Assignment 20: Infinite Series (8.2-.7)   **Name_____**
Please provide a handwritten response.

1a. To find the partial sum S_{10} of the infinite series $\sum_{k=1}^{\infty} \dfrac{1}{k^{0.9}}$ execute $\sum (1/k\wedge.9, k, 1, 10)$ and

record the result in the table below. By pressing **SECOND ENTRY**, changing the **10** to **50**, etc. and pressing **ENTER**, complete the second column of the table below.

n	$S_n = \sum_{k=1}^{n} \dfrac{1}{k^{0.9}}$	$S_n = \sum_{k=1}^{n} \dfrac{5}{k^{1.1}}$
10		
50		
100		
500		
1000		

1b. Likewise modify the command in part **a** to find the partial sums of the infinite series $\sum_{k=1}^{\infty} \dfrac{5}{k^{1.1}}$ and complete the third column in the table above. Notice that in each row, the entry in the second column is smaller than that in the third. Can this be the case for all n? Why?

2a. The harmonic series $\sum_{k=1}^{\infty} \dfrac{1}{k}$ is a divergent series. To see how fast it diverges plot the partial sums on the graph provided below. Switch **MODE** to **Parametric** and enter
$\begin{cases} x_{t1} = t \\ y_{t1} = sum\left(seq\left(1/n, n, 1, t, 1\right)\right) \end{cases}$. Set the **Style** to **DrawDot (dot)** and set the window:

TI-89	Voyage 200
$tMin = 0$, $tMax = 50$, $tStep = 1$	$tMin = 0$, $tMax = 50$, $tStep = 1$
$xMin = 0$, $xMax = 50$, $xscl = 1$	$xMin = 0$, $xMax = 50$, $xStep = 1$
$yMin = 1$, $yMax = 5$, $yscl = .2$	$yMin = 1$, $yMax = 5$, $yStep = .2$

Sketch the result on the axes provided. Repeat the graphing with **tMax = 100** and **xMax = 100**. Record these results on the same graph in different colors. Would you say that the partial sums are approaching ∞ quickly?
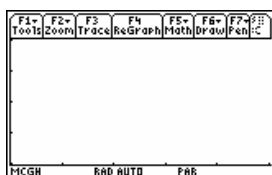
$0 \le x \le 50, 1 \le y \le 5$
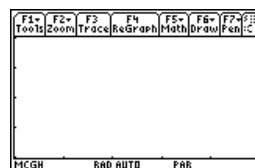
$0 \le x \le 100, 1 \le y \le 5$

2b. Construct a list of ordered pairs, $\left(n, \sum_{k=1}^{n} \dfrac{1}{k}\right)$ where the "y-value" is the nth partial sum of the harmonic series. Record values for $n = $ **1, 10, 20, 30, 40, 50** in the table provided below.

This can be achieved from the graph by pressing **F5 1 (Value).** Enter 1 at the prompt and record the value below. Repeat to find and evaluate the other values.

n	1	10	20	30	40	50
Sum						

3a. The Taylor polynomial for $f(x) = \cos x$ expanded about $c = \dfrac{\pi}{2}$ is given by $Taylor(\cos(x), x, 4, \pi/2)$. Execute this command and record the result below.

3b. Now plot $y1 = Taylor(\cos(x), x, 4, \pi/2)$ over $-\pi \le x \le 2\pi, -12 \le y \le 12$ sketch the results on the axes below. Sketch $y = \cos x$ (graph it as **y2**) on the same set of axes. On roughly what interval are the two graphs indistinguishable?
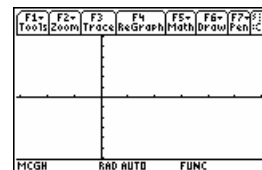
$$-\pi \le x \le 2\pi, -12 \le y \le 12$$

3c. Repeat parts **3a** and **3b** with $n = 8$ instead of $n = 4$. For this new Taylor polynomial sketch the graph on the same axes (use a different color to distinguish between the graphs). Again, answer the question in **part b**.

3d. To measure the error in this Taylor approximation on $-\pi \le x \le 2\pi$, graph $y3 = y2 - y1$ over the interval $-\pi \le x \le 2\pi, -2 \le y \le 2$ and sketch the result on the axes below. How large does the absolute value of the error become and for what value(s) of x is the error the greatest?
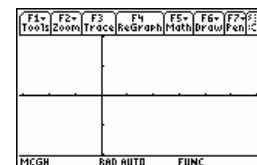
$$-\pi \le x \le 2\pi, -3 \le y \le 3$$

3e. By increasing the number of terms still further while keeping everything else the same, can you reduce the maximum error in part **d** to less than 0.1? Experiment to find how many terms are needed and record your results below.

3f. Try to answer part **e** with $\cos x$ changed to $\tan^{-1} x$ $(\arctan x)$ expanded about $c = 0$ and the interval changed to $-1.5 \le x \le 1.5, -8 \le y \le 8$. Can you add enough terms? Why?