_{20} \cong 5.44447$, 0.00%

9-189

After 10 steps with h=0.2: $y_{10} = 1.86469$, 0.00%.

After 20 steps with h=0.1: $y_{20} \cong 1.86466$, 0.00%

9-191

After 10 steps with h=0.2: $y_{10} = 59.0020$, 0.00%.

After 20 steps with h=0.1: $y_{20} \cong 59.50130$, 0.00%.

9-193

After 10 steps with h=0.2: $y_{10} = 0.33334$, 0.00%.

After 20 steps with h=0.1: $y_{20} \cong 0.33373$, 0.12%.

9-205 After one step: 5.633270, 0.00%. **After two steps:** 5.928240, 0.00%.

9-207 After one step: -0.313114, 1.28%. **After two steps:** -0.103270, 3.94%.

9-209 After one step: -20.38632, 6.52%. **After two steps:** -86.83465, 13.69%.

9-211 After one step: -1.468573, 0.01%. **After two steps:** -1.003938, 0.39%.

9-213 After one step: 2.143301, 0.00%. **After two steps:** 2.218951, 0.00%.

9-215

After 10 steps with h=0.2: $y_{10} = 3084.421$, 46.88%.

After 20 steps with h=0.1: $y_{20} \cong 5228.891$, 9.94%.

9-217

After 10 steps with h=0.2: $y_{10} = 127.1810$, 0.32%.

After 20 steps with h=0.1: $y_{20} \cong 127.5772$, 0.01%.

9-218

After 10 steps with h=0.2: $y_{10} = 7.388599$, 0.01%.

After 20 steps with h=0.1: $y_{20} \cong 7.389046$, 0.00%.

9-219

After 10 steps with h=0.2: $y_{10} \cong 17.77719$, 0.00%.

After 20 steps with h=0.1: $y_{20} \cong 17.77811$, 0.00%.

9-221

After 10 steps with h=0.2: $y_{10} = 5.443662$, 0.01%.

After 20 steps with h=0.1: $y_{20} \cong 5.444487$, 0.00%.

9-223

After 10 steps with h=0.2: $y_{10} = 1.86468$, 0.00%.

After 20 steps with h=0.1: $y_{20} \cong 1.86466$, 0.00%.

9-225

After 10 steps with h=0.2: $y_{10} = 59.0020$, 0.00%.

After 20 steps with h=0.1: $y_{20} \cong 59.5018$, 0. 00%.

9-227

After 10 steps with h=0.2: $y_{10} = 0.333191$, 0.04%.

After 20 steps with h=0.1: $y_{20} \cong 0.333326$, 0.12%.

9-231 After one step: $y_1 = 0.6$, $z_1 = 0.6$: **After two steps:** $y_2 = 0.436254$, $z_2 = 1.16749$.

9-233 After one step: $y_1 = 0.20$: **After two steps:** $y_2 = 0.32$

9-235

After one step: $y_1 = 0.38520$, $z_1 = 1.49267$,

After two steps: $y_1 = 0.761160$, , $z_1 = 1.14579$

9-237 After one step: $y_1 = 0.2734$: **After two steps:** $y_2 = 0.66803$.

9-239

For N=10 and h=0.2: $y\_Euler_{10} := 2.164159$, $z\_Euler_{10} := 2.133926$

For N=20 and h=0.1: $y\_Euler_{20} := 2.380172$, $z\_Euler_{20} := 2.322527$

9-241.

For N=10 and h=0.2: $y\_Euler_{10} := 6.833296$, $z\_Euler_{10} := 17.39652$

For N=20 and h=0.1: $y\_Euler_{20} := 7.975250$, $z\_Euler_{20} := 20.14064$

9-243 For N=10 and h=0.2: $y_{10} = 3.389388$, **For N=20 and h=0.1** $y_{20} = 3.311436$

9-245

For N=10 and h=0.2: $y\_RungeKutta_{10} := 8.176204$, $z\_RungeKutta_{10} := 2.839624$

For N=20 and h=0.1: $y\_RungeKutta_{20} := 8.177077$, $z\_RungeKutta_{20} := 2.839957$

9-247 For N=10 and h=0.2: $y_{10} = 4.666445$, **For N=20 and h=0.1:** $y_{20} = 4.666412$

9-287 (a) $t = 521$ minutes