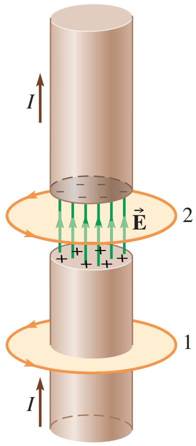

Figure 22.34 A long cylindrical wire carrying a constant current I has a small gap. The faces of the gap act as capacitor plates and charge accumulates on them. Two circular loops, one around the wire and one around the gap, are used in Ampère's law. No current cuts through the interior of loop 2, but electric field lines in the gap do cut through it. As the electric field in the gap increases in magnitude, the electric flux through the interior of loop 2 increases.

CHAPTER 22

The Ampère-Maxwell Law

Here is an example to show that a changing electric field *must* give rise to a magnetic field. Imagine a long straight wire of radius R carrying a constant current I. At one point, the wire has a tiny gap (Fig. 22.34). The surfaces of the gap act as a capacitor; as current flows, the lower surface accumulates positive charge at a rate $\Delta q/\Delta t = I$ and the upper surface accumulates negative charge at the same rate. Ampère's law says that the circulation of $\vec{\mathbf{B}}$ around a loop must equal μ_0 times the current that crosses the interior of the loop. Applying Ampère's law to circular loop 1 gives the usual result for the magnetic field near a long, straight wire. However, the interior of circular loop 2 has *no current cutting through it.* Could the magnetic field at points just outside the gap be zero, no matter how tiny the gap?

Maxwell recognized that, although no current cuts through the interior of loop 2, there is a *changing electric flux* through it. The surface cuts through the electric field lines between the capacitor plates; as more and more charge accumulates on the plates, the field is getting stronger, and therefore the electric flux is increasing. The electric field in the gap is

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} = \frac{q}{\epsilon_0 \pi R^2} \qquad \text{(17-13)}$$

The rate of change of the electric flux is

$$\frac{\Delta \Phi_E}{\Delta t} = \frac{\Delta E \times \pi R^2}{\Delta t} = \frac{\Delta q}{\epsilon_0 \pi R^2} \times \frac{\pi R^2}{\Delta t} = \frac{I}{\epsilon_0}$$

The rate of change of the flux is proportional to the current! Maxwell recognized that the contradiction is resolved if Ampère's law is modified as

$$\sum B_\parallel \, \Delta l = \mu_0 \left(I + \epsilon_0 \frac{\Delta \Phi_E}{\Delta t} \right) \qquad \text{(22-2)}$$

Using this modified form of Ampère's law, the magnetic field at a point on loop 2 is the same as the magnetic field at the corresponding point on loop 1.