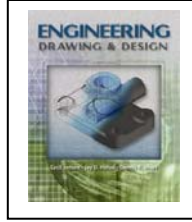


# Chapter 28

## Applied Mechanics



### UNIT 28-1

#### Forces

Weight (mass) and density have many applications in industry and construction. A few examples are

- Defining quantities of materials, such as bags of mortar and tons of steel.
- Defining physical characteristics of materials like steel beams (W24 x 94), 210-lb asphalt shingles, and 3.5-lb density board.
- Defining load capacity for supports, bridges, cranes, and elevators.

A *force* is that which changes, or tends to change, the state of rest or uniform motion of a body.

#### U.S. Customary Units

In the U.S. Customary System, the force values are given in pounds, kips, and tons. The methods for solving problems in either metric or U.S. Customary units are identical.

#### Metric Units

The term mass (not weight) is used to refer to the quantity of matter in an object (rather than to the force of gravity acting on it). Mass is always measured in terms of the kilogram, gram, or some related unit; that is, a multiple or submultiple of the gram. Whenever a quantity is specified in such a unit, mass, not force, is the quantity under consideration.

The force of gravity, pulling downwards, acts on each mass on the Earth. This "mass" and the "force of gravity," while having entirely different characteristics, are linked. The word "weight" in the metric system is avoided, since it is ambiguous; it sometimes means "mass" and sometimes "force of gravity," depending on the context.

Force is measured in newtons. The preferred units are newtons (N) for small forces, kilonewtons (kN) for intermediate forces and meganewtons (MN) for large forces. One newton is approximately one-quarter of a one-pound force.

Using a spring scale, it would take about 10 N to lift a 1-kg stone, perhaps 4 N to pull it along the floor, and maybe 6.5 N to pull it up a ramp.

In this unit, the force of gravity acting on the part, rather than the mass of the part is given in the examples and problems.

## GRAPHIC REPRESENTATION OF FORCES

The graphical method of solving mechanical problems involving forces is often used because it is quick and accurate. The force is shown graphically. To describe completely the force, the following particulars must be given:

1. Its magnitude
2. Its point of application
3. Its direction
4. Its sense, i.e., whether it is pushing or pulling

A line is drawn to a given length to represent the magnitude of the force. The direction of this line is parallel to the direction of the force. The sense of the force is indicated by an arrow on the line indicating whether it is acting toward or away from the point of application. The graphical representation of the force is called a *vector*.

Thus a pull of 6 tons (T) acting at a point A at  $45^\circ$  to the horizontal would be represented by the vector  $AB$ , as shown in Fig. 28-1-1. Using the scale .25 in. = 1 T, the length of the vector would be 1.50 in.

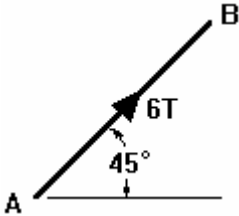


Fig. 28-1-1 A vector.

A body is said to be in equilibrium if the forces acting at a point balance one another. If two equal and opposite forces act at a point in a straight line, the body is in equilibrium. Examples are tie bars, which are bars under pull or tension, and struts or columns, which are bars under push or compression (Fig. 28-1-2).

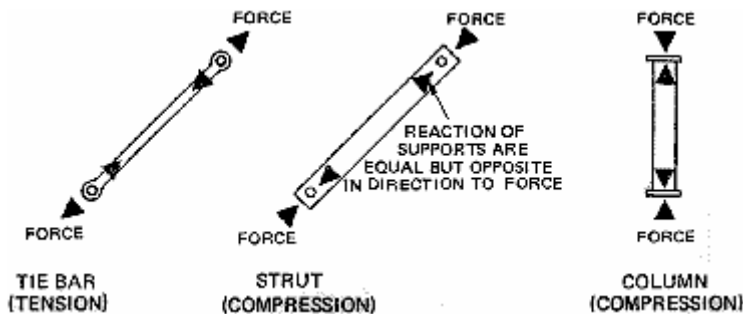


Fig. 28-1-2 Types of forces acting on supports.

## TWO FORCES ACTING AT A POINT

Two or more forces acting at a point may be replaced by one force that will produce the same effect. This force is called the *resultant* of the forces. If two opposite forces of 8 and 5 T act at a point  $O$  in a straight line, as in Fig. 28-1-3, a resultant force of 3 T acting in the same direction as the 8 T force could replace the two original forces.

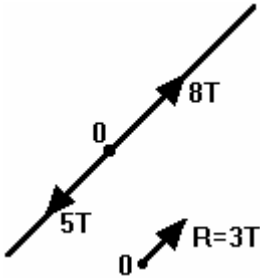


Fig. 28-1-3 Resultant when two forces act in a straight line.

If two opposite forces  $F_1$  and  $F_2$  act at point  $O$  at angles of  $120^\circ$  to each other, as in Fig. 28-1-4, the resultant force  $R$  may be found by drawing the two forces to scale and completing the parallelogram. The diagonal  $OC$  would be the resultant, and the magnitude of the force could be measured. This method is called the *parallelogram of forces*. Accuracy of direction and distance is important in laying out forces.

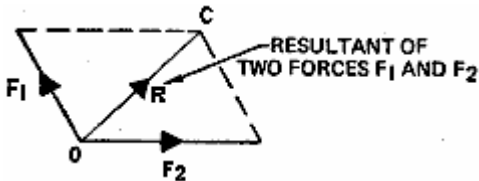
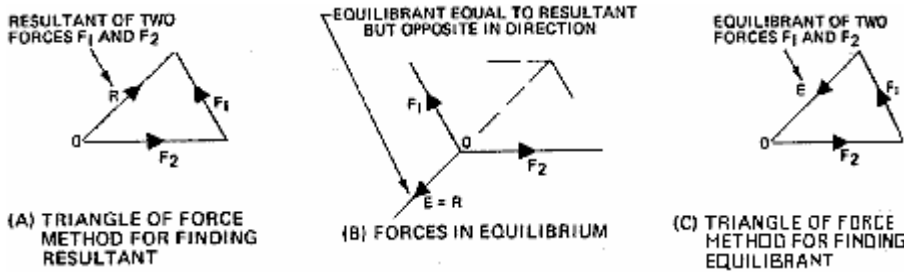


Fig. 28-1-4 Parallelogram of forces.

Another way of finding the resultant is the *triangle of force* method. The known force vectors are laid end to end with the forces traveling in the same direction. The resultant  $R$  is found by joining the beginning of the first vector to the end of the last vector, as shown in Fig. 28-1-5A, and the direction of the resultant force is in the combined direction of the other two forces.

If a force equal to the resultant of forces  $F_1$  and  $F_2$ , but acting in the opposite direction, was to act at  $O$ , as shown in Fig. 28-1-5B, the object would be in equilibrium, since the forces acting at point  $O$  tend to balance one another. This force balancing the other forces is known as the *equilibrant*.



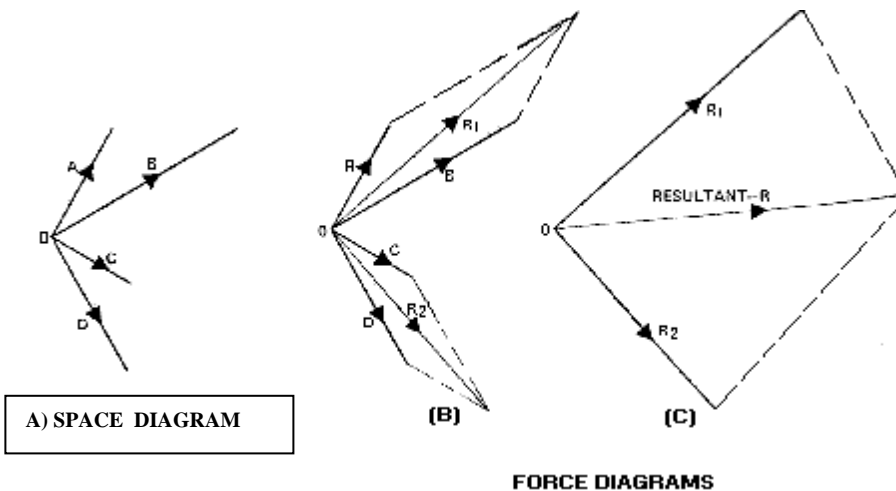
**Fig. 28-1-5** Resultant and equilibrant of two forces acting at a point using the triangle of force method.

The equilibrant is found in a similar manner to the resultant, by using the triangle of force method. Note that the arrows representing the direction of the forces are pointing the same way around the triangle.

### MORE THAN TWO FORCES ACTING AT A POINT

Resultants or equilibrants may be found for any number of forces acting at a point and in one plane. Let  $A$ ,  $B$ ,  $C$ , and  $D$  represent forces acting at a point  $O$ , shown in Fig. 28-1-6A.

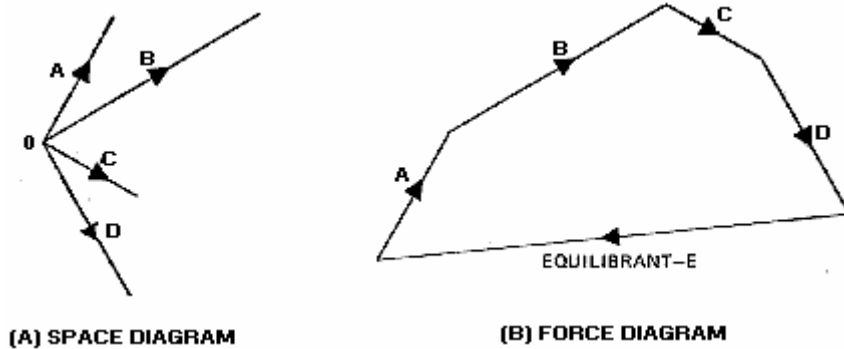
Using the parallelogram of forces method shown in Fig. 28-1-6B, we find the resultant  $R_1$  for forces  $A$  and  $B$  and resultant  $R_2$  for forces  $C$  and  $D$ . Using resultants  $R_1$  and  $R_2$  in Fig. 28-1-6C instead of the forces  $A$ ,  $B$ ,  $C$ , and  $D$ , we find the resultant  $R$  of the four forces. The equilibrant or force required to keep the forces  $A$ ,  $B$ ,  $C$ , and  $D$  in equilibrium would be equal to  $R$  but would act in the opposite direction.



**Fig. 28-1-6** Parallelogram of forces method of finding resultant of  $R$  for more than two forces acting at a point.

## POLYGON OF FORCES

Using the polygon of forces method shown in Fig. 28-1-7, which is the extension of the triangle method, join the forces  $A$ ,  $B$ ,  $C$ , and  $D$  end to end to form a polygon. Be careful to keep the arrows pointing the same way around. The line joining the beginning of the first force and the end of the last force is the equilibrant.

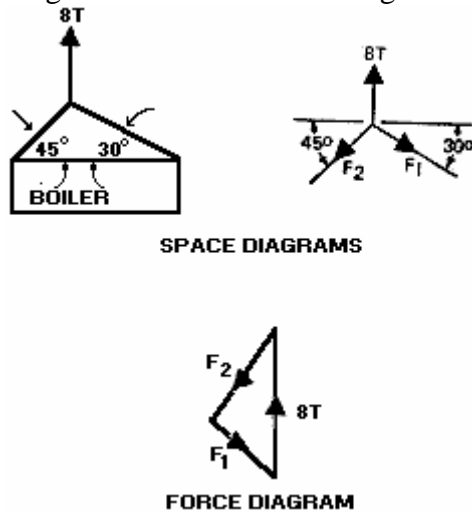


**Fig. 28-1-7** Polygon of forces method for finding the equilibrant for more than two forces acting at a point.

The following examples illustrate how vector diagrams are applied to practical problems.

**EXAMPLE 1** A crane lifts a steel boiler by means of a chain sling. The sling makes angles of  $30^\circ$  and  $45^\circ$  with the boiler. Find the forces acting on the sling if the weight of the boiler is 8 tons.

**Solution** The force on the crane's chain supporting the sling is equal to the force created by the weight of the boiler. The angles on the sling indicate the direction of the sling forces  $F_1$  and  $F_2$ , as shown in the space diagram in Fig. 28-1-8. The force diagram is then drawn to a suitable scale and the lengths of lines  $F_1$  and  $F_2$  are measured to find the magnitude of the forces acting on the sling.



**Fig.28-1-8** Solution to Example I by the force diagram method.

**EXAMPLE 2** In Fig. 28-1-9, a simple wall crane has a 400-lb load applied at the end of the jib. Neglecting the weight of the crane parts, calculate the forces acting on the tie and jib.

**Solution** A space diagram is drawn first to find the direction of the force acting on the tie. With the direction of the three forces and the magnitude of one force  $W$  known, a force diagram is then drawn to a suitable scale. The length of lines  $F_1$  and  $F_2$  can now be measured to find the magnitude of the forces acting on the jib and tie.

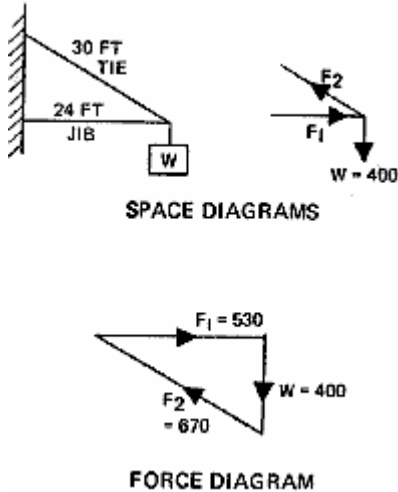


Fig. 28-1-9 Solution to Example 2 by the force diagram method.

**EXAMPLE 3** In Fig. 28-1-10, a machine having a mass of 12 tonnes (t) is lifted by a jib crane. The length of the jib is 5500 mm and an 8500-mm tie is fastened to a point 4300 mm behind the base of the jib. The chain that lifts the machine passes over a pulley at the top of the jib and connects to a winch located 1800 mm behind the base of the jib. Find the forces acting on the jib and tie.

**Solution** A 1-t mass creates a gravitational force of 9806.65 N. The 12-t mass exerts a force of  $12 \times 9806.65 \text{ N} = 117\,700 \text{ N}$  or 117.7 kN on the chain. A space diagram is drawn first to find the direction of the forces acting on the tie, jib, and chain connected to the winch. With the direction of the four forces and the magnitude of two forces known, a force diagram is then drawn to a suitable scale. The length of lines  $F_1$  and  $F_2$  can now be measured to find the magnitude of the forces acting on the jib and tie.

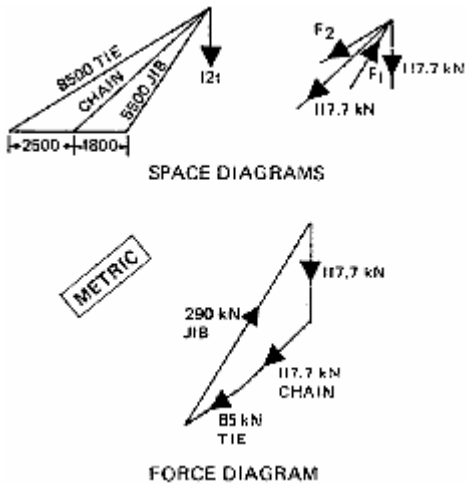


Fig.28-1-10 Solution to Example 3 by the force diagram method.

**EXAMPLE 4** In Fig. 28-1-11, a simple roof truss has a force  $W$  applied at the top. Find the forces acting on the rafters, tie bar, and walls.

**Solution** A space diagram is drawn first to find the direction of the force acting on the rafters. A force diagram taken at point  $A$  is drawn as the rafters  $AB$  and  $AC$  support the load  $W$ . The forces acting on rafters  $AB$  and  $AC$  may be found by making a force diagram taken at point  $A$ , drawing  $W$  to scale, and measuring the values of  $AB$  and  $AC$ .

Since the force acting on the wall is equal to  $W/2$  (the roof design and load being symmetrical) and the forces acting at points  $B$  and  $C$  being equal, only one force diagram needs be drawn to find the force acting on the tie.

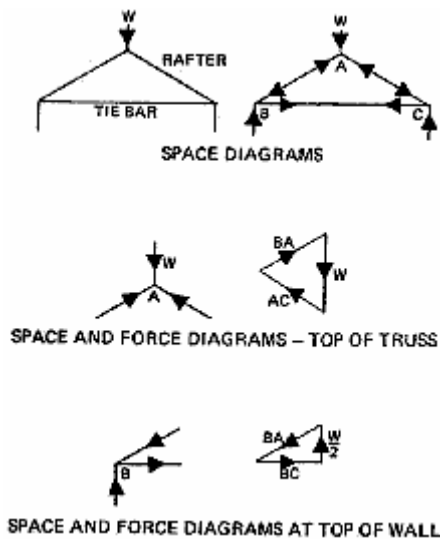
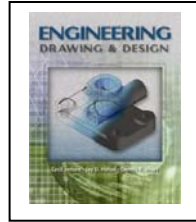


Fig.28-1-11 Solution to Example 4 by the force diagram method.

# Chapter 28 Applied Mechanics



## UNIT 28-2 Beams

### BOW'S NOTATION

In the previous illustrations, the forces have been identified as  $F_1$ ,  $F_2$ ,  $R$ , etc. Another system of identifying forces, called *Bow's notation*, is helpful in solving force problems. In the space diagram (Fig. 28-2-1), a boldface capital letter, **A**, **B**, **C**, etc., is placed in the space between two forces and the force is referred to by the two boldface capital letters in the adjoining spaces. The force **AB** in the space diagram is represented by the vector  $ab$  in the force diagram, the letters  $a$  and  $b$  being placed at the beginning and end, respectively, of the vector. The letters in the space diagram are usually given in alphabetical order and in a clockwise direction.

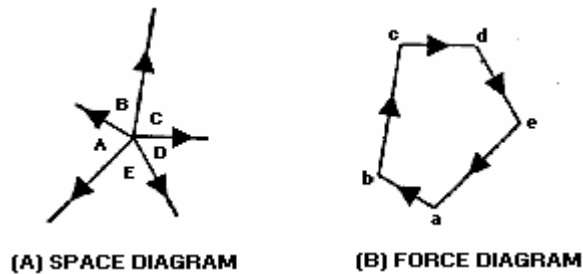


Fig. 28-2-1 Bow's notation.

### EQUILIBRIUM POLYGON

*Equilibrium* or *funicular polygons* are used in the graphical solutions for finding the magnitude, direction, and point of application of resultants, equilibrants, and reactions. They are also used to check whether or not a number of forces are in equilibrium.

### Graphic Method of Finding Resultant and Reactions of Vertical Forces Acting on a Beam

**EXAMPLE 1** A number of parallel forces are acting on a beam as shown in Fig. 28-2-2A. It is required to find, graphically, the magnitude and point of application of the resultant and the magnitude of the reactions.



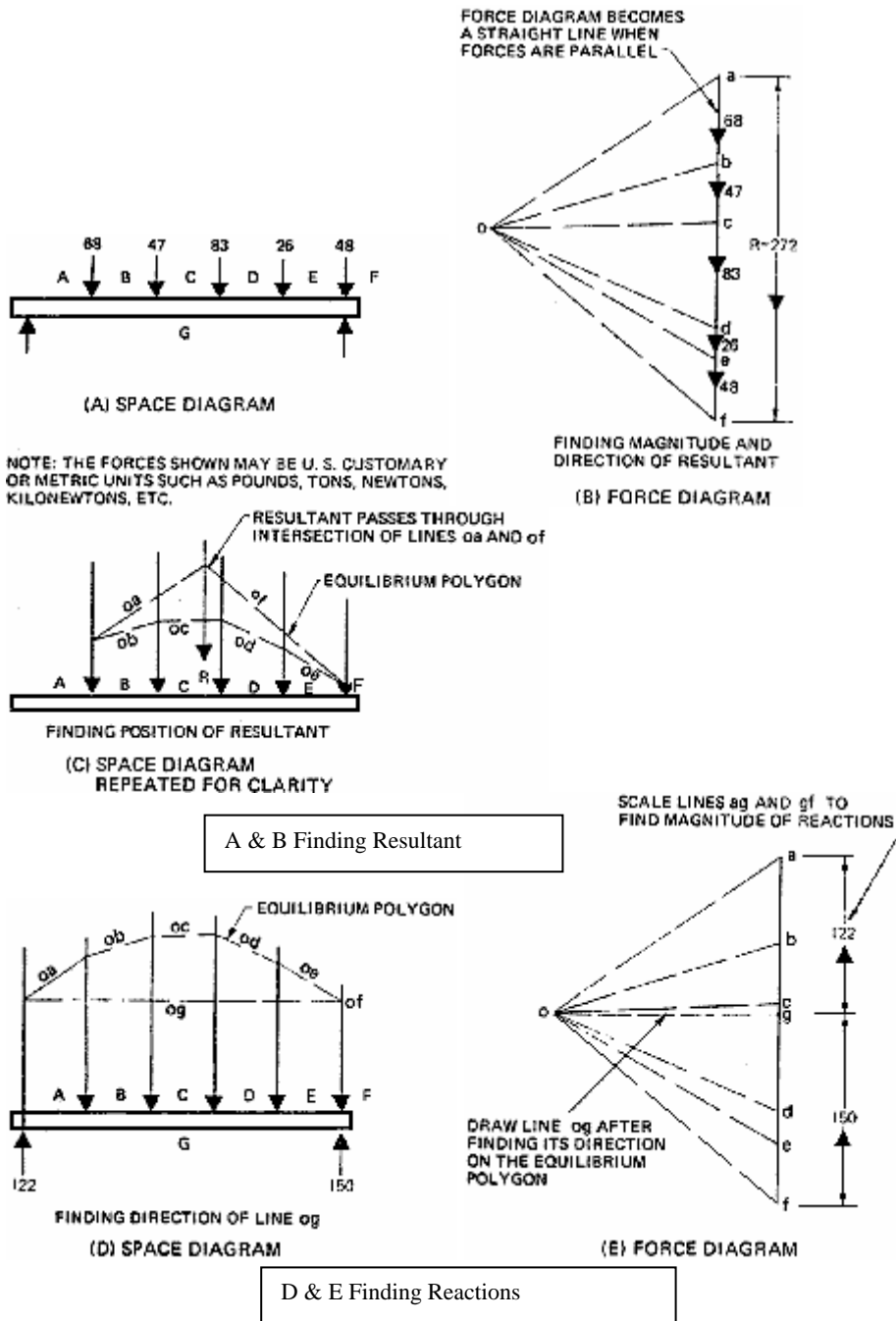


Fig. 28-2-2. Graphic method of finding resultant and reactions of vertical forces acting on a beam.

**Solution** First draw a space and force diagram using Bow's notation. Note that the force diagram is a straight line and not a polygon when all the forces are parallel. The magnitude of the resultant is found by measuring the distance from  $a$  to  $f$  on the force diagram. The direction of the resultant is also established, but its position with respect to the six forces is still unknown. To find its position, locate a point  $o$  anywhere on the force diagram and join  $o$  to each letter with a line.

Draw a line anywhere in space **B** of the space diagram (Fig. 28-2-2C) parallel to line *ob* in the force diagram, until it intersects forces **AB** and **BC**. Next draw a line parallel to *oc* in space **C** but starting where line *ob* intersects force **BC**. This is continued until the equilibrium polygon is completed. The resultant *R* passes through the intersection of lines *oa* and *of*, thus determining its position. The polygon constructed in the space diagram is called an equilibrium or funicular polygon.

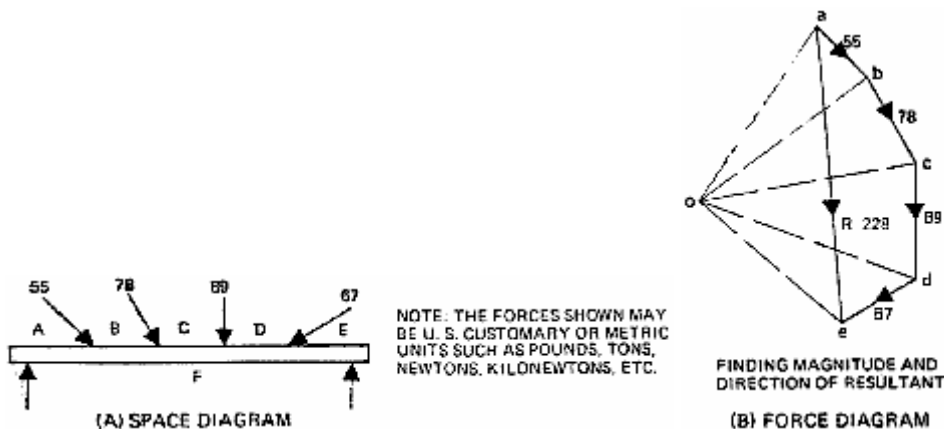
The magnitude of the reaction forces is found by making an equilibrium polygon (see Fig. 28-2-2D), which includes reaction forces **AG** and **FG**. These forces were not needed to find the resultant. To construct the equilibrium polygon, draw a line *oa* anywhere in space **A** and parallel to line *oa* in the force diagram, as shown in Fig. 28-2-2E. Repeat for lines *ob*, *oc*, *od*, and *oe*, having each of these lines touch each other as shown.

Line *of* will not have any length on the equilibrium polygon since forces **EF** and **FG** act in the same line. Draw line *og* in the space diagram by joining the start of line *oa* to *of*. Now draw line *og* on the force diagram parallel to line *og* in the space diagram. The magnitude of the reactions (two vertical upward forces supporting the beam) **GA** of 122 units and **FG** of 150 units may be found by measuring lines *ag* and *gf* on the force diagram. Note, the units for the forces may be U.S. Customary or metric such as pounds, tons, newtons, and kilonewtons.

### Graphic Method of Finding Resultant of Nonparallel Forces Acting on a Beam

**EXAMPLE 2** A number of nonparallel forces are acting on a beam as shown in Fig. 28-2-3A. It is required to find, graphically, the magnitude and the point of application of the resultant.

**Solution** To find the resultant, first draw a space and force diagram and label the forces, using Bow's notation. The magnitude and the direction of the resultant are shown by line *ae* on the force diagram (Fig. 28-2-3B). The location of the resultant with respect to the four forces must now be found. Locate a point *o* anywhere on the force diagram and join *o* to each letter with a line. Draw a line *ob* anywhere in space **B** of the space diagram (Fig. 28-2-3C), parallel to line *ob* in the force diagram. Next draw a line parallel to *oc* in space **C** but starting where line *ob* intersects force **BC**. This is continued until the equilibrium polygon is completed. The resultant *R* is then drawn through the intersection of lines *oa* and *oe*, thus determining its position.



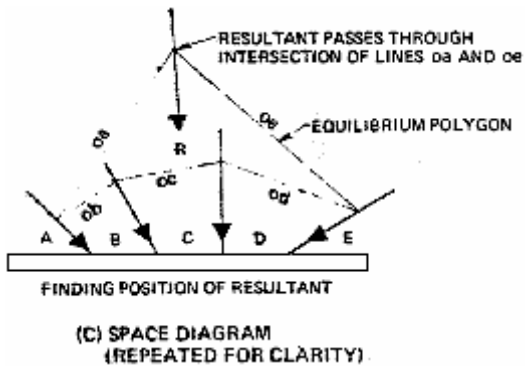
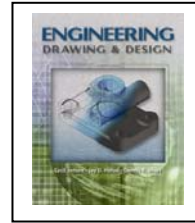


Fig. 28-2-3 Graphic method of finding resultant of nonparallel forces acting on a beam.

# Chapter 28 Applied Mechanics



## Unit 28-3 Truss Reactions When Loads Are Parallel

### BRIDGE AND ROOF TRUSSES

The graphical solution offers a quick and convenient method of checking or determining truss calculations. Some of the more common types of roof and bridge trusses are shown in Fig. 28-3-1.

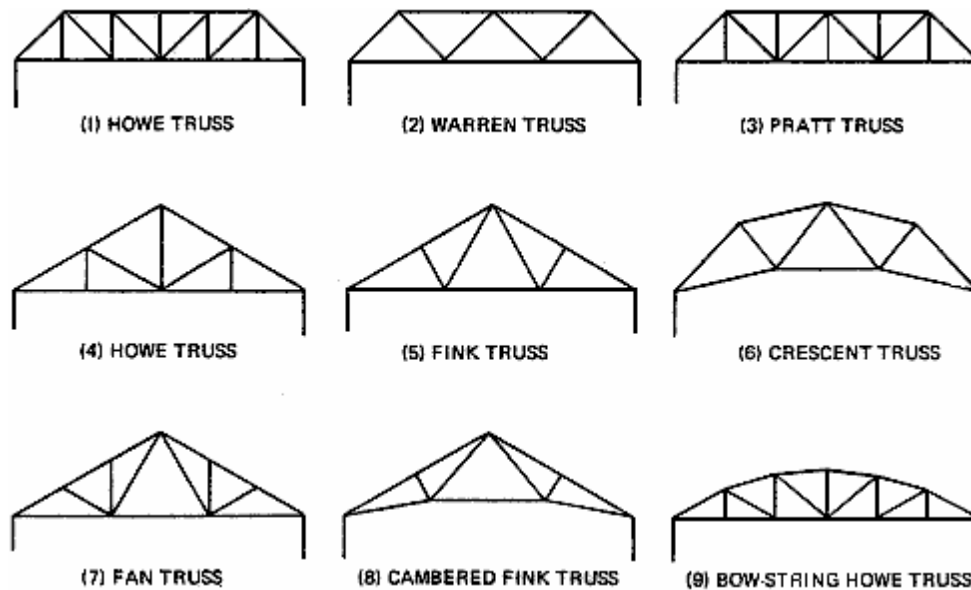


Fig. 28-3-1 Common types of trusses.

The loads that the truss and abutments support are combinations of the mass of the truss and the mass of the materials placed on the roof or truss, for example, snow, wind, and live loads such as cars and trains. The upward forces of the abutments or walls are called the *reactions*. Rollers are often used at one support of the truss to allow for expansion and contraction. Truss terminology is shown in Fig. 28-3-2.

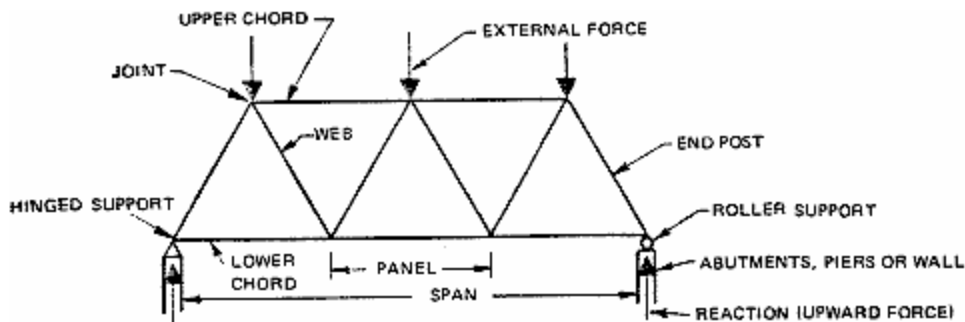


Fig 28-3-2 Truss terminology.

## GRAPHIC METHOD OF FINDING TRUSS REACTIONS WHEN LOADS ARE VERTICAL

Consider the truss in Fig. 28-3-3A. The forces acting on the truss are vertical downward forces, while the reaction forces are vertical upward forces, since they must be equal and opposite to the truss forces.

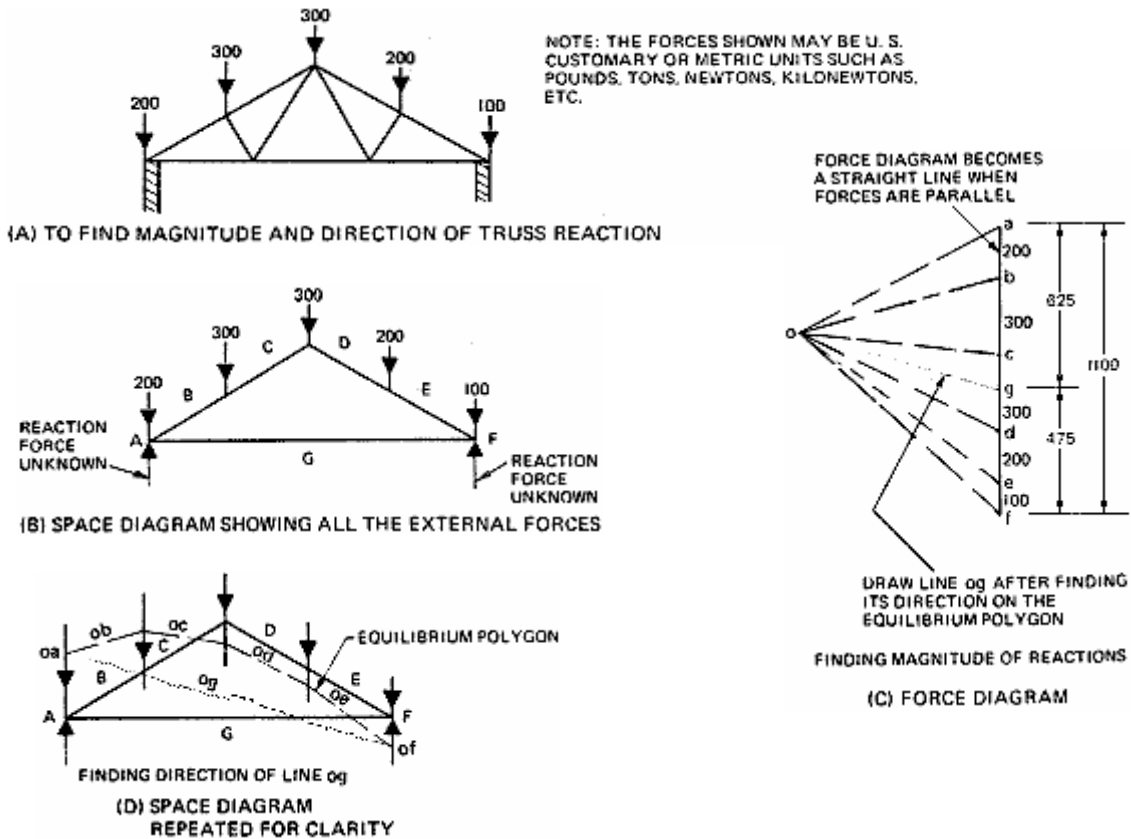


Fig 28-3-3 Graphic method of finding truss reactions when loads are vertical.

The lines of action and directions of all forces are known, and only the magnitude of the reactions must be computed. Draw a space diagram and label the forces using Bow's notation. Next draw the force diagram. Note that the force diagram becomes a straight line when all the forces are parallel. The magnitudes of the reactions **FG** and **GA** are not yet known, but their combined magnitude is equal to 1100 units, the length of line **af** on the force diagram. Locate a point **o** anywhere on the force diagram and join **o** to points **a** and **f** with a broken line.

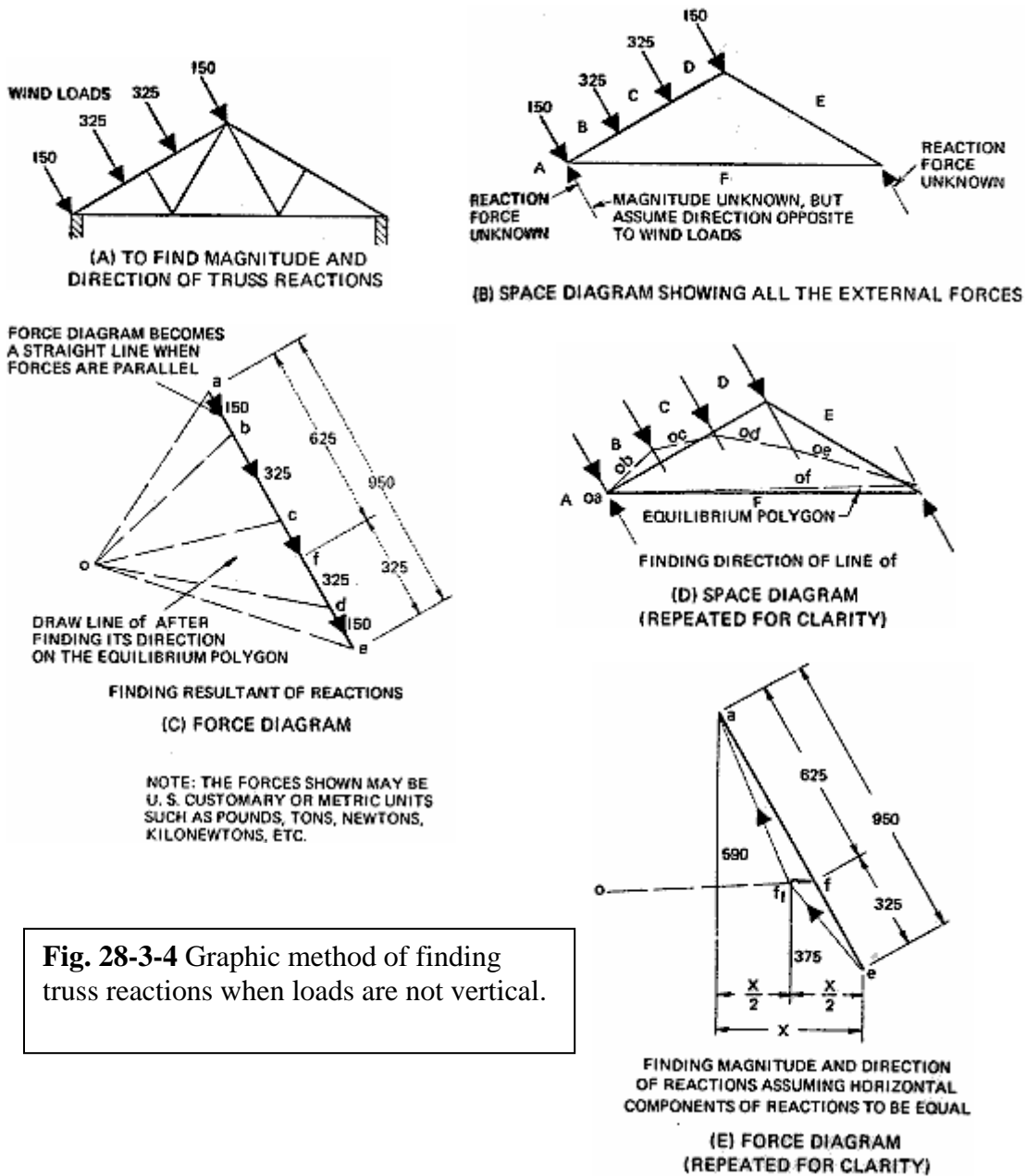
Draw a line **ob** parallel to line **ob** in the force diagram, anywhere in space **B** of the space diagram (Fig. 28-3-3D) until it intersects forces **AB** and **BC**. Draw a line parallel to **oc** in space **C**, but start where the line **ob** intersects force **BC**. Repeat for lines **od** and **oe**. Line **oa** will not have any length on the equilibrium polygon since forces **AB** and **GA** act in the same line. The same is true for line **of** since forces **EF** and **FG** act in the same line. Close the polygon by joining **oa** to **of** with line **og**. Draw line **og** on the force diagram

parallel to the line *og* on the space diagram. The values of the reactions **GA** of 625 units and **FG** of 475 units may be found by measuring lines *ga* and *fg* on the force diagram.

**GRAPHIC METHOD OF FINDING TRUSS REACTIONS WHEN LOADS ARE NOT VERTICAL**

When the direction of the resultant of the roof and wind load is not vertical, the reactions are not normally parallel.

The solution for finding the magnitude and direction of the reactions when the truss has fixed supports is based on the assumption that the horizontal components of the reactions are equal, since the truss is assumed to be rigid and rigidly held to the supports.



**Fig. 28-3-4** Graphic method of finding truss reactions when loads are not vertical.

Consider the truss shown in Fig. 28-3-4A. The forces acting on the truss are inclined downward forces, while the reaction forces are inclined upward forces. The lines of action and direction of all the downward forces are known and only the magnitude and direction of the reactions need to be computed. Draw a space diagram and label the forces using Bow's notation. Next draw the force diagram as shown in Fig. 28-3-4C. The resultant of the downward forces, line  $ae$  on the force diagram, is equal but opposite to the resultant of the reaction forces.

The magnitude and direction of the reactions **EF** and **FA** are not yet known, but the magnitude of their resultant is equal to 950 units. Locate a point  $o$  anywhere on the force diagram and join  $o$  to points  $a$  and  $e$  with a line. Draw a line  $ob$  parallel to line  $ob$  on the force diagram anywhere in space **B** of the space diagram (Fig. 28-3-4D) until it intersects forces **AB** and **BC**.

Draw a line parallel to  $oc$  in space **C** starting where line  $ob$  intersects force **BC**. Repeat for lines  $od$  and  $oe$ . Line  $oa$  will not have any length on the equilibrium polygon since forces **AB** and **FA** act in the same line. Close the equilibrium polygon by joining  $oa$  and  $oe$  with line  $of$ . Draw line  $of$  on the force diagram parallel to line  $of$  on the space diagram. The values of 625 units and 325 units may be found by measuring lines  $fa$  and  $ef$  on the force diagram. These values are the individual resultants of reactions **EF** and **FA**. Since the horizontal components of the reactions are assumed to be equal, the magnitude and direction of reactions **FA** and **EF** can be found by completing the force diagram as shown in Fig. 28-3-4E.

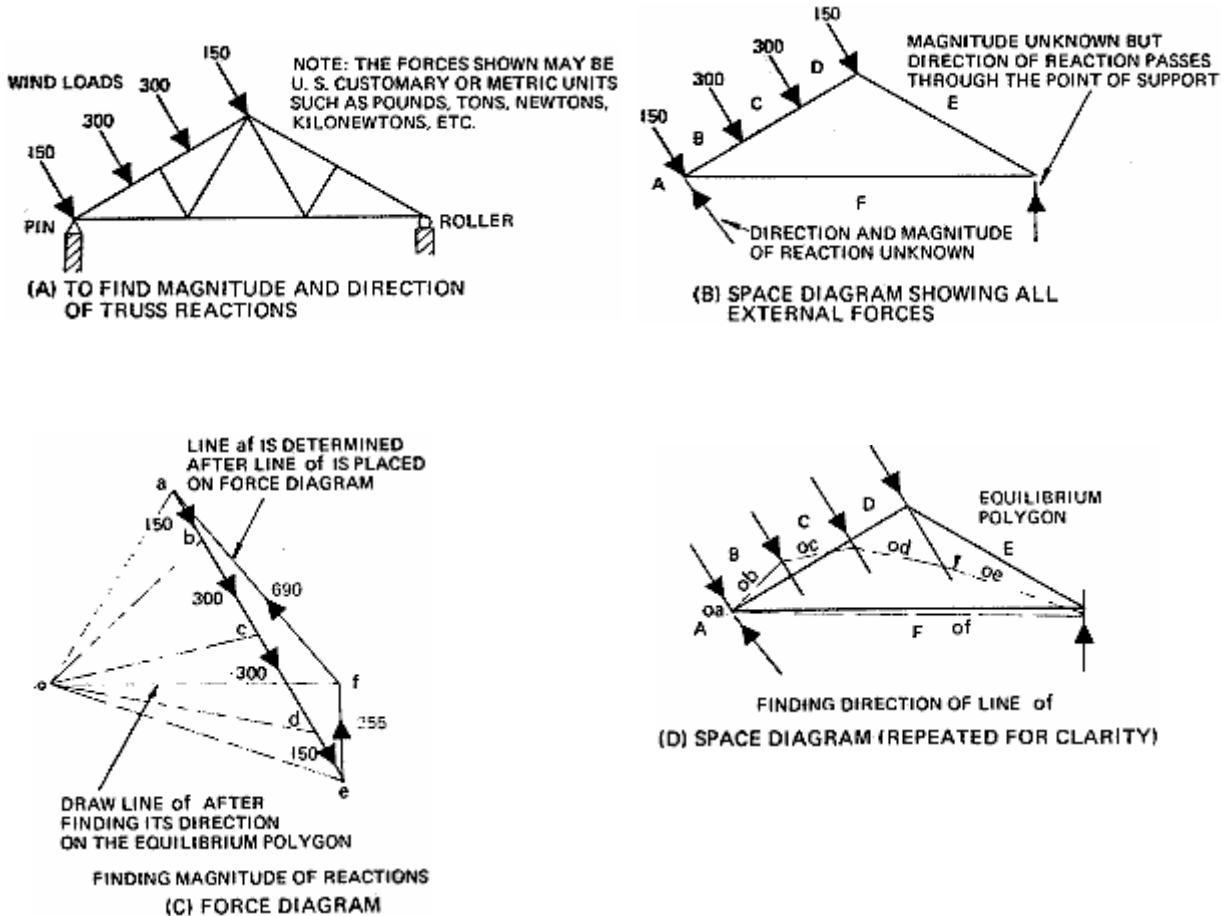
Distance  $X$  represents the horizontal component of the combined reaction forces. Since the truss is rigidly held to the supports, we may assume that each support will take half the horizontal force. Therefore  $X/2$  represents the horizontal component of each reaction. From point  $f$  on line  $ae$ , extend a horizontal line until it intersects the vertical line bisecting distance  $X$  at point  $f_1$ . The values of the reactions **FA** of 590 units and **EF** of 375 units may be found by measuring lines  $f_1a$  and  $ef_1$  on the force diagram. The directions of the reaction forces will be parallel to lines  $f_1a$  and  $ef_1$ .

## REACTION OF HINGED-PIN AND ROLLER SUPPORTS

Normally in bridge and roof truss design, one end of the truss is supported by a hinged pin and the other end rests on a roller. This provides for changes in the length of the truss because of temperature changes. The reaction of the roller support is taken through the roller and is usually perpendicular to the path of the roller. The reaction of the hinged-pin support is equal to the direction and size of force required to keep the structure in equilibrium. If the forces acting on the bridge or roof are vertical, then we can assume that the reactions of the hinge and rollers are also vertical. If the resultant of the forces acting on the bridge or roof is inclined due to wind loads and the reaction at the roller support is vertical, then the reaction at the hinged-pin support must be inclined.

## GRAPHIC METHOD OF FINDING TRUSS REACTIONS WHEN LOADS ARE PARALLEL FOR HINGED-PIN AND ROLLER SUPPORT

Consider the truss shown in Fig. 28-3-5A. The forces acting on the truss are inclined downward forces, caused by wind loads; a vertical upward force through the center of the roller; and an inclined upward force at the hinged-pin support. Now, find the magnitude of the reactions and the direction of the reaction at the hinged-pin support.



**Fig. 28-3-5** Graphic method of finding truss reaction when loads are parallel for hinged-pin and roller support.

Draw a space diagram (Fig. 28-3-5B), and label all the forces using Bow's notation. Draw wind forces **AB**, **BC**, **CD**, and **DE** on the force diagram. Since the direction of reaction force **EF** is known, its direction can be drawn on the force diagram. Its length or magnitude is not known. Reaction force **FA** cannot be drawn since both its magnitude and direction are not known, but its line of action passes through the point of support. Thus the equilibrium polygon is started at the pin support. Locate a point *o* anywhere on the force diagram and join *o* to points *a* and *e*. Draw a line *ob* parallel to line *ob* on the force diagram in space **B** of the space diagram (Fig. 28-3-5D) starting at the pin support until it intersects force **BC**. Draw a line parallel to *oc* in space **C** but starting where line *ob* intersects force **BC**. Repeat for lines *od* and *oe*. Line *oa* will not have any length on



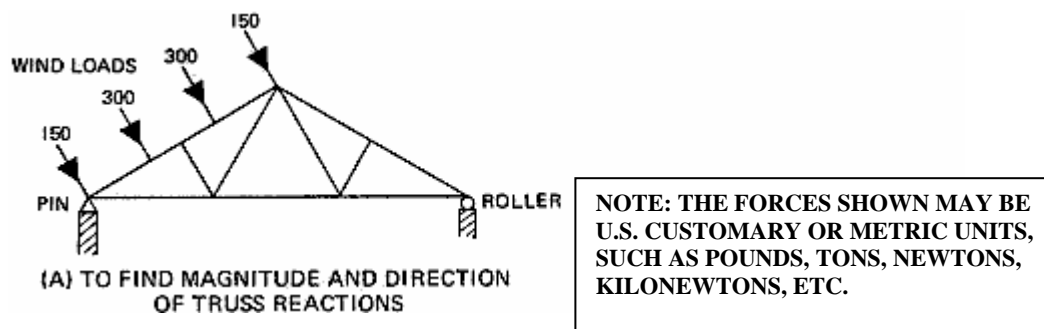
the equilibrium polygon since forces **AB** and **FA** act at the same point. Close the equilibrium polygon by joining *oa* to *oe* with line *of*. Draw line *of* on the force diagram parallel to line *of* on the space diagram until it intersects reaction force *ef* at point *f*. Close the force polygon with line *fa*. The direction of the hinged reaction force **FA** is parallel to line *fa*. The values of the reactions **EF** of 255 units and **FA** of 690 units may be found by measuring lines *ef* and *fa* on the force diagram.

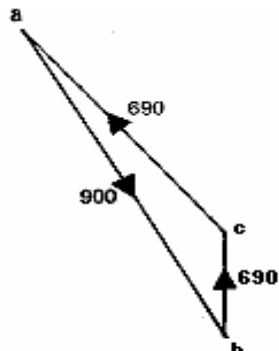
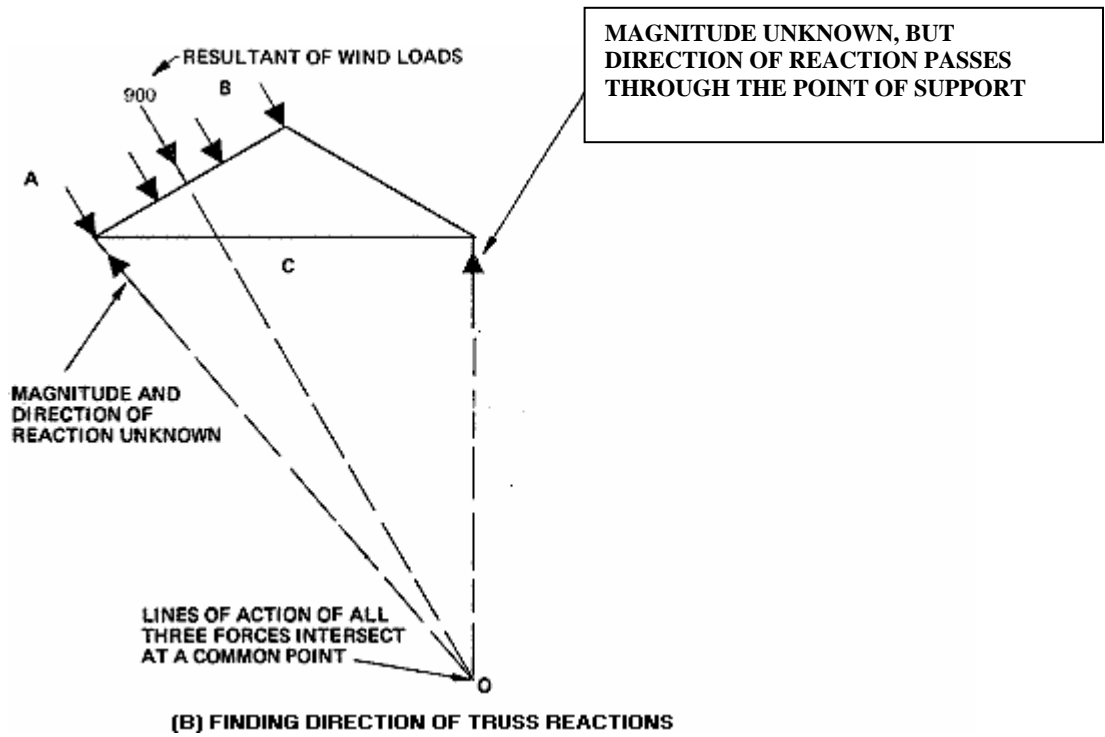
### ALTERNATIVE GRAPHIC METHOD OF FINDING TRUSS REACTIONS WHEN LOADS ARE PARALLEL FOR HINGED-PIN AND ROLLER SUPPORT

An alternative method can be employed for finding the truss reactions when the reaction forces at the pin and roller and the load force are not parallel. Consider the forces acting in Fig. 28-3-6A. The four wind forces of 150, 300, 300, and 150 units can be replaced by a resultant force of 900 units acting midway on the truss since the loads are symmetrical. The direction of the reaction at the hinge support is not known, but the direction of the reaction at the roller support will be vertical. Since the lines of action of any three nonparallel forces in equilibrium intersect at a common point, the direction of the reaction at the hinged support can be found. The lines of action of the resultant wind loads and the vertical reaction force are extended until they intersect at point *O* (Fig. 28-3-6B). Since the lines of action of all three forces must pass through point *O*, a line joining point *O* to the point of intersection at the hinged support determines the direction of the left reaction.

Knowing the direction of the three forces and the magnitude of the resultant, the magnitudes of the truss reactions can readily be found by making a force diagram and measuring lines *ca* and *bc*.

**Fig. 28-3-6** Alternative graphic method of finding truss reactions when loads are parallel for hinged-pin and roller support.





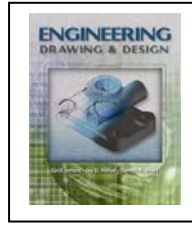
FINDING MAGNITUDE OF TRUSS REACTIONS AFTER FINDING DIRECTION OF FORCE *oa*

**(C) FORCE DIAGRAM**

**Fig. 28-3-6** Alternative graphic method of finding truss reactions when loads are parallel for hinged-pin and roller support. (Continued)

# Chapter 28

## Applied Mechanics



### UNIT 28-4

### Truss Reactions When Loads Are Not Parallel

#### GRAPHIC METHOD OF FINDING TRUSS REACTIONS WHEN WIND AND TRUSS LOADS ARE NOT PARALLEL

Consider the truss shown in Fig. 28-4-1A. The forces acting on the truss are a vertical downward force, an inclined downward force, and the reaction forces, which are inclined upward forces and normally do not act in the same direction. Since the truss is rigid and rigidly held to the supports, we may assume that the horizontal components of the reactions are equal.

The lines of action and direction of the downward forces are known, and only the magnitude and direction of the reactions need be computed. Draw a space diagram and label the forces using Bow's notation. Next draw a force diagram (Fig. 28-4-1B). The combined resultant of the two reaction forces is equal to  $ac$ . The individual magnitudes and directions of the reaction forces **CD** and **DA** are not yet known, but the magnitude of their combined resultant is equal to 970 units. Locate a point  $o$  anywhere on the force diagram and join  $o$  to points  $a$ ,  $b$ , and  $c$  with a line, as shown in Fig. 28-4-1D. Draw a line  $oa$  parallel to line  $oa$  on the force diagram anywhere in space **A** of the space diagram (Fig. 28-4-1C) until it intersects forces **DA** and **AB**. Draw a line parallel to  $ob$  in space **B** but start where line  $oa$  intersects force **AB**. Repeat for line  $oc$ . Close the equilibrium polygon with line  $od$ .

Draw line  $od$  on the force diagram (Fig. 28-4-1D) parallel to line  $od$  on the equilibrium polygon. The values of 560 units and 410 units may be found by measuring lines  $ad$  and  $dc$  on the force diagram.

These values are the individual resultants of reactions **CD** and **DA**. Since the horizontal components of the reactions are assumed to be equal, the magnitude and direction of reactions **CD** and **DA** can be found by drawing the force diagram as shown in Fig. 28-4-1F. Distance  $X$  represents the horizontal component of the combined reaction forces, therefore  $X/2$  represents the horizontal component of each reaction. From point  $d$  on line  $ac$ , extend a horizontal line until it intersects the vertical line bisecting distance  $X$  at point  $d_1$ . The values of the reactions **CD** of 425 units and **DA** of 550 units may be found by measuring lines  $cd_1$ , and  $d_1a$  on the force diagram. The directions of the reaction forces will be parallel to lines  $cd_1$ , and  $d_1a$ .

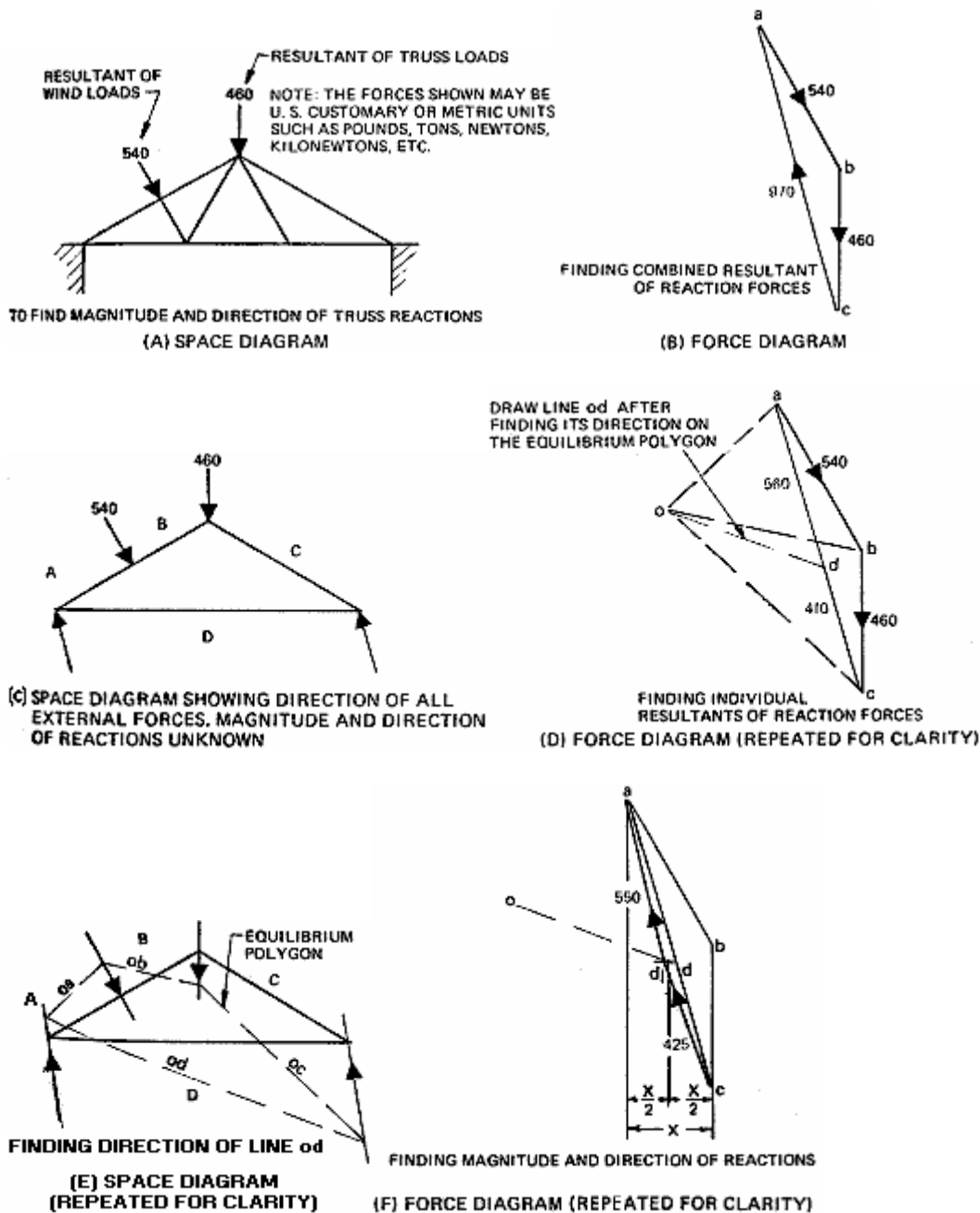
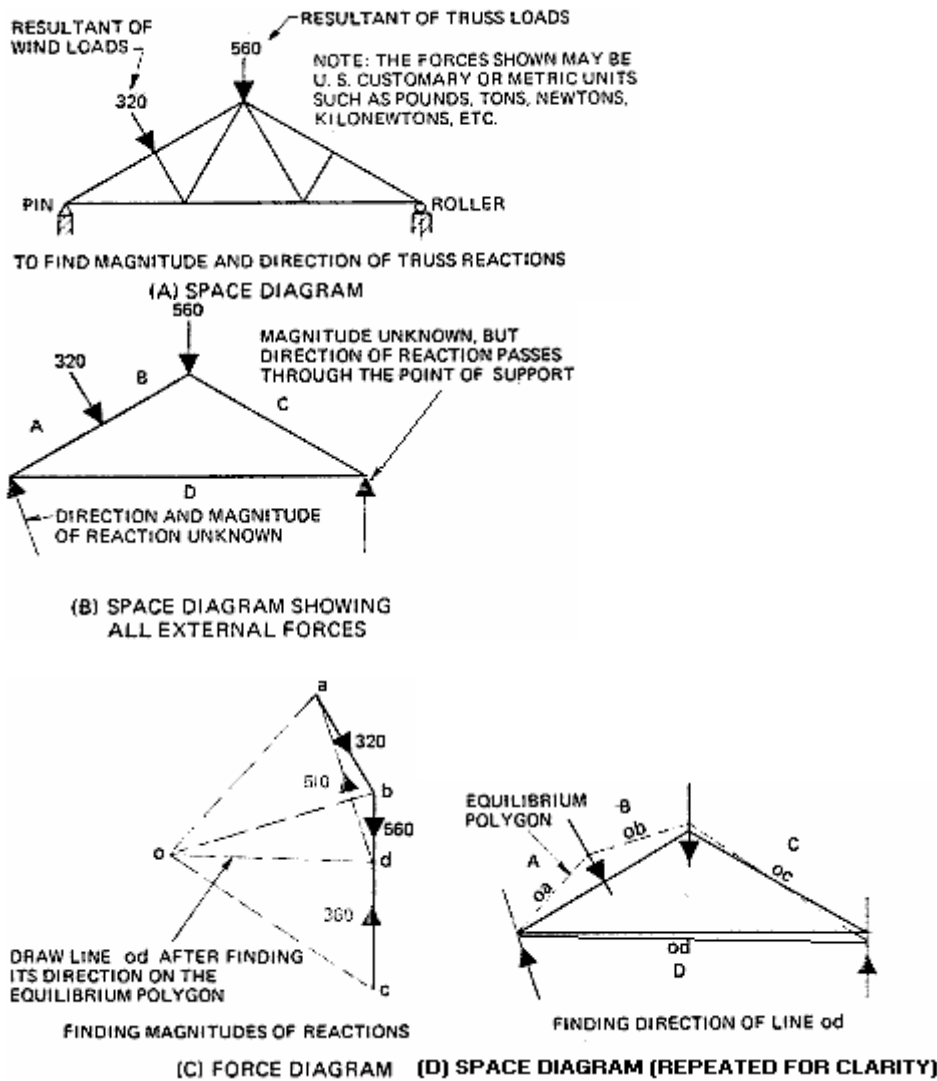


Fig. 28-4-1 Graphic method of finding truss reactions when wind and mass loads are not parallel.

### GRAPHIC METHOD OF FINDING TRUSS REACTIONS, ROLLER AT ONE END, WHEN WIND AND TRUSS LOADS ARE NOT PARALLEL

Consider the truss shown in Fig. 28-4-2A. The forces acting on the truss are a vertical downward force, an inclined downward force, a vertical upward force through the center of the roller, and an inclined upward force at the hinged-pin support. Draw a space diagram and label all the forces using Bow's notation. Next, partially draw the force diagram showing the known forces **AB** and **BC**. Force **CD** is a vertical upward force, but its magnitude is not

known. Locate a point  $o$  anywhere on the force diagram and join  $o$  to points  $a$ ,  $b$ , and  $c$ . Draw a line  $oa$  parallel to line  $oa$  on the force diagram anywhere in space **A** of the space diagram (Fig. 28-4-2D), until it intersects forces **DA** and **AB**. Draw a line parallel to  $ob$  in space **B** but start where line  $oa$  intersects force **AB**. Repeat for line  $oc$ . Close the equilibrium polygon by joining  $oa$  and  $oc$  with line  $od$ . Draw line  $od$  on the force diagram parallel to line  $od$  on the space diagram until it intersects line  $bc$  at point  $d$ . Join  $a$  to  $d$  with a line which represents the direction of force **DA**. The values of the reactions **CD** of 360 units and **DA** of 510 units may be found by measuring lines  $cd$  and  $da$  on the force diagram.



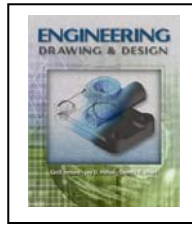
**Fig. 28-4-2** Graphic method of finding truss reactions, roller at one end, when wind and mass loads are not parallel.

# Chapter 28

## Applied Mechanics

### UNIT 28-5

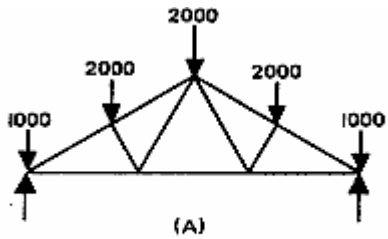
#### Internal Forces or Stresses in a Truss



#### GRAPHIC METHOD OF FINDING INTERNAL FORCES IN A ROOF TRUSS

The previous examples relating to roof trusses have been confined to finding the external forces acting on the roof and walls. Once the external forces have been calculated, the forces acting in the truss members can be determined graphically by two methods. Consider the truss shown in Fig. 28-5-1A. Since the forces are symmetrical, the reaction forces will be equal; namely 4000 units each. Using Bow's notation and stating all the forces, draw a space diagram to scale, as in Fig. 28-5-1B. Consider joint  $ABHG$ , the left support. Two of the four forces are known (Fig. 28-5-1C). Draw the force diagram (Fig. 28-5-1G) to a convenient scale, starting with vectors  $ga$  and  $ab$ . The next force in order is  $BH$ . Draw a line parallel to  $BH$  through point  $b$ . Point  $h$ , which is one of vector  $bh$ , has not yet been established. Draw a line parallel to the last force  $HG$  through point  $g$ . Since point  $h$  is on this line as well as on line  $bh$ ,  $h$  must be their point of intersection. The arrows must travel in the same direction around the polygon, thus indicating the direction of the forces acting at the joint. We find that truss member  $BH$  is under compression and truss member  $HG$  is under tension. Next, consider joint  $BCJH$  in Fig. 28-5-1D. There are two known forces,  $BC$  of 2000 units and  $HB$  which was found to be 6000 units and under compression. The directions of forces  $CJ$  and  $JH$  are known, but their magnitude is not.

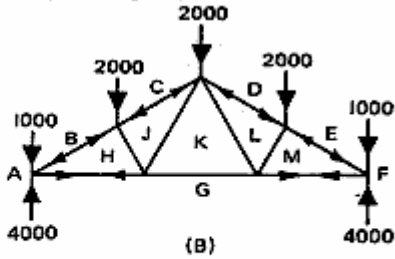
The member  $CJ$  is under compression, but we do not know whether member  $JH$  is a tie or a strut. Draw the force diagram (Fig. 28-5-1H), to a convenient scale by first drawing the vectors  $hb$  and  $bc$ . Draw a line parallel to  $CJ$  through point  $c$ . Point  $J$ , which is one end of the vector, has not yet been established. Draw a line parallel to the last force  $JH$  through point  $h$ . Since point  $j$  is on this line as well as on line  $cj$ ,  $j$  must be their point of intersection. The arrows must travel in the same direction around the polygon, thus indicating the direction of the force acting at the joint. We find that both truss members  $CJ$  and  $JH$  are under compression. Because of this, they are struts. Next consider joint  $CDLKJ$  (Fig. 28-5-1E). The magnitude of three of the five forces is known and the magnitude of the two unknown forces is equal since the loads are symmetrical about the center of the truss. Draw the force diagram (Fig. 28-5-1J) to a convenient scale by first drawing the vectors  $jc$ ,  $cd$  and  $dl$ . The next force in order is  $LK$ . Draw a line parallel to  $LK$  through point  $L$ . Point  $k$ , which is one end of the vector, has not yet been established. Draw a line parallel to the last force  $KL$  through point  $j$ . Since point  $k$  is also on this line as well as on line  $jk$ , it must be at their point of intersection. The arrows must travel around the polygon in the same direction, thus indicating the direction of the forces acting at the joint. We find that truss members  $LK$  and  $KJ$  are under tension and are tie bars. Only one member remains to be calculated. It is not known whether the truss member  $GK$  is under compression or tension, nor is its magnitude known. From the space diagram shown in Fig. 28-5-1E, draw the force diagram in Fig. 28-5-1K.  $GK$  is found to have a magnitude of 3500 units and is under tension.



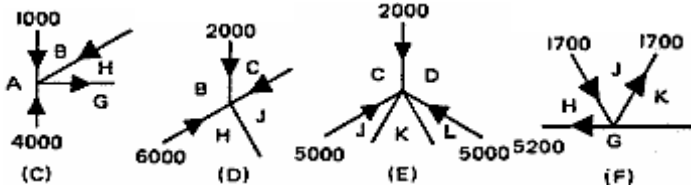
NOTE: THE FORCES SHOWN MAY BE U. S. CUSTOMARY OR METRIC UNITS SUCH AS POUNDS, TONS, NEWTONS, KILONEWTONS, ETC.

PROBLEM- TO FIND FORCES ACTING ON TRUSS MEMBERS

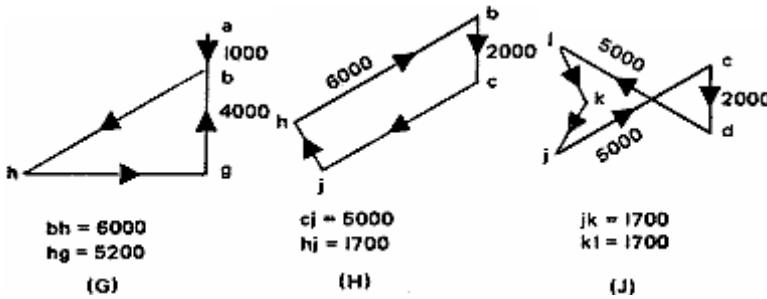
SPACE DIAGRAMS



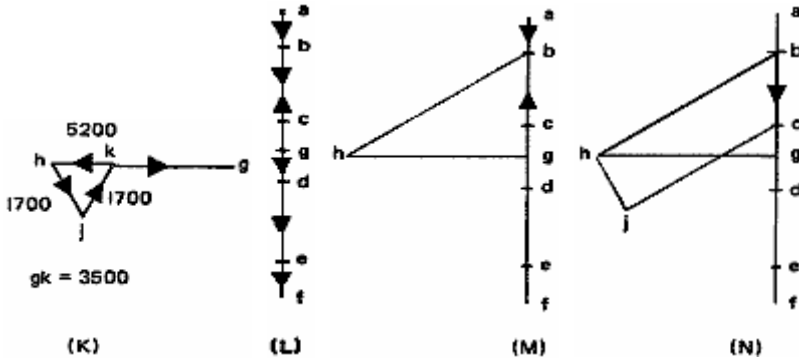
USE OF BOW'S NOTATION TO IDENTIFY MEMBERS  
NOTE - DIRECTION OF FORCES HJ, JK, GK UNKNOWN  
REACTIONS EQUAL AS LOADS ARE SYMMETRICAL

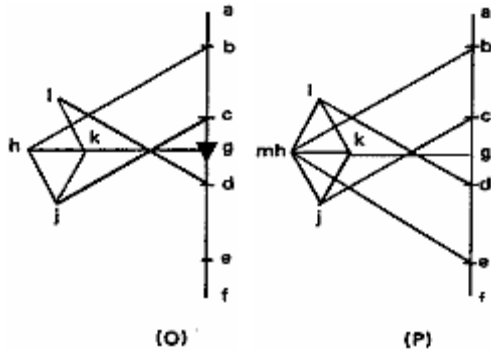


SPACE DIAGRAMS OF JOINTS



FORCE DIAGRAMS — USING POLYGON OF FORCES TAKEN AT EACH JOINT (G, H, J, AND K)





CONSTRUCTION OF A LOADING DIAGRAM (L, M, N, O AND P)

**Fig. 28-5-1** Graphic method of finding internal forces in a roof truss.

The second method for finding the forces in the truss members is much faster. A single diagram, called a *loading diagram*, that combines all the separate force polygons is used. The first step in constructing a loading diagram is to draw the force diagram of the external forces, as in Fig. 28-5-1L. Now consider a joint where only one or two forces are unknown, such as joint *ABHG*. The force diagram *ab, bh, hg, ga* can be drawn on the stress diagram in a similar manner to that previously explained. Next, a force diagram similar to Fig. 28-5-1H for joint *BCJH* is constructed on the loading diagram, adding vectors *jh* and *cj*. The same procedure is used for joints *CDLKJ* and *GHJK* until the loading diagram is complete. Only the directions of the external forces are shown on the loading diagram, since the directions of the other forces would alternate at the different joints.