Outerbridge Crossing between Staten Island and New Jersey

Constructed in 1928, the Outerbridge Crossing is a cantilever truss bridge consisting of a 750-ft main span, two 375-ft anchor arms, and a 300-ft through truss span at either end. The 143-ft clearance at the suspended midspan permits large ships to pass under the bridge. It is the outermost crossing in the district of The Port Authority of New York and New Jersey. Replaced by newer, stronger materials and structural systems, truss bridges have diminished in popularity in recent years.

C H A P T E R 4

Trusses

Chapter Objectives

- Study the characteristics and behavior of trusses. Since truss members carry only axial loads, the configuration of the bars is key to a truss' efficiency and use.
- Analyze determinate trusses by method of joints and method of sections to determine bar forces. Also learn to visually identify bars with zero force.
- Classify determinate and indeterminate truss structures, and determine the degree of indeterminacy.
- Determine if a truss structure is stable or unstable.

4.1 Introduction

A truss is a structural element composed of a stable arrangement of slender interconnected bars (see Figure 4.1*a*). The pattern of bars, which often subdivides the truss into triangular areas, is selected to produce an efficient, lightweight, load-bearing member. Although joints, typically formed by welding or bolting truss bars to gusset plates, are rigid (see Figure 4.1*b*), the designer normally assumes that members are connected at joints by frictionless pins, as shown in Figure 4.1*c*. (Example 4.9 on page 149 clarifies the effect of this assumption.) Since no moment can be transferred through a frictionless pin joint, truss members are assumed to carry only axial force—either tension or compression. Because truss members act in direct stress, they carry load efficiently and often have relatively small cross sections.

As shown in Figure 4.1*a*, the upper and lower members, which are either horizontal or sloping, are called the top and bottom chords. The chords are connected by vertical and diagonal members.

The structural action of many trusses is similar to that of a beam. As a matter of fact, a truss can often be viewed as a beam in which excess material has been removed to reduce weight. The chords of a truss correspond to the flanges of a beam. The forces that develop in these members make up the internal couple that carries the moment produced by the applied loads. The

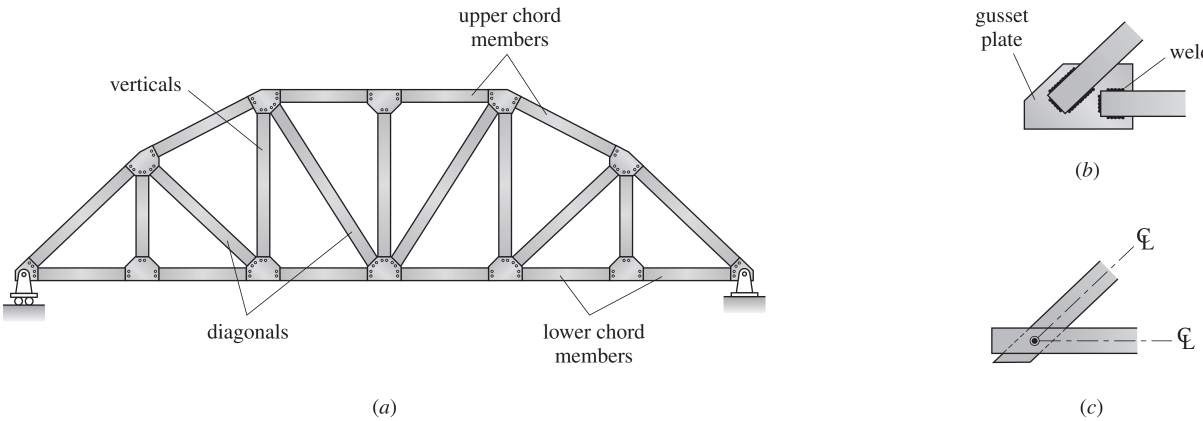


Figure 4.1: (*a*) Details of a truss; (*b*) welded joint; (*c*) idealized joint, members connected by a frictionless pin.

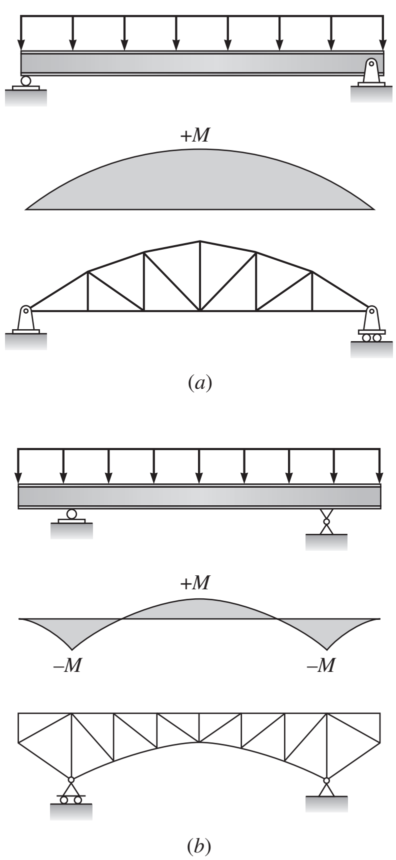


Figure 4.2: (*a*) and (*b*) depth of truss varied to conform to ordinates of moment curve.

primary function of the vertical and diagonal members is to transfer vertical force (shear) to the supports at the ends of the truss. Generally, on a per pound basis it costs more to fabricate a truss than to roll a steel beam; however, the truss will require less material because the material is used more efficiently. In a long-span structure, say 200 ft or more, the weight of the structure can represent the major portion (on the order of 75 to 85 percent) of the design load to be carried by the structure. By using a truss instead of a beam, the engineer can often design a lighter, stiffer structure at a reduced cost.

Even when spans are short, shallow trusses called bar joists are often used as substitutes for beams when loads are relatively light. For short spans these members are often easier to erect than beams of comparable capacity because of their lighter weight. Moreover, the openings between the web members provide large areas of unobstructed space between the floor above and the ceiling below the joist through which the mechanical engineer can run heating and air-conditioning ducts, water and waste pipes, electrical conduit, and other essential utilities.

In addition to varying the area of truss members, the designer can vary the truss depth to reduce its weight. In regions where the bending moment is large—at the center of a simply supported structure or at the supports in a continuous structure—the truss can be deepened (see Figure 4.2).

The diagonals of a truss typically slope upward at an angle that ranges from 45 to 60°. In a long-span truss the distance between panel points should not exceed 15 to 20 ft (5 to 7 m) to limit the unsupported length of the compression chords, which must be designed as columns. As the slenderness of a compression chord increases, it becomes more susceptible to buckling. The slenderness of tension members must be limited also to reduce vibrations produced by wind and live load.

If a truss carries equal or nearly equal loads at all panel points, the direction in which the diagonals slope will determine if they carry tension or compression forces. Figure 4.3, for example, shows the difference in forces set up in the diagonals of two trusses that are identical in all respects (same span, same loads, and so forth) except for the direction in which the diagonals slope (T represents tension and C indicates compression).

Although trusses are very stiff in their own plane, they are very flexible out of plane and must be braced or stiffened for stability. Since trusses are often used in pairs or spaced side by side, it is usually possible to connect several trusses together to form a rigid-box type of structure. For example, Figure 4.4 shows a bridge constructed from two trusses. In the horizontal planes of the top

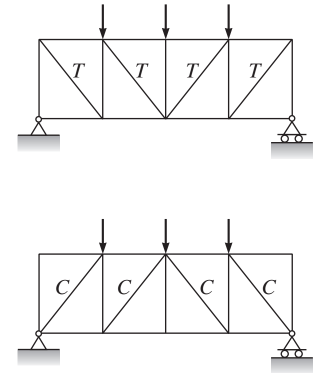


Figure 4.3: T represents tension and C compression.

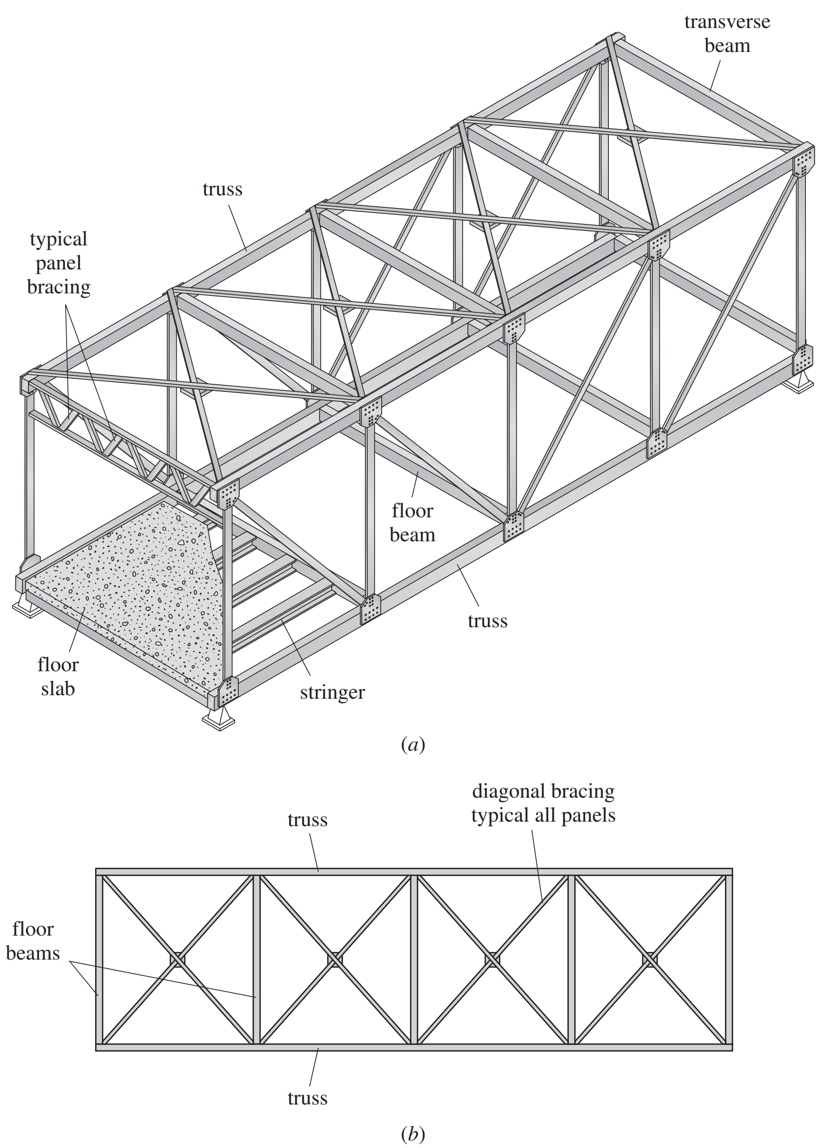


Figure 4.4: Truss with floor beams and secondary bracing: (*a*) perspective showing truss interconnected by transverse beams and diagonal bracing; diagonal bracing in bottom plane, omitted for clarity, is shown in (*b*); (*b*) bottom view showing floor beams and diagonal bracing. Lighter beams and bracing are also required in the top plane to stiffen trusses laterally.

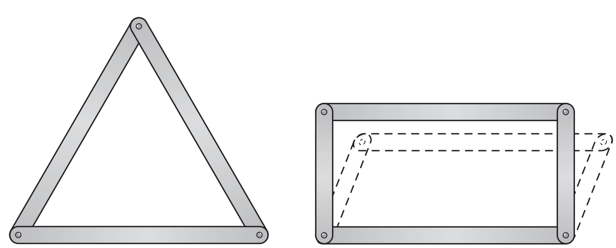
Photo 4.1: Massive roof trusses with bolted joints and gusset plates.

Photo 4.2: Reconstructed Tacoma Narrows bridge showing trusses used to stiffen the roadway floor system. See original bridge in Photo 2.1.

and bottom chords, the designer adds transverse members, running between panel points, and diagonal bracing to stiffen the structure. The upper and lower chord bracing together with the transverse members forms a truss in the horizontal plane to transmit lateral wind load into the end supports. Engineers also add diagonal knee bracing in the vertical plane at the ends of the structure to ensure that the trusses remain perpendicular to the top and bottom planes of the structure.

4.2 Types of Trusses

The members of most modern trusses are arranged in triangular patterns because even when the joints are pinned, the triangular form is geometrically stable and will not collapse under load (see Figure 4.5*a*). On the other hand, a pin-connected rectangular element, which acts like an unstable linkage (see Figure 4.5*b*), will collapse under the smallest lateral load.



(*a*) (*b*)

Figure 4.5: Pin-jointed frames: (*a*) stable; (*b*) unstable.

One method to establish a stable truss is to construct a basic triangular unit (see the shaded triangular element *ABC* in Figure 4.6) and then establish additional joints by extending bars from the joints of the first triangular element. For example, we can form joint *D* by extending bars from joints *B* and *C*. Similarly, we can imagine that joint *E* is formed by extending bars from joints *C* and *D*. Trusses formed in this manner are called *simple trusses*.

If two or more simple trusses are connected by a pin or a pin and a tie, the resulting truss is termed a *compound truss* (see Figure 4.7). Finally, if a truss—usually one with an unusual shape—is neither a simple nor a compound truss, it is termed a *complex truss* (see Figure 4.8). In current practice, where computers are used to analyze, these classifications are not of great significance.

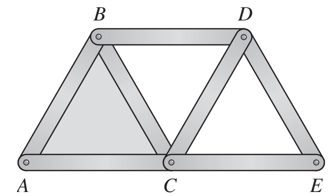


Figure 4.6: Simple truss.

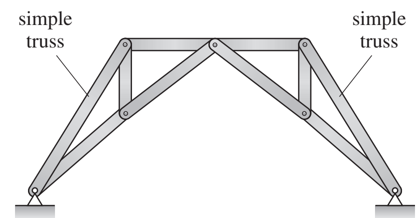


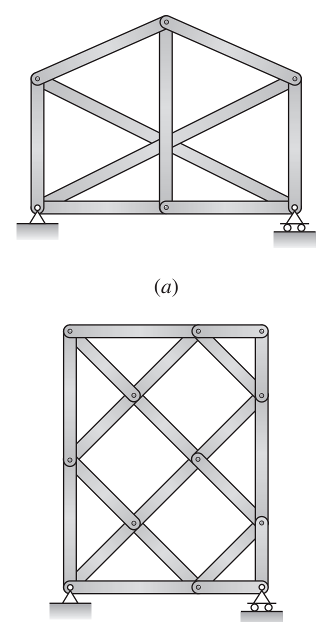
Figure 4.7: Compound truss is made up of simple trusses.

4.3 Analysis of Trusses

A truss is completely analyzed when the magnitude and sense (tension or compression) of all bar forces and reactions are determined. To compute the reactions of a determinate truss, we treat the entire structure as a rigid body and, as discussed in Section 3.6, apply the equations of static equilibrium together with any condition equations that may exist. The analysis used to evaluate the bar forces is based on the following three assumptions:

1. *Bars are straight and carry only axial load* (i.e., bar forces are directed along the longitudinal axis of truss members). This assumption also implies that we have neglected the deadweight of the bar. If the weight of the bar is significant, we can approximate its effect by applying one-half of the bar weight as a concentrated load to the joints at each end of the bar.
2. *Members are connected to joints by frictionless pins*. That is, no moments can be transferred between the end of a bar and the joint to which it connects. (If joints are rigid and members stiff, the structure should be analyzed as a rigid frame.)
3. *Loads are applied only at joints*.

As a sign convention (after the sense of a bar force is established) we label a *tensile force positive* and a *compression force negative*. Alternatively, we can



(*a*)

(*b*)

Figure 4.8: Complex trusses.

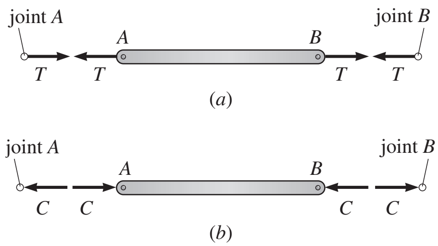


Figure 4.9: Free-body diagrams of axially loaded bars and adjacent joints: (*a*) bar *AB* in tension; (*b*) bar *AB* in compression.

denote the sense of a force by adding after its numerical value a *T* to indicate a tension force or a *C* to indicate a compression force.

If a bar is in tension, the axial forces at the ends of the bar act outward (see Figure 4.9*a*) and tend to elongate the bar. The equal and opposite forces on the ends of the bar represent the action of the joints on the bar. Since the bar applies equal and opposite forces to the joints, a tension bar will apply a force that acts outward from the center of the joint.

If a bar is in compression, the axial forces at the ends of the bar act inward and compress the bar (see Figure 4.9*b*). Correspondingly, a bar in compression pushes against a joint (i.e., applies a force directed inward toward the center of the joint).

Bar forces may be analyzed by considering the equilibrium of a joint—the *method of joints*—or by considering the equilibrium of a section of a truss—the *method of sections*. In the latter method, the section is produced by passing an imaginary cutting plane through the truss. The method of joints is discussed in Section 4.4; the method of sections is treated in Section 4.6.

4.4 Method of Joints

To determine bar forces by the method of joints, we analyze free-body diagrams of joints. The free-body diagram is established by imagining that we cut the bars by an imaginary section just before the joint. For example, in Figure 4.10*a* to determine the bar forces in members *AB* and *BC*, we use the free body of joint *B* shown in Figure 4.10*b*. Since the bars carry axial force, the line of action of each bar force is directed along the longitudinal axis of the bar.

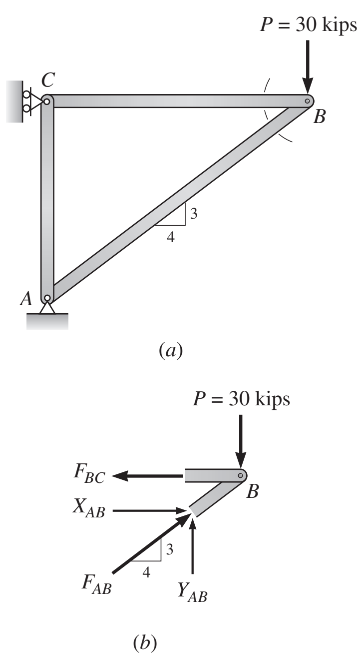


Figure 4.10: (*a*) Truss (dashed lines show location of circular cutting plane used to isolate joint *B*); (*b*) free body of joint *B*.

Because all forces acting at a joint pass through the pin, they constitute a concurrent force system. For this type of force system, only two equations of statics (that is, $\Sigma F_x = 0$ and $\Sigma F_y = 0$) are available to evaluate unknown bar forces. Since only two equations of equilibrium are available, we can only analyze joints that contain a maximum of two unknown bar forces.

The analyst can follow several procedures in the method of joints. For the student who has not analyzed many trusses, it may be best initially to write the equilibrium equations in terms of the components of the bar forces. On the other hand, as one gains experience and becomes familiar with the method, it is possible, without formally writing out the equilibrium equations, to determine bar forces at a joint that contains only one sloping bar by observing the magnitude and direction of the components of the bar forces required to produce equilibrium in a particular direction. The latter method permits a more rapid analysis of a truss. We discuss both procedures in this section.

To determine bar forces by writing out the equilibrium equations, we must assume a direction for each *unknown* bar force (*known* bar forces must be shown in their correct sense). The analyst is free to assume either tension or compression for any unknown bar force (many engineers like to assume that all bars are in tension, that is, they show all unknown bar forces acting outward from the center of the joint). Next, the forces are resolved into their

X and Y (rectangular) components. As shown in Figure 4.10*b*, the force or the components of a force in a particular bar are subscripted with the letters used to label the joints at each end of the bar. To complete the solution, we write and solve the two equations of equilibrium.

If only one unknown force acts in a particular direction, the computations are most expeditiously carried out by summing forces in that direction. After a component is computed, the other component can be established by setting up a proportion between the components of the force and the slope of the bar (the slope of the bar and the bar force are obviously identical).

If the solution of an equilibrium equation produces a positive value of force, the direction initially assumed for the force was correct. On the other hand, if the value of force is negative, its magnitude is correct, but the direction initially assumed is incorrect, and the direction of the force must be reversed on the sketch of the free-body diagram. After the bar forces are established at a joint, the engineer proceeds to adjacent joints and repeats the preceding computation until all bar forces are evaluated. This procedure is illustrated in Example 4.1.

Determination of Bar Forces by Inspection

Trusses can often be analyzed rapidly by inspection of the bar forces and loads acting on a joint that contains one sloping bar in which the force is unknown. In many cases the direction of certain bar forces will be obvious after the resultant of the known force or forces is established. For example, since the applied load of 30 kips at joint B in Figure 4.10*b* is directed downward, the y-component, Y_{AB} of the force in member AB—the only bar with a vertical component—must be equal to 30 kips and directed upward to satisfy equilibrium in the vertical direction. If Y_{AB} is directed upward, force F_{AB} must act upward and to the right, and its horizontal component X_{AB} must be directed to the right. Since X_{AB} is directed to the right, equilibrium in the horizontal direction requires that F_{BC} act to the left. The value of X_{AB} is easily computed from similar triangles because the slopes of the bars and the bar forces are identical (see Section 3.2).

$$\frac{X_{AB}}{4} = \frac{Y_{AB}}{3}$$

and

$$X_{AB} = \frac{4}{3}Y_{AB} = \frac{4}{3}(30)$$

$$X_{AB} = 40 \text{ kips} \qquad \textbf{Ans.}$$

To determine the force F_{BC}, we mentally sum forces in the x direction.

$$\rightarrow+ \quad \Sigma F_x = 0$$

$$0 = -F_{BC} + 40$$

$$F_{BC} = 40 \text{ kips} \qquad \textbf{Ans.}$$

EXAMPLE 4.1

Analyze the truss in Figure 4.11*a* by the method of joints. Reactions are given.

Solution

The slopes of the various members are computed and shown on the sketch. For example, the top chord *ABC*, which rises 12 ft in 16 ft, is on a slope of 3 : 4.

To begin the analysis, we must start at a joint with a maximum of two bars. Either joint *A* or *C* is acceptable. Since the computations are simplest at a joint with one sloping member, we start at *A*. On a free body of joint *A* (see Figure 4.11*b*), we arbitrarily assume that bar forces F_{AB} and F_{AD} are tensile forces and show them acting outward on the joint. We next replace F_{AB} by its rectangular components X_{AB} and Y_{AB}. Writing the equilibrium equation in the *y*-direction, we compute Y_{AB}.

$$\overset{+}{\uparrow} \quad \Sigma F_y = 0$$

$$0 = -24 + Y_{AB} \qquad \text{and} \qquad Y_{AB} = 24 \text{ kips} \qquad \textbf{Ans.}$$

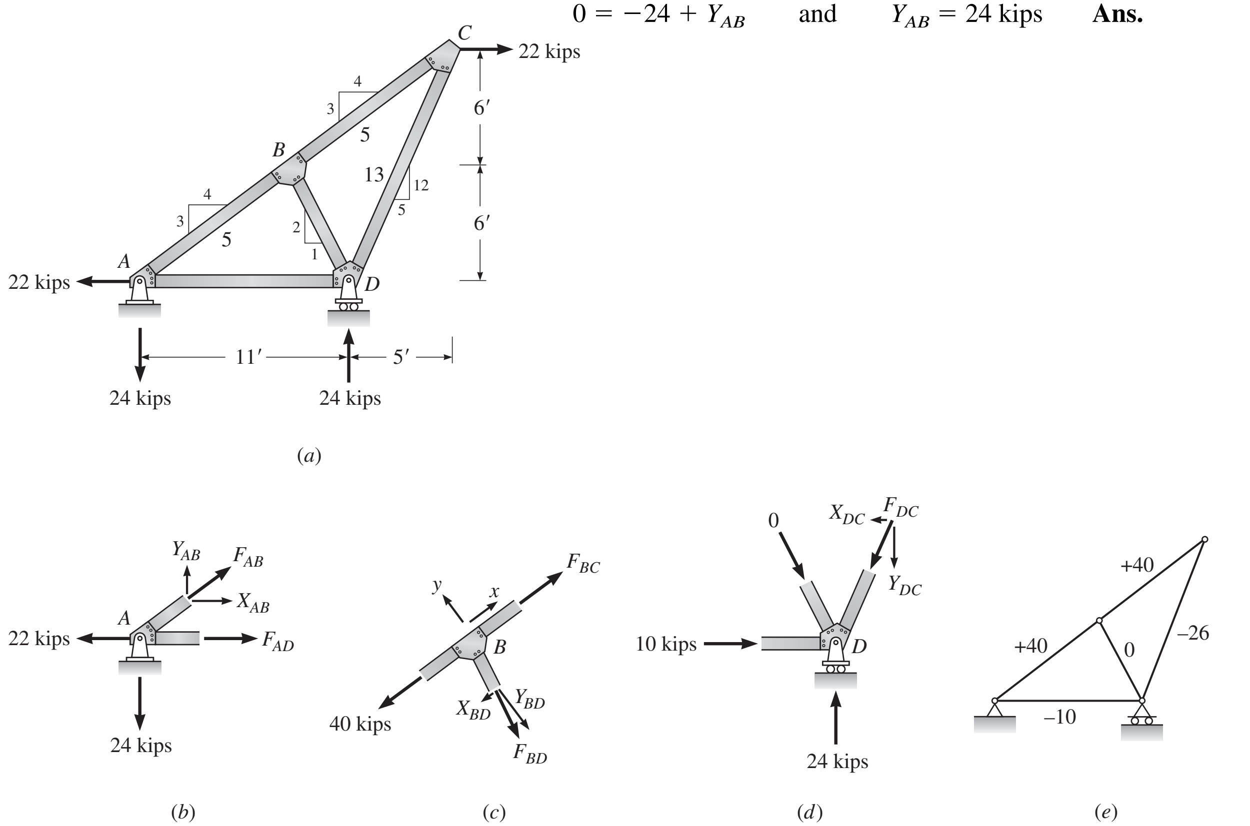


Figure 4.11: (*a*) Truss; (*b*) joint *A*; (*c*) joint *B*; (*d*) joint *D*; (*e*) summary of bar forces (units in kips).

Since Y_{AB} is positive, it is a tensile force, and the assumed direction on the sketch is correct. Compute X_{AB} and F_{AB} by proportion, considering the slope of the bar.

$$\frac{Y_{AB}}{3} = \frac{X_{AB}}{4} = \frac{F_{AB}}{5}$$

and

$$X_{AB} = \frac{4}{3}Y_{AB} = \frac{4}{3}(24) = 32 \text{ kips}$$

$$F_{AB} = \frac{5}{3}Y_{AB} = \frac{5}{3}(24) = 40 \text{ kips} \qquad \textbf{Ans.}$$

Compute F_{AD}.

$$\xrightarrow{+} \quad \Sigma F_x = 0$$

$$0 = -22 + X_{AB} + F_{AD}$$

$$F_{AD} = -32 + 22 = -10 \text{ kips} \qquad \textbf{Ans.}$$

Since the minus sign indicates that the direction of force F_{AD} was assumed incorrectly, the force in member AD is compression, not tension.

We next isolate joint B and show all forces acting on the joint (see Figure 4.11*c*). Since we determined $F_{AB} = 40$ kips tension from the analysis of joint A, it is shown on the sketch acting outward from joint B. Superimposing an x-y coordinate system on the joint and resolving F_{BD} into rectangular components, we evaluate Y_{BD} by summing forces in the y direction.

$$\overset{+}{\uparrow} \quad \Sigma F_y = 0$$

$$Y_{BD} = 0$$

Since $Y_{BD} = 0$, it follows that $F_{BD} = 0$. From the discussion to be presented in Section 4.5 on zero bars, this result could have been anticipated.

Compute F_{BC}.

$$\xrightarrow{+} \quad \Sigma F_x = 0$$

$$0 = F_{BC} - 40$$

$$F_{BC} = 40 \text{ kips tension} \qquad \textbf{Ans.}$$

Analyze joint D with $F_{BD} = 0$ and F_{DC} shown as a compressive force (see Figure 4.11*d*).

$$\xrightarrow{+} \quad \Sigma F_x = 0 \quad 0 = 10 - X_{DC} \quad \text{and} \quad X_{DC} = 10 \text{ kips}$$

$$\overset{+}{\uparrow} \quad \Sigma F_y = 0 \quad 0 = 24 - Y_{DC} \quad \text{and} \quad Y_{DC} = 24 \text{ kips}$$

As a check of the results, we observe that the components of F_{DC} are proportional to the slope of the bar. Since all bar forces are known at this point, we can also verify that joint C is in equilibrium, as an alternative check. The results of the analysis are summarized in Figure 4.11*e* on a sketch of the truss. A tension force is indicated with a plus sign, a compressive force with a minus sign.

4.5 Zero Bars

Trusses, such as those used in highway bridges, typically support moving loads. As the load moves from one point to another, forces in truss members vary. For one or more positions of the load, certain bars may remain unstressed. The unstressed bars are termed *zero bars*. The designer can often speed the analysis of a truss by identifying bars in which the forces are zero. In this section we discuss two cases in which bar forces are zero.

Case 1. If No External Load Is Applied to a Joint That Consists of Two Bars, the Force in Both Bars Must Be Zero

To demonstrate the validity of this statement, we will first assume that forces F_1 and F_2 exist in both bars of the two-bar joint in Figure 4.12*a*, and then we demonstrate that the joint cannot be in equilibrium unless both forces equal zero. We begin by superimposing on the joint a rectangular coordinate system with an x axis oriented in the direction of force F_1, and we resolve force F_2 into components X_2 and Y_2 that are parallel to the x and y axes of the coordinate system, respectively. If we sum forces in the y direction, it is evident that the joint cannot be in equilibrium unless Y_2 equals zero because no other force is available to balance Y_2. If Y_2 equals zero, then F_2 is zero, and equilibrium requires that F_1 also equal zero.

A second case in which a bar force must equal zero occurs when a joint is composed of three bars—two of which are collinear.

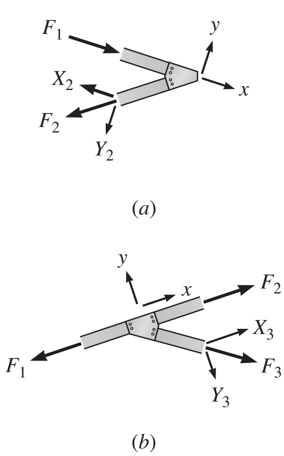


Figure 4.12: Conditions that produce zero forces in bars: (*a*) two bars and no external loads, F_1 and F_2 equal zero; (*b*) two collinear bars and no external loads, force in third bar (F_3) is zero.

Case 2. If No External Load Acts at a Joint Composed of Three Bars—Two of Which Are Collinear—the Force in the Bar That Is Not Collinear Is Zero

To demonstrate this conclusion, we again superimpose a rectangular coordinate system on the joint with the x axis oriented along the axis of the two collinear bars. If we sum forces in the y direction, the equilibrium equation can be satisfied only if F_3 equals zero because there is no other force to balance its y-component Y_3 (see Figure 4.12*b*).

Although a bar may have zero force under a certain loading condition, under other loadings the bar may carry stress. Thus the fact that the force in a bar is zero does not indicate that the bar is not essential and may be eliminated.

EXAMPLE 4.2

Based on the earlier discussion in Section 4.5, label all the bars in the truss of Figure 4.13 that are unstressed when the 60-kip load acts.

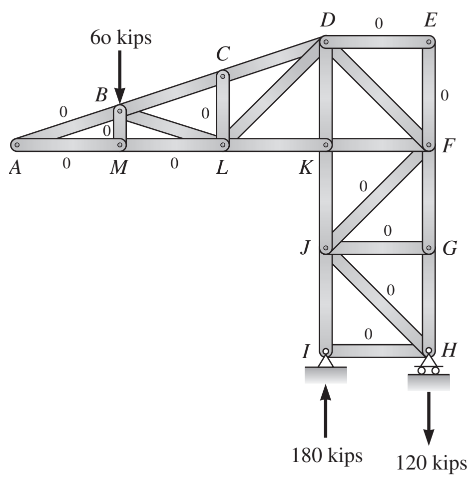


Figure 4.13

Solution

Although the two cases discussed in this section apply to many of the bars, we will examine only joints *A*, *E*, *I*, and *H*. The verification of the remaining zero bars is left to the student. Since joints *A* and *E* are composed of only two bars and no external load acts on the joints, the forces in the bars are zero (see Case 1).

Because no horizontal loads act on the truss, the horizontal reaction at *I* is zero. At joint *I* the force in bar *IJ* and the 180-kip reaction are collinear; therefore, the force in bar *IH* must equal zero because no other horizontal force acts at the joint. A similar condition exists at joint *H*. Since the force in bar *IH* is zero, the horizontal component of bar *HJ* must be zero. If a component of a force is zero, the force must also be zero.

4.6 Method of Sections

To analyze a stable truss by the method of sections, we imagine that the truss is divided into two free bodies by passing an imaginary cutting plane through the structure. The cutting plane must, of course, pass through the bar whose force is to be determined. At each point where a bar is cut, the internal force in the bar is applied to the face of the cut as an external load. Although there is no restriction on the number of bars that can be cut, we often use sections that cut three bars since three equations of static equilibrium are available to analyze a free body. For example, if we wish to determine the bar forces in the chords and diagonal of an interior panel of the truss in Figure 4.14*a*, we can pass a vertical section through the truss, producing the free-body diagram shown in Figure 4.14*b*. As we saw in the method of joints, the engineer is free to assume the direction of the bar force. If a force is assumed in the correct direction, solution of the equilibrium equation will produce a positive value of force. Alternatively, a negative value of force indicates that the direction of the force was assumed incorrectly.

If the force in a diagonal bar of a truss with parallel chords is to be computed, we cut a free body by passing a vertical section through the diagonal bar to be analyzed. An equilibrium equation based on summing forces in the y-direction will permit us to determine the vertical component of force in the diagonal bar.

If three bars are cut, the force in a particular bar can be determined by extending the forces in the other two bars along their line of action until they intersect. By summing moments about the axis through the point of intersection, we can write an equation involving the third force or one of its components. Example 4.3 illustrates the analysis of typical bars in a truss with parallel chords. Example 4.4 on page 136, which covers the analysis of a determinate truss with four restraints, illustrates a general approach to the analysis of a complicated truss using both the method of sections and the method of joints.

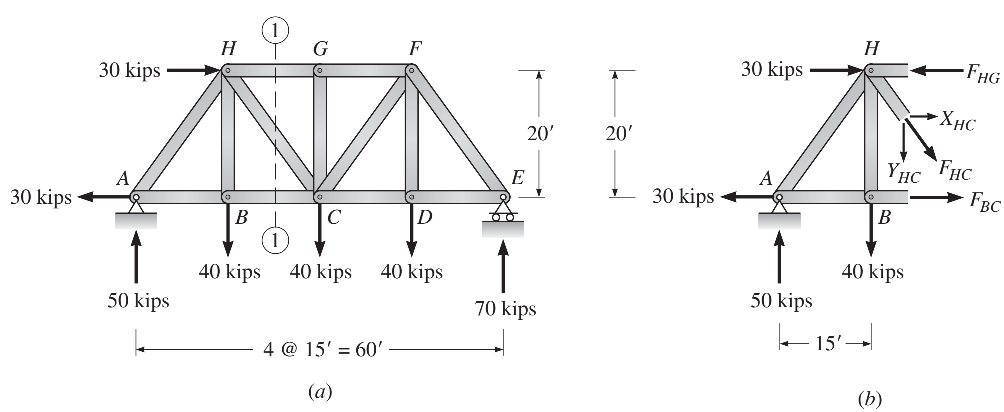


Figure 4.14

EXAMPLE 4.3

Using the method of sections, compute the forces or components of force in bars *HC*, *HG*, and *BC* of the truss in Figure 4.14*a*.

Solution

Pass section 1-1 through the truss cutting the free body shown in Figure 4.14*b*. The direction of the axial force in each member is arbitrarily assumed. To simplify the computations, force F_{HC} is resolved into vertical and horizontal components.

Compute Y_{HC} (see Figure 4.14*b*).

$$\overset{+}{\uparrow} \quad \Sigma F_y = 0$$

$$0 = 50 - 40 - Y_{HC}$$

$$Y_{HC} = 10 \text{ kips tension} \qquad \textbf{Ans.}$$

From the slope relationship,

$$\frac{X_{HC}}{3} = \frac{Y_{HC}}{4}$$

$$X_{HC} = \frac{3}{4} Y_{HC} = 7.5 \text{ kips} \qquad \textbf{Ans.}$$

Compute F_{BC}. Sum moments about an axis through *H* at the intersection of forces F_{HG} and F_{HC}.

$$\circlearrowright^{+} \quad \Sigma M_H = 0$$

$$0 = 30(20) + 50(15) - F_{BC}(20)$$

$$F_{BC} = 67.5 \text{ kips tension} \qquad \textbf{Ans.}$$

Compute F_{HG}.

$$\xrightarrow{+} \quad \Sigma F_x = 0$$

$$0 = 30 - F_{HG} + X_{HC} + F_{BC} - 30$$

$$F_{HG} = 75 \text{ kips compression} \qquad \textbf{Ans.}$$

Since the solution of the equilibrium equations above produced positive values of force, the directions of the forces shown in Figure 4.14*b* are correct.

EXAMPLE 4.4

Analyze the determinate truss in Figure 4.15*a* to determine all bar forces and reactions.

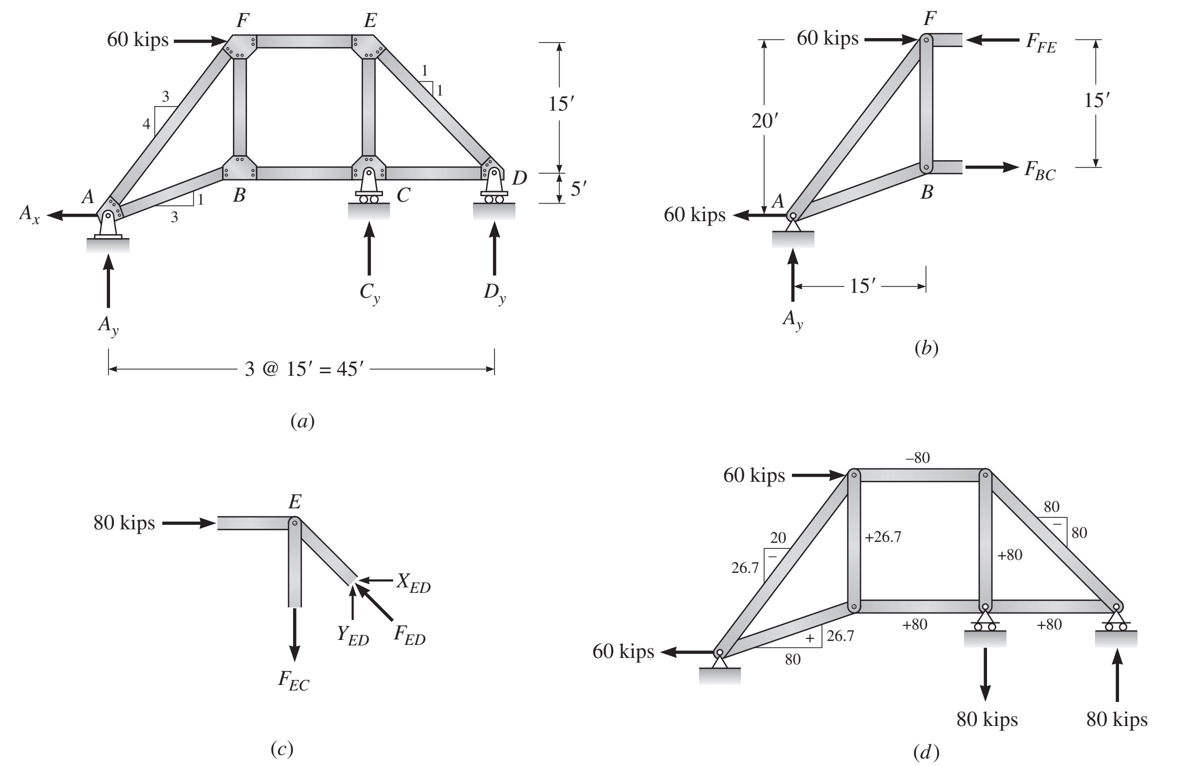


Figure 4.15

Solution

Since the supports at *A*, *C*, and *D* supply four restraints to the truss in Figure 4.15*a*, and only three equations of equilibrium are available, we cannot determine the value of all the reactions by applying the three equations of static equilibrium to a free body of the entire structure. However, recognizing that only one horizontal restraint exists at support *A*, we can determine its value by summing forces in the *x*-direction.

$$\xrightarrow{+} \quad \Sigma F_x = 0$$

$$-A_x + 60 = 0$$

$$A_x = 60 \text{ kips} \qquad \textbf{Ans.}$$

Since the remaining reactions cannot be determined by the equations of statics, we must consider using the method either of joints or of sections. At this stage the method of joints cannot be applied because three or more unknown forces act at each joint. Therefore, we will pass a vertical section through the center panel of the truss to produce the free body shown in Figure 4.15*b*. We must use the free body to the left of the section because the free body to the right of the section cannot be analyzed since the reactions at *C* and *D* and the bar forces in members *BC* and *FE* are unknown.

Compute A_y (see Figure 4.15*b*).

$$\overset{+}{\uparrow} \quad \Sigma F_y = 0$$

$$A_y = 0 \qquad \textbf{Ans.}$$

Compute F_{BC}. Sum moments about an axis through joint *F*.

$$\circlearrowright^{+} \quad \Sigma M_F = 0$$

$$60(20) - F_{BC}(15) = 0$$

$$F_{BC} = 80 \text{ kips (tension)} \qquad \textbf{Ans.}$$

Compute F_{FE}.

$$\rightarrow+ \quad \Sigma F_x = 0$$

$$+60 - 60 + F_{BC} - F_{FE} = 0$$

$$F_{FE} = F_{BC} = 80 \text{ kips (compression)} \qquad \textbf{Ans.}$$

Now that several internal bar forces are known, we can complete the analysis using the method of joints. Isolate joint *E* (Figure 4.15*c*).

$$\rightarrow+ \quad \Sigma F_x = 0$$

$$80 - X_{ED} = 0$$

$$X_{ED} = 80 \text{ kips (compression)} \qquad \textbf{Ans.}$$

Since the slope of bar *ED* is 1:1, $Y_{ED} = X_{ED} = 80$ kips.

$$\overset{+}{\uparrow} \quad \Sigma F_y = 0$$

$$F_{EC} - Y_{ED} = 0$$

$$F_{EC} = 80 \text{ kips (tension)} \qquad \textbf{Ans.}$$

The balance of the bar forces and the reactions at *C* and *D* can be determined by the method of joints. Final results are shown on a sketch of the truss in Figure 4.15*d*.

EXAMPLE 4.5

Determine the forces in bars HG and HC of the truss in Figure 4.16a by the method of sections.

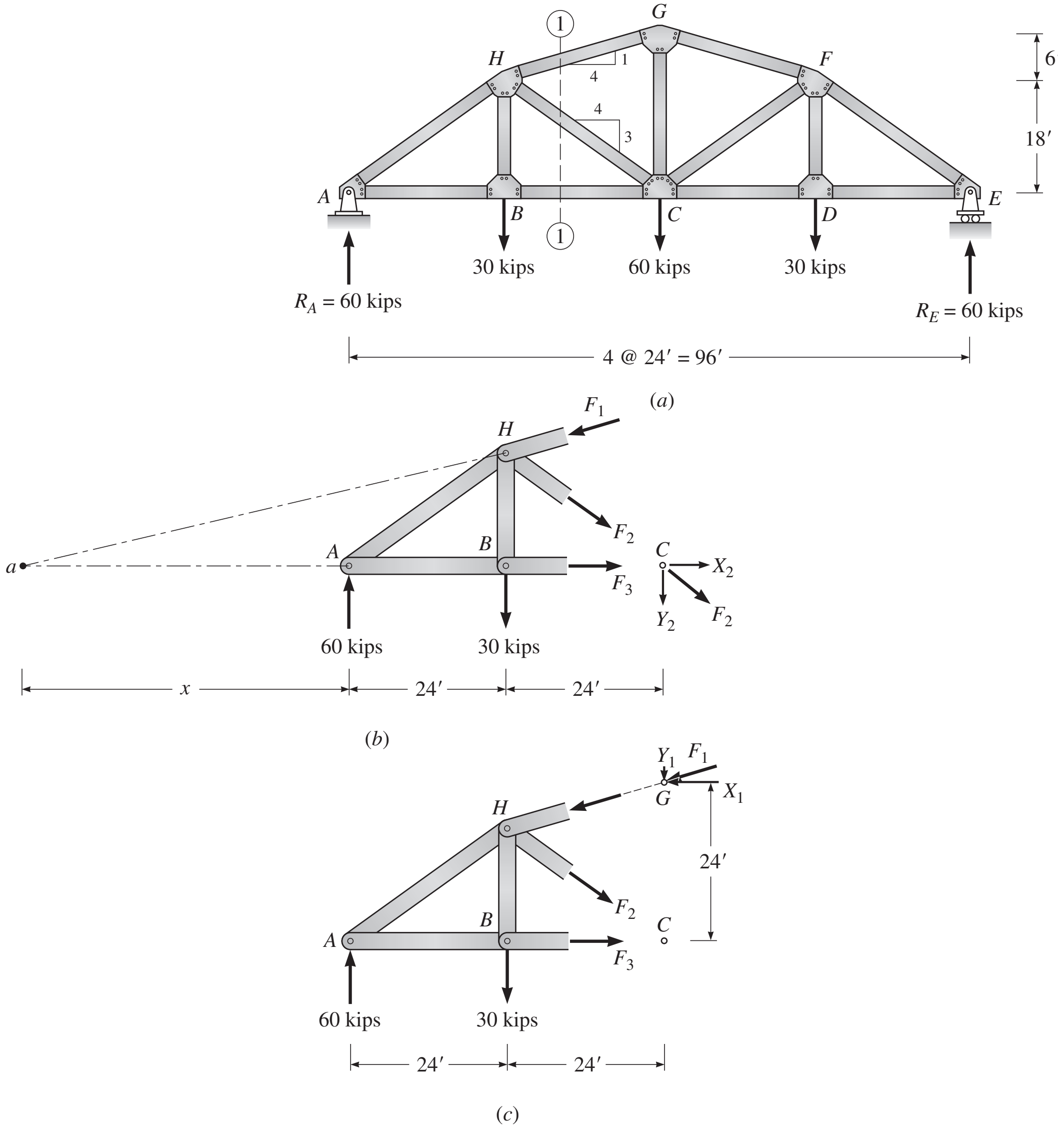


Figure 4.16: (a) Details of truss; (b) free body to compute force in bar HC; (c) free body to compute force in bar HG.

Solution

First compute the force in bar *HC*. Pass vertical section 1-1 through the truss, and consider the free body to the left of the section (see Figure 4.16*b*). The bar forces are applied as external loads to the ends of the bars at the cut. Since three equations of statics are available, all bar forces can be determined by the equations of statics. Let F_2 represent the force in bar *HC*. To simplify the computations, we select a moment center (point *a* that lies at the intersection of the lines of action of forces F_1 and F_3). Force F_2 is next extended along its line of action to point *C* and replaced by its rectangular components X_2 and Y_2. The distance *x* between *a* and the left support is established by proportion using similar triangles, that is, *aHB* and the slope (1: 4) of force F_1.

$$\frac{1}{18} = \frac{4}{x + 24}$$
$$x = 48 \text{ ft}$$

Sum moments of the forces about point *a* and solve for Y_2.

$$\circlearrowright^{+} \quad \Sigma M_a = 0$$
$$0 = -60(48) + 30(72) + Y_2(96)$$
$$Y_2 = 7.5 \text{ kips tension} \qquad \textbf{Ans.}$$

Based on the slope of bar *HC*, establish X_2 by proportion.

$$\frac{Y_2}{3} = \frac{X_2}{4}$$
$$X_2 = \frac{4}{3}Y_2 = 10 \text{ kips} \qquad \textbf{Ans.}$$

Now compute the force F_1 in bar *HG*. Select a moment center at the intersection of the lines of action of forces F_2 and F_3, that is, at point *C* (see Figure 4.16*c*). Extend force F_1 to point *G* and break into rectangular components. Sum moments about point *C*.

$$\circlearrowright^{+} \quad \Sigma M_c = 0$$
$$0 = 60(48) - 30(24) - X_1(24)$$
$$X_1 = 90 \text{ kips compression} \qquad \textbf{Ans.}$$

Establish Y_1 by proportion.

$$\frac{X_1}{4} = \frac{Y_1}{1}$$
$$Y_1 = \frac{X_1}{4} = 22.5 \text{ kips} \qquad \textbf{Ans.}$$

EXAMPLE 4.6

Using the method of sections, compute the forces in bars *BC* and *JC* of the *K* truss in Figure 4.17*a*.

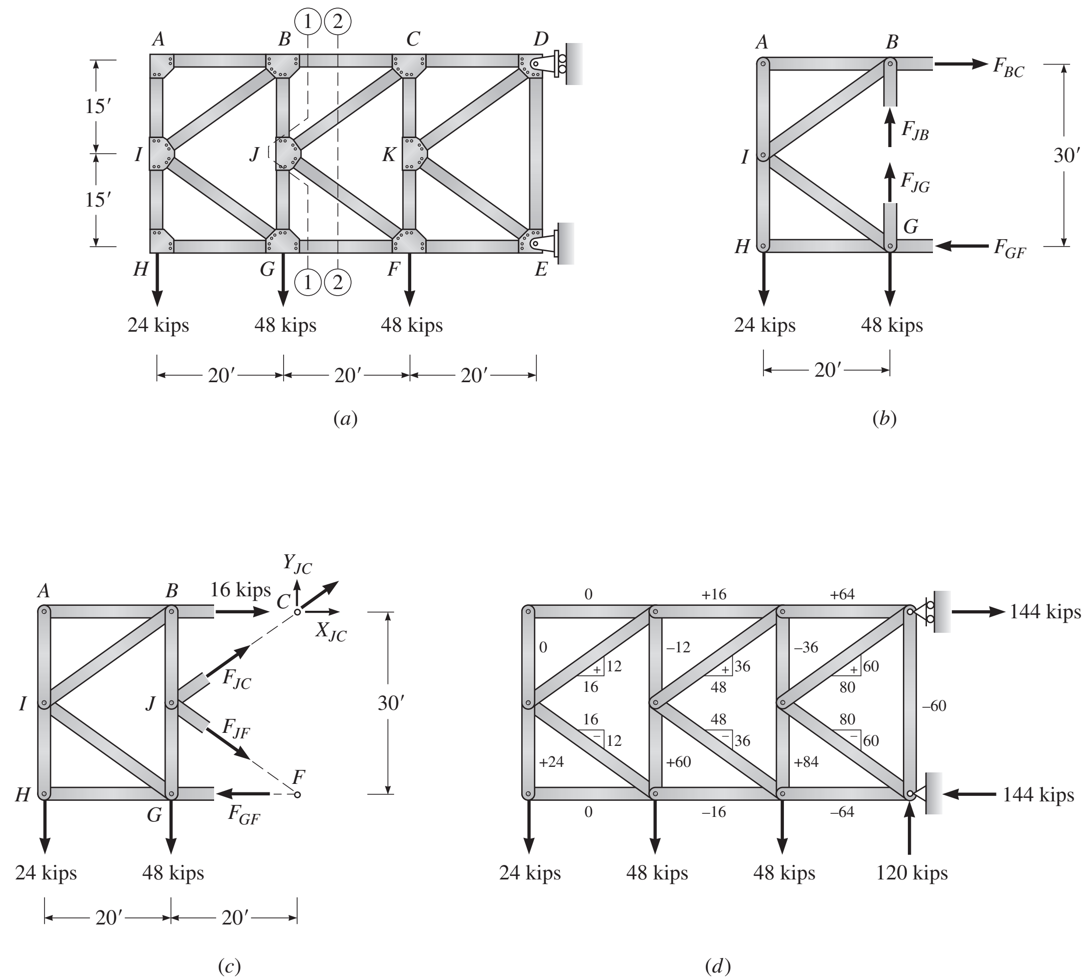


Figure 4.17: (*a*) *K* truss; (*b*) free body to the left of section 1-1 used to evaluate F_{BC}; (*c*) free body used to compute F_{JC}; (*d*) bar forces.

Solution

Since any *vertical* section passing through the panel of a *K* truss cuts four bars, it is not possible to compute bar forces by the method of sections because the number of unknowns exceeds the number of equations of statics. Since no moment center exists through which three of the bar forces pass, not even a partial solution is possible using a standard vertical section. As we illustrate in this example, it is possible to analyze a *K* truss by using two sections in sequence, the first of which is a special section curving around an interior joint.

To compute the force in bar *BC*, we pass Section 1–1 through the truss in Figure 4.17*a*. The free body to the left of the section is shown in Figure 4.17*b*. Summing moments about the bottom joint *G* gives

$$\circlearrowright^{+} \quad \Sigma M_G = 0$$

$$30F_{BC} - 24(20) = 0$$

$$F_{BC} = 16 \text{ kips tension} \qquad \textbf{Ans.}$$

To compute F_{JC}, we pass section 2-2 through the panel and consider again the free body to the left (see Figure 4.17*c*). Since the force in bar *BC* has been evaluated, the three unknown bar forces can be determined by the equations of statics. Use a moment center at *F*. Extend the force in bar *JC* to point *C* and break into rectangular components.

$$\circlearrowright^{+} \quad \Sigma M_F = 0$$

$$0 = 16(30) + X_{JC}(30) - 20(48) - 40(24)$$

$$X_{JC} = 48 \text{ kips}$$

$$F_{JC} = \frac{5}{4} X_{JC} = 60 \text{ kips tension} \qquad \textbf{Ans.}$$

NOTE. The *K* truss can also be analyzed by the method of joints by starting from an outside joint such as *A* or *H*. The results of this analysis are shown in Figure 4.17*d*. The *K* bracing is typically used in deep trusses to reduce the length of the diagonal members. As you can see from the results in Figure 4.17*d*, the shear in a panel divides equally between the top and bottom diagonals. One diagonal carries compression, and the other carries tension.

4.7 Determinacy and Stability

Thus far the trusses we have analyzed in this chapter have all been stable determinate structures; that is, we knew in advance that we could carry out a complete analysis using the equations of statics alone. Since indeterminate trusses are also used in practice, an engineer must be able to recognize a structure of this type because indeterminate trusses require a special type of analysis. As we will discuss in Chapter 11, compatibility equations must be used to supplement equilibrium equations.

If you are investigating a truss designed by another engineer, you will have to establish if the structure is determinate or indeterminate before you begin the analysis. Further, if you are responsible for establishing the configuration of a truss for a special situation, you must obviously be able to select an arrangement of bars that is stable. The purpose of this section is to extend to trusses the introductory discussion of stability and determinacy in Sections 3.8 and 3.9—topics you may wish to review before proceeding to the next paragraph.

If a loaded truss is in equilibrium, all members and joints of the truss must also be in equilibrium. If load is applied only at the joints and if all truss members are assumed to carry only axial load (an assumption that implies the dead load of members may be neglected or applied at the joints as an equivalent concentrated load), the forces acting on a free-body diagram of a joint will constitute a concurrent force system. To be in equilibrium, a concurrent force system must satisfy the following two equilibrium equations:

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

Since we can write two equilibrium equations for each joint in a truss, the total number of equilibrium equations available to solve for the unknown bar forces b and reactions r equals $2n$ (where n represents the total number of joints). Therefore, it must follow that if a truss is *stable* and *determinate*, the relationship between bars, reactions, and joints must satisfy the following criteria:

$$r + b = 2n \tag{4.1}$$

In addition, as we discussed in Section 3.8, *the restraints exerted by the reactions must not constitute either a parallel or a concurrent force system.*

Although three equations of statics are available to compute the reactions of a determinate truss, these equations are not independent and they cannot be added to the $2n$ joint equations. Obviously, if all joints of a truss are in equilibrium, the entire structure must also be equilibrium; that is, the resultant of the external forces acting on the truss equals zero. If the resultant is zero, the equations of static equilibrium are automatically satisfied when applied to the entire structure and thus do not supply additional independent equilibrium equations.

If

$$r + b > 2n$$

then the number of unknown forces exceed the available equations of statics and the truss is indeterminate. The degree of indeterminacy D equals

$$D = r + b - 2n \tag{4.2}$$

Finally, if

$$r + b < 2n$$

there are insufficient bar forces and reactions to satisfy the equations of equilibrium, and the structure is unstable.

Moreover, as we discussed in Section 3.8, you will always find that the analysis of an unstable structure leads to an inconsistent equilibrium equation. Therefore, if you are uncertain about the stability of a structure, analyze the structure for any arbitrary loading. If a solution that satisfies statics results, the structure is stable.

To illustrate the criteria for stability and determinacy for trusses introduced in this section, we will classify the trusses in Figure 4.18 as stable or unstable. For those structures that are stable, we will establish whether they are determinate or indeterminate. Finally, if a structure is indeterminate, we will also establish the degree of indeterminacy.

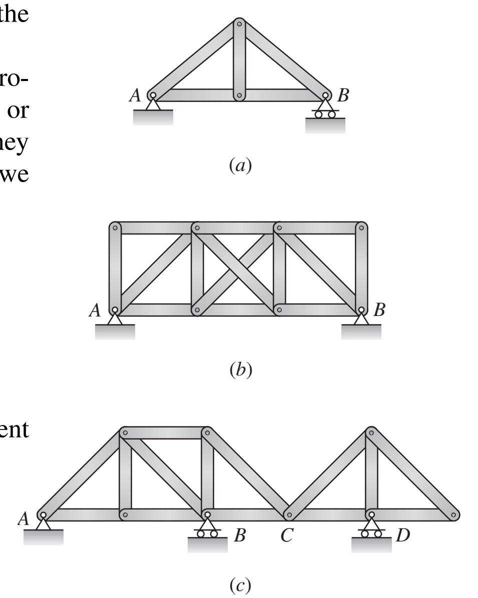


Figure 4.18: Classifying trusses: (*a*) stable determinate; (*b*) indeterminate second degree; (*c*) determinate (*continues*).

Figure 4.18*a*

$$b + r = 5 + 3 = 8 \qquad 2n = 2(4) = 8$$

Since $b + r = 2n$ and the reactions are not equivalent to either a concurrent or a parallel force system, the truss is stable and determinate.

Figure 4.18*b*

$$b + r = 14 + 4 = 18 \qquad 2n = 2(8) = 16$$

Since $b + r$ exceeds $2n$ ($18 > 16$), the structure is indeterminate to the second degree. The structure is one degree *externally* indeterminate because the supports supply four restraints, and *internally* indeterminate to the first degree because an extra diagonal is supplied in the middle panel to transmit shear.

Figure 4.18*c*

$$b + r = 14 + 4 = 18 \qquad 2n = 2(9) = 18$$

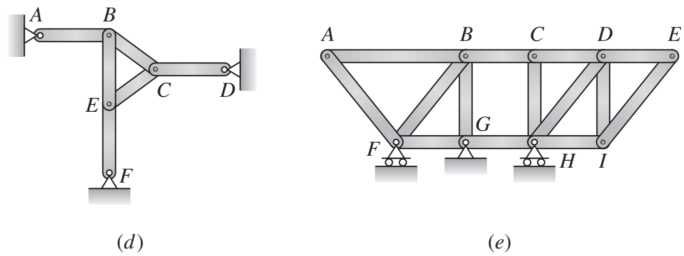


Figure 4.18: *Continued*: (*d*) determinate; (*e*) determinate.

Because $b + r = 2n = 18$, and the supports are not equivalent to either a parallel or a concurrent force system, the structure appears stable. We can confirm this conclusion by observing that truss *ABC* is obviously a stable component of the structure because it is a simple truss (composed of triangles) that is supported by three restraints—two supplied by the pin at *A* and one supplied by the roller at *B*. Since the hinge at *C* is attached to the stable truss on the left, it, too, is a stable point in space. Like a pin support, it can supply both horizontal and vertical restraint to the truss on the right. Thus we can reason that truss *CD* must also be stable since it, too, is a simple truss supported by three restraints, that is, two supplied by the hinge at *C* and one by the roller at *D*.

Figure 4.18*d* Two approaches are possible to classify the structure in Figure 4.18*d*. In the first approach, we can treat triangular element *BCE* as a three-bar truss ($b = 3$) supported by three links—*AB*, *EF*, and *CD* ($r = 3$). Since the truss has three joints (*B*, *C*, and *E*), $n = 3$. And $b + r = 6$ equals $2n = 2(3) = 6$, and the structure is determinate and stable.

Alternatively, we can treat the entire structure as a six-bar truss ($b = 6$), with six joints ($n = 6$), supported by three pins ($r = 6$), $b + r = 12$ equals $2n = 2(6) = 12$. Again we conclude that the structure is stable and determinate.

Figure 4.18*e*

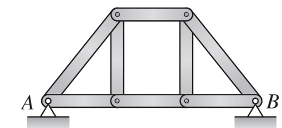
$$b + r = 14 + 4 = 18 \qquad 2n = 2(9) = 18$$

Since $b + r = 2n$, it appears the structure is stable and determinate; however, since a rectangular panel exists between joints *B*, *C*, *G*, and *H*, we will verify that the structure is stable by analyzing the truss for an arbitrary load of 4 kips applied vertically at joint *F* (see Example 4.7). Since analysis by the method of joints produces unique values of bar force in all members, we conclude that the structure is both stable and determinate.

Figure 4.18*f*

$$b + r = 8 + 4 = 12 \qquad 2n = 2(6) = 12$$

Although the bar count above satisfies the necessary condition for a stable determinate structure, the structure appears to be unstable because the center panel, lacking a diagonal bar, cannot transmit vertical force. To confirm this conclusion, we will analyze the truss, using the equations of statics. (The analysis is carried out in Example 4.8.) Since the analysis leads to an inconsistent equilibrium equation, we conclude that the structure is unstable.

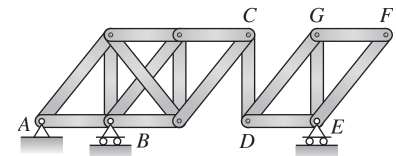


(*f*)

Figure 4.18*g*

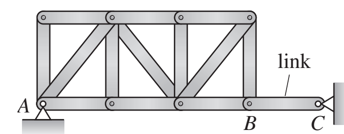
$$b = 16 \qquad r = 4 \qquad n = 10$$

Although $b + r = 2n$, the small truss on the right (*DEFG*) is unstable because its supports—the link *CD* and the roller at *E*—constitute a parallel force system.



(*g*)

Figure 4.18*h* Truss is geometrically unstable because the reactions constitute a concurrent force system; that is, the reaction supplied by the link *BC* passes through the pin at *A*.

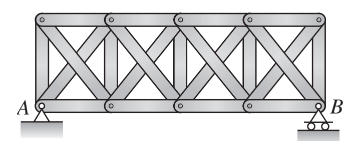


(*h*)

Figure 4.18*i*

$$b = 21 \qquad r = 3 \qquad n = 10$$

And $b + r = 24$, $2n = 20$; therefore, truss is indeterminate to the fourth degree. Although the reactions can be computed for any loading, the indeterminacy is due to the inclusion of double diagonals in all interior panels.

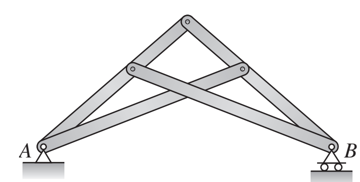


(*i*)

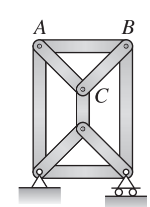
Figure 4.18*j*

$$b = 6 \qquad r = 3 \qquad n = 5$$

And $b + r = 9$, $2n = 10$; the structure is unstable because there are fewer restraints than required by the equations of statics. To produce a stable structure, the reaction at *B* should be changed from a roller to a pin.



(*j*)



(*k*)

Figure 4.18*k* Now $b = 9$, $r = 3$, and $n = 6$; also $b + r = 12$, $2n = 12$. However, the structure is unstable because the small triangular truss *ABC* at the top is supported by three parallel links, which provide no lateral restraint.

Figure 4.18: Classifying trusses: (*f*) unstable; (*g*) unstable; (*h*) unstable; (*i*) indeterminate fourth degree; (*j*) unstable; (*k*) unstable.

EXAMPLE 4.7

Verify that the truss in Figure 4.19 is stable and determinate by demonstrating that it can be completely analyzed by the equations of statics for a force of 4 kips at joint *F*.

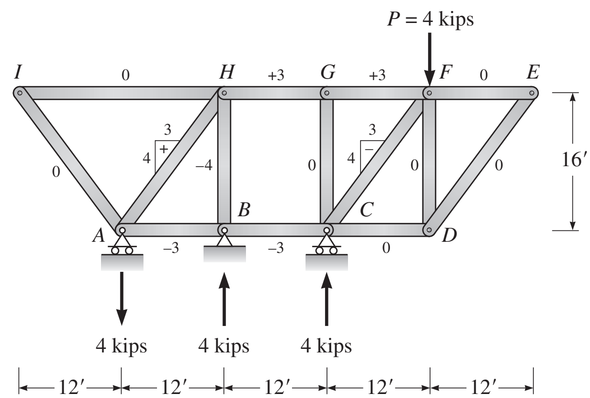


Figure 4.19: Analysis by *method of joints* to verify that truss is stable.

Solution

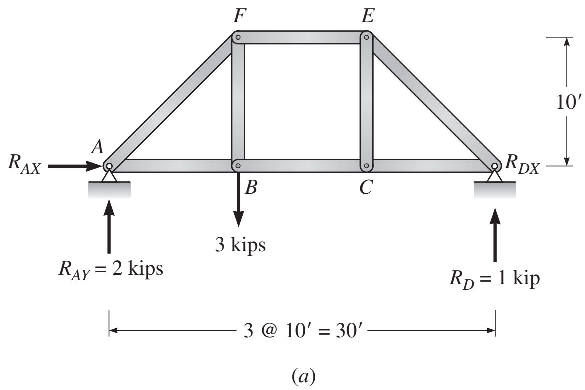
Since the structure has four reactions, we cannot start the analysis by computing reactions, but instead must analyze it by the method of joints. We first determine the zero bars.

Since joints *E* and *I* are connected to only two bars and no external load acts on the joints, the forces in these bars are zero (see Case 1 of Section 4.5). With the remaining two bars connecting to joint *D*, applying the same argument would indicate that these two members are also zero bars. Applying Case 2 of Section 4.5 to joint *G* would indicate that bar *CG* is a zero bar.

Next we analyze in sequence joints *F*, *C*, *G*, *H*, *A*, and *B*. Since all bar forces and reactions can be determined by the equations of statics (results are shown on Figure 4.19), we conclude that the truss is stable and determinate.

Prove that the truss in Figure 4.20*a* is unstable by demonstrating that its analysis for a load of arbitrary magnitude leads to an inconsistent equation of equilibrium.

EXAMPLE 4.8



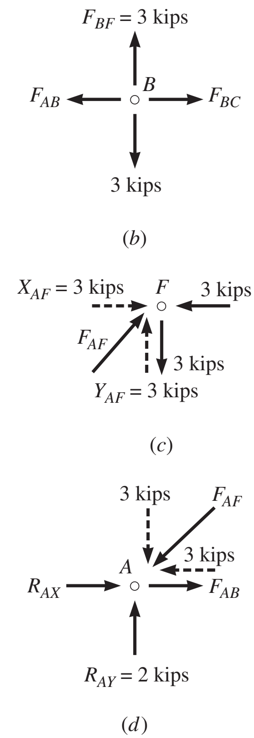


Figure 4.20: Check of truss stability: (*a*) details of truss; (*b*) free body of joint *B*; (*c*) free body of joint *F*; (*d*) free body of support *A*.

Solution

Apply a load at joint *B*, say 3 kips, and compute the reactions, considering the entire structure as a free body.

$$\circlearrowright^{+} \quad \Sigma M_A = 0$$

$$3(10) - 30R_D = 0 \qquad R_D = 1 \text{ kip}$$

$$\overset{+}{\uparrow} \quad \Sigma F_y = 0$$

$$R_{AY} - 3 + R_D = 0 \qquad R_{AY} = 2 \text{ kips}$$

Equilibrium of joint *B* (see Figure 4.20*b*) requires that $F_{BF} = 3$ kips tension. Equilibrium in the *x* direction is possible if $F_{AB} = F_{BC}$.

We next consider joint *F* (see Figure 4.20*c*). To be in equilibrium in the *y*-direction, the vertical component of F_{AF} must equal 3 kips and be directed upward, indicating that bar *AF* is in compression. Since the slope of bar *AF* is 1:1, its horizontal component also equals 3 kips. Equilibrium of joint *F* in the *x* direction requires that the force in bar *FE* equal 3 kips and act to the left.

We now examine support *A* (Figure 4.20*d*). The reaction R_A and the components of force in bar *AF*, determined previously, are applied to the joint. Writing the equation of equilibrium in the *y*-direction, we find

$$\overset{+}{\uparrow} \quad \Sigma F_y = 0$$

$$2 - 3 \neq 0 \qquad \text{(inconsistent)}$$

Since the equilibrium equation is not satisfied, the structure is not stable.

4.8 Computer Analysis of Trusses

The preceding sections of this chapter have covered the analysis of trusses based on the assumptions that (1) members are connected at joints by frictionless pins and (2) loads are applied at joints only. When design loads are conservatively chosen, and deflections are not excessive, over the years these simplifying assumptions have generally produced satisfactory designs.

Since joints in most trusses are constructed by connecting members to gusset plates by welds, rivet, or high-strength bolts, joints are usually *rigid*. To analyze a truss with rigid joints (a highly indeterminate structure) would be a lengthy computation by the classical methods of analysis. That is why, in the past, truss analysis has been simplified by allowing designers to assume pinned joints. Now that computer programs are available, we can analyze both determinate and indeterminate trusses as a rigid-jointed structure to provide a more precise analysis, and the limitation that loads must be applied at joints is no longer a restriction.

Because computer programs require values of cross-sectional properties of members—area and moment of inertia—members must be initially sized. Procedures to estimate the approximate size of members are discussed in Chapter 15 of the text. In the case of a truss with rigid joints, the assumption of pin joints will permit you to compute axial forces that can be used to select the initial cross-sectional areas of members.

To carry out the computer analyses, we will use the RISA-2D computer program that is located on the website of this textbook; that is, http://www.mhhe.com/leet. Although a tutorial is provided on the website to explain, step by step, how to use the RISA-2D program, a brief overview of the procedure is given below.

1. Number all joints and members.
2. After the RISA-2D program is opened, click **Global** at the top of the screen. Insert a descriptive title, your name, and the number of sections.
3. Click **Units**. Use either Standard Metric or Standard Imperial for U.S. Customary System units.
4. Click **Modify**. Set the scale of the grid so the figure of the structure lies within the grid.
5. Fill in tables in **Data Entry Box**. These include Joint Coordinates, Boundary Conditions, Member Properties, Joint Loads, etc. Click **View** to label members and joints. The figure on the screen permits you to check visually that all required information has been supplied correctly.
6. Click **Solve** to initiate the analysis.
7. Click **Results** to produce tables listing bar forces, joint defections, and support reactions. The program will also plot a deflected shape.

EXAMPLE 4.9

Using the RISA-2D computer program, analyze the determinate truss in Figure 4.21, and compare the magnitude of the bar forces and joint displacements, assuming (1) joints are *rigid* and (2) joints are *pinned*. Joints are denoted by numbers in a circle; members, by numbers in a rectangular box. A preliminary analysis of the truss was used to establish initial values of each member's cross-sectional properties (see Table 4.1). For the case of pinned joints, the member data are similar, but the word *pinned* appears in the columns titled **End Releases**.

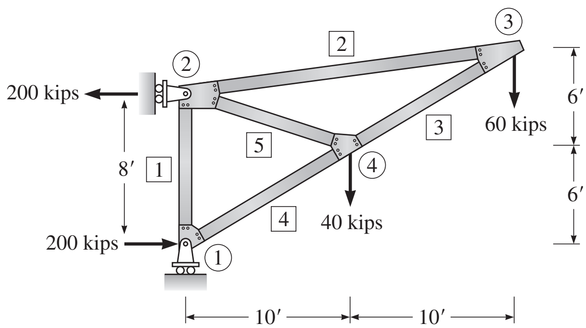


Figure 4.21: Cantilever truss.

TABLE 4.1
Member Data for Case of Rigid Joints

Member Label	I Joint	J Joint	Area (in^2)	Moment of Inertia (in^4)	Elastic Modulus (ksi)	End Releases I-End	End Releases J-End	Length (ft)
1	1	2	5.72	14.7	29,000			8
2	2	3	11.5	77	29,000			20.396
3	3	4	11.5	77	29,000			11.662
4	4	1	15.4	75.6	29,000			11.662
5	2	4	5.72	14.7	29,000			10.198

TABLE 4.2
Comparision of Joint Displacements

Rigid Joints: Joint Label	Rigid Joints: X Translation (in)	Rigid Joints: Y Translation (in)	Pinned Joints: Joint Label	Pinned Joints: X Translation (in)	Pinned Joints: Y Translation (in)
1	0	0	1	0	0
2	0	0.011	2	0	0.012
3	0.257	−0.71	3	0.266	−0.738
4	0.007	−0.153	4	0	−0.15

[*continues on next page*]

Example 4.9 continues . . .

TABLE 4.3

Comparison of Member Forces

Rigid Joints						Pin Joints		
Member Label	Section*	Axial (kips)	Shear (kips)	Moment (kip · ft)		Member Label	Section*	Axial (kips)
1	1	−19.256	−0.36	0.918		1	1	−20
	2	−19.256	−0.36	−1.965			2	−20
2	1	−150.325	0.024	−2.81		2	1	−152.971
	2	−150.325	0.024	−2.314			2	−152.971
3	1	172.429	0.867	−2.314		3	1	174.929
	2	172.429	0.867	7.797			2	174.929
4	1	232.546	−0.452	6.193		4	1	233.238
	2	232.546	−0.452	0.918			2	233.238
5	1	−53.216	−0.24	0.845		5	1	−50.99
	2	−53.216	−0.24	−1.604			2	−50.99

*Sections 1 and 2 refer to member ends.

To facilitate the connection of the members to the gusset plates, the truss members are often fabricated from pairs of double angles oriented back to back. The cross-sectional properties of these structural shapes, tabulated in the *AISC Manual of Steel Construction*, are used in this example.

CONCLUSIONS: The results of the computer analysis shown in Tables 4.2 and 4.3 indicate that the magnitude of the axial forces in the truss bars, as well as the joint displacements, are approximately the same for both pinned and rigid joints. The axial forces are slightly smaller in most bars when *rigid* joints are assumed because a portion of the load is transmitted by shear and bending.

Since members in direct stress carry axial load efficiently, cross-sectional areas tend to be small when sized for axial load alone. However, the flexural stiffness of small compact cross sections is also small. Therefore, when joints are *rigid*, bending stress in truss members may be *significant even when the magnitude of the moments is relatively small*. If we check stresses in member M3, which is constructed from two $8 \times 4 \times \frac{1}{2}$ in angles, at the section where the moment is 7.797 kip·ft, the axial stress is $P/A = 14.99$ kips/in^2 and the bending stress $Mc/I = 6.24$ kips/in^2. In this case, we conclude that bending stresses are significant in several truss members when the analysis is carried out assuming joints are *rigid*, and the designer must verify that the combined stress of 21.23 kips/in^2 does not exceed the allowable value specified by the AISC design specifications.

Summary

- Trusses are composed of slender bars that are assumed to carry only axial force. Joints in large trusses are formed by welding or bolting members to gusset plates. If members are relatively small and lightly stressed, joints are often formed by welding the ends of vertical and diagonal members to the top and bottom chords.
- Although trusses are stiff in their own plane, they have little lateral stiffness; therefore, they must be braced against lateral displacement at all panel points.
- To be *stable* and *determinate*, the following relationship must exist among the number of bars b, reactions r, and joints n:

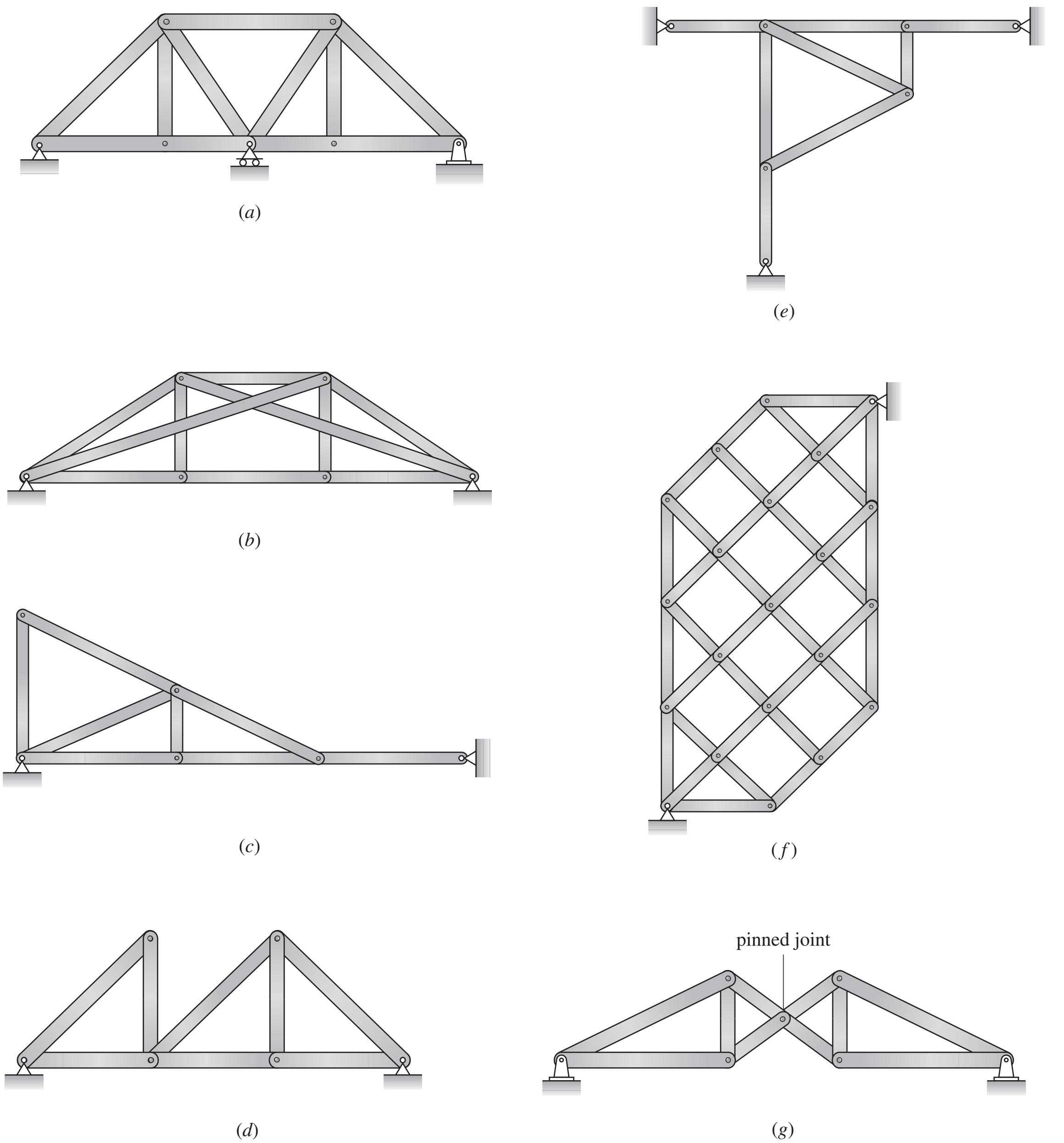
  $$b + r = 2n$$

  In addition, the restraints exerted by the reactions must *not* constitute either a parallel or a concurrent force system.

  If $b + r < 2n$, the truss is unstable. If $b + r > 2n$, the truss is indeterminate.
- Determinate trusses can be analyzed either by the method of joints or by the method of sections. The method of sections is used when the force in one or two bars is required. The method of joints is used when all bar forces are required.
- If the analysis of a truss results in an inconsistent value of forces, that is, one or more joints are not in equilibrium, then the truss is unstable.

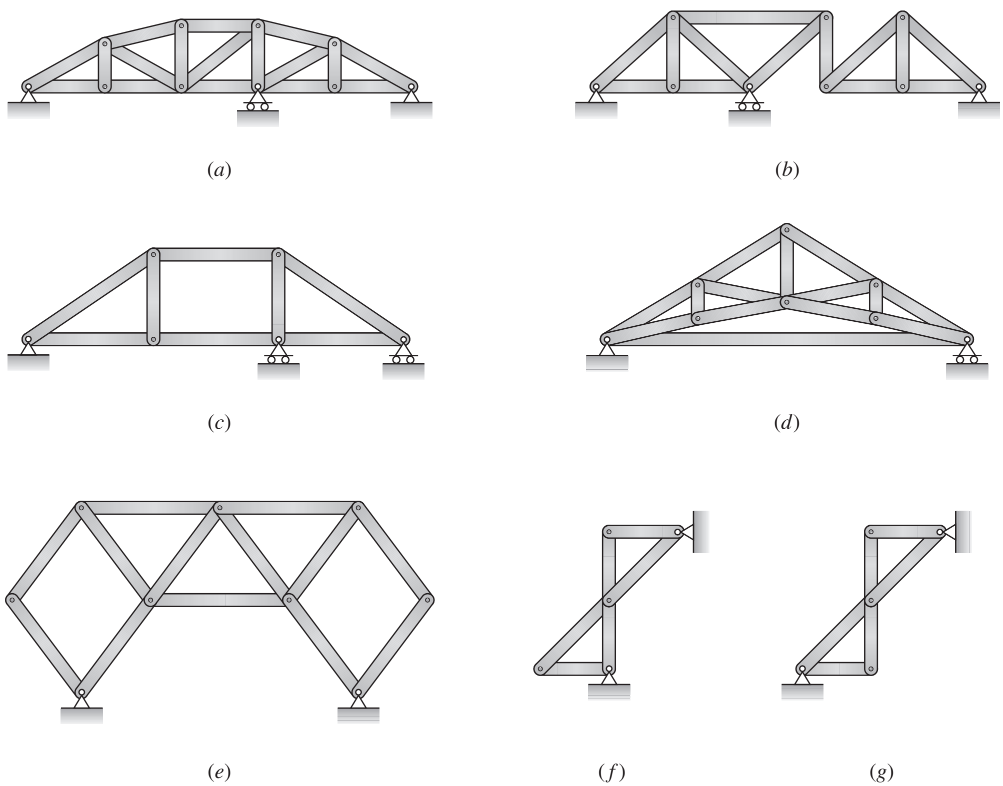
PROBLEMS

P4.1. Classify the trusses in Figure P4.1 as stable or unstable. If stable, indicate if determinate or indeterminate. If indeterminate, indicate the degree of indeterminacy.



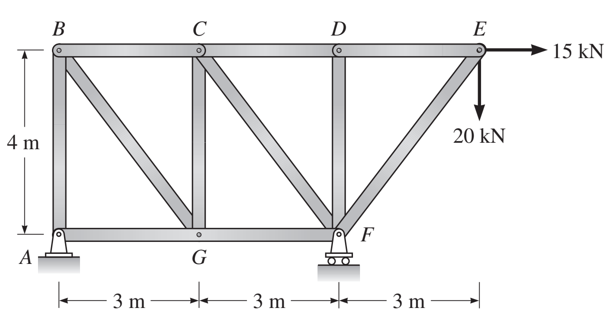
P4.1

P4.2. Classify the trusses in Figure P4.2 as stable or unstable. If stable, indicate if determinate or indeterminate. If indeterminate, indicate the degree.

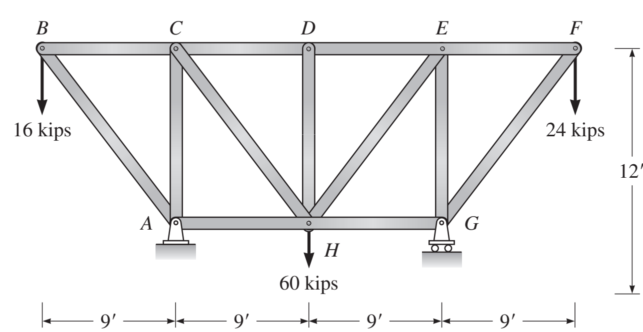


P4.2

P4.3 and **P4.4.** Determine the forces in all bars of the trusses. Indicate tension or compression.

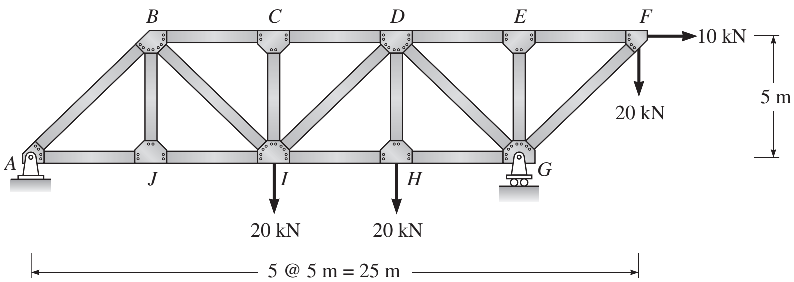


P4.3

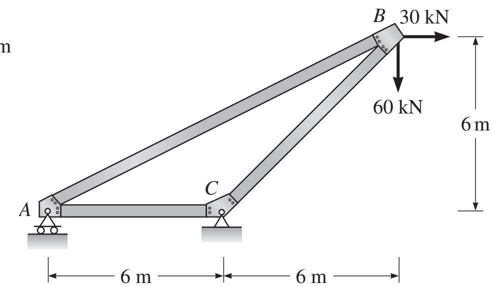


P4.4

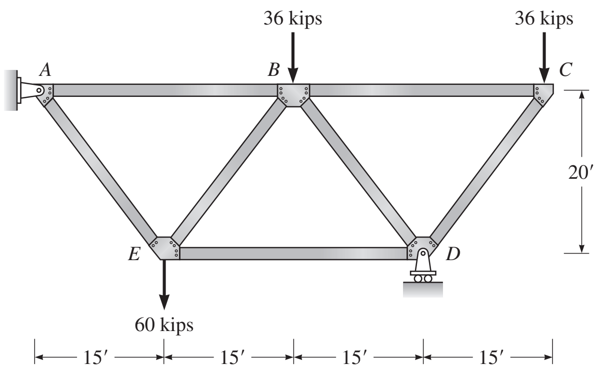
P4.5 to **P4.10.** Determine the forces in all bars of the trusses. Indicate tension or compression.



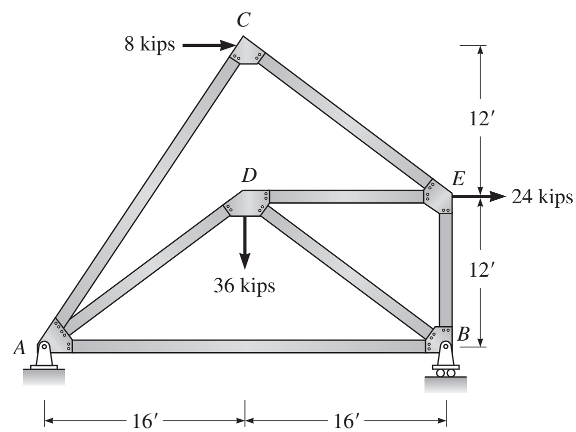
P4.5



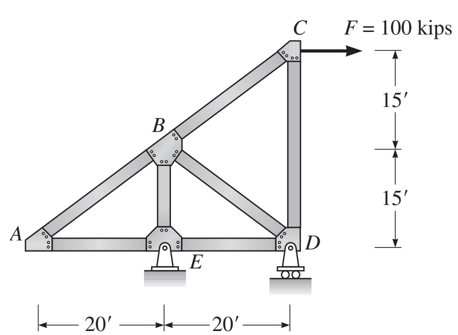
P4.8



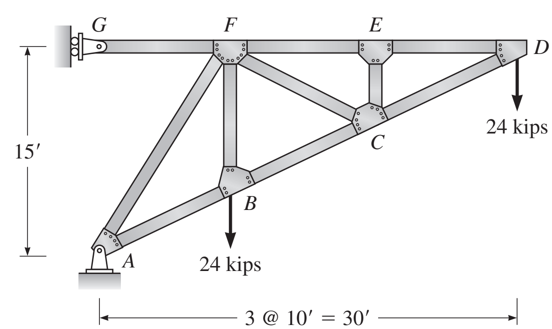
P4.6



P4.9

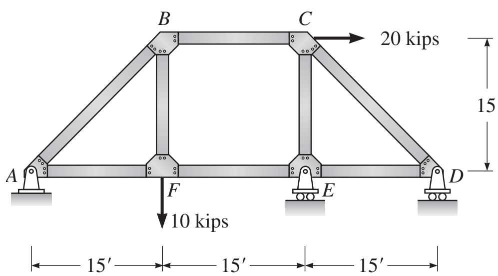


P4.7

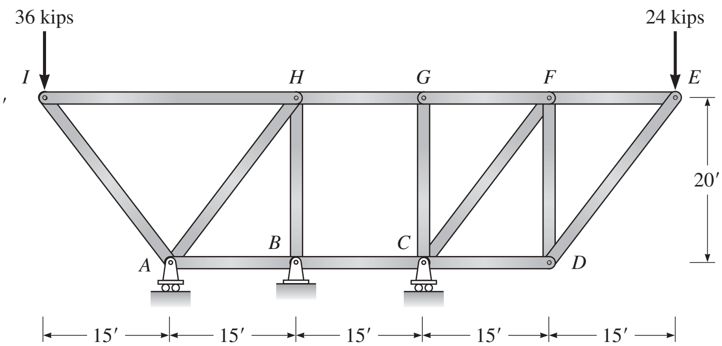


P4.10

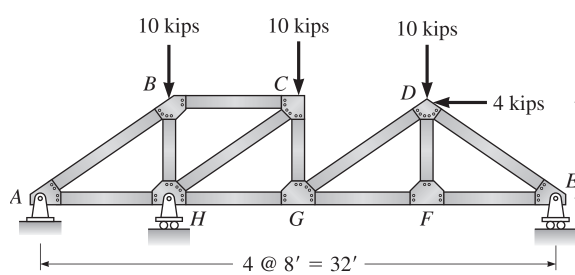
P4.11 to **P4.15.** Determine the forces in all bars of the trusses. Indicate if tension or compression.



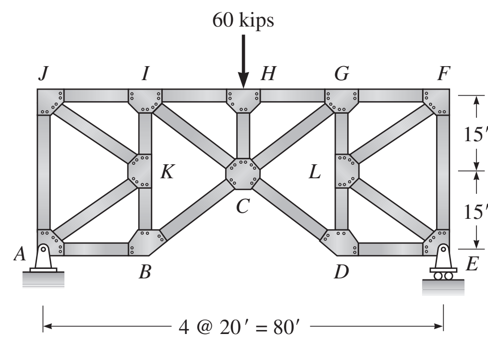
P4.11



P4.12

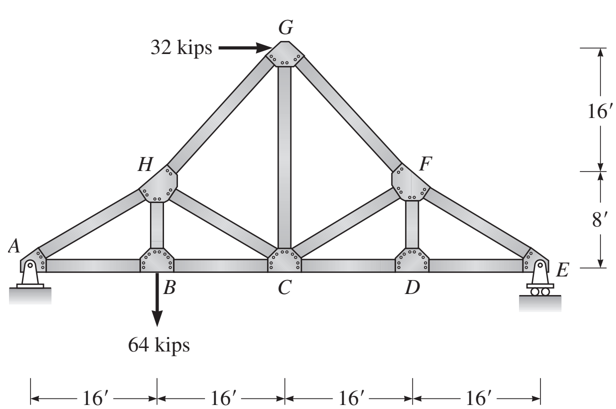


P4.13

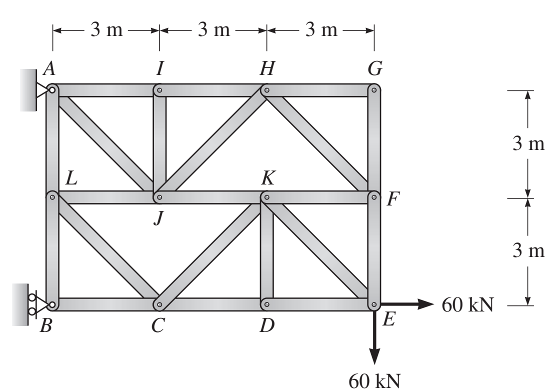


P4.14

P4.16. Determine the forces in all bars of the truss. *Hint*: If you have trouble computing bar forces, review *K* truss analysis in Example 4.6.

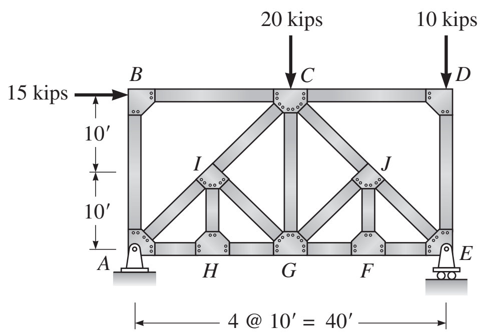


P4.15

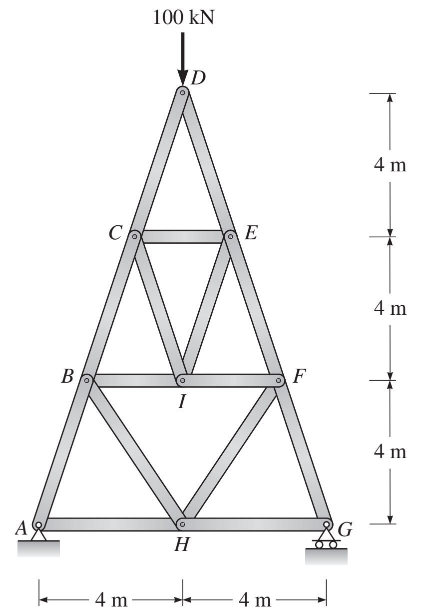


P4.16

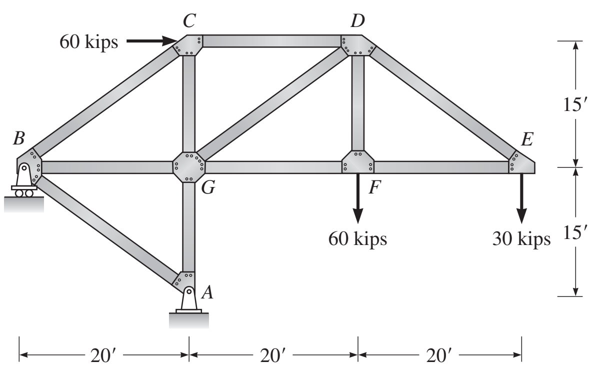
P4.17 to **P4.21.** Determine the forces in all bars of the trusses. Indicate if tension or compression.



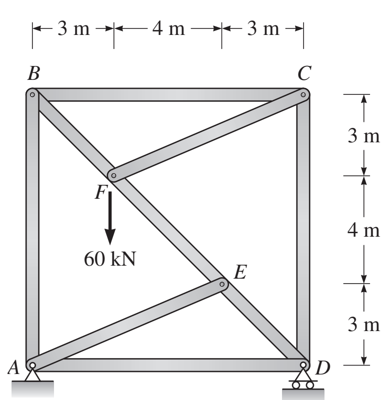
P4.17



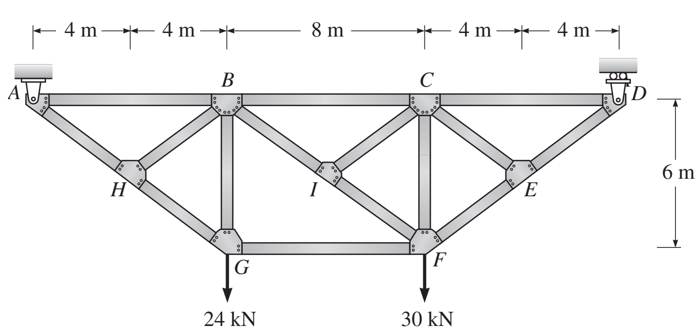
P4.20



P4.18

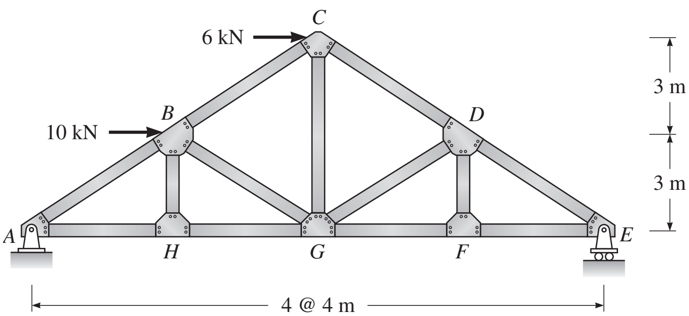


P4.19

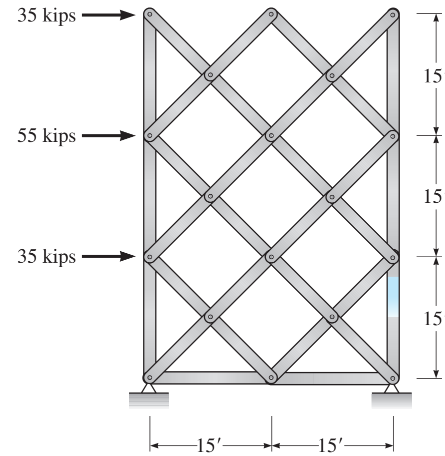


P4.21

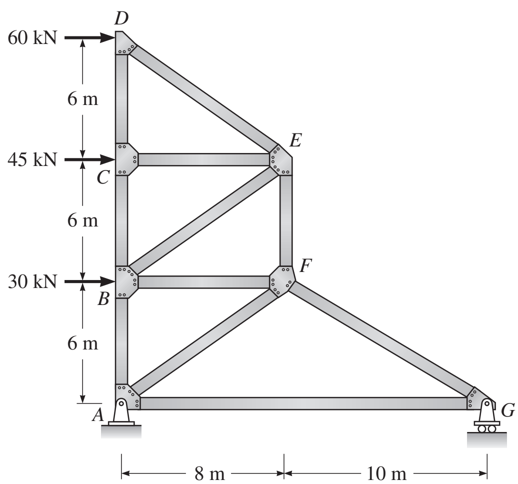
P4.22 to **P4.26.** Determine the forces in all truss bars. Indicate tension or compression.



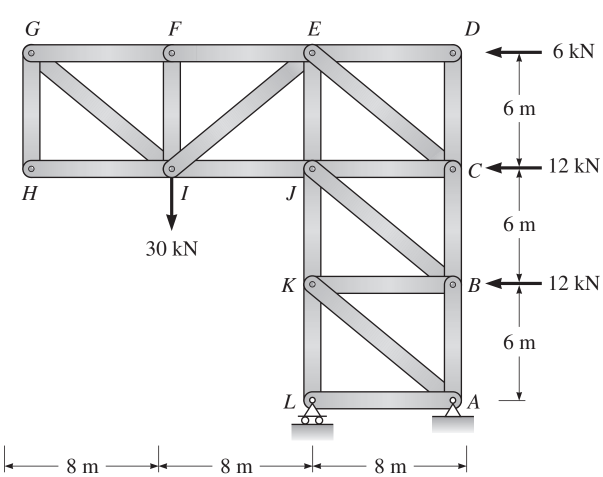
P4.22



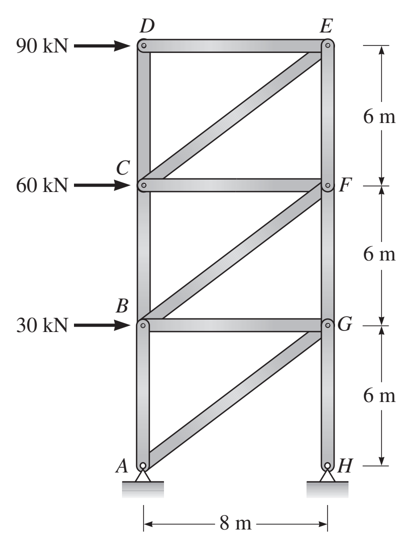
P4.25



P4.23

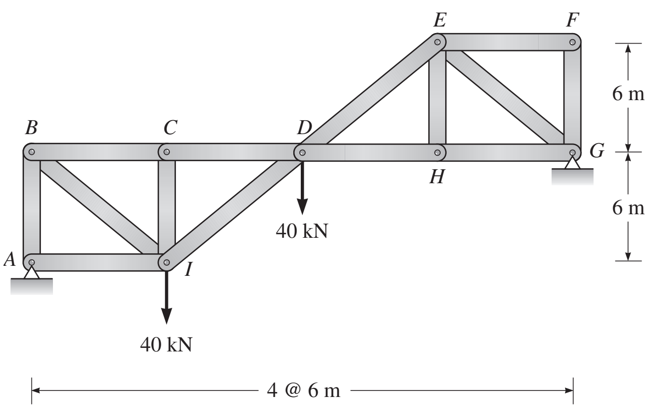


P4.24



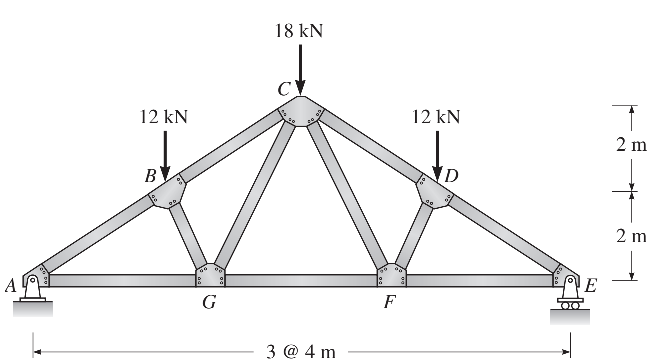
P4.26

P4.27. Determine the forces in all bars of the truss in Figure P4.27. If your solution is statically inconsistent, what conclusions can you draw about the truss? How might you modify the truss to improve its behavior? Also, analyze the truss with your computer program. Explain your results.

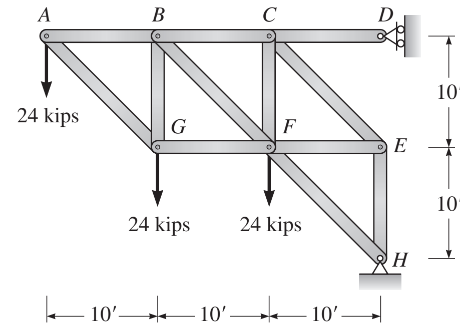


P4.27

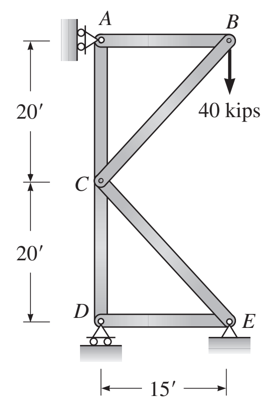
P4.28 to **P4.31.** Determine the forces in all bars. Indicate tension or compression.



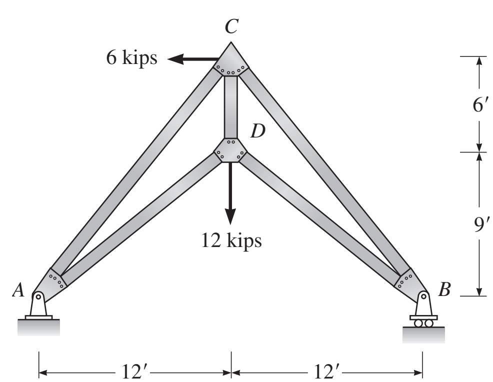
P4.28



P4.29

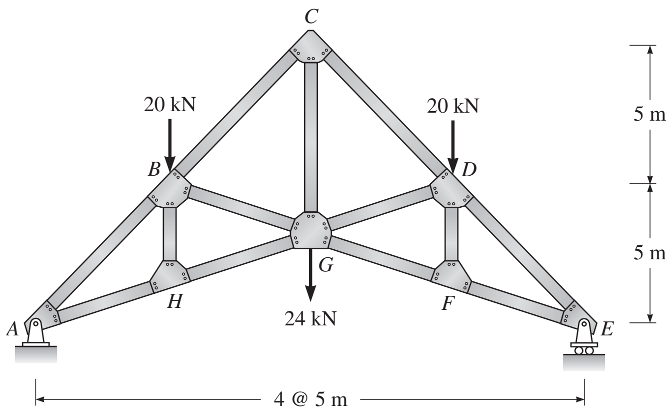


P4.30

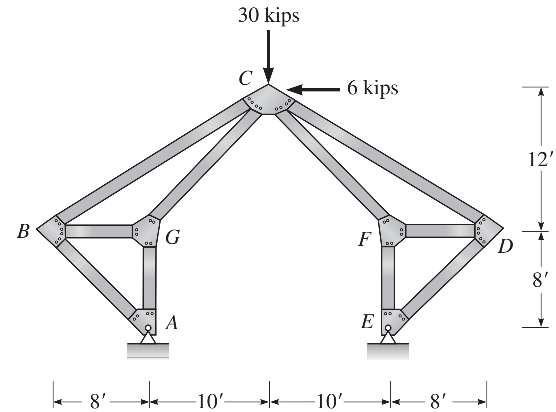


P4.31

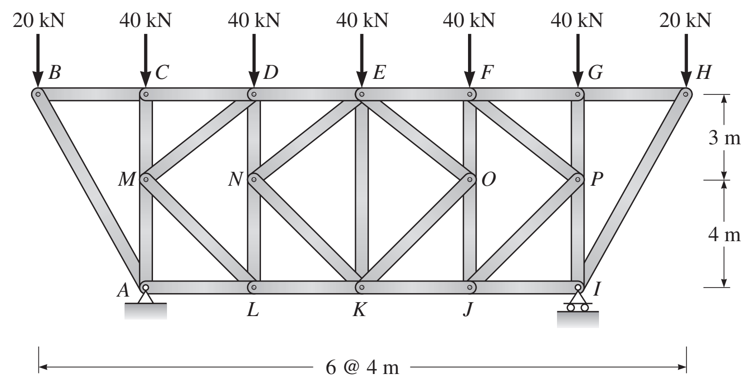
P4.32 to **P4.34.** Determine all bar forces. Indicate tension or compression.



P4.32

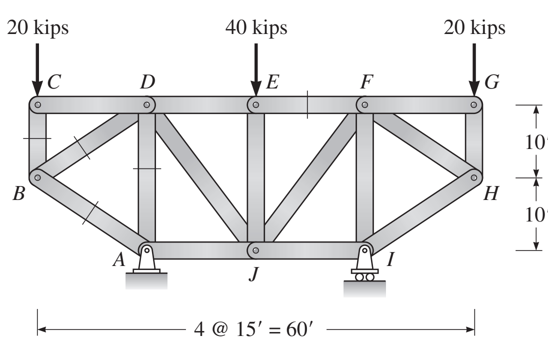


P4.33



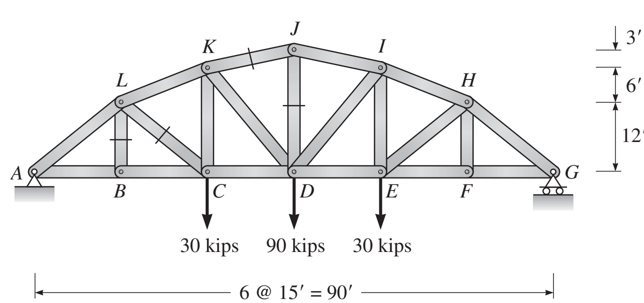
P4.34

P4.35 to **P4.36.** Using the method of sections, determine the forces in the bars listed below each figure.



AB, *BD*, *AD*, *BC*, and *EF*

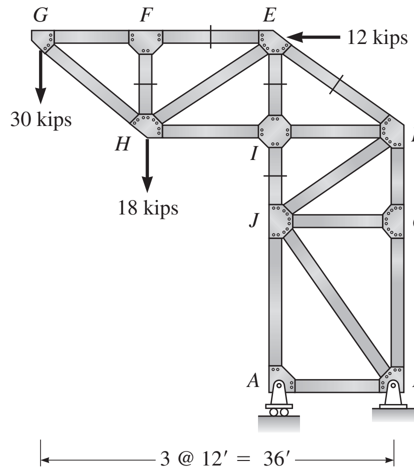
P4.35



BL, *KJ*, *JD*, and *LC*

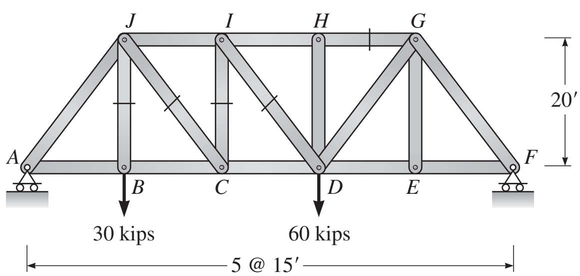
P4.36

P4.37 and **P4.39.** Using the method of sections, determine the forces in the bars listed below each figure.



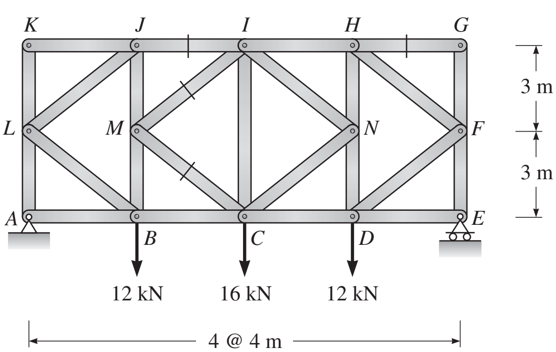
EF, *EI*, *ED*, *FH*, and *IJ*

P4.37



BJ, *CJ*, *CI*, *HG*, and *DI*

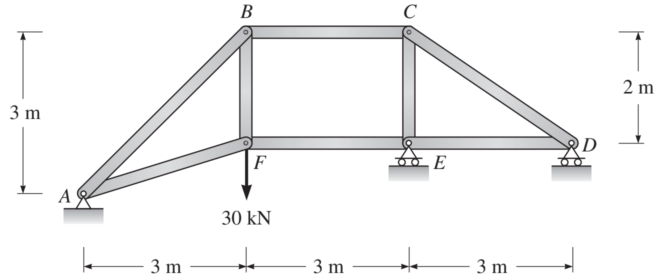
P4.38



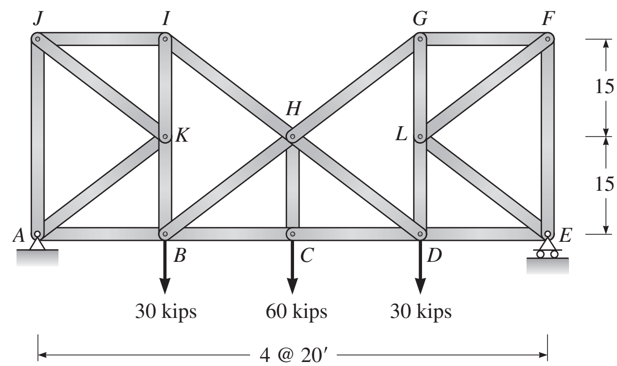
IJ, *MC*, and *MI*

P4.39

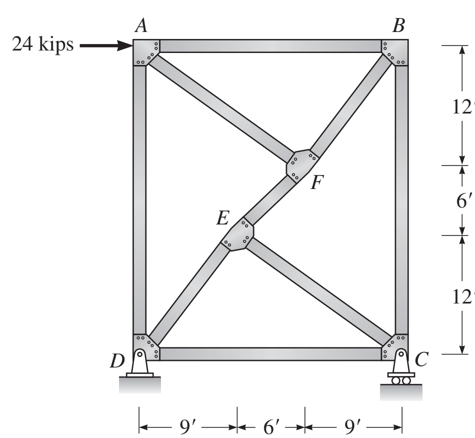
P4.40 to **P4.42.** Determine the forces in all bars of the trusses in Figures P4.40 to P4.42. Indicate if bar forces are tension or compression. *Hint*: Start with the method of sections.



P4.40

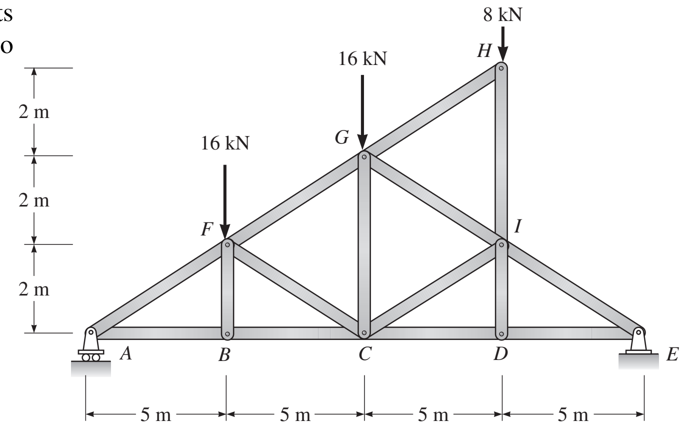


P4.41

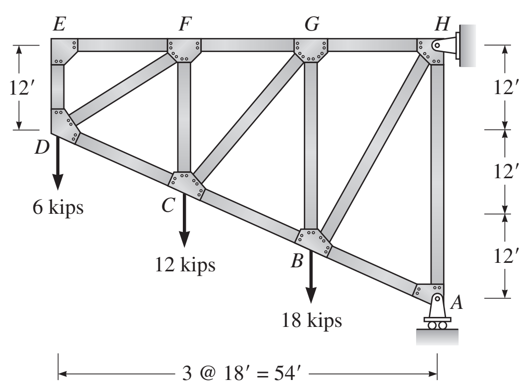


P4.42

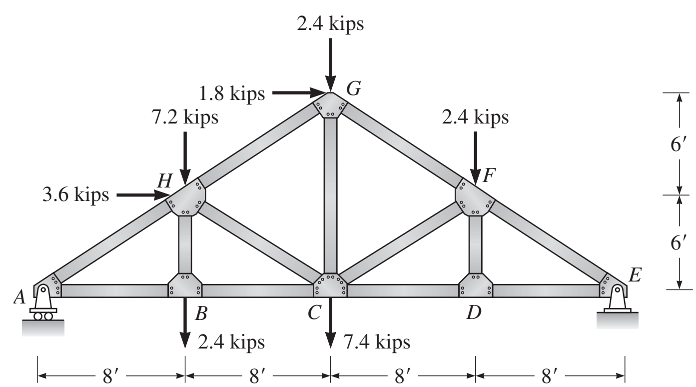
P4.43 to **P4.47.** Determine the forces or components of force in all bars of the trusses in Figures P4.43 to P4.47. Indicate tension or compression.



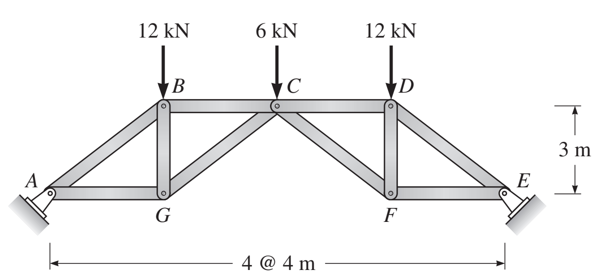
P4.43



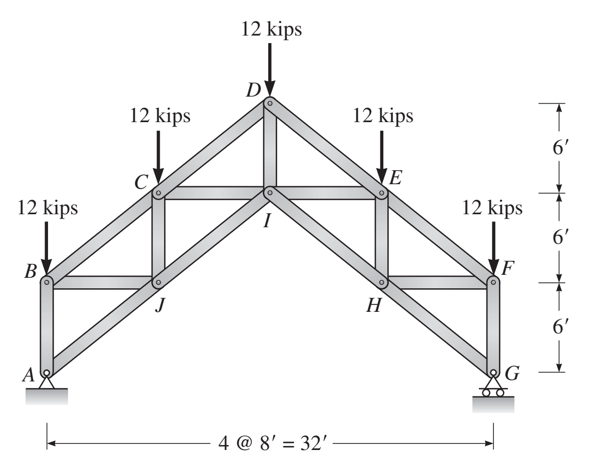
P4.44



P4.46

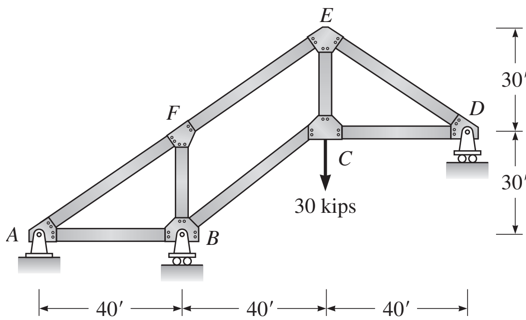


P4.45

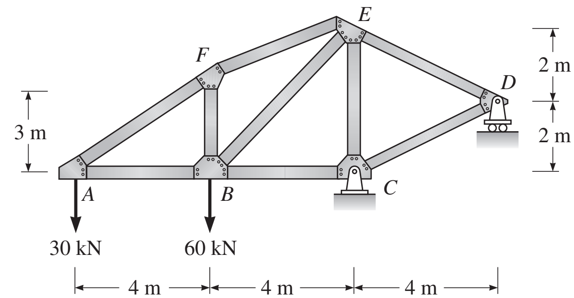


P4.47

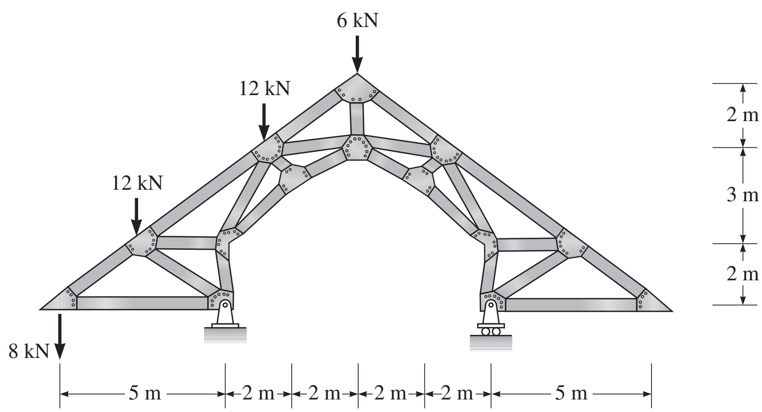
P4.48 to **P4.51.** Determine the forces or components of force in all bars of the trusses in Figures P4.48 to P4.51. Indicate tension or compression.



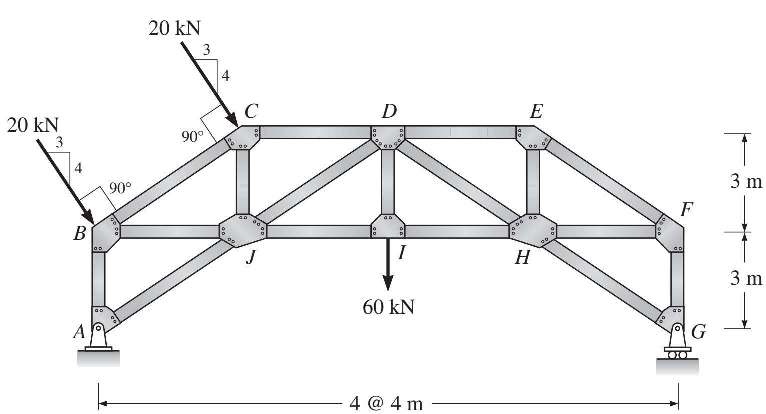
P4.48



P4.49

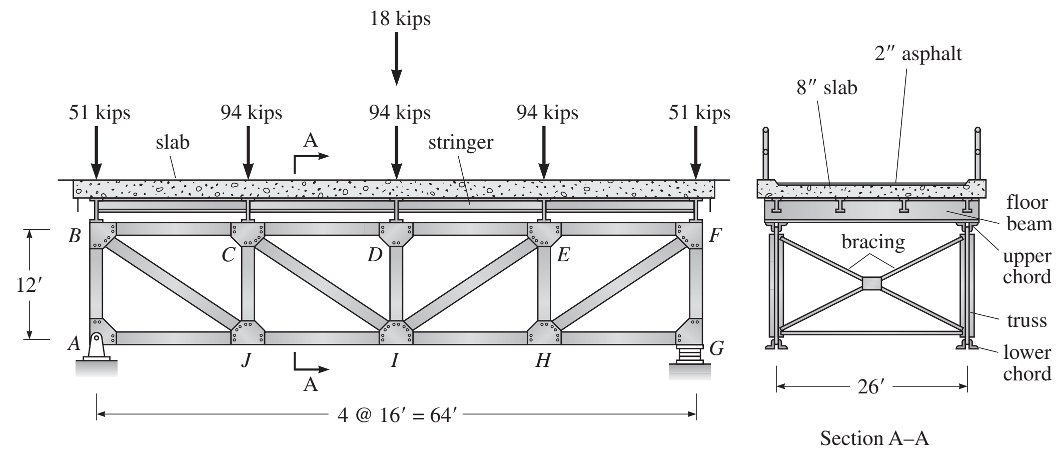


P4.50



P4.51

P4.52. A two-lane highway bridge, supported on two deck trusses that span 64 ft, consists of an 8-in reinforced concrete slab supported on four steel stringers. The slab is protected by a 2-in wearing surface of asphalt. The 16-ft-long stringers frame into the floor beams, which in turn transfer the live and dead loads to the panel points of each truss. The truss, bolted to the left abutment at point A, may be treated as pin supported. The right end of the truss rests on an elastomeric pad at G. The elastomeric pad, which permits only horizontal displacement of the joint, can be treated as a roller. The loads shown represent the total dead and live loads. The 18-kip load is an additional live load that represents a heavy wheel load. Determine the force in the lower chord between panel points I and J, the force in member JB, and the reaction applied to the abutment at support A.



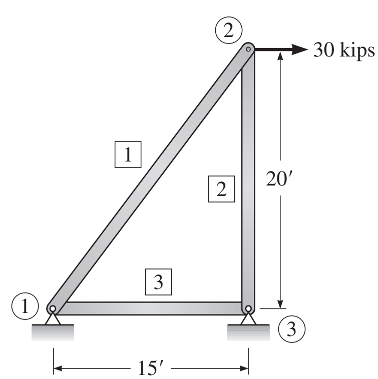
P4.52

P4.53. *Computer analysis of a truss*. The purpose of this study is to show that the *magnitude of the joint displacements* as well as the magnitude of the forces in members may control the proportions of structural members. For example, building codes typically specify maximum permitted displacements to ensure that excessive cracking of attached construction, such as exterior walls and windows, does not occur (see Photo 1.1 in Section 1.3).

A preliminary design of the truss in Figure P4.53 produces the following bar areas: member 1, 2.5 in^2; member 2, 3 in^2; and member 3, 2 in^2. Also $E = 29{,}000$ kips/in^2.

Case 1: Determine all bar forces, joint reactions, and joint displacements, assuming pin joints. Use the computer program to plot the deflected shape.

Case 2: If the maximum horizontal displacement of joint 2 is not to exceed 0.25 in, determine the minimum required area of the truss bars. For this case assume that all truss members have the *same* cross-sectional area. Round the area to the nearest whole number.



P4.53

P4.54. *Computer study*. The objective is to compare the behavior of a determinate and an indeterminate structure.

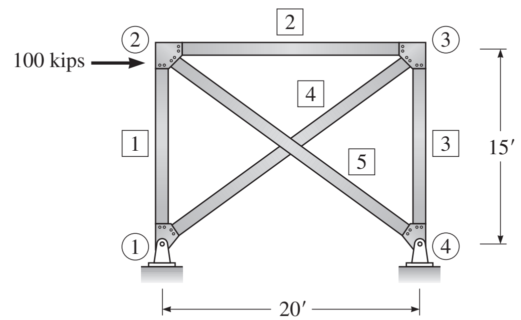
The forces in members of *determinate* trusses are not affected by member stiffness. Therefore, there was no need to specify the cross-sectional properties of the bars of the determinate trusses we analyzed by hand computations earlier in this chapter. In a *determinate* structure, for a given set of loads, only one load path is available to transmit the loads into the supports, whereas in an *indeterminate structure*, multiple load paths exist (see Section 3.10). In the case of trusses, the axial stiffness of members (a function of a member's cross-sectional area) that make up each load path will influence the magnitude of the force in each member of the load path. We examine this aspect of behavior by varying the properties of certain members of the indeterminate truss shown in Figure P4.54. Use $E = 29{,}000$ kips/in^2.

Case 1: Determine the reactions and the forces in members 4 and 5 if the area of all bars is 10 in^2.

Case 2: Repeat the analysis in **Case 1**, this time increasing the area of member 4 to 20 in^2. The area of all other bars remains 10 in^2.

Case 3: Repeat the analysis in **Case 1**, increasing the area of member 5 to 20 in^2. The area of all other bars remains 10 in^2.

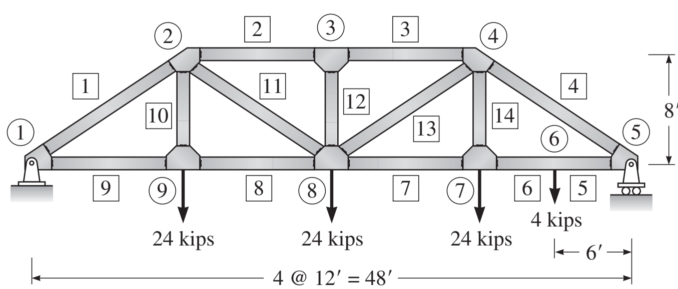
What conclusions do you reach from the above study?



P4.54

Practical Application

P4.55. *Computer analysis of a truss with rigid joints*. The truss in Figure P4.55 is constructed of square steel tubes *welded* to form a structure with rigid joints. The top chord members 1, 2, 3, and 4 are $4 \times 4 \times {}^1/_4$ inch square tubes with $A = 3.59$ in^2 and $I = 8.22$ in^4. All other members are $3 \times 3 \times {}^1/_4$ inch square tubes with $A = 2.59$ in^2 and $I = 3.16$ in^4. Use $E = 29{,}000$ kips/in^2.



P4.55

(*a*) Considering all joints as rigid, compute the axial forces and moments in all bars and the deflection at midspan when the three 24-kip design loads act at joints 7, 8, and 9. (Ignore the 4-kip load.)

(*b*) If a hoist is also attached to the lower chord at the midpoint of the end panel on the right (labeled joint 6*) to raise a concentrated load of 4 kips, determine the forces and moments in the lower chord (members 5 and 6). If the maximum stress is not to exceed 25 kips/in^2, can the lower chord support the 4-kip load safely in addition to the three 24-kip loads? Compute the maximum stress, using the equation

$$\sigma = \frac{F}{A} + \frac{Mc}{I}$$

where $c = 1.5$ in (one-half the depth of the lower chord).

**Note*: If you wish to compute the forces or deflection at a particular point of a member, designate the point as a joint.

Practical Application

P4.56. Analyze and compare two trusses, namely the Pratt Truss and the Howe Truss in Figures P4.56 (a) and (b), respectively. The trusses have the same depth, length, panel spacing, loading and supports. All joints are pinned. For each truss, determine the following:

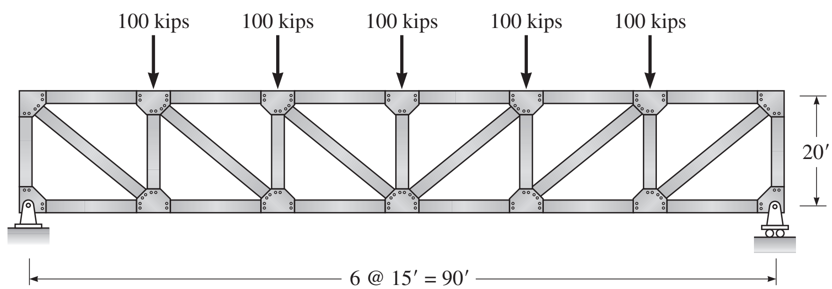
a) All bar forces, indicate tension or compression.

b) The required cross sectional areas for each bar, given an allowable tensile stress of 45 ksi, and an allowable compressive stress of 24 ksi. Note that allowable compressive stress is lower due to buckling.

c) Tabulate your results showing bar forces, cross sectional areas, and lengths.

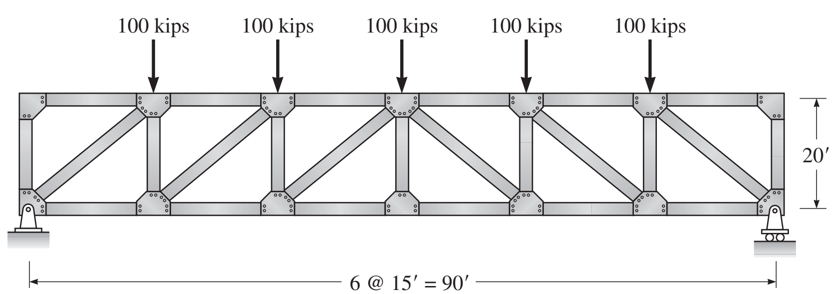
d) Calculate the total weight of each truss and determine which truss has a more efficient configuration. Explain your results.

e) What other conclusions can you draw from the study?



Pratt Truss

(a)



Howe Truss

(b)

P4.56