## CHAPTER ONE WEB APPENDIX <br> Graphs and Their Meaning

## Graphs and Their Meaning

If you glance quickly through this text, you will find many graphs. These graphs are included to help you visualize and understand economic relationships. Most of our principles or models explain relationships between just two sets of economic data, which can be conveniently represented with two-dimensional graphs.

## Construction of a Graph

A graph is a visual representation of the relationship between two variables. The table in Figure 1 is a hypothetical illustration showing the relationship between income and consumption for the economy as a whole. Because people tend to buy more goods and services when their incomes go up, it is not surprising to find in the table that total consumption in the economy increases as total income increases.

The information in the table is also expressed graphically in Figure 1. Here is how it is done: We want to show visually or graphically how consumption changes as income changes. Since income is the determining factor, we follow mathematical custom and represent it on the horizontal axis of the graph. And because consumption depends on income, it is represented on the vertical axis of the graph.

The vertical and horizontal scales of the graph reflect the ranges of values of consumption and income, marked in convenient increments. As you can see, the values on the scales cover all the values in the table.

FIGURE 1
Graphing the direct relationship between consumption and income. Two sets of data that are positively or directly related, such as consumption and income, graph as an upsloping line.

| Income <br> per Week | Consumption <br> per Week | Point |
| :---: | :---: | :---: |
| $\$ 0$ | $\$ 50$ | $a$ |
| 100 | 100 | b |
| 200 | 150 | c |
| 300 | 200 | d |
| 400 | 250 | e |



Because the graph has two dimensions, each point within it represents an income value and its associated consumption value. To find a point that represents one of the five income-consumption combinations in the table, we draw lines from the appropriate values on the vertical and horizontal axes. For example, to plot point $c$ (the $\$ 200$ income- $\$ 150$ consumption point), lines are drawn up from the horizontal (income) axis at $\$ 200$ and across from the vertical (consumption) axis at $\$ 150$. These lines intersect at point $c$, which represents this particular income-consumption combination. You should verify that the other income-consumption combinations shown in the table in Figure 1 are properly located in the graph that is there.

Finally, by assuming that the same general relationship between income and consumption prevails for all other incomes, we draw a line or smooth curve to connect these points. That line or curve represents the income-consumption relationship.

If the graph is a straight line, as in Figure 1, the relationship is said to be linear.

## Direct and Inverse Relationships

The line in Figure 1 slopes upward to the right, so it depicts a direct relationship between income and consumption. A direct relationship, or positive relationship, means that two variables (here, consumption and income) change in the same direction. An increase in consumption is associated with an increase in income; a decrease in consumption accompanies a decrease in income. When two sets of data are positively or directly related, they always graph as an upsloping line, as in Figure 1.

In contrast, two sets of data may be inversely related. Consider the table in Figure 2, which shows the relationship between the price of basketball tickets and game attendance for Big Time University (BTU). Here there is an inverse relationship, or negative relationship, because the two variables change in opposite directions. When ticket prices for the games decrease, attendance increases. When ticket prices increase, attendance decreases. The six data points in the table are plotted in the graph in Figure 2.
direct relationship The (positive) relationship between two variables that change in the same direction.
inverse relationship
The (negative) relationship between two variables that change in opposite directions. This inverse relationship graphs as a downsloping line.

FIGURE 2
Graphing the inverse relationship between ticket prices and game attendance. Two sets of data that are negatively or inversely related, such as ticket price and the attendance at basketball games, graph as a downsloping line.

| Ticket <br> Price | Attendance, <br> Thousands | Point |
| :---: | :---: | :---: |
| $\$ 50$ | 0 | $a$ |
| 40 | 4 | $b$ |
| 30 | 8 | c |
| 20 | 12 | d |
| 10 | 16 | e |
| 0 | 20 | $f$ |


independent variable The variable causing a change in some other (dependent) variable; the "causal variable."
dependent variable The variable that changes as a result of a change in some other (independent) variable; the "outcome variable."

## slope (of a straight

line)
The ratio of the vertical change (the rise or fall) to the horizontal change (the run) between any two points on a line.

## Dependent and Independent Variables

Economists seek to determine which variable is the "cause" and which the "effect." Or, more formally, they seek the independent variable and the dependent variable. The independent variable is the cause or source; it is the variable that changes first. The dependent variable is the effect or outcome; it is the variable that changes because of the change in the independent variable. As in our income-consumption example, income generally is the independent variable and consumption, the dependent variable. Income causes consumption to be what it is rather than the other way around. Similarly, ticket prices (set in advance of the season and printed on the ticket) determine attendance at BTU basketball games; attendance at games does not determine the printed ticket prices for those games. Ticket price is the independent variable and the quantity of tickets purchased is the dependent variable.

Mathematicians always put the independent variable (cause) on the horizontal axis and the dependent variable (effect) on the vertical axis. Economists are less tidy; their graphing of independent and dependent variables is more arbitrary. Their conventional graphing of the income-consumption relationship is consistent with mathematical convention, but economists historically put price and cost data on the vertical axis of their graphs. Contemporary economists have followed the tradition. So economists' graphing of BTU's ticket price-attendance data differs from normal mathematical procedure. This does not present a problem, but we want you to be aware of this fact to avoid any possible confusion.

## Other Things Equal

Our simple two-variable graphs purposely ignore many other factors that might affect the amount of consumption occurring at each income level or the number of people who attend BTU basketball games at each possible ticket price. When economists plot the relationship between any two variables, they employ the ceteris paribus (other-things-equal) assumption. Thus, in Figure 1 all factors other than income that might affect the amount of consumption are presumed to be constant or unchanged. Similarly, in Figure 2 all factors other than ticket price that might influence attendance at BTU basketball games are assumed constant. In reality, "other things" are not equal; they often change, and when they do, the relationship represented in our two tables and graphs will change. Specifically, the lines we have plotted would shift to new locations.

Consider a stock market "crash." The dramatic drop in the value of stocks might cause people to feel less wealthy and therefore less willing to consume at each level of income. The result might be a downward shift of the consumption line. To see this, you should plot a new consumption line in Figure 1, assuming that consumption is, say, $\$ 20$ less at each income level. Note that the relationship remains direct; the line merely shifts downward to reflect less consumption spending at each income level.

Similarly, factors other than ticket prices might affect BTU game attendance. If BTU loses most of its games, attendance at BTU games might be less at each ticket price. To see this, redraw Figure 2, assuming that 2000 fewer fans attend BTU games at each ticket price.

## Slope of a Line

Lines can be described in terms of their slopes. The slope of a straight line is the ratio of the vertical change (the rise or drop) to the horizontal change (the run) between any two points of the line.

Positive Slope Between point $b$ and point $c$ in the graph in Figure 1, the rise or vertical change (the change in consumption) is $+\$ 50$ and the run or horizontal change (the change in income) is $+\$ 100$. Therefore:

$$
\text { Slope }=\frac{\text { vertical change }}{\text { horizontal change }}=\frac{+50}{+100}=\frac{1}{2}=.5
$$

Note that our slope of $\frac{1}{2}$ or .5 is positive because consumption and income change in the same direction; that is, consumption and income are directly or positively related.

Negative Slope Between any two of the identified points in the graph of Figure 2, say, point $c$ and point $d$, the vertical change is -10 (the drop) and the horizontal change is +4 (the run). Therefore:

$$
\text { Slope }=\frac{\text { vertical change }}{\text { horizontal change }}=\frac{-10}{+4}=-2 \frac{1}{2}=-2.5
$$

This slope is negative because ticket price and attendance have an inverse relationship.

Slopes and Marginal Analysis Economists are largely concerned with changes in values. The concept of slope is important in economics because it reflects marginal changes-those involving 1 more (or 1 fewer) unit. For example, in Figure 1 the .5 slope shows that $\$ .50$ of extra or marginal consumption is associated with each $\$ 1$ change in income. In this example, people collectively will consume $\$ .50$ of any $\$ 1$ increase in their incomes and reduce their consumption by $\$ .50$ for each $\$ 1$ decline in income. Careful inspection of Figure 2 reveals that every $\$ 1$ increase in ticket price for BTU games will decrease game attendance by 400 people and every $\$ 1$ decrease in ticket price will increase game attendance by 400 people.

Infinite and Zero Slopes Many variables are unrelated or independent of one another. For example, the quantity of wristwatches purchased is not related to the price of bananas. In Figure 3a the price of bananas is measured on the vertical axis and the quantity of watches demanded on the horizontal axis. The graph of their relationship is the line parallel to the vertical axis, indicating that the same quantity of watches is purchased no matter what the price of bananas. The slope of such a line is infinite.

Similarly, aggregate consumption is completely unrelated to the nation's divorce rate. In Figure 3b we put consumption on the vertical axis and the divorce rate on the horizontal axis. The line parallel to the horizontal axis represents this lack of relatedness. This line has a slope of zero.

Slope of a Nonlinear Curve We now move from the simple world of linear relationships (straight lines) to the somewhat more complex world of nonlinear relationships. The slope of a straight line is the same at all its points. The slope of a line representing a nonlinear relationship changes from one point to another. Such lines are always referred to as curves.

Consider the downsloping curve in Figure 4. Its slope is negative throughout, but the curve flattens as we move down along it. Thus, its slope constantly changes; the curve has a different slope at each point.

## FIGURE 3

Infinite and zero slopes. (a) A line parallel to the vertical axis has an infinite slope. Here, purchases of watches remain the same no matter what happens to the price of bananas. (b) A line parallel to the horizontal axis has a slope of zero. In this case, total consumption remains the same no matter what happens to the divorce rate. In both (a) and (b), the two variables are totally unrelated to one another.


To measure the slope at a specific point, we draw a straight line tangent to the curve at that point. A line is tangent at a point if it touches, but does not intersect, the curve at that point. So line $a a$ is tangent to the curve in Figure 4 at point $A$. The slope of the curve at that point is equal to the slope of the tangent line. Specifically, the total vertical change (drop) in the tangent line $a a$ is -20 and the total horizontal change (run) is +5 . Because the slope of the tangent line $a a$ is $-20 /+5$, or -4 , the slope of the curve at point $A$ is also -4.

Line $b b$ in Figure 4 is tangent to the curve at point $B$. Using the same procedure, we find the slope at $B$ to be $-5 /+15$, or $-\frac{1}{3}$. Thus, in this flatter part of the curve, the slope is less negative.

Several of the Appendix Problems are of a "workbook" variety, and we urge you to go through them carefully to check your understanding of graphs and slopes.

FIGURE 4
Determining the slopes of curves. The slope of a nonlinear curve changes from point to point on the curve. The slope at any point (say, B) can be determined by drawing a straight line that is tangent to that point (line bb) and calculating the slope of that line.


## Appendix Summary

1. Graphs are a convenient and revealing way to represent economic relationships.
2. Two variables are positively or directly related when their values change in the same direction. The line (curve) representing two directly related variables slopes upward.
3. Two variables are negatively or inversely related when their values change in opposite directions. The curve representing two inversely related variables slopes downward.
4. The value of the dependent variable (the "effect") is determined by the value of the independent variable (the "cause").

## Appendix Terms and Concepts

direct relationship
inverse relationship
independent variable

## Appendix Questions <br> 

1. Briefly explain the use of graphs as a way to represent economic relationships. What is an inverse relationship? How does it graph? What is a direct relationship? How does it graph? LO6
2. Describe the graphical relationship between ticket prices and the number of people choosing to visit amusement parks. Is that relationship consistent with the fact that, historically, park attendance and ticket prices have both risen? Explain. LO6

## Appendix Problems

1. Graph and label as either direct or indirect the relationships you would expect to find between (a) the number of inches of rainfall per month and the sale of umbrellas, (b) the amount of tuition and the level of enrollment at a university, and (c) the popularity of an entertainer and the price of her concert tickets. LO6
2. Indicate how each of the following might affect the data shown in the table and graph in Figure 2 of this appendix: LO6
a. BTU's athletic director hires away the coach from a perennial champion.
b. An NBA team locates in the city where BTU plays.
c. BTU contracts to have all its home games televised.
3. The following table contains data on the relationship between saving and income. Rearrange these data into a
4. When the "other factors" that might affect a two-variable relationship are allowed to change, the graph of the relationship will likely shift to a new location.
5. The slope of a straight line is the ratio of the vertical change to the horizontal change between any two points. The slope of an upsloping line is positive; the slope of a downsloping line is negative.
6. The slope of a line or curve is especially relevant for economics because it measures marginal changes.
7. The slope of a horizontal line is zero; the slope of a vertical line is infinite.
8. The slope of a curve at any point is determined by calculating the slope of a straight line tangent to the curve at that point.

## dependent variable

slope of a straight line
3. Look back at Figure 2, which shows the inverse relationship between ticket prices and game attendance at Big Time University. (a) Interpret the meaning of the slope. (b) If the slope of the line were steeper, what would that say about the amount by which ticket sales respond to increases in ticket prices? LO6
meaningful order and graph them on the accompanying grid. What is the slope of the line? What would you predict saving to be at the $\$ 12,500$ level of income? LO6

| Income per Year | Saving per Year |
| :---: | :---: |
| $\$ 15,000$ | $\$ 1,000$ |
| 0 | -500 |
| 10,000 | 500 |
| 5,000 | 0 |
| 20,000 | 1,500 |


4. Construct a table from the data shown on the graph below. Which is the dependent variable and which the independent variable?

LO6

5. Suppose that when the price of gold is $\$ 100$ an ounce, gold producers find it unprofitable to sell gold. However, when the price is $\$ 200$ an ounce, 5000 ounces of output (production) is profitable. At $\$ 300$, a total of 10,000 ounces of output is profitable. Similarly, total production increases by 5000 ounces for each successive $\$ 100$ increase in the price of gold. Describe the relevant relationship between the price of gold and the production of
gold in a table and on a graph. Put the price of gold on the vertical axis and the output of gold on the horizontal axis. LO6
6. The accompanying graph shows curve $X X^{\prime}$ and tangents to the curve at points $A, B$, and $C$. Calculate the slope of the curve at each of these three points. LO6

7. In the accompanying graph, is the slope of curve $A A^{\prime}$ positive or negative? Does the slope increase or decrease as we move along the curve from $A$ to $A^{\prime}$ ? Answer the same two questions for curve $B B^{\prime}$. LO6


