Chapter 10

ELASTICITY AND OSCILLATIONS

Problems

1. Strategy The stress is proportional to the strain. Use Eq. (10-4).

Solution Find the vertical compression of the beam.

$$Y\frac{\Delta L}{L}=\frac{F}{A},\ \text{so}\ \Delta L=\frac{FL}{YA}=\frac{(5.8\times 10^{4}\ \text{N})(2.5\ \text{m})}{(200\times 10^{9}\ \text{Pa})(7.5\times 10^{-3}\ \text{m}^2)}=\boxed{0.097\ \text{mm}}.$$

9. Strategy Refer to Fig. 10.4c. The stress is proportional to the strain.

Solution Calculate Young's moduli for tension and compression of bone.
Tension:
For tensile stress and strain, the graph is far from being linear, but for relatively small values of stress and strain, it is approximately linear. So, for small values of tensile stress and strain, Young's Modulus is

$$Y=\frac{\text{stress}}{\text{strain}}=\frac{5.0\times 10^{7}\ \text{N/m}^2}{0.0033}=\boxed{1.5\times 10^{10}\ \text{N/m}^2}.$$

Compression:

Similarly, for small values of compressive stress and strain, $Y=\dfrac{-4.5\times 10^{7}\ \text{N/m}^2}{-0.0050}=\boxed{9.0\times 10^{9}\ \text{N/m}^2}$.

21. Strategy Use Hooke's law for volume deformations.

Solution Find the change in volume of the sphere.

$$\Delta P=-B\frac{\Delta V}{V},\ \text{so}\ \Delta V=-\frac{V\Delta P}{B}=-\frac{(1.00\ \text{cm}^3)(9.12\times 10^{6}\ \text{Pa})}{160\times 10^{9}\ \text{Pa}}=-57\times 10^{-6}\ \text{cm}^3.$$

$\boxed{\text{The volume of the steel sphere would decrease by } 57\times 10^{-6}\ \text{cm}^3.}$

35. Strategy The angular frequency of oscillation is inversely proportional to the square root of the mass. Form a proportion.

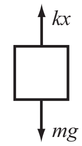
Solution Find the new value of ω.

$$\omega\propto\sqrt{\frac{1}{m}},\ \text{so}\ \frac{\omega_\text{f}}{\omega_\text{i}}=\frac{\sqrt{1/m_\text{f}}}{\sqrt{1/m_\text{i}}}=\sqrt{\frac{m_\text{i}}{m_\text{f}}}=\sqrt{\frac{1}{4.0}}=\frac{1}{2.0}.\ \text{Therefore,}\ \omega_\text{f}=\frac{\omega_\text{i}}{2.0}=\frac{10.0\ \text{rad/s}}{2.0}=\boxed{5.0\ \text{rad/s}}.$$

43. (a) Strategy The speed is maximum when the spring and mass system is at its equilibrium point. Use Newton's second law.

Solution Find the extension of the spring.

$$\Sigma F_y = kx - mg = 0,\ \text{so}\ x = \frac{mg}{k} = \frac{(0.60\ \text{kg})(9.80\ \text{N/kg})}{15\ \text{N/m}} = \boxed{0.39\ \text{m}}.$$



(b) Strategy Use Eqs. (10-20a) and (10-21).

Solution Find the maximum speed of the body.

$$v_\text{m} = \omega A = \sqrt{\frac{k}{m}}x = \sqrt{\frac{15\ \text{N/m}}{0.60\ \text{kg}}}(0.39\ \text{m}) = \boxed{2.0\ \text{m/s}}$$

53. (a) Strategy Use Eq. (10-20a) to find the spring constant. Then, find the elastic potential energy using $\frac{1}{2}kx^2$.

Solution Find the spring constant.

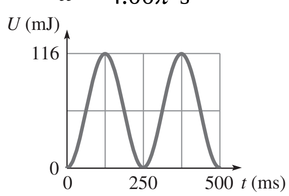
$$\omega = \sqrt{\frac{k}{m}},\ \text{so}\ k = \omega^2 m = (2.00\ \text{Hz})^2(2\pi\ \text{rad/cycle})^2(0.2300\ \text{kg}) = 36.3\ \text{N/m}.$$

The equation for the elastic potential energy is

$$U(t) = \frac{1}{2}(36.3\ \text{N/m})(0.0800\ \text{m})^2\sin^2[(2.00\ \text{Hz})(2\pi\ \text{rad/cycle})t] = (116\ \text{mJ})\sin^2\left[(4.00\pi\ \text{s}^{-1})t\right].$$

Since the sine function is squared, the period of $U(t)$ is half that of a sine function or

$$T = \frac{\pi}{\omega} = \frac{\pi}{4.00\pi\ \text{s}^{-1}} = 250\ \text{ms}.\ \text{Graph}\ U(t).$$



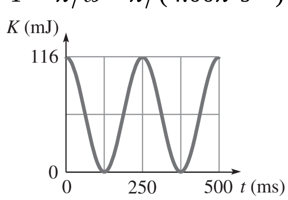
(b) Strategy Find the kinetic energy using $\frac{1}{2}mv_x^2$.

Solution The equation for the kinetic energy is

$$K(t) = \frac{1}{2}(0.2300\ \text{kg})(2.00\ \text{Hz})^2\left(\frac{2\pi\ \text{rad}}{\text{cycle}}\right)^2(0.0800\ \text{m})^2\cos^2\left[(2.00\ \text{Hz})\left(\frac{2\pi\ \text{rad}}{\text{cycle}}\right)t\right]$$
$$= (116\ \text{mJ})\cos^2\left[(4.00\pi\ \text{s}^{-1})t\right].$$

Since the cosine function is squared, the period of $K(t)$ is half that of a cosine function or

$T = \pi/\omega = \pi/(4.00\pi\ \text{s}^{-1}) = 250\ \text{ms},$ which is the same as $U(t)$. Graph $K(t)$.

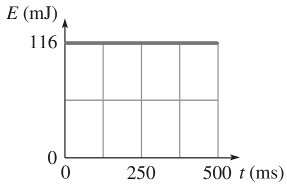


(c) Strategy Add $U(t)$ and $K(t)$ and graph the result.

Solution

$$\begin{aligned}E(t) &= U(t)+K(t) = (116\text{ mJ})\sin^2\left[(4.00\pi\text{ s}^{-1})t\right]+(116\text{ mJ})\cos^2\left[(4.00\pi\text{ s}^{-1})t\right]\\ &= (116\text{ mJ})\left\{\sin^2\left[(4.00\pi\text{ s}^{-1})t\right]+\cos^2\left[(4.00\pi\text{ s}^{-1})t\right]\right\} = (116\text{ mJ})(1) = 116\text{ mJ}\end{aligned}$$

Graph $E(t) = U(t)+K(t)$.



(d) Strategy and Solution Friction does nonconservative work on the object, thus, $\boxed{U,\ K,\text{ and } E \text{ would gradually be reduced to zero}}$.

69. Strategy $E = \frac{1}{2}m\omega^2A^2 \propto A^2$ for a pendulum.

Solution Find the percent decrease of the oscillator's energy in ten cycles.

$$\frac{\Delta E}{E}\times 100\% = \frac{\Delta A^2}{A^2}\times 100\% = \frac{(1-0.0500)^2-1^2}{1^2}\times 100\% = \frac{0.9500^2-1^2}{1^2}\times 100\% = \boxed{-9.75\%}$$

85. Strategy Use Eq. (10-2) to find the tensile stress. Then compare the tensile stress to the elastic limit of steel piano wire.

Solution Find the tensile stress in the piano wire in Problem 90.

$$\text{tensile stress} = \frac{F}{A} = \frac{T}{\frac{1}{4}\pi d^2} = \frac{4T}{\pi d^2} = \frac{4(402\text{ N})}{\pi(0.80\times 10^{-3}\text{ m})^2} = 8.0\times 10^8\text{ Pa} < 8.26\times 10^8\text{ Pa}$$

The tensile stress is $\boxed{8.0\times 10^8\text{ Pa; it is just under the elastic limit.}}$