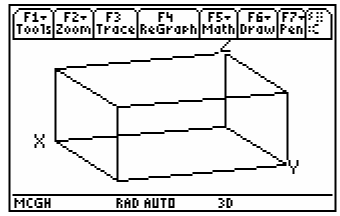
Assignment 32: Vector Fields in Space (14.6-8)   Name______________________
Please provide a handwritten response.

1a. Evaluate the surface integral $\iint_S g(x,y,z)\,dS$ for $\iint_S \sqrt{x^2+y^2}\,dS$ where S is the hemisphere $z=\sqrt{9-x^2-y^2}$. You can begin by graphing the hemisphere over $-3 \le x \le 3$, $-3 \le y \le 3$, $0 \le z \le 3$. Show your graph on the axes below.



1b. To evaluate the integral it is easier to parameterize the surface $g(x,y,z)=\sqrt{x^2+y^2}$ using cylindrical coordinates by letting $x = r\cos(\theta)$, $y = r\sin(\theta)$ and $z = r$ where $0 \le r \le 3$ and $0 \le \theta \le 2\pi$ than to evaluate it directly. To find dS you can use $dS = \|t_r \times t_\theta\| dA$ where $dA = r\,dr\,d\theta$. Begin by defining $t(r,\theta) = \left\langle r\cos\theta, r\sin\theta, \sqrt{9-r^2}\right\rangle$. Then $dS = \|t_r(r,\theta)\times t_\theta(r,\theta)\|$ where $n(r,\theta) = t_r(r,\theta)\times t_\theta(r,\theta)$ is a vector normal to the surface $g(x,y,z)=\sqrt{x^2+y^2}$. **Define** $n(r,\theta) = crossP\big(d\big(t(r,\theta),r\big), d\big(t(r,\theta),\theta\big)\big)$. Then $dS = norm\big(n(r,\theta)\big) * r\,dr\,d\theta$ and $\int\Big(\int\Big(\sqrt{}\big(9-r\wedge 2\big)*\|n(r,\theta)\|, r, 0, 3\Big), \theta, 0, 2\pi\Big)$. Evaluate this integral and record the result below.

2a. The flux integral is $\iint_s F\cdot n\,dS$ where $F(x,y,z)$ is the vector field $\langle y,-x,1\rangle$ and n is a unit normal vector. **Define** $f(x,y,z)=[y,-x,1]$. Parameterize S over $0 \le u \le 10$ and $0 \le v \le 4\pi$ by defining $r(u,v) = [u*\cos(v), u*\sin(v), v]$ and normal vector $nv(u,v) = crossP\big(d\big(r(u,v),u\big), d\big(r(u,v),v\big)\big)$. **Define** $n(u,v) = \dfrac{-nv(u,v)}{\|nv(u,v)\|}$. Calculate $n(u,v)$ and record your result below.

2b. Taking the unit normal n to have positive z-component, would you expect $\iint_s F\cdot n\,dS$ to be positive, negative or zero? Why?

2c. In order to find the integrand $\boldsymbol{F}\cdot\boldsymbol{ndS}$, you first need to find $\boldsymbol{f(x,y,z) \mid x = u*cos(v)\ and\ y = u*sin(v)\ and\ z = v \rightarrow k(u,v)}$ and then **define** $\boldsymbol{fn(u,v) = dotP(k(u,v), nv(u,v))}$. The flux integral $\boldsymbol{\int\left(\int\left(fn(u,v),u,1,10\right),v,0,4\pi\right)}$ can now be evaluated[1]. Record your result below. Were your expectations in **2b** borne out?

3a. The Divergence Theorem can be used to compute $\iint_{\partial Q} \boldsymbol{F}\cdot\boldsymbol{n\,ds}$ where $\boldsymbol{F}(x,y,z) = \langle x^3, y^3 - z, xy^2\rangle$ and $\boldsymbol{Q}$ is bounded by $\boldsymbol{z = x^2 + y^2}$ and $\boldsymbol{z = 4}$ where $\boldsymbol{-2 \le x \le 2}$ and $\boldsymbol{0 \le y \le 2}$. The curl of F, $\boldsymbol{curl\,F} = \nabla\times\boldsymbol{F}$ can be readily calculated once you **Define** $\boldsymbol{curlf = [d(x*y\wedge 2, y) - d(y\wedge 3 - z, z), d(x\wedge 3, z) - d(x*y\wedge 2, x), d(y\wedge 3 - z, x) - d(x\wedge 3, y)]}$ Record the result below. The divergence of F, $\boldsymbol{div\,F} = \nabla\cdot\boldsymbol{F}$ is also readily calculated as $\boldsymbol{divF = d(x\wedge 3, x) + d(y\wedge 3 - z, y) + d(x*y\wedge 2, z)}$. Record the result below. Are these results correct?

3b. Now set up (by hand) an iterated integral giving $\iiint_{Q} \nabla\cdot\boldsymbol{F}(x,y,z)\boldsymbol{dV}$ and use your calculator to evaluate it. Record the answer below.

3c. Stokes' Theorem tells you that $\iint_{S} (\nabla\times\boldsymbol{F})\cdot\boldsymbol{n\,ds}$ is the same whether $\boldsymbol{S}$ is the bottom "bowl" or the top "lid" of $\partial\boldsymbol{Q}$. In **3a** you found the curl of $\boldsymbol{F}$, $\nabla\times\boldsymbol{F}$, into which you can substitute the components of $\boldsymbol{\vec{r}(u,v) = [u*cos(v), u*sin(v), u^2]}$, $\boldsymbol{0 \le u \le 2, 0 \le v \le 2\pi}$.
Define $\boldsymbol{\vec{r}(u,v) = [u*cos(v), u*sin(v), u^2]}$ and
Define $\boldsymbol{n\vec{v}(u,v) = crossP(d(r(u,v),u), d(r(u,v),v))}$. You can now calculate $\boldsymbol{curlf \mid x = u*cos(v)\ and\ y = u*sin(v)\ and\ z = u\wedge 2 \rightarrow h(u,v)}$. Execute the double integral $\boldsymbol{\int\left(\int\left(dotP(h(u,v), nv(u,v)),u,0,2\right),v,0,2\pi\right)}$ and record your result below. Now you can make slight modifications in the above to calculate $\iint_{S} (\nabla\times\boldsymbol{F})\cdot\boldsymbol{n\,ds}$ for the lid. Record the result below. Do the two results agree?

[1] This integral evaluates very slowly.