

The least-cost rule

Suppose a firm wishes to produce its given output with the least-cost combination of resources. What input levels should it choose? We can devise the required conditions by the mathematical technique of minimizing cost with respect to the inputs, subject to the constraint that output is fixed at some pre-determined level, say Q^* .

The technique we will use is to form a “Lagrange” equation, where λ is an unspecified “Lagrange multiplier.” (The role of this multiplier will be apparent shortly.) We begin by noting that the firm’s cost is the sum of its expenditures on labor and capital: $C(L, K) = wL + rK$ where w and r are the (constant) prices of labor and capital, respectively. Output is a function of the two inputs as well: $Q = g(L, K)$. Now form the Lagrangian: Let $C^*(L, K, \lambda) = wL + rK + \lambda(Q^* - g(L, K))$. We wish to minimize C^* with respect to L , K and λ .

The first-order conditions for a maximum are (we will assume that the second-order conditions are fulfilled):

$$\frac{\partial C^*}{\partial L} = w - \lambda \frac{\partial g}{\partial L} = 0, \text{ or } \frac{\left(\frac{\partial g}{\partial L}\right)}{w} = \lambda$$

$$\frac{\partial C^*}{\partial K} = r - \lambda \frac{\partial g}{\partial K} = 0, \text{ or } \frac{\left(\frac{\partial g}{\partial K}\right)}{r} = \lambda$$

$$\frac{\partial C^*}{\partial \lambda} = Q^* - g(L, K) = 0, \text{ or } Q^* = g(L, K)$$

Here we see the role played by the Lagrange multiplier, λ : provided $\lambda \neq 0$, note that the last condition simply requires that our constraint on output is fulfilled.

Our interest lies in the first two conditions. Since both $\frac{\left(\frac{\partial g}{\partial L}\right)}{w}$ and $\frac{\left(\frac{\partial g}{\partial K}\right)}{r}$ must both equal λ , they must of course equal each other. This is the result as stated in the text as the **least-cost rule**: production at least cost requires the ratio of labor’s marginal product to its price equals the ratio of capital’s marginal product to its price. The amounts of labor and capital employed must be adjusted, all the while keeping output constant, until this condition is achieved.