Elasticity

The price elasticity of demand is defined as the percentage change in quantity demanded divided by the percentage change in price: $E_d = \dfrac{\%\Delta Q}{\%\Delta P} = \dfrac{\Delta Q}{Q_0} \Big/ \dfrac{\Delta P}{P_0}$ where Q_0 and P_0 are the original quantity and price, respectively.[1] A problem arises in that, given any two values X and Y, $\left|\dfrac{\mathrm{Y}-\mathrm{X}}{\mathrm{X}}\right| \neq \left|\dfrac{\mathrm{X}-\mathrm{Y}}{\mathrm{Y}}\right|$. That is, the percentage *increase* going from a low value X to a high value Y is not the same (in absolute value) as the percentage *decrease* in going from Y to X. In general, we will not want to specify whether the price change is positive or negative, or in other words, we won't know which of the two prices is the "original" price. Hence, we use the "midpoint" formula: $E_d = \dfrac{\Delta Q}{\overline{Q}} \Big/ \dfrac{\Delta P}{\overline{P}}$ where $\overline{Q}$ and $\overline{P}$ represent the average quantity and price. To simplify notation, we will substitute "Q" and "P" for their respective average values. In that case, after inverting and multiplying the denominator and rearranging, we can rewrite the formula as $E_d = \dfrac{\Delta Q}{\Delta P} \times \dfrac{P}{Q}$.

The first term in this product, $\dfrac{\Delta Q}{\Delta P}$, is the inverse of the slope of the demand curve as it is traditionally drawn. Along a straight-line demand curve, this value is constant. However the second term, $\dfrac{P}{Q}$, clearly declines as price falls and quantity increases (moving southeast along a demand curve.) Accordingly, the elasticity of demand declines along a straight-line demand curve.

In fact, suppose demand is given by the equation $P = a - bQ$ so that a is the intercept on the price axis and b is the (absolute value of the) slope. The elasticity of demand is then $E_d = \dfrac{1}{b} \times \dfrac{P}{Q}$. Consider a segment of the demand curve centered at the midpoint of the curve. At this point, $P = a/2$ and $Q = a/2b$. Inserting these values into the formula for the elasticity gives $E_d = \dfrac{1}{b} \times \dfrac{a/2}{a/2b} = 1$. That is, demand is unit elastic at the midpoint of a straight-line demand curve. Since we know that elasticity continuously declines along the demand curve, these results also imply that demand is elastic above the midpoint and inelastic below.

[1] Along a demand curve, price and quantity move in opposite directions, so that the price change and quantity change have opposite signs. This is usually ignored, so that demand elasticities are presented as positive numbers.