3 Marginal Analysis for Optimal Decisions

Learning Objectives

After reading Chapter 3 and working the problems for Chapter 3 in the textbook and in this Workbook, you should be able to:

- Employ marginal analysis to find the optimal levels of activities in unconstrained maximization problems.
- Explain why sunk costs, fixed costs, and average costs are irrelevant for determining the optimal levels of activities.
- Employ marginal analysis to find the optimal levels of two or more activities in constrained maximization and minimization problems.

Essential Concepts

1. Formulating an optimization problem involves specifying three things: (1) the objective function to be either maximized or minimized, (2) the activities or choice variables that determine the value of the objective function, and (3) any constraints that may restrict the range of values that the choice variables may take.

2. *Marginal analysis* involves changing the value of a choice variable by a small amount to see if the objective function can be further increased (in the case of maximization problems) or further decreased (in the case of minimization problems).

3. *Net benefit* from an activity (*NB*) is the difference between total benefit (*TB*) and total cost (*TC*) for the activity: $NB = TB - TC$. The net benefit function is the objective function to be maximized in unconstrained maximization problems. The optimal level of the activity (A^*) is the level of activity that maximizes net benefit.

4. The *choice variables* determine the value of the objective function. Choice variables can be either *continuous* or *discrete*. A choice variable is continuous if the decision maker can choose from an uninterrupted span (or continuum) of values. A discrete choice variable is one for which the decision maker chooses from a span of values that is interrupted by gaps.

5. *Marginal benefit* (*MB*) is the change in total benefit caused by an incremental change in the level of activity. *Marginal cost* (*MC*) is the change in total cost caused by an incremental change in the level of activity. An "incremental change" in activity is a small positive or negative change in activity, usually a one-unit

increase or decrease in activity. Marginal benefit and marginal cost can be expressed mathematically as

$$MB = \frac{\text{change in total benefit}}{\text{change in activity}} = \frac{\Delta TB}{\Delta A}$$

$$MC = \frac{\text{change in total cost}}{\text{change in activity}} = \frac{\Delta TC}{\Delta A}$$

where the symbol) means *the change in* and A denotes the level of activity.

6. Because "marginal" variables measure rates of change in corresponding "total" variables, marginal benefit and marginal cost are also slopes of total benefit and total cost curves, respectively. Marginal benefit (cost) of a particular unit of activity is measured by the slope of the line tangent to the total benefit (total cost) curve at that point of activity.

7. If, at a given level of activity, a small increase or decrease in activity causes net benefit to increase, then this level of activity is not optimal. The activity must then be increased (if marginal benefit exceeds marginal cost) or decreased (if marginal cost exceeds marginal benefit) to reach the highest net benefit. The optimal level of the activity is attained when no further increases in net benefit are possible for any changes in the activity. This point occurs at the activity level for which marginal benefit equals marginal cost: $MB = MC$.

8. When a manager faces an unconstrained maximization problem and must choose among discrete levels of an activity, the manager should increase the activity if $MB > MC$ and decrease the activity if $MB < MC$. The optimal level of activity is the last level for which MB exceeds MC.

9. *Sunk costs* are costs that have previously been paid and cannot be recovered. *Fixed costs* are costs that are constant and must be paid no matter what level of activity is chosen. *Average (or unit) cost* is the cost per unit of activity, which is computed by dividing total cost by the number of units of activity. Decision makers wishing to maximize net benefit should ignore any sunk costs, any fixed costs, and the average costs associated with the activity because none of these costs affect the marginal cost of the activity, and so are irrelevant for making optimal decisions.

10. The ratio of marginal benefit divided by the price of an activity (MB/P) tells the decision maker the additional benefit of that activity per additional dollar spent on that activity. In constrained optimization problems, the ratios of marginal benefits to prices of the various activities are used by managers to determine how to allocate a fixed number of dollars among activities.

11. To maximize or minimize an objective function subject to a constraint, the ratios of the marginal benefit to price must be equal for all activities,

$$\frac{MB_A}{P_A} = \frac{MB_B}{P_B} = \cdots = \frac{MB_Z}{P_Z}$$

and the values of the choice variables must meet the constraint.

Matching Definitions

activities or choice variables	marginal cost
average (or unit) cost	maximization problem
constrained optimization	minimization problem
continuous variable	objective function
discrete variable	optimal level of activity
fixed costs	sunk costs
marginal analysis	unconstrained optimization
marginal benefit	

1. ____________________ Function to be either maximized or minimized.
2. ____________________ Optimization problem in which the decision maker is trying to maximize some activity.
3. ____________________ Optimization problem in which the decision maker is trying to minimize some activity
4. ____________________ The determinants of the values of objective functions.
5. ____________________ A variable that cannot take a continuum of values.
6. ____________________ A variable that can take any value between two end points.
7. ____________________ A situation in which a manager may choose the optimal levels of activities from an unrestricted set of values.
8. ____________________ A situation in which a manager may choose the optimal levels of activities from a restricted set of values.
9. ____________________ The analytical process of making incremental changes to the level of the choice variables to arrive at the point where no further improvements in the objective function are possible.
10. ____________________ The level of activity that maximizes net benefit.
11. ____________________ The additional benefits derived per unit increase in activity.
12. ____________________ The additional cost realized per unit increase in activity.
13. ____________________ Costs that have already been paid and cannot be recovered.
14. ____________________ Costs that are constant and must be paid no matter what level of activity is chosen.
15. ____________________ Total cost divided by the number of units of activity.

Study Problems

1. a. Fill in the missing numbers below.

A	*Total Benefit*	*Total Cost*	*Marginal Benefit*	*Marginal Cost*	*Net Benefit*
0	0	_____	xx	xx	0
1	_____	_____	10	_____	8
2	_____	5	9	_____	_____
3	25	_____	_____	4	_____
4	_____	_____	_____	6	15
5	34	22	_____	_____	_____

b. Define "optimal level of activity." In part *a*, what is the optimal level of activity? Why?

c. In part *a*, marginal benefit does not equal marginal cost for any quantity. Does this mean there is no optimal level of activity? Why or why not?

d. At the optimal level of activity, could you increase the level of activity and get an increase in total benefit? If so, why should the manager *not* increase the activity further?

2. The manager of a firm estimates that the sales of her firm are related to radio and newspaper advertising in the following way:

$S = 12{,}000 + 1{,}800AR$, where

S = the number of units sold,
A = the number of quarter-page newspaper advertisements, and
R = the number of minutes of radio spots.

a. Derive the marginal benefit of newspaper and radio advertising. [Hint: The marginal benefit of advertising can be found by determining how much S changes for each one-unit change in A, holding R constant.]

$$\frac{\Delta S}{\Delta A} = \_\_\_\_\_\_\_\_\_\_ \qquad \text{and} \qquad \frac{\Delta S}{\Delta R} = \_\_\_\_\_\_\_\_\_\_$$

b. If the newspaper ads cost $600 per quarter-page ad ($P_A = \$600$) and the radio ads cost $200 per minute ($P_R = \$200$), find the combination of radio and television ads that maximizes sales when the advertising budget is $7,200. Also compute the optimal level of sales.
[Hint: Set $MB_A / P_A = MB_R / P_R$, then solve for either A or R and substitute this expression into the budget constraint $600A + 200R = \$7{,}200$ to solve for A^* and R^*.]

$$A^* = \_\_\_\_\_\_\_\_$$
$$R^* = \_\_\_\_\_\_\_\_$$

$S^* =$ ________

c. Suppose the advertising budget is cut so that only $4,800 can be spent on advertising. Now what are the sales-maximizing values of A, R, and S?

$A^* =$ ________

$R^* =$ ________

$S^* =$ ________

d. Based on parts *b* and *c*, what is the effect of changing the advertising budget constraint on the optimal level of sales?

$$\frac{\Delta S^*}{\Delta B} = \underline{\qquad\qquad},$$

where ΔB is the change in the advertising budget. Do you expect this number to be positive or negative?

3. A life-insurance salesman spends 9 hours a week on the telephone soliciting new clients. From past experience, the salesman estimates that each hour spent calling students, blue-collar workers, and professionals will produce the following number of additional sales:

	Number of Additional Sales		
Hours Calling	*Students*	*Blue-Collar Workers*	*Professionals*
1	10	8	14
2	8	6	11
3	6	4	8
4	4	3	6
5	1	1	4
6	0	0	1

a. How should the life-insurance salesman allocate his phone-calling time to maximize the number of sales?

Hours spent calling students = ______

Hours spent calling blue-collar workers = ______

Hours spent calling professionals = ______

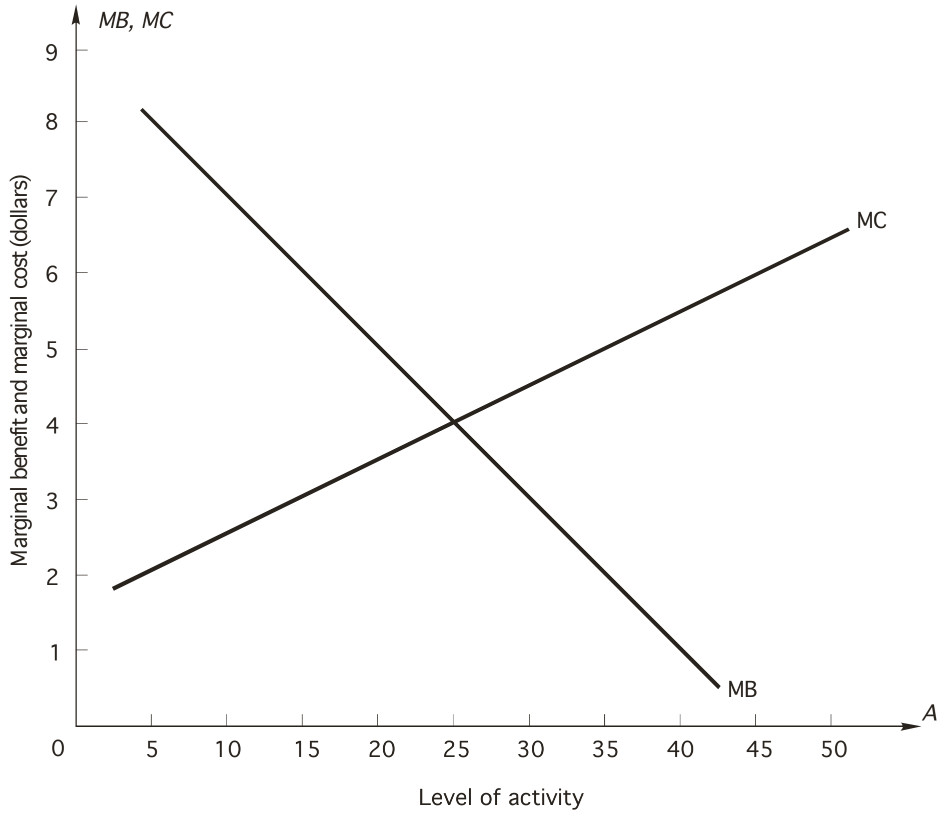
b. Now suppose the salesman decides to spend 16 hours a week soliciting new clients. How should he allocate his time?

Hours spent calling students = ______

Hours spent calling blue-collar workers = ______

Hours spent calling professionals = ______

4. Use the figure below to answer the following questions



a. At 15 units of the activity, marginal benefit is $______ and marginal cost is $______.

b. Adding the 15th unit of activity causes net benefit to __________ (increase, decrease) by $______.

c. At 35 units of the activity, marginal benefit is $______ and marginal cost is $______.

d. Subtracting the 35th unit of activity causes net benefit to __________ (increase, decrease) by $______.

e. The optimal level of activity is ______ units, *MB* = $______ and *MC* = $______.

f. Can you compute total benefit, total cost, and net benefit for the optimal level of activity? If so, how? If not, why not?

5. Activity A has the following marginal benefit (MB) and marginal cost (MC) functions:

$$MB = 10 - 0.05A \text{ and } \qquad MC = 2 + 0.05A$$

where MB and MC are measured in dollars.

a. The 70th unit of activity increases total benefit by $______ and increases total cost by $______. Since marginal benefit is __________ (greater, less) than marginal cost, adding the 70th unit of the activity __________ (increases, decreases) net benefit by $______.

b. The 110th unit of activity increases total benefit by $______ and increases total cost by $______. Since marginal benefit is __________ (greater, less) than marginal cost, adding the 110th unit of the activity __________ (increases, decreases) net benefit by $______.

c. The optimal level of activity is ______ units. At the optimal level of activity, marginal benefit is $______ and marginal cost is $______.

The total benefit (TB) and total cost (TC) functions for the activity are

$$TB = 10A - 0.025A^2 \quad \text{and} \qquad TC = 2A + 0.025A^2$$

where TB and TC are measured in dollars.

d. For the optimal level of activity in part *c*, the total benefit is $______, the total cost is $______, and the net benefit is $______.

e. Compute the net benefit for one unit more and one unit less than the level of activity found to be optimal in part *c* (i.e., compute NB for $A^* + 1$ and $A^* - 1$). Are your results consistent with the definition of "optimal"? Explain.

6. Evaluate the following statements:

a. "The optimal level of an activity is that level for which marginal benefit exceeds marginal cost by the greatest possible amount."

b. "The ratio of marginal benefit to marginal cost of an activity measures the additional benefit attributable to increasing the activity by one unit."

c. "At the optimal level of activity, further increases in the activity necessarily decrease total benefit."

d. "This is a lousy vacation resort, and it's been raining the whole time. I'd leave but I've already paid for the hotel for the week, so I guess I will stay."

e. "I hate golf, but I paid so much for the clubs that I can't give it up."

f. "The cost of my yearly business license is doubling next year, so I must plan to increase output next year in order to cover the additional cost of doing business."

g. "Now is the perfect time to buy more television ads because the TV networks are offering us lower prices on any extra ads we purchase."

7. Suppose a manager wishes to find the optimal level of two activities *X* and *Y*, which yield the total benefits presented in the table below. The price of *X* is $40 per unit, and the price of *Y* is $100 per unit.

Level of Activity	*Total Benefit of Activity X (TB_X)*	*Total Benefit of Activity Y (TB_Y)*
0	0	0
1	800	1,000
2	1,440	1,900
3	2,000	2,700
4	2,360	3,400
5	2,680	4,000
6	2,960	4,500
7	3,200	4,900
8	3,400	5,200

a. The manager faces a budget constraint of $500 for expenditures on activities *X* and *Y*. The optimal levels of activities of *X* and *Y* when the manager can spend only $500 are $X^* =$ __________ and $Y^* =$ __________.

b. In part *a*, the total benefit associated with the optimal level of *X* and *Y* is $________________.

c. Now let the budget constraint increase to $780. The optimal levels of activities of *X* and *Y* when the manager can spend $780 are $X^* =$ _________ and $Y^* =$ __________.

d. In part *c*, the total benefit associated with the optimal level of *X* and *Y* is $_____________.

Multiple Choice / True-False

1. For an unconstrained optimization problem with a continuous choice variable, the optimal level of an activity is that level of activity for which
   a. total benefit exceeds total cost by the greatest amount possible.
   b. marginal benefit minus marginal cost equals zero.
   c. total benefit equals total cost.
   d. marginal benefit is zero.
   e. both *a* and *b*.

In Questions 2 – 6, consider an activity *A* that has the following marginal benefit (*MB*) and marginal cost (*MC*) functions:

$$MB = 50 - 0.025A \quad \text{and} \quad MC = 40 + 0.025A$$

and the following total benefit (*TB*) and total cost (*TC*) functions:

$$TB = 50A - 0.0125A^2 \quad \text{and} \quad TC = 40A + 0.0125A^2 .$$

2. Undertaking the 100^{th} unit of the activity
   a. reduces total benefit by \$2.50 and increases total cost by \$2.50.
   b. increases total benefit by \$2.50 and reduces total cost by \$2.50.
   c. causes net benefit to fall.
   d. maximizes net benefit because *MB* = *MC* at 100 units.
   e. none of the above.

3. Undertaking the 400^{th} unit of the activity
   a. reduces total benefit by \$10 and increases total cost by \$10.
   b. increases total benefit by \$40 and increases total cost by \$50.
   c. causes net benefit to fall.
   d. both *a* and *c*.
   e. both *b* and *c*.

4. What is the optimal level of activity?
   a. 100
   b. 200
   c. 300
   d. 400
   e. 500

5. For the optimal activity level in question 4, total benefit, total cost, and net benefit are respectively
   a. \$9,500, \$8,500, and \$1,000
   b. \$21,875, \$23,125, and –\$1,250
   c. \$23,125, \$21,875, and –\$1,250
   d. \$13,875, \$13,125, and \$750
   e. none of the above.

6. What level of activity maximizes TOTAL benefit?
   a. 500
   b. 1,500
   c. 1,750
   d. 1,850
   e. 2,000

In questions 7 – 9, a firm can spend $1,150 monthly on advertising in either the newspaper or on the radio. Marketing experts estimate that monthly sales can be increased by the following amounts:

	Additional Units Sold Monthly	
Number of Ads Monthly	*Newspaper*	*Radio*
1	1,000	1,800
2	750	1,500
3	500	1,200
4	400	1,000
5	250	600

The prices of newspaper and radio ads are $250 and $300 respectively.

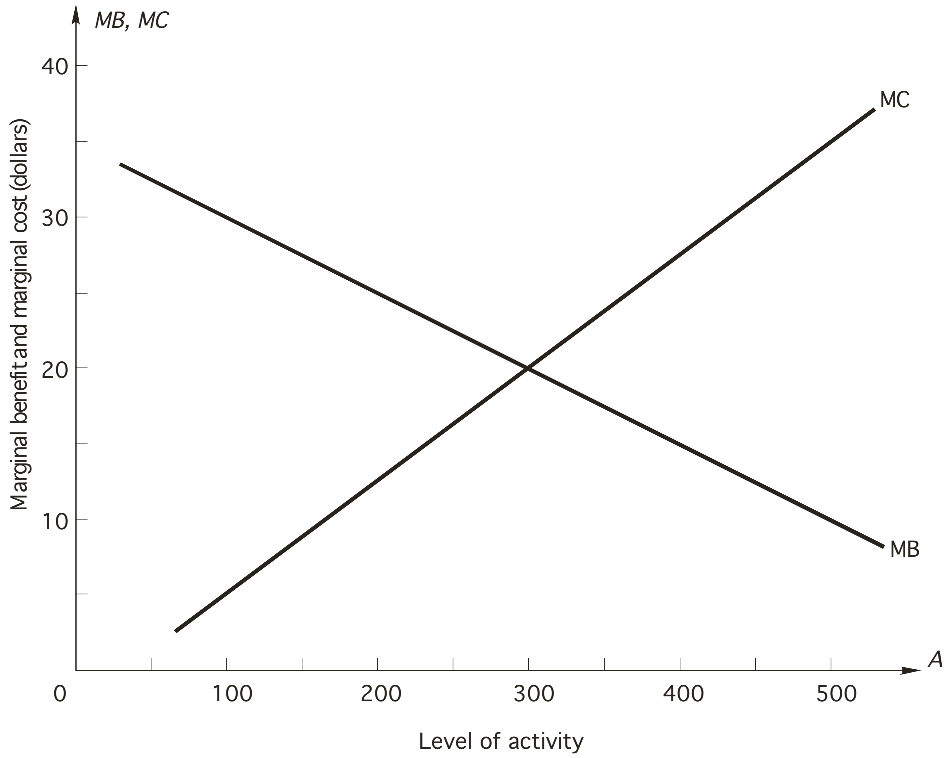
7. In order to maximize monthly sales, the advertising budget should be allocated so that
   a. 1 newspaper ad and 1 radio ad are purchased monthly.
   b. 1 newspaper ad and 2 radio ads are purchased monthly.
   c. 2 newspaper ads and 2 radio ads are purchased monthly.
   d. 3 newspaper ads and 1 radio ad are purchased monthly.
   e. none of the above.

8. If the advertising budget is increased to $2,250 per month, how should the budget be allocated to maximize sales?
   a. 2 newspaper ads and 2 radio ads are purchased monthly.
   b. 3 newspaper ads and 3 radio ads are purchased monthly.
   c. 3 newspaper ads and 4 radio ads are purchased monthly.
   d. 3 newspaper ads and 5 radio ads are purchased monthly.
   e. none of the above.

9. In question 8 above, the values of MB_N / P_N and MB_R / P_R are both equal to
   a. 1
   b. 2
   c. 3
   d. 4
   e. none of the above

10. If the marginal benefit of the last unit of activity *X* is 100 and the price of *X* is $25,
    a. the last unit of *X* causes total benefit to rise by 4.
    b. the last unit of *X* causes total benefit to rise by 100.
    c. spending one more dollar on *X* causes total benefit to rise by 100.
    d. spending one more dollar on *X* causes total benefit to rise by 4.
    e. both *b* and *d*.

Use the following graph showing the marginal benefit and marginal cost curves for activity *A* to answer questions 11 – 14.



11. If the firm is using 100 units of the activity, marginal benefit is $______ and marginal cost is $______.
    a. $20; $20
    b. $30; $5
    c. $5; $30
    d. $25; $25
    e. $30; 25

12. If the firm is using 100 units of the activity, the firm can _________ (increase, decrease) the activity by one unit and increase net benefit by $________.
    a. decrease; $25
    b. decrease; $35
    c. increase; $30
    d. increase; $25

13. If the firm is using 500 units of the activity, it can ______________ (increase, decrease) the activity by one unit and increase net benefits by $________.
    a. decrease; $25
    b. decrease; $35
    c. increase; $25
    d. increase; $30

14. To maximize net benefits the firm should use ________ units of the activity, at which point _______________.
    a. 300; $MB > MC$
    b. 200; $MB = MC =$ $25
    c. 400; $MB < MC$
    d. 300; $MB = MC =$ $20
    e. 400; $MB = MC$

15. When marginal cost is greater than marginal benefit at the current activity level, the decision maker can increase net benefit by decreasing the activity because
    a. total benefit will rise by more than total cost will rise.
    b. total cost will fall by more than total benefit will fall.
    c. net benefit is upward sloping at this point.
    d. marginal cost is rising faster than marginal benefit is falling.

16. A firm is currently buying 30 TV ads at $200 each and 20 newspaper ads at $100 each for a total advertising expenditure of $8,000. The additional sales from the last TV ad were 300 units and from the last newspaper ad were 200 units. If the firm buys 2 more newspaper ads it can increase sales by ______ units and keep advertising cost the same by reducing TV ads by _____ units.
    a. 300; 1
    b. 100; 2
    c. 100; 1
    d. 400; 2
    e. 500; 3

17. T F At the optimal level of an activity, total cost is minimized and total benefit is maximized.

18. T F The solution to an unconstrained minimization problem is the same as the solution to the unconstrained maximization problem for which the net benefit function is the negative of the net cost function.

19. T F As a rule, the optimal number of product defects is zero.

20. T F When a manager faces an objective function with more than one choice variable, the firm attains a maximum of its objective function when the marginal benefit from each activity equals its marginal cost.

Answers

MATCHING DEFINITIONS

1. objective function
2. maximization problem
3. minimization problem
4. activities or choice variables
5. discrete variable
6. continuous variable
7. unconstrained optimization
8. constrained optimization
9. marginal analysis
10. optimal level of activity
11. marginal benefit
12. marginal cost
13. sunk costs
14. fixed costs
15. average (or unit) costs

STUDY PROBLEMS

1. a.

A	*Total Benefit*	*Total Cost*	*Marginal Benefit*	*Marginal Cost*	*Net Benefit*
0	0	0	*xx*	*xx*	0
1	10	2	10	2	8
2	19	5	9	3	14
3	25	9	6	4	16
4	30	15	5	6	15
5	34	22	4	7	12

b. The level of activity that maximizes the objective function (net benefit) is called the *optimal level of activity*. $A^* = 3$ because at this value, net benefit is maximized.

c. When the choice variable (in this case A) is not continuous, the objective function may reach its maximum value at a level of activity where marginal benefit does *not* equal marginal cost. Marginal analysis still leads to the optimal value, however. At a quantity of 3, marginal benefit (= \$6) exceeds marginal cost (= \$4). Clearly, producing the third unit increases profit. For the fourth unit of activity, marginal cost (= \$6) exceeds marginal benefit (= \$5), and net benefit would decrease if it were produced. Thus, 3 units is the optimal level.

d. Because *MB* is positive at the optimal level, *TB* could be further increased by increasing the level of activity. This is not desirable because the increase in *TB* would be accompanied by a decrease in *NB* (since $MB < MC$).

2. a. $\Delta S / \Delta A = 1800R$; $\Delta S / \Delta R = 1800A$

b. The optimal levels of R and A must satisfy two conditions:

(i) $MB_A / P_A = MB_R / P_R$
$(1800R / 600) = (1800A / 200) \Rightarrow R = 3A$

(ii) $P_R R + P_A A = 7{,}200$
$200R + 600A = 7{,}200$

These two conditions are simultaneously satisfied if
$200(3A) + 600A = 7{,}200 \Rightarrow A^* = 6, R^* = 18,$ and $S^* = 206{,}400$

c. $R^* = 12;\quad A^* = 4;\quad S^* = 98{,}400$

d. $\Delta S^* / \Delta B = 108{,}000 / 2{,}400 = 45$. Positive because an increase (decrease) in the advertising budget will increase (decrease) sales.

3. a. 3; 2; 4 (*MB* per hour = *MB*/1 = 6 for all 3 and total number of hours = 9)
   b. 5; 5; 6 (*MB*/1 = 1 for all 3 and total number of hours = 16)

4. a. MB_{15} = \$6; MC_{15} = \$3 [Note: These numbers are read from the figure.]
   b. increase; \$3
   c. MB_{35} = \$2; MC_{35} = \$5
   d. increase; \$3
   e. 25; \$4; \$4
   f. No, because you are only given marginal benefit and marginal cost. You cannot compute total benefit, total cost, and net benefit using the information given.

5. a. \$6.50 [= 10 – 0.05(70)]; \$5.50 [= 2 + 0.05(70)]; greater; increases; \$1 (= 6.50 – 5.50)
   b. \$4.50 [= 10 – 0.05(110)]; \$7.50 [= 2 + 0.05(110)]; less; decreases; \$3 (= 7.50 – 4.50)
   c. A^* = 80 units (set *MB* = *MC* and solve for A^*);
      \$6; \$6 [Note: $MB_{80} = MC_{80} = 10 - 0.05(80) = 2 + 0.05(80) = 6$.]
   d. *TB* = \$640; *TC* = \$320; *NB* = \$320
   e. $A^* + 1 = 81$ and NB_{81} = \$319.95;
      $A^* - 1 = 79$ and NB_{79} = \$319.95;
      Since NB_{80} (= \$320) exceeds both NB_{79} and NB_{81}, these computations are consistent with *NB* being maximized at 80 units.

6. a. The optimal level of an activity is that level for which *total* benefit exceeds *total* cost by the greatest possible amount.
   b. *MB*/*MC* gives the additional benefit for *spending one more dollar* on the activity, not for increasing the activity by one more unit. Only if *MC* = \$1 would spending one more dollar on the activity also result in one more unit of the activity being undertaken.
   c. This statement is generally incorrect. At the optimal level of activity, further increases in the activity cause total benefit to decrease *only* if *MB* is negative. *MB* equals *MC* at the optimal level activity level. Because *MC* cannot be negative, *MB* cannot be negative at the optimal activity level. Since *MB* cannot be negative, total benefit cannot fall as activity increases beyond X^*. It is true (by definition) that net benefit *falls* as *X* increases beyond the optimal level of activity.
   d. Bad reasoning. What has already been paid for the room is a sunk cost and is irrelevant. This person should weigh the expected marginal benefits and costs, then make the decision about whether to stay.
   e. What was paid for the golf clubs is a sunk cost and should be ignored.
   f. After paying for the more expensive business license, presumably a once-a-year fee paid at the beginning of the year, the cost is sunk and has no effect on marginal cost (or marginal revenue, for that matter). Thus, ignore the sunk cost and plan to produce the same level of output, unless something else causes a change in either *MB* or *MC*.
   g. Extra ads are only worth purchasing if, at the lower price, the marginal cost of buying more ads is less than the marginal benefit of running more ads. It is indeed possible that by lowering the price of additional advertising spots, ad buyers will find it optimal to buy more ads. Of course, it is also possible that even at "low" prices, extra ads are not worth the added cost.

7. a. $X^* = 5$; $Y^* = 3$
   b. \$5,380 since $TB_{X=5}$ = \$2,680 and $TB_{Y=3}$ = \$2,700
   c. $X^* = 7$; $Y^* = 5$
   d. \$7,200 since $TB_{X=7}$ = \$3,200 and $TB_{Y=5}$ = \$4,000

MULTIPLE CHOICE / TRUE-FALSE

1. e — Since $NB = TB - TC$, maximizing NB implies TB exceeds TC by the greatest amount possible. For a continuous choice variable, $MB = MC$ at the optimal level of an activity. Thus $MB - MC = 0$ at the optimal level of activity.

2. e — Undertaking the 100th unit of activity increases total benefit by \$47.50 (= 50 – 0.025 × 100) and increases total cost by \$42.50 (= 40 + 0.025 × 100). This would cause net benefit to rise by \$5. Thus, none of the choices are correct.

3. e — Undertaking the 400th unit of activity increases total benefit by \$40 (= 50 – 0.025 × 400) and increases total cost by \$50 (= 40 + 0.025 × 400). This would cause net benefit to fall (by \$10). Thus, both *b* and *c* are correct.

4. b — At 200 units, MB = \$45 (= 50 – 0.025 × 200) and MC = \$45 (= 40 + 0.025 × 200).

5. a — TB = \$9,500 (= $50 \times 200 - 0.025 \times 200^2$); TC = \$8,500 (= $40 \times 200 + 0.025 \times 200^2$); NB = \$1,000 (= \$9,500 – \$8,500).

6. e — Total benefit is maximized at the activity level for which $MB = 0$. Thus, solve $0 = 50 - 0.025A$ to get the activity level where MB reaches its peak: $A = 2{,}000$.

7. e — The optimal number of newspaper and radio ads are $N^* = 1$ and $R^* = 3$.

8. d — At $N^* = 3$ and $R^* = 5$, $MR/P = 2$ for both newspaper and radio ads; \$2,250 is spent.

9. b — See the answer to question 8 above.

10. e — Since $MB = 100$, the last unit of X increases total benefit by 100. Since $MB/P = 100/25 = 4$, spending one more dollar of activity X increases total benefit by 4.

11. b — From the graph at $A = 100$, $MB = 30$, $MC = 5$.

12. d — Total benefit rises by \$30 and total cost rises by \$5, so net benefit rises by \$25.

13. a — Total benefit falls by \$30 and total cost falls by \$5, so net benefit falls by \$25.

14. d — $MB = MC$ = \$20 at $A = 300$.

15. b — Decreasing A causes TC and TB to fall. In this case, $MC > MB$, so TC falls by more than TB falls, and NB rises.

16. c — If 2 more newspaper adds are purchased sales will rise by 400 units (200 more units at the margin for each extra newspaper ad), and if 1 less TV ad is purchased sales will fall by 300 units. Thus, if 2 more newspaper ads are purchased while 1 less TV ad is purchased, then sales will rise by 100 units (= 400 – 300), and total cost remains \$8,000 because the increased expenditure on newspaper ads is exactly equal to the reduced expenditure on TV ads.

17. F — Neither TB nor TC is maximized or minimized at the optimal level of activity. It is NB that is being maximized.

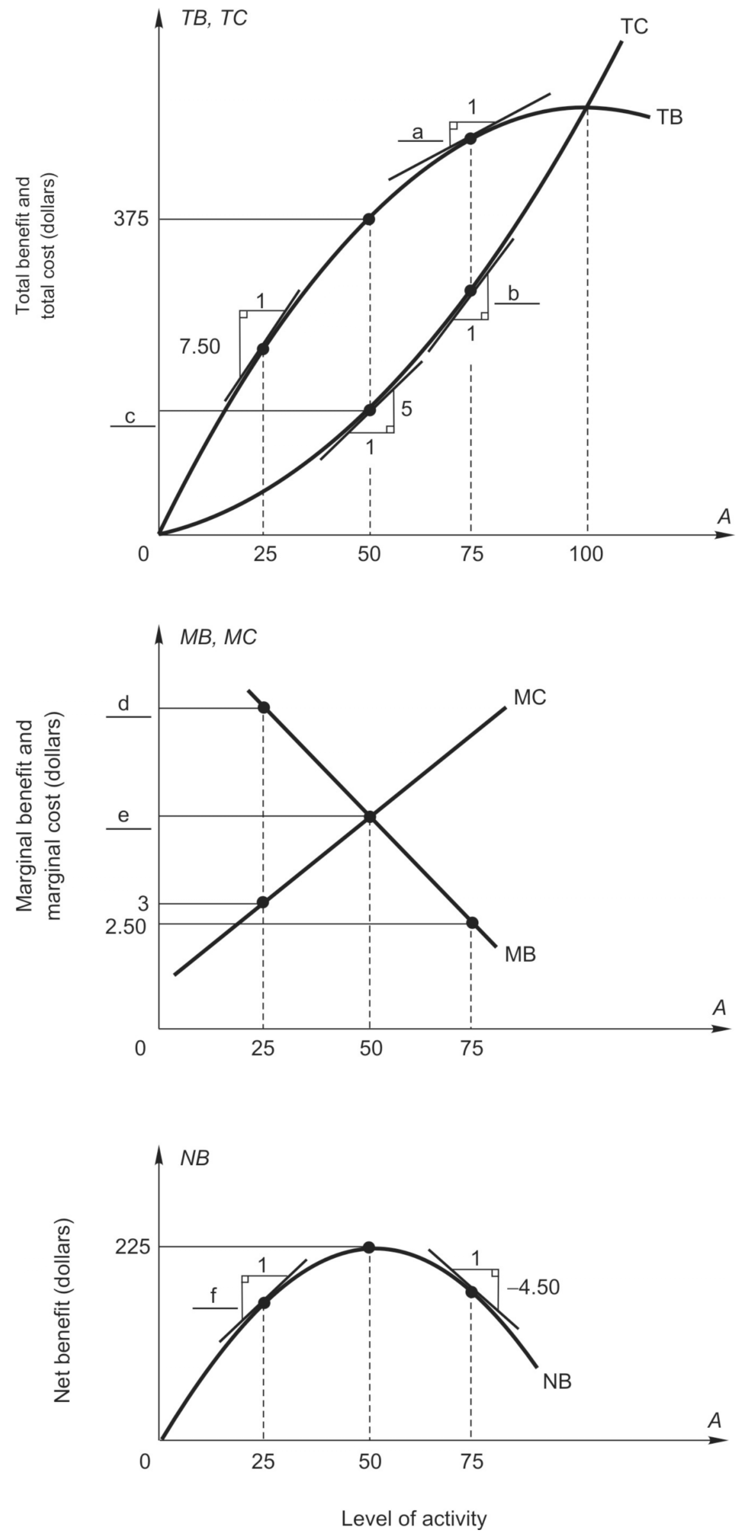
18. T — Unconstrained minimization involves multiplying net cost (NC) by –1 to get net benefit (NB), which is then maximized by finding the level of activity for which $MB = MC$.

19. F — MB generally does not equal MC at zero defects because the marginal cost of eliminating the last defective product is generally expensive.

20. T — $MB_X = MC_X$; $MB_Y = MC_Y$; and so on.

Homework Exercises

1. Use the figure below to fill in the blanks in the following questions.



a. The values that belong in the blanks *a* – *f* in the figure on the previous page are:

a.	______	d.	______
b.	______	e.	______
c.	______	f.	______

b. The optimal level of activity is ______ units of activity where ______ (*TB*, *TC*, *MB*, *MC*, *NB*) reaches its maximum value of $_________.

c. Because *TB* is maximized at 100 units of activity, a line drawn tangent to the *TB* curve at 100 units of activity (*not* shown in the figure) has a slope of ______. *NB* at 100 units of activity is $_________, which is identical to the value of *NB* at ______ units of activity. Beyond 100 units, *NB* is _____________ (positive, rising, zero, negative).

d. At 25 units of activity, ______ (*TB*, *TC*, *MB*, *MC*, *NB*) is rising faster than ______ (*TB*, *TC*, *MB*, *MC*, *NB*) is rising, and thus ______ (*TB*, *TC*, *MB*, *MC*, *NB*) is rising.

e. At 75 units of activity, decreasing the activity causes ______ (*TB*, *TC*, *MB*, *MC*, *NB*) to fall faster than ______ (*TB*, *TC*, *MB*, *MC*, *NB*) falls, and thus ______ (*TB*, *TC*, *MB*, *MC*, *NB*) rises.

2. Consider some activity *A* for which the total benefits and total costs associated with the different levels of *A* are measurable in dollars as tabulated below.

Level of Activity A	*Total Benefit*	*Total Cost*	*Marginal Benefit*	*Marginal Cost*	*Net Benefit*
0	0	0	xx	xx	______
1	$31.75	$2.50	______	______	______
2	60.75	6.25	______	______	______
3	87.25	11.25	______	______	______
4	111.40	18.00	______	______	______
5	133.60	26.30	______	______	______
6	154.60	35.40	______	______	______
7	174.10	49.20	______	______	______
8	192.10	66.95	______	______	______
9	209.20	88.95	______	______	______
10	223.45	114.95	______	______	______
11	235.90	145.45	______	______	______
12	247.10	180.05	______	________	______

a. Calculate marginal cost and marginal benefit. The optimal level of activity *A* is $A^* =$ ________.

b. Explain carefully, using the logic of marginal analysis, why $A = 4$ is *not* the optimal level of activity A.

c. Explain carefully, using the logic of marginal analysis, why $A = 12$ is *not* the optimal level of activity A.

d. Now calculate the net benefit for each level of A and tabulate the values in the last column of the table. Inspect net benefit and determine the level of A that maximizes net benefit.

$A^* =$ __________.

Is this the same value as in part *a*?

3. A manager can spend \$7,000 on two activities (X and Y) that generate benefits for the firm. The price of X is \$1,000 per unit and the price of Y is \$2,000 per unit. The table below gives the total benefits for various levels of activities X and Y.

Level of Activity X	*Total Benefit of Activity X*	*Total Benefit of Activity Y*
0	0	0
1	\$ 30,000	\$ 70,000
2	55,000	110,000
3	75,000	140,000
4	90,000	164,000
5	100,000	186,000
6	105,000	206,000

a. The optimal levels of activities X and Y when the manager faces a budget constraint of \$7,000 are $X^* =$ ____________ and $Y^* =$ ____________. The total benefit to the firm of engaging in the optimal levels of activities X and Y is \$____________.

b. Now let the budget increase to \$17,000. The optimal levels of activities X and Y when the manager faces a budget constraint of \$17,000 are $X^* =$ ___________ and $Y^* =$ ____________. The total benefit to the firm of engaging in the optimal levels of activities X and Y is \$____________.

4. Circle the correct words to go in the blanks below.

   a. If marginal benefit exceeds marginal cost, then _______________ (increasing, decreasing) the level of activity by one unit increases ______________(total, marginal, net) benefit by more than it _________________(increases, decreases) ______________ (total, marginal) cost. Therefore __________________ (increasing, decreasing) the level of activity by one unit must increase net benefit. The decision maker should continue to _______________ (increase, decrease) the level of activity until marginal benefit and marginal cost are both ___________________ (zero, equal).

   b. If marginal cost exceeds marginal benefit, then _______________ (increasing, decreasing) the level of activity by one unit decreases _________________(total, marginal, net) benefit by less than it _______________(increases, decreases) ______________ (total, marginal) cost. Therefore, _______________ (increasing, decreasing) the level of activity by one unit must increase net benefit. The decision maker should continue to _______________ (increase, decrease) the activity until marginal benefit and marginal cost are both _________________ (zero, equal).

5. Several years ago, Nabisco spent $330 million building a facility in Brazil to produce Oreo cookies and Ritz crackers for sale in their South American markets. At a recent board meeting, managers at Nabisco were discussing closing the Brazilian plant because profits from South American sales declined sharply last year. One senior vice president opposed shutting down the Brazilian plant saying, "We spent so much money getting our Brazilian facility going, we just can't quit now." Evaluate the vice president's advice. Explain why you agree or disagree.

6. Consider this statement: “Conservationists want to save too many spotted owls.” Use graphical analysis accompanied by a concise narrative discussion to explain circumstances under which this statement would be true. Make sure your graphs have clearly labeled axes and curves. [Note: Good answers are dispassionate and logical.]

7. A publisher of a new novel has spent $250,000 setting the type. The publisher must spend $1 million advertising the new book. It is now ready to print the book. For practical purposes, as many books as they like can be printed. In deciding how many copies to run,
   a. does the cost of typesetting have any influence on the publisher’s decision? Explain.

   b. does the cost of advertising have any influence on the publisher’s decision? Explain.

8. The southeast sales manager for Manufacturer's Aluminum Supply makes most of her outside sales by playing either golf or tennis with potential clients. From much past experience, she estimates that various levels of playing golf and tennis generate the following amounts of additional sales (*MB*):

Number of rounds of golf (G)	*Additional sales generated (MB_G)*	*Number of tennis matches (T)*	*Additional sales generated (MB_T)*
1	$2,500	1	$2,400
2	2,000	2	2,250
3	1,750	3	2,100
4	1,375	4	1,800
5	1,250	5	1,500
6	1,200	6	1,050
7	1,125	7	750
8	1,100	8	600

One round of golf (18 holes) takes 5 hours to play (including a half-hour stop at the clubhouse), and one tennis match (best two out of three sets) takes 3 hours to complete. The sales manager can get away from the office to play golf and tennis only 20 hours each week.

a. What is the optimal number of rounds of golf to play and the optimal number of tennis matches to play?

b. Suppose the sales manager gets sick one week and can only play 12 hours of golf and tennis. What level of golf and tennis play is optimal?