



Demand Estimation and Forecasting

Learning Objectives

After reading Chapter 7 and working the problems for Chapter 7 in the textbook and in this Workbook, you should be able to:

- Specify an empirical demand function—both linear and nonlinear functional forms.
- For price-setting firms with market power, you will learn to how to use least-squares regression methodology to estimate a firm's demand function.
- Forecast sales and prices using time-series regression analysis.
- Employ dummy variables to account for cyclical or seasonal variation in sales.
- Discuss and explain several important problems that arise when using statistical methods to forecast demand.

Essential Concepts

1. *Empirical demand functions* are demand equations derived from actual market data. Empirical demand functions are extremely useful in making pricing and production decisions.
2. In *linear* form, an empirical demand function can be specified as

$$Q = a + bP + cM + dP_R$$

where Q is the quantity demanded, P is the price of the good or service, M is consumer income, and P_R is the price of some related good R .

In the linear form, $b = \Delta Q / \Delta P$, $c = \Delta Q / \Delta M$, and $d = \Delta Q / \Delta P_R$. The expected signs of the coefficients are: (1) b is expected to be negative, (2) if good X is normal (inferior), c is expected to be positive (negative), (3) if related good R is a substitute (complement), d is expected to be positive (negative).

The estimated elasticities of demand are computed as

$$\hat{E} = \hat{b} \frac{P}{Q} \quad \hat{E}_M = \hat{c} \frac{M}{Q} \quad \hat{E}_{XR} = \hat{d} \frac{P_R}{Q}$$

3. When demand is specified in *log-linear* form, the demand function can be written as

$$Q = aP^b M^c P_R^d$$

To estimate a log-linear demand function, the above equation must be converted to logarithms:

$$\ln Q = \ln a + b \ln P + c \ln M + d \ln P_R$$

In this log-linear form, the elasticities of demand are constant: $\hat{E} = \hat{b}$, $\hat{E}_M = \hat{c}$, and $\hat{E}_{XR} = \hat{d}$

4. When a firm possesses some degree of market power, which makes it a price-setting firm, the demand curve for the firm can be estimated using the method of least-squares estimation set forth in Chapter 4. The following steps can be followed to estimate the demand function for a price-setting firm:

Step 1: Specify the price-setting firm's demand function.

Step 2: Collect data for the variables in demand function.

Step 3: Estimate the firm's demand using least-squares regression.

5. A time-series model shows how a time-ordered sequence of observations on a variable, such as price or output, is generated. The simplest form of time-series forecasting is linear trend forecasting. In a linear trend model, sales in each time period (Q_t) are assumed to be linearly related to time (t):

$$Q_t = a + bt$$

and regression analysis is used to estimate the values of a and b . If $b > 0$, sales are increasing over time, and if $b < 0$, sales are decreasing. If $b = 0$, then sales are constant over time. The statistical significance of a trend is determined testing \hat{b} for statistical significance or by examining the p -value for \hat{b} .

6. *Seasonal or cyclical variation* can bias the estimation of a and b in linear trend models. In order to account for seasonal variation in trend analysis, *dummy variables* are added to the trend equation. Dummy variables serve to shift the trend line up or down according to the particular seasonal pattern encountered. The significance of seasonal behavior is determined by using a t -test or p -value for the estimated coefficient on the dummy variable.
7. When using dummy variables to account for N seasonal time periods, $N - 1$ dummy variables are added to the linear trend. Each dummy variable accounts for one of the seasonal time periods. The dummy variable takes a value of 1 for those observations that occur during the season assigned to that dummy variable, and a value of 0 otherwise.
8. The following problems and limitations are inherent in forecasting:
 - i. The further into the future the forecast is made, the wider is the confidence interval or region of uncertainty.
 - ii. If the model is misspecified by either excluding an important variable or by using an inappropriate functional form, then the reliability of the forecast will be reduced.
 - iii. Forecasts are incapable of predicting sharp changes that occur because of structural changes in the market itself.

Matching Definitions

dummy variable
empirical demand functions
representative sample
response bias
seasonal or cyclical variation
time-series model

1. _____ Demand equations derived from actual market data.
2. _____ A sample that has the same characteristics as the population as a whole.
3. _____ The difference between the response given by an individual to a hypothetical question and the action the individual takes when the situation actually occurs.
4. _____ A statistical model that uses only a time-series of observations on a variable to make forecasts of future values of the variable.
5. _____ The regular variation that time-series data frequently exhibits.
6. _____ A variable that can take only the value of 0 or 1.

Study Problems

1. Under what circumstances is it appropriate to estimate demand using the (ordinary) method of least-squares regression presented in Chapter 4?
2. The following demand function for a price-setting firm selling good X was estimated using standard regression analysis:

$$Q = a + bP + cM + dP_R$$

The estimation results are:

DEPENDENT VARIABLE: QX				
OBSERVATIONS: 275				
VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT	9500.4	3350.6	2.84	0.0049
P	-12.75	4.30	-2.87	0.0044
M	-0.0163	0.0066	-2.49	0.0135
PR	5.05	1.10	4.59	0.0001

- a. Is the sign of \hat{b} as we would have predicted? Why or why not?
- b. Is this good normal or inferior good? Explain.

- c. Are goods X and R substitutes or complements? Explain.
 - d. Which coefficients are significant at the 5 percent level of significance? Explain.
 - e. Using the values $P = \$20$, $M = \$50,000$, and $P_R = \$100$, calculate estimates of the following:
 - (i) The price elasticity of demand is _____.
 - (ii) The cross-price elasticity of demand is _____.
 - (iii) The income elasticity of demand is _____.
3. Consider the following nonlinear demand function, which is estimated for a price-setting firm. The method of least-squares is used to estimate the parameters.

$$Q = aP^b M^c P_R^d$$

The results of the estimation are:

DEPENDENT VARIABLE:	LNQ	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	26	0.9248	90.18	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT		3.04	1.01	3.01	0.0064
LNP		-1.90	0.48	-3.96	0.0007
LN M		2.16	0.675	3.20	0.0041
LN P _R		0.78	0.169	4.62	0.0001

- a. Before the nonlinear demand equation can be estimated using regression analysis, the demand equation must be transformed into the following linear form:
 $\ln Q =$ _____.
- b. Are the parameter estimates statistically significant at the 5 percent level of significance?
- c. The estimated value of a is equal to _____.
- d. Is this good a normal or inferior good?
- e. Is this good a substitute or complement with respect to related good R ?
- f. Compute estimates of the following elasticities:
 - (i) The price elasticity of demand is _____.
 - (ii) The income elasticity of demand is _____.
 - (iii) The cross-price elasticity of demand is _____.
- g. A 23.15 percent decrease in household income, holding all other things constant, will cause quantity demanded to _____ (increase, decrease) by _____ percent.
- h. All else constant, a 4 percent increase in price causes quantity demanded to _____ (increase, decrease) by _____ percent.
- i. A 12.82 percent decrease in the price of R , holding all other things constant, will cause quantity demanded to _____ (increase, decrease) by _____ percent.

4. For the past 12 months you have been the night manager of Dixie Fried Chicken. In order to evaluate your performance as a manager, your boss estimates the following linear trend equation for nighttime sales (Q_t) over the last 12 months ($t = 1, \dots, 12$):

$$Q_t = a + bt$$

where Q_t is the number of pieces of chicken sold nightly. The results of the regression are as follows:

DEPENDENT VARIABLE:	QT	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	12	0.8991	89.108	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT		175.0	38.88	4.50	0.0011
T		16.0	6.4	2.50	0.0314

- Evaluate the statistical significance of the estimated coefficients. Does this estimation indicate a significant trend, either upward or downward, in sales during your tenure as night manager?
- Perform an F -test for significance of the trend equation at the 5 percent level of significance.
- If your boss uses the estimated linear trend to forecast your sales for months 14 and 16, how many units does he expect you to sell in these months?

$$\hat{Q}_{t=14} = \underline{\hspace{2cm}} \text{ and } \hat{Q}_{t=16} = \underline{\hspace{2cm}}$$

- Comment on the precision of these two forecasts.

5. Suppose you manage the pro shop at a golf club in Miami and would like to be able to forecast the number of golf cart rentals on a quarterly basis. A simple linear trend model must account for the fact that demand for golf carts is always higher in the winter quarters (quarters 1 and 4) as tourists from northern states vacation in Florida. You decide to estimate the following equation:

$$Q_t = a + bt + cD$$

where D is a dummy variable equal to 1 for quarters 1 and 4, and zero otherwise. Using quarterly data from 2004–2007 ($t = 1, \dots, 16$), you obtain the following estimation results:

DEPENDENT VARIABLE:	QT	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	16	0.9510	126.153	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT		3471.23	901.61	3.85	0.0020
T		-6.10	1.80	-3.39	0.0048
D		870.18	133.87	6.50	0.0001

- Perform t - and F - tests to check for statistical significance (at the 99 percent confidence level) of the individual parameter estimates and the equation.
- Is the downward trend in golf cart rentals statistically significant?

- c. Calculate the intercept for winter quarters and summer quarters. What do the values imply?
- d. Use the estimated equation to forecast golf cart rentals in the four quarters of 2008.

Multiple Choice / True-False

1. Which of the following is NOT a major problem inherent in forecasting?
 - a. The further a forecast variable is from its sample mean value, the less precise the forecast.
 - b. Predicted values of exogenous variables are very difficult to find.
 - c. Misspecifying the empirical demand equation can seriously reduce forecast accuracy.
 - d. Structural changes in the economy can cause forecasts to completely miss abrupt changes in the value of the predicted variable.
2. Demand equations derived from actual market data are
 - a. empirical demand functions.
 - b. never estimated using consumer interviews.
 - c. generally estimated using regression analysis.
 - d. both *a* and *c*.
 - e. all of the above.

In questions 3–8, suppose the following nonlinear demand function is estimated using regression analysis:

$$Q = aP^b M^c P_y^d$$

The estimation results are:

DEPENDENT VARIABLE: LNQ				
OBSERVATIONS: 44				
VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT	–2.00	0.40	–5.00	0.0001
LNP	–1.10	0.44	–2.50	0.0166
LN M	2.40	0.60	4.00	0.0003
LN P _R	–0.20	0.05	–4.00	0.0003

3. These estimates indicate that the demand for the good is
 - a. price inelastic since $\hat{E} = -1.10$.
 - b. price elastic since $\hat{E} = -1.10$.
 - c. price inelastic since $\hat{E} = -2.5$.
 - d. price elastic since $\hat{E} = -2.5$.

4. The estimated own price elasticity of demand is
 - a. statistically significant at the 5 percent significance level since $-2.5^* > 2.021$.
 - b. *not* statistically significant at the 5 percent significance level since $0.004 < 2.021$.
 - c. statistically significant at the 5 percent significance level since $-2.5^* > 2.000$.
 - d. *not* statistically significant at the 5 percent significance level since $2.6 < 2.721$.

5. The estimates indicate that the income elasticity of demand is
 - a. -0.20 .
 - b. -2.0 .
 - c. 2.4 .
 - d. 4.0 .

6. The estimate of the cross-price elasticity indicates that the two goods are
 - a. normal goods.
 - b. substitute goods.
 - c. inferior goods.
 - d. complementary goods.
 - e. none of the above.

7. According to the estimated demand equation, if the price of Y rises by 25 percent, then quantity demanded will
 - a. increase by 5 percent.
 - b. decrease by 5 percent.
 - c. increase by 6 percent.
 - d. decrease by 6 percent.

8. From the above estimates, we would expect quantity demanded to rise by 30 percent if income
 - a. rises by 12.5 percent.
 - b. rises by 8 percent.
 - c. falls by 0.08 percent.
 - d. falls by 125 percent.

9. If demand is estimated using the empirical specification

$$\ln Q = \ln a + b \ln P + c \ln M + d \ln P_R$$
 then an equivalent expression for demand is
 - a. $\ln Q = a + bP + cM + dP_R$.
 - b. $Q = a + bP + cM + dP_R$.
 - c. $Q = e^a + bP + cM + dP_R$.
 - d. $Q = abPcMdP_R$.
 - e. none of the above.

10. For a linear demand function, $Q = a + bP + cM + dP_R$, the income elasticity is
- c .
 - $c(M/Q)$.
 - $c(Q/M)$.
 - $-c$.
 - $-c(Q/P_R)$.
11. A representative sample
- eliminates the problem of response bias.
 - reflects the characteristics of the population.
 - is frequently a random sample.
 - both b and c .
 - all of the above.

For questions 12–22 refer to the following:

The manufacturer of Cabbage Patch dolls used quarterly price data for the period 2000I – 2008IV ($t = 1, \dots, 36$) and the regression equation

$$P_t = a + bt + c_1D_1 + c_2D_2 + c_3D_3$$

to forecast doll prices in the year 2009. P_t is quarterly prices of dolls, and D_1 , D_2 , and D_3 are dummy variables for quarters I, II, and III, respectively.

DEPENDENT VARIABLE:	PT	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	36	0.9078	76.34	0.0001	
VARIABLE	PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE	
INTERCEPT	24.0	6.20	3.87	0.0005	
T	0.800	0.240	3.33	0.0022	
D1	–8.00	2.60	–3.08	0.0043	
D2	–6.00	1.80	–3.33	0.0022	
D3	–4.00	0.60	–6.67	0.0001	

12. At the 2 percent level of significance, is there a statistically significant trend in price of dolls?
- Yes, because $0.0022 < 0.02$.
 - No, because $0.0022 > 0.02$.
 - Yes, because $0.800 > 0.02$.
 - Yes, because $0.240 > 0.02$.
 - Yes, because $3.33 > 0.02$.

13. The estimated *quarterly* increase in price is _____, and the estimated *annual* increase in price is _____.
- \$1.50; \$6.00
 - \$1.40; \$4.00
 - \$0.60; \$2.40
 - \$0.80; \$3.20
 - none of the above
14. What is the estimated intercept of the trend line in the first quarter?
- 24
 - 8
 - 32
 - 16
 - none of the above
15. What is the estimated intercept of the trend line in the fourth quarter?
- 22.8
 - 16
 - 18
 - 20
 - none of the above
16. At the 2 percent level of significance, the estimation results indicate that price in the _____ quarter is significantly higher than in any other quarter.
- first
 - second
 - third
 - fourth
17. At the 2 percent level of significance, the results indicate that price in the _____ quarter is significantly lower than in any other quarter.
- first
 - second
 - third
 - fourth
18. In any given year price tends to vary as follows:
- $P_I > P_{II} > P_{III} > P_{IV}$
 - $P_I > P_{IV} > P_{III} > P_{II}$
 - $P_{II} > P_{III} > P_{IV} > P_I$
 - $P_{III} > P_I > P_{II} > P_{IV}$
 - $P_{IV} > P_{III} > P_{II} > P_I$

19. Using the estimated time-series regression, predicted price in the first quarter of 2009 is
- a. \$53.60.
 - b. \$45.60.
 - c. \$56.00.
 - d. \$37.60.
 - e. none of the above
20. Using the estimated time-series regression, predicted price in the second quarter of 2009 is
- a. \$48.40
 - b. \$54.40
 - c. \$40.40
 - d. \$51.40
 - e. none of the above
21. Using the estimated time-series regression, predicted price in the third quarter of 2009 is
- a. \$55.20.
 - b. \$47.20.
 - c. \$58.00.
 - d. \$56.00.
 - e. none of the above
22. Using the estimated time-series regression, predicted price in the fourth quarter of 2009 is
- a. \$48.
 - b. \$50.
 - c. \$52.
 - d. \$56.
 - e. none of the above
23. T F Dummy variables can only be used for adjusting time-series models for cyclical variations.
24. T F In the linear trend model of sales $Q_t = a + bt$, a negative value of b is, by itself, evidence of a downward trend in sales.
25. T F Time-series forecasting is too simple to be useful in real-world forecasting.

Answers

MATCHING DEFINITIONS

1. empirical demand functions
2. representative sample
3. response bias
4. time-series model
5. seasonal or cyclical variation
6. dummy variable

STUDY PROBLEMS

1. OLS is appropriate when the firm is a price-setting firm. In this case, movement along demand is caused not by shifts in an industry supply curve, but rather by the decisions made by the firm. Consequently, P is not an endogenous variable in a system of demand and supply equations.
2.
 - a. Yes, Q should be inversely related to P along a demand curve.
 - b. Inferior. Since \hat{c} is negative, X is an inferior good.
 - c. Substitutes. Since $\hat{d} (= \Delta Q / \Delta P_R)$ is positive, goods X and R must be substitutes.
 - d. The critical value of t for $n - k = 275 - 4 = 271$ degrees of freedom and the 5 percent level of significance is (approximately) 1.96. Since $|t| > 1.96$ for all four t -ratios, all four parameters are statistically significant. Also, the p -values are all smaller than 0.05, which indicates exact levels of significance smaller than 5%.
 - e. $\hat{Q} = 8,942$ at $P = 20$, $M = 50,000$, $P_R = 100$
 - (i) $\hat{E} = -0.028 [= -12.35 \times (20/8,942)]$
 - (ii) $\hat{E}_{XR} = 0.056 [= 5.05 \times (100/8,942)]$
 - (iii) $\hat{E}_M = -0.091 [= -0.0163 \times (50,000/8,942)]$
3.
 - a. $\ln Q = \ln a + b \ln P + c \ln M + d \ln P_R$
 - b. Yes, the absolute values of all t -ratios are greater than 2.074.
 - c. $\hat{a} = e^{3.04} = 20.905$
 - d. The estimated value of c is positive, and significant, indicating this good is a normal good.
 - e. The estimated value of d is positive, and significant, indicating the two goods are substitutes.
 - (i) $\hat{E} = \hat{b} = -1.90$, (ii) $\hat{E}_M = \hat{c} = 2.16$, (iii) $\hat{E}_{XR} = \hat{d} = 0.78$
 - f. decrease; 50%
 - g. decrease; 7.60%
 - h. decrease; 10%
4.
 - a. \hat{a} : p -value is 0.0011, so \hat{a} is statistically significant at better than the 1% level of significance (or 99% level of confidence). The probability that $a = 0$ (i.e., a Type I error) is quite small, about one-tenth of 1 percent.
 \hat{b} : p -value is 0.0314, so \hat{b} is statistically significant at the 5% level of significance

(or 95% level of confidence). The probability that $b = 0$ is small, about a 3 percent chance.

Conclusion: Sales exhibit a statistically significant positive trend over time (i.e., $\hat{b} > 0$ and its p -value is acceptably small). The model as a whole is also explaining a statistically significant amount of the total variation in sales, as indicated by the very small p -value for the F -ratio.

- b. From the F -table, $F_{1,10} = 4.96$. Since the calculated F -ratio is 89.108, the equation is significant at the 95 percent confidence level or 5 percent significance level.

c. $\hat{Q}_{14} = 175 + 16 \times 14 = 399$

$$\hat{Q}_{16} = 175 + 16 \times 16 = 431$$

- d. We would expect the confidence interval to be smaller for the forecast of sales in the 14th month because 14 is closer to the sample mean value of t ($\bar{t} = 6.5$) than is 16.

5. a. $t_{\hat{a}} = 3.85 > 3.012 \Rightarrow \hat{a}$ is statistically significant.

$$|t_{\hat{b}}| = |-3.39| > 3.012 \Rightarrow \hat{b} \text{ is statistically significant.}$$

$$t_{\hat{c}} = 6.5 > 3.012 \Rightarrow \hat{c} \text{ is statistically significant.}$$

The calculated value of F of 126.153 $> F_{2,13} = 6.70 \Rightarrow$ the equation as a whole is statistically significant.

- b. Yes; the exact level of significance is 0.48% \Rightarrow virtually no chance of committing a Type I error (finding significance when there is none).

- c. The winter intercept = $\hat{a} + \hat{c} = 4341.41$.

The summer (i.e., “other” months) intercept = $\hat{a} = 3471.23$.

Since \hat{c} is positive and statistically significant, the regression indicates that golf cart rentals increase in winter months (despite the overall downward trend in rentals).

- d. First quarter 2008: $Q_{17} = 3,471.23 - 6.10(17) + 870.18(1) = 4,237.71$

$$\text{Second quarter 2008: } Q_{18} = 3,471.23 - 6.10(18) + 870.18(0) = 3,361.43$$

$$\text{Third quarter 2008: } Q_{19} = 3,471.23 - 6.10(19) + 870.18(0) = 3,355.33$$

$$\text{Fourth quarter 2008: } Q_{20} = 3,471.23 - 6.10(20) + 870.18(1) = 4,219.41$$

MULTIPLE CHOICE / TRUE-FALSE

1. b Only answer b is *not* a major problem in forecasting. It is usually possible to obtain predicted values for exogenous variables.
2. d Empirical demand equations are generally estimated using regression analysis.
3. b Demand is elastic because $\hat{E} = \hat{b} = -1.10$ and $|-1.10| > 1$.
4. a $|t_{\hat{b}}| = 1.10/0.44 = 2.5 > 2.021 (= t_{\text{critical}})$
5. c $\hat{E}_M = \hat{c} = 2.4$
6. d $\hat{E}_{XY} = \hat{d} = -0.20 < 0 \Rightarrow X$ and Y are complements
7. b $\% \Delta Q / \% \Delta P_y = -0.20 \Rightarrow \% \Delta Q / +25\% = -0.20 \Rightarrow \% \Delta Q = -5\%$
8. a $\% \Delta Q / \% \Delta M = 2.4 \Rightarrow 30\% / \% \Delta M = 2.4 \Rightarrow \% \Delta M = +12.5\%$
9. e none of the choices are correct: $Q = aP^bM^cP_R^d$.

10. b For linear specifications, elasticities vary. The income elasticity is $c(M/Q)$, where c is constant but M/Q varies.
11. d Both are true by the definition of a representative sample.
12. a The p -value for the estimated trend parameter \hat{b} is less than 0.02, so it is significant at the 2 percent level.
13. d \hat{b} is the estimated quarterly trend in price ($= \$0.80$), so the annual rate of increase in price is four times \hat{b} , $\$3.20$.
14. d The estimated intercept in quarter 1 $= \hat{a} + \hat{c} = 24 + (-8) = 16$.
15. e The estimated intercept in quarter 4 $= \hat{a} = 24$. Remember, the base quarter is the fourth quarter in this problem.
16. d Since all three dummies are negative, the base quarter, which is the fourth quarter, is estimated to have the largest intercept.
17. a Since the first quarter dummy is the most negative of the three negative dummies, the first quarter has the smallest intercept value.
18. e This pattern follows directly from the pattern of the dummy variables. See answers 14 and 15.
19. b In the first quarter of 2009, $t = 37$, and $\hat{P}_{37} = 24.0 + 0.8 \times 37 - 8.0 \times 1 = 45.60$.
20. a In the second quarter of 2009, $t = 38$, and $\hat{P}_{38} = 24.0 + 0.8 \times 38 - 6.0 \times 1 = 48.40$.
21. e In the third quarter of 2009, $t = 39$, and $\hat{P}_{39} = 24.0 + 0.8 \times 39 - 4.0 \times 1 = 51.20$.
22. d In the fourth quarter of 2009, $t = 40$, and $\hat{P}_{40} = 24.0 + 0.8 \times 40 = 56$.
23. F Dummy variables can be used to adjust for many types of occurrences that influence sales, not just seasonal effects. For example, if the sales data include some time periods during which a war was being fought, then a dummy variable for war years and peace years could be added to account for the effect of the war on sales.
24. F By itself, a negative value of \hat{b} is not sufficient evidence of a downward trend in sales. Unless \hat{b} is shown to be statistically significant at an acceptable level of significance, then it is not possible to reject the hypothesis that $b = 0$ (i.e., there is no trend up or down) without taking an unacceptable risk of committing a Type I error (i.e., reject $b = 0$ when, in fact, b really is zero). Remember, just because the estimate of b is not zero does not mean, by itself, that the true value of b is not zero. You do a t -test or evaluate the p -value for \hat{b} .
25. F Time-series forecasting may be easier to do than econometric forecasting, but for short-run forecasts the time-series method may be more accurate than some of the more complicated techniques.

Homework Exercises

1. Suppose a savings and loan association wants to forecast the delinquency rate on home mortgages. Using monthly data, the following trend model is estimated

$$DR_t = a + bt$$

where DR_t = the percentage of mortgage payments delinquent in time period t , and $t = 1, \dots, 48$ (January 2004 through December 2007). The following estimation results are obtained:

DEPENDENT VARIABLE:	DRT	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	48	0.7982	181.93	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT		2.60	0.74	3.51	0.0010
T		0.030	0.010	3.00	0.0043

- a. Does this estimate indicate a significant upward trend in the delinquency rate on home mortgages? In the space below, perform the appropriate t -test at the 5 percent significance level.
- b. Perform an F -test to determine if the model is statistically significant at the 5 percent level of significance.
- c. Calculate the predicted delinquency rates for January 2008 and March 2008.

$$DR_{Jan\ 2008} = \underline{\hspace{2cm}} \text{ percent}$$

$$DR_{Mar\ 2008} = \underline{\hspace{2cm}} \text{ percent}$$

Show your work here:

The trend analysis above indicates that the delinquency rate is rapidly becoming a serious problem for this savings and loan. A management consultant suggests that the trend analysis may be overstating the upward trend because it fails to account for the fact that the months of December and January are consistently much worse months than the rest. The consultant suggests estimating the following model:

$$DR_t = a + bt + cD$$

where D is equal to one for the months of December and January, and zero otherwise. The estimation results are:

DEPENDENT VARIABLE:	DRT	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	48	0.9876	1794.0	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT		2.200	0.780	2.82	0.0071
T		0.018	0.004	4.50	0.0001
D		0.810	0.210	3.86	0.0004

- d. Does the new model indicate a significant upward trend in the delinquency rate? Perform the appropriate t -test.
- e. Perform a t -test to determine whether December and January are significantly worse months for mortgage payments.
- f. Predict the delinquency rates for January 2008 and March 2008.
- $DR_{Jan\ 2008} = \underline{\hspace{1cm}}$ percent
- $DR_{Mar\ 2008} = \underline{\hspace{1cm}}$ percent
- Show your work here:*
- g. Compare your forecasts in parts *c* and *f*. Explain why they differ.

COMPUTER HOMEWORK EXERCISES

Use a computer regression package, such as the Student Edition of Statistix 8, to work these two computer exercises.

2. Ozark Bottled Water Products, Inc. hired a marketing consulting firm to perform a test marketing of its new brand of spring water called Liquid Ozarka. The marketing experts selected 15 small and medium-sized towns in Arkansas and Missouri for a one-month-long sales test. For one month, Liquid Ozarka was sold at a variety of prices ranging from \$3 per gallon to \$4 per gallon. Specifically, in three of the markets, price was set by the marketing experts at \$3 per gallon. In three more markets, price was set at \$3.25 per gallon, and so on. The prices charged in each market (P) are shown in the table below. For each of the 15 market areas, the marketing consultants collected data on average household income (M),

the population of the marketing area (N), and the price of a rival brand of bottled water (P_R). At the end of the month, total sales of Liquid Ozarka (Q) were tabulated to provide the following data from which the consultants estimated an empirical demand function for the new product.

<i>Market</i>	<i>P</i>	<i>M</i>	<i>PR</i>	<i>N</i>	<i>Q</i>
1	\$3.00	\$45,586	\$2.75	274,000	7,952
2	3.00	37,521	3.50	13,450	8,222
3	3.00	41,333	2.64	54,150	7,166
4	3.25	47,352	2.35	6,800	6,686
5	3.25	51,450	2.75	11,245	7,715
6	3.25	27,655	3.15	54,500	6,643
7	3.50	30,265	2.55	26,600	5,155
8	3.50	39,542	3.00	158,000	7,127
9	3.50	41,596	2.75	22,500	5,834
10	3.75	42,657	2.45	46,150	5,093
11	3.75	36,421	2.89	8,200	5,828
12	3.75	47,624	2.49	38,500	6,590
13	4.00	50,110	3.15	105,000	6,228
14	4.00	57,421	2.80	92,000	7,218
15	4.00	38,450	2.90	38,720	5,846

Using the marketing data from the 15 test markets shown above, estimate the parameters of the linear empirical demand function:

$$Q = a + bP + cM + dPR + eN$$

If any of the parameter estimates are *not* significant at the 2 percent level of significance, drop the associated explanatory variable from the model and estimate the demand function again.

- a. Your estimated linear demand function for Liquid Ozarka is

$$\hat{Q} = \underline{\hspace{10em}}.$$

- b. What percentage of the variation in sales of Liquid Ozarka is explained by your estimated demand function?

The marketing consultants describe a “typical” market as one in which the price of Liquid Ozarka is \$3.50 per gallon, average household income is \$45,000, the price of rival bottled water is \$3 per gallon, and the population is 75,000. Answer the following questions for this “typical” market scenario.

- c. What is the estimated elasticity of demand for Liquid Ozarka? Is demand elastic or inelastic? What would be the percentage change in price required to increase sales of Liquid Ozarka by 10 percent?

- d. What is the estimated income elasticity of demand? Is Liquid Ozarka a normal or inferior good? A 6 percent increase in average household income would be predicted to cause what percentage change in sales of Liquid Ozarka?
- e. What is the estimated cross-price elasticity of demand for Liquid Ozarka with respect to changes in price of its rival brand of bottled water? Does the estimated cross-price elasticity have the expected algebraic sign? Why or why not? If the price of the rival brand of water rises by 8 percent, what is the estimated percentage change in sales of Liquid Ozarka?

Using the marketing data from the preceding 15 test markets, estimate the parameters for the log-linear empirical demand function:

$$Q = aP^b M^c P_R^d N^e$$

If any of the parameter estimates are *not* significant at the 2 percent level of significance, drop the associated explanatory variable from the model and estimate the demand function again.

- f. Your estimated log-linear demand function for Liquid Ozarka is

$$\hat{Q} = \underline{\hspace{10cm}}.$$

- g. Does a log-linear specification work better than a linear specification of demand for Liquid Ozarka? Explain by comparing F -ratios, R^2 s, and t -ratios (or p -values).
- h. Using the estimated log-linear demand function, compute the price, income, and cross-price elasticities of demand. How do they compare to the estimated elasticities for the linear demand specification?

3. For 2004–2006, Gallaway, Inc. has collected the following data on monthly sales of its Titan II driving club, where Q = the number of units sold per month.

Year	Month	Q	Year	Month	Q	Year	Month	Q
2004	January	6,942	2005	January	8,007	2006	January	7,925
	February	7,348		February	7,698		February	7,326
	March	7,328		March	7,417		March	8,037
	April	8,350		April	8,897		April	9,087
	May	8,619		May	8,607		May	9,303
	June	9,282		June	9,314		June	9,139
	July	8,183		July	8,686		July	8,105
	August	8,317		August	8,539		August	8,321
	September	8,552		September	8,967		September	8,960
	October	7,993		October	8,507		October	7,580
	November	8,198		November	8,359		November	8,562
	December	8,082		December	8,157		December	8,072

- a. Management at Gallaway is concerned about sales. They would like to know if there is an upward trend in sales of the Titan II. Use the data above to estimate the monthly trend in sales using a linear trend model of the form: $Q_t = a + bt$. Does your statistical analysis indicate a trend? If so, is it an upward or downward trend and how great is it? Is it a statistically significant trend (use the 5 percent level of significance)?
- b. Now adjust your statistical model to account for seasonal variation in club sales. Estimate the following model of sales:

$$Q_t = a + bt + c_1 D1_t + c_2 D2_t + c_3 D3_t$$

where $D1_t = 1$ for the months of January–March or 0 otherwise, $D2_t = 1$ for the months of April–June or 0 otherwise, and $D3_t = 1$ for the months of July–September or 0 otherwise. Do the data indicate a statistically significant seasonal pattern (use the 5 percent level of significance)? If so, what is the seasonal pattern of sales of Titan II clubs?

- c. Comparing your estimates of the trend in sales in parts *a* and *b*, which estimate is likely to be more accurate? Why?

- d. Using the estimated forecast equation from part *b*, forecast sales of Titan II clubs for January 2007, January 2008, July 2007, and July 2008.