CHAPTER 8

Four principles can be used along with Table 8.1 to find the rotational inertias for rigid bodies about various axes: the sum and stretch rules and parallel- and perpendicular-axis theorems.

Sum Rule

Rotational inertia is defined as a sum [Eq. (8-2)]. Therefore, an object can be broken into parts and the rotational inertia of the object is the sum of the rotational inertias of the parts.

Example 8.16

Rotational Inertia of a Rod About Its Midpoint

Table 8.1 gives the rotational inertia of a thin rod with the axis of rotation perpendicular to its length and passing through one end ($I = \frac{1}{3}ML^2$). From this expression, derive the rotational inertia of a rod with mass M and length L that rotates about a perpendicular axis through its *midpoint* (Fig. 8.43).

Strategy In general, the same object rotating about a different axis has a different rotational inertia. With the axis at the midpoint, the rotational inertia is smaller than for the axis at the end, since the mass is closer to the axis, on average. Imagine performing the sum $\sum_{i=1}^{N} m_i r_i^2$; for the axis at the end, the values of r_i range from 0 to L, whereas with the axis at the midpoint, r_i is never larger than $\frac{1}{2}L$.

Imagine cutting the rod in half; then there are two rods, each with its axis of rotation at one of its *ends*. Then, since rotational inertia is additive, the rotational inertia for two such rods is just twice the value for one rod.

Solution Each of the halves has mass $\frac{1}{2}M$ and length $\frac{1}{2}L$ and rotates about an axis at its endpoint. Table 8.1 gives $I = \frac{1}{3}ML^2$ for a rod with mass M and length L rotating about its end, so each of the halves has

$$I_{\text{half}} = \tfrac{1}{3} \times \text{mass of half} \times (\text{length of half})^2$$
$$= \tfrac{1}{3} \times (\tfrac{1}{2}M) \times (\tfrac{1}{2}L)^2 = \tfrac{1}{3} \times \tfrac{1}{2} \times \tfrac{1}{4} \times ML^2 = \tfrac{1}{24}ML^2$$

Since there are two such halves, the total rotational inertia is twice that:

$$I = I_{\text{half}} + I_{\text{half}} = \tfrac{1}{12}ML^2$$

Discussion The rotational inertia is less than $\frac{1}{3}ML^2$, as expected. That it is $\frac{1}{4}$ as much is a result of the distances r_i being *squared* in the definition of rotational inertia. The various particles that compose the rod are at distances that range from 0 to $\frac{1}{2}L$ from the rotation axis, instead of from 0 to L. Think of it as if the rod were compressed to half its length, still pivoting about the endpoint. All the distances r_i are half as much as before; since each r_i is squared in the sum, the rotational inertia is $(\frac{1}{2})^2 = \frac{1}{4}$ its former value.

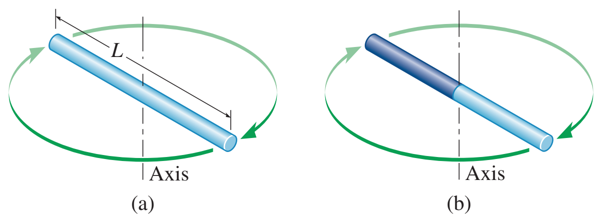

Figure 8.43 (a) A rod rotating about a vertical axis through its center. (b) The same rod, viewed as two rods, each half as long, rotating about an axis through an end.

Stretch Rule

The rotational inertia of a rigid object is not changed if the object is stretched or compressed in a direction parallel to the rotation axis. This rule follows directly from the definition of rotational inertia [Eq. (8-2)] because moving a bit of mass parallel to the axis does not change its distance r_i from the rotation axis.

For example, you might want to calculate the rotational inertia of a door about the axis through its hinges. Mentally "compress" the door vertically (parallel to the axis) into a horizontal rod with the same mass, as in Fig. 8.44. The rotational inertia is unchanged by the compression, since every particle maintains the same distance from the axis of rotation. Thus, the formula for the rotational inertia of a rod (listed in Table 8.1) can be used for the door.

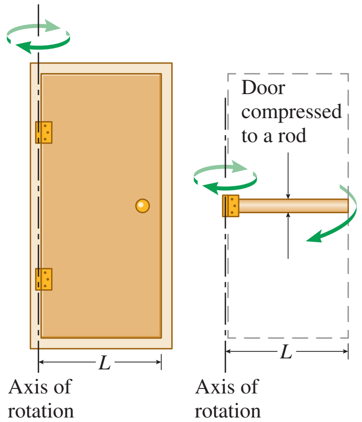

Figure 8.44 The rotational inertias of a door and a rod are both given by $\frac{1}{3}ML^2$.

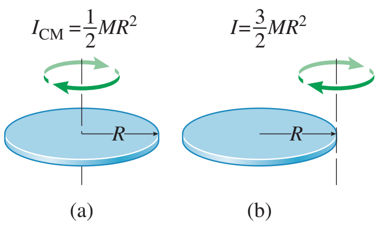

Figure 8.45 From Table 8.1, the rotational inertia of a disk about an axis through the center of mass and perpendicular to the disk is $I_{CM} = \frac{1}{2}MR^2$. Then, using the parallel-axis theorem, the rotational inertia of the disk about an axis through the edge and perpendicular to the disk is $I = I_{CM} + Md^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$.

Parallel Axis Theorem

Suppose the rotational inertia of an object about an axis that passes through the center of mass is I_{CM}. Then the rotational inertia I about any axis *parallel* to that axis is

$$I = I_{CM} + Md^2 \tag{8-16}$$

where M the object's mass and d is the perpendicular distance between the two axes (Fig. 8.45).

Perpendicular Axis Theorem

Suppose a flat rigid object lies entirely within the xy-plane. Define three mutually-perpendicular rotation axes that pass through a single point in the xy-plane, one parallel to each of the x-, y-, and z-axes (Fig. 8.46). Call the rotational inertias of the object about these axes I_x, I_y, and I_z. Then

$$I_z = I_x + I_y \tag{8-17}$$

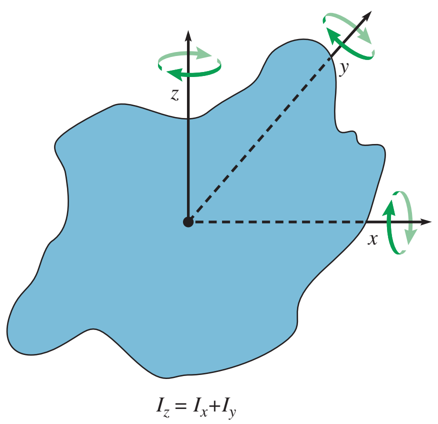

Figure 8.46 The perpendicular-axis theorem relates the rotational inertias of a flat object about three mutually perpendicular axes that all pass through the same point in the plane of the object and one of which is perpendicular to the plane of the object.