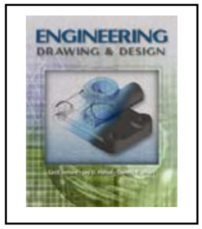
CHAPTER 29

# Strength of Materials




## UNIT 29-1
## Stresses and Strain

### Relationship Between Mass and Force

*Mass* is the quantity of matter in a body. The mass of an object remains constant, regardless of its location on earth. Mass is measured in ounces, pounds, and tons (U.S. Customary), or grams, kilograms, and metric tons (metric).

Some examples of the use of these units of measurement are in

- Defining quantities of material, packaged in bulk, such as bags of mortar and tons of sand
- Defining physical characteristics of material such as 210-lb asphalt shingles and 18-oz, 24-oz, or 32-oz glass
- Defining load capacities for building elements, elevator cranes, hoists, bridges, roads, supports, and bearing surfaces
- Specifying application of materials such as 20-lb roofing asphalt per mopping per 100 $ft^2$ (U.S. Customary) or 10-kg roofing asphalt per mopping per 100 $m^2$ (metric)
- Establishing costs for materials, unit prices, and rates on an ounce, pound, or ton (U.S. Customary) or gram, kilogram, or metric ton basis (metric)

*Force* is the external agent that changes or tends to change the condition of rest of a body. Force is measured in ounce-force, pound-force, ton-force and kip-force (U.S. Customary) or in newtons (N) for light forces, kilonewtons (kN) for intermediate forces, and meganewtons (MN) for heavy forces (metric).

Forces related to the design and construction processes are numerous: bearing capacity, applied weight (mass under the influence of gravity) of live, dead, and mobile loads, connection load, etc. Force may be concentrated on a tiny spot or applied over an immense area.

To convert kilograms to a force value, multiply the mass value (in kilograms) by 9.806 65 to obtain the force in newtons.

### STRESSES

When a force acts on a piece of material, internal resistance or forces are set up in the material to resist the external force. The resisting forces are called *stresses* and are measured in pounds per square inch or square foot (U.S. Customary) or pascals (metric). *A pascal* (Pa) is a pressure or stress produced when a force of one newton (N) is applied to an area of one square meter ($m^2$).

$$\text{Pa} = \frac{\text{N}}{\text{m}^2}$$

The pascal is a very small unit of measure. It is used for very low-stress applications. In most instances the kilopascal (kPa) and megapascal (MPa) are used.

In solving stress problems the following formulas can be used:

$$\text{stress} = \frac{\text{force}}{\text{area}} \text{ or area} = \frac{\text{force}}{\text{stress}}$$

force = stress x area

There are three kinds of stresses: *tension*, *compression*, and *shear*. Tension, or tensile stress, is caused by an external force that tends to pull apart or stretch the material. Tie bars supporting heating units or fans from ceiling members are examples of parts subject to tensile stress. See Fig. 29-1-1.

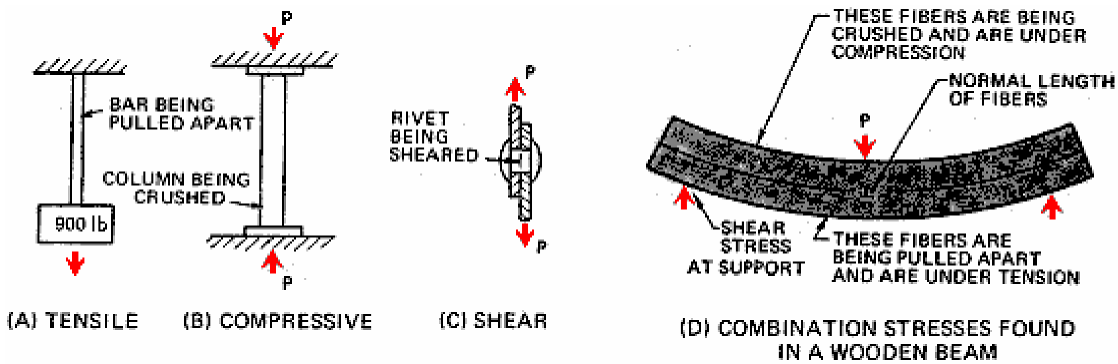



**Fig. 29-1-1** Stresses.

Compression, or compressive stress, is caused by external forces that tend to crush or push the material together. Basement posts and walls are parts subject to compressive stress.

Shear stress is caused by external forces that tend to cause the particles within the material to slide past one another. Rivets holding metal plates together are subject to shear stress.

These stresses often appear in combination. In a simple beam supporting a load, all three stresses occur. There is a tensile stress along the bottom of the beam, a compressive stress along the top of the beam, and a shear stress at each side at the abutments.

The *ultimate strength* of a material is the highest unit of stress that the material can withstand without breaking.

## LOADS

The external forces acting on a body, called *loads* and measured in pounds, tons, and kips (U.S. Customary) or newtons, kilonewtons, and meganewtons (metric), are classified according to the manner in which they are applied.

A *static* load is one that is applied gradually to a part and that remains practically constant once the maximum load is reached. The weight (mass) of a building acting on its foundation is an example of a static load. Static loads are also referred to as *dead* loads.

An *impact* or *shock load* is one that is applied suddenly on an object for a short time. When a nail is struck by a hammer or a train passes over a portion of track, the loads resulting from these actions are known as impact loads.

*Repeated loads* are loads that are alternately applied and removed many times. An example of a part that is subjected to this type of load is a connecting rod in an automobile engine.

Only static and impact loads will be dealt with in this unit.

## TYPES OF STRESS

Since there are three types of stresses, a material will have three different ultimate strengths. When machine parts are designed, it is not feasible to work precisely to the ultimate strength of the material since the addition of any shock or unforeseen load would cause breaking.

Therefore, when parts are designed, another stress known as the *safe working stress* (allowable unit stress) is used. This stress is obtained by dividing the ultimate strength of the material by a number called the *factor of safety*. Hence

$$\text{Safe working stress (S)} = \frac{\text{ultimate strength (Su)}}{\text{factor of safety (FS)}}$$

| Term | Symbol | Formula |
|---|---|---|
| Force or Load | F | $F = A \times S$ |
| Area | A | $A = \frac{F}{S}$ |
| Stress | S | $S = \frac{F}{A}$ |
| Ultimate Strength | Su | $Su = S \times FS$ |
| Factor of Safety | FS | $FS = \frac{Su}{S}$ |
| Deformation (Unit Strain) | Du | $Du = \frac{Dt}{L} = \frac{S}{E}$ |
| Deformation (Total Strain) | Dt | $Dt = Du \times L$ |
| Coefficients of Linear Expansion | N | |
| Modulus of Elasticity | E | $E = \frac{S}{Du}$ |
| Length of Part being Deformed | L | $L = \frac{Dt}{Du}$ |

**Fig. 29-1-2** Common terms, symbols, and formulas.

Other common terms and formulas are shown in Fig. 29-1-2. The number used for the factor of safety varies according to the material, the proposed location of the part, and the type of force that it must withstand. For example, a wooden part that is subjected to a shock force would have a greater factor of safety than a steel part that is subjected to a dead load. Factors of safety are not used as frequently as they were in the past since many of today's codes for structures and machines list the allowable unit or working stresses to be used. However, in certain applications, such as aircraft design, the ultimate strength and factors of safety are often used. Allowable unit stresses for steel will be covered in greater detail later in the chapter.

Figure 29-1-3 shows the average values of the ultimate strengths of various materials.

| Material | | Ultimate Strength | | | Allowable Unit Stress | | | Modulus of Elasticity |
|---|---|---|---|---|---|---|---|---|
| | | Tension | Compression | Shear | Tension | Compression | Shear | Tension |
| Aluminum | $10^3$ lb/in.$^2$ | **15** | **12** | **12** | | | | **11 000** |
| | MPa | 103 | 83 | 83 | | | | 78 500 |
| Brass | $10^3$ lb/in.$^2$ | **21** | **30** | **36** | | | | **9 000** |
| | MPa | 145 | 207 | 248 | | | | 62 000 |
| Copper | $10^3$ lb/in.$^2$ | **34** | **32** | **36** | | | | **15 000** |
| | MPa | 235 | 220 | 248 | | | | 103 500 |
| Cast Iron | $10^3$ lb/in.$^2$ | **21** | **90** | **24** | | **15** | **4** | **14 000** |
| Gray | MPa | 145 | 620 | 165 | | 103 | 28 | 96 500 |
| Cast Iron | $10^3$ lb/in.$^2$ | **31** | **46** | **40** | **5.2** | **7.5** | **6.6** | **25 000** |
| Malleable | MPa | 214 | 317 | 275 | 35 | 52 | 45 | 172 000 |
| Cast Iron | $10^3$ lb/in.$^2$ | **48** | **48** | **40** | **12** | **12** | **10** | **28 000** |
| Wrought | MPa | 330 | 330 | 275 | 83 | 83 | 69 | 193 000 |
| Steel | | | | | | | | |
| A572-50 | $10^3$ lb/in.$^2$ | **65** | **65** | **50** | **30** | **30** | **20** | **29 000** |
| A572M-350 | MPa | 450 | 450 | 345 | 210 | 210 | 140 | 200 000 |
| (See Fig. 29-1-8) | | | | | | | | |

Bold figures denote U.S. Customary system values.

**Fig. 29-1-3** Physical properties of common materials.

**EXAMPLE 1** A 2-ton weight is suspended from a 1.25 x 2.00-in. steel bar. What is the unit stress in pounds per square inch (psi)?

**Solution** The unit stress will be the force divided by the area.

$$\text{stress} = \frac{\text{force}}{\text{area}}$$

$$\mathbf{S} = \frac{\mathbf{F}}{\mathrm{A}}$$

$$= \frac{2 \times 2000}{1.25 \times 2.00}$$

$$= 1600 \text{ psi}$$

**EXAMPLE 2** What tensile force would cause a ϕ2.00 A572-50 steel rod to fail?

**Solution** The cross-sectional area of the rod is equal to $\pi R^2$ = 3.1416 x 1.00 x 1.00 = 3.1416 in.$^2$. From the table shown in Fig. 29-1-3, A572-50 steel has an ultimate strength of 65 000 psi. Therefore, the tensile force that would cause the rod to fail is

Area x ultimate strength = 3.1416 x 65 000 = 204 204 lb

**EXAMPLE 3** What is the maximum permissible tensile load an A572M-350 structural steel column, having a cross-sectional area of 300 mm$^2$, can carry if the factor of safety is given as 5?
**Solution** From the table shown in Fig. 29-1-3, A572M-350 structural steel has an ultimate tensile strength of 450 MPa.

$$\text{Working stress} = \frac{\text{ultimate strength}}{\text{factor of safety}}$$

$$= \frac{450}{5} = 90 \text{ MPa}$$

Therefore, maximum allowable load = S x A = working stress x area = $(90 \times 10^6)$ Pa x $(300 \times 10^{-6})$ = 27 kN.

**EXAMPLE 4** A 10" x 10" x 8'-0 Western hemlock construction-grade post supports a weight of 75 000 lb. Does this meet the minimum requirements as recommended by the Institute of Timber Construction (ITC)?
**Solution** From the table shown in Fig. 29-1-4 under the headings Carrying Load Independently, Compression, and Parallel to Grain, we find the allowable unit stress for Western hemlock, construction grade, is 1100 psi. Therefore

Maximum allowable load = A x S = 10 x 10 x 1100 = 110 000 lb

The load is acceptable.

Next check for buckling, using the formula $1/d \leq 10$, where $L$ = length in inches and $d$ = least dimension of compression member in inches. $1/d = (8 \times 12) \div 10 = 9.6$. Therefore, the post carrying this load would meet ITC requirements.

| Group | Grade of Lumber | | Carrying Load Independently Working Stress | | | | |
|---|---|---|---|---|---|---|---|
| | | | Bending | | Compression | | |
| | | | Stress at Extreme Fiber | Longitudinal Shear | Parallel to Grain $l/d \leq 10$ | Perpendicular to Grain | Tension Parallel to Grain |
| Douglas Fir | Construction | lb/in.$^2$ | **1500** | **120** | **1200** | **415** | **1500** |
| | | MPa | 10 | 0.8 | 8 | 2.9 | 10 |
| | Standard | lb/in.$^2$ | **1200** | **95** | **1000** | **390** | **1200** |
| | | MPa | 8 | 0.7 | 7 | 2.7 | 8 |
| Western Hemlock | Construction | lb/in.$^2$ | **1500** | **100** | **1100** | **365** | **1500** |
| | | MPa | 10 | 0.7 | 8 | 2.5 | 10 |
| | Standard | lb/in.$^2$ | **1200** | **80** | **1000** | **365** | **1200** |
| | | MPa | 8 | 0.6 | 7 | 2.5 | 8 |
| Spruce (All) | Structural | lb/in.$^2$ | **1050** | **90** | **750** | **300** | **1050** |
| | | MPa | 7 | 0.6 | 6 | 2.1 | 9 |
| | Construction | lb/in.$^2$ | **840** | **70** | **600** | **300** | **840** |
| | | MPa | 6 | 0.5 | 4 | 2.1 | 6 |
| Red Cedar & Pine | Structural | lb/in.$^2$ | **900** | **80** | **750** | **260** | **900** |
| | | MPa | 6 | 0.6 | 5 | 1.8 | 8 |
| | Construction | lb/in.$^2$ | **720** | **65** | **600** | **260** | **720** |
| | | MPa | 5 | 0.5 | 4 | 1.8 | 5 |

Note: Values shown are for teaching purposes only. Consult your local building codes for exact values.
Bold figures denote U.S. Customary system values.

**Fig. 29-1-4** Allowable unit stresses for sawn timber members.

**EXAMPLE 5** What force is required to punch a 1.50 in. diameter hole in a no. 12 USS gage sheet?

**Solution** The sheared area will be equal to the circumference of the circle multiplied by the thickness of the sheet. Circumference of a 1.50 in. diameter hole = 4.71 in. Thickness of a no. 12 USS plate (see Appendix) = .109 in. Sheared area = 4.71 x .109 = .513 in$^2$. Ultimate shear strength of steel (see Fig. 29-1-3) is 50 000 psi. Therefore the force required to punch the hole = $A$ x S = .513 x 50 000 = 25 650 lb.

## DEFORMATION

When an object is subjected to a load or force, the shape of the material is changed slightly. This change in length is called *strain,* or *deformation.* See Fig. 29-1-5. The length of an object is shortened by a compressive force or lengthened by a tensile force. The change in size is called *total elongation* and is normally measured in inches (U.S. Customary) or millimeters (metric) while the change in length per inch or millimeter is called *unit elongation* and is normally measured in inches per inch (U.S. Customary), or millimeters per millimeter (metric). Normally the deformation is so small that it cannot be detected by the naked eye. The deformation or sag that occurs on a beam when a load is applied is called *deflection.*

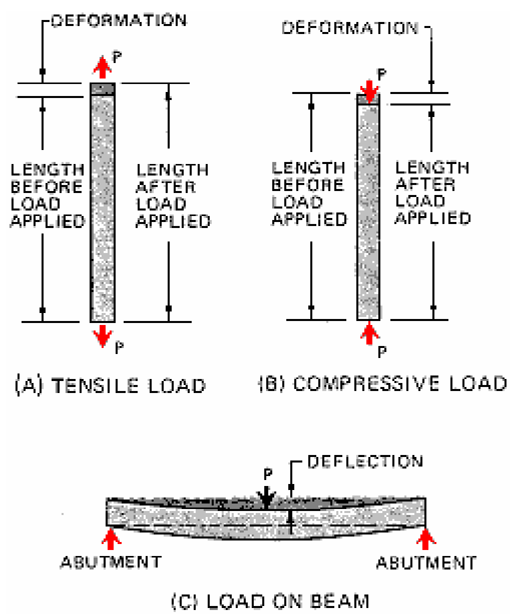



**Fig. 29-1-5** Deformation due to loads.

### Stress-Strain Diagram

The relationship between stress and strain for any material is best shown by a diagram; see Fig. 29-1-6. A piece of carbon steel 1.00 in. x 1.00 in. having an area of 1.00 in.$^2$ was subjected to a tensile load which was increased each time by 5000 lb, and the results were recorded. Up to point *A* on the graph, the elongation of the bar was proportional to the stress. Point *A*, which was recorded at 29 000 lb, was the elastic limit for that steel. After point *A*, the elongation increased at a faster rate. At a stress slightly higher than the elastic limit, deformation occurred without an increase in stress. This is known as the *yield point* of the material. As the tension increased, the bar elongated until point *B* was reached. This was the largest load applied, which was recorded at 65 000 lb. Beyond this point the bar continued to stretch or elongate with less tension. Point *B* was the *ultimate strength* of the material. The breaking point of the bar was point *C*, which was recorded at 48 000 lb.

The strength of any material may be plotted and calculated in a similar manner, although not all materials act in the same way. An example of this would be a cast-iron part. Since the ultimate strength and the breaking point would be the same, the part would break at the maximum load.

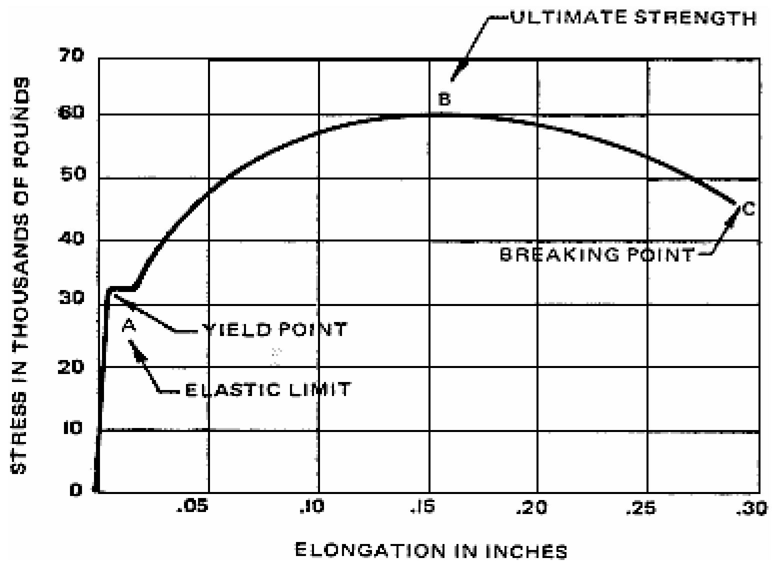



**Fig. 29-1-6** Stress-Strain diagram for A572-42 carbon steel.

If a force acting on an object is not great, the material will return to its original shape when the force is removed. This tendency to return to the original shape after being deformed is called *elasticity* and varies greatly in different materials. For example, lead is said to have little or no elasticity, while spring steel has a great amount. If, however, the material does not return to its original shape after it has been subjected to a force, it is said to be stressed beyond its elastic limit. Up to this elastic limit the deformation is proportional to the load; that is, the unit stress is proportional to the unit strain at any point in a material up to its elastic limit. This is known as *Hooke's law*. Beyond the elastic limit, the deformation ceases to be proportional to the load. The elastic limit of a material is difficult to determine accurately.

The *modulus of elasticity* of a material is defined as the ratio of unit stress to unit deformation (the stress in 1 in. divided by the deformation in 1 in.) and is denoted by the letter *E*. It may be used for finding the elongation per inch or millimeter caused by any given load.

In the metric system, the modulus of elasticity is the stress in pascals divided by the deformation in one millimeter.

**EXAMPLE 6** A steel bar 10 ft long elongates .075 in. under a tensile force. Calculate the unit deformation.

**Solution**

Unit elongation (*D*u)

$$= \frac{\text{total strain or deformation (Dt)}}{\text{length of part (L)}}$$

$$= \frac{.075}{10 \times 12} = .000\,63 \text{ in. per in.}$$

**EXAMPLE 7** Find the unit deformation on a piece of steel produced by a stress of 45 000 psi.

**Solution** From Fig. 29-1-3 we find the modulus of elasticity for steel is 29 000 000. Therefore,

$$\text{Du} = \frac{\mathbf{S}}{\mathbf{E}} = \frac{45\,000}{290\,00\,000} = .001\,552 \text{ in. per in.}$$

**EXAMPLE 8** A .25 x 1.00 in. steel bar 15 ft in length supports a tensile load of 5000 lb. Find the total deformation.

**Solution**

$$\mathbf{S} = \frac{\mathbf{P}}{\text{A}} = \frac{5000}{.25 \times 1.00} = 20\,000 \text{ psi}$$

The modulus of elasticity for steel (see Fig. 29-1-3) = 29 000 000.
Du = **S**/**E** = 20 000 ÷ 29 000 000 = .000 69 in. per in. Therefore,

*Dt* = *Du* x *L* = .000 69 x 15 x 12 =.1242 in.

**Temperature Stresses.** When the temperature of a piece of metal is changed, the length of the metal will be either decreased or increased, depending on whether the temperature of the metal is lowered or raised. If, however, the part is rigidly held and is restrained from changing its length, stresses known as *temperature stresses* will result. The main factors concerning temperature stress are (1) amount of heat involved, (2) material undergoing temperature change (aluminum, iron, etc.), and (3) length of part. In order to avoid these stresses, trusses or girders of long spans frequently have one end placed on a roller or a sliding plate.

The linear change per inch or millimeter of length of a part for a degree of change in temperature is called the *coefficient of linear expansion or contraction.* The coefficients of common materials are shown in Fig. 29-1-7. Thus, the total deformation resulting from temperature change can be found as follows. Let total strain or deformation be *Dt*, the coefficient of linear expansion *Ce*, temperature change (°F)*T*, and length of part (in.) *L*. Then *Dt* = *Ce* x *T* x *L*. In the metric system, degrees Celsius (°C) is used.

| Material | Ce Coefficient of Linear Expansion Inches per Inch per °F [Millimeters per millimeter per °C] |
|---|---|
| Aluminum | .000 012 8 [0.000 023 0] |
| Brass | .000 010 4 [0.000 018 7] |
| Bronze | .000 010 1 [0.000 018 2] |
| Copper | .000 009 3 [0.000 016 7] |
| Iron—Cast | .000 006 2 [0.000 011 2] |
| Iron—Wrought | .000 006 8 [0.000 012 2] |
| Steel—Hard | .000 007 4 [0.000 013 3] |
| Steel—Medium | .000 006 7 [0.000 012 1] |
| Steel—Soft | .000 006 1 [0.000 011 0] |

Bracketed figures denote metric values.

**Fig. 29-1-7** Coefficients of expansion.

**EXAMPLE 9** A medium steel bar 100 in. long is raised from 70 to 170°F. How much does it expand?
**Solution** The coefficient of linear expansion for medium steel (see Fig. 29-1-7) is .000 006 7. Therefore

Total deformation ($Dt$) = $Ce \times T \times L$ = .000 006 7 x 100 x 100 = .067 in.

**EXAMPLE 10** If the steel bar in Example 9 is restrained and is 2 in. square, what compressive stress is placed on the bar and what load is placed on the restraining members?
**Solution**

$$\text{Total strain} = 0.67\ \text{in.}$$

$$\text{Unit strain} = \frac{.067}{100}$$

$$= .000\,67\ \text{in. per in.}$$

$Du$ (unit strain) = $\dfrac{S}{E}$

Where $S$ = stress
$E$ = modulus of elasticity.
See Fig. 29-1-3.

Therefore
$S$ = $Du \times E$
= .000 67 x 29 000 000
= 19 430 psi

Therefore
Load = stress x area
= 19 430 x 2.00 x 2.00
= 77 720 lb

| | U.S. CUSTOMARY | | | | | | | METRIC | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Allowable Unit Stress – kips/in. | | | | | | | Allowable Unit Stress – MPa | | | |
| Description | Steel Standard | Ultimate Tensile Strength — kips/in | Yield Point — kips/in.² | Tension | Compression | Bending | Shear | Steel Standard | Ultimate Tensile Strength — MPa | Yield Point — MPa | Tension | Compression | Bending | Shear |
| General Construction | A36 | 58 | 36 | 22 | 22 | 24 | 14.5 | A36 | 400 | 250 | 150 | 150 | 165 | 100 |
| | A572 -60 | 75 | 60 | 36 | 36 | 40 | 24 | A572M -410 | 520 | 410 | 245 | 245 | 270 | 165 |
| | -55 | 70 | 55 | 33 | 33 | 36 | 22 | -380 | 480 | 380 | 230 | 230 | 250 | 150 |
| | -50 | 65 | 50 | 30 | 30 | 33 | 20 | -350 | 450 | 350 | 210 | 210 | 230 | 140 |
| | -45 | 60 | 45 | 27 | 27 | 30 | 18 | -310 | 410 | 310 | 185 | 185 | 205 | 125 |
| | -42 | 60 | 42 | 25 | 25 | 28 | 16.8 | -290 | 410 | 290 | 175 | 175 | 190 | 115 |

Notes 1. Metric designations and values were not available at time of printing. They are soft converted.
2. Values shown are for steel having a maximum thickness of 50 mm (2.00 in.).

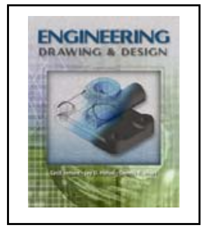
**Fig. 29-1-8** Allowable working stress for steel.

## UNIT STRESSES FOR STEEL

The increasing use of high-strength steels no longer permits the continuation of a standard design specification based on the exclusive use of one grade of steel. These high-strength steels afford as much as a 50 percent increase in strength as compared to common structural carbon steel.

To simplify matters, permissible unit stresses for the various grades of steel are given in terms of a percentage of a specified minimum yield point. These unit stresses are not to exceed 61 percent of the yield point. For steel having a yield point of 36 kips/in.$^2$, the permissible unit stress would be 22 kips/in.$^2$, which provides for a factor of safety of 1.64. Figure 29-1-8 lists the various grades of steels and their allowable unit stresses. In keeping with the inclusion of steels of several strength grades, a number of corresponding specifications for cast-steel forgings and other materials such as rivets, welding electrodes, and high-strength bolts have been introduced.

CHAPTER 29
# Strength of Materials




## UNIT 29-2
## Bolted and Riveted Joints

It is assumed that the reader has a full understanding of the many advantages of bolted and riveted construction and possesses a knowledge of this type of working drawing and terminology.

The factors of safety for fasteners used in tension are preferably based upon ultimate strength rather than yield point since ultimate strength is of much greater significance for fasteners. The permissible working stresses as shown in Fig. 29-2-1 represent working loads that are approximately one-third to one-half of the value of the ultimate loads observed in tests.

For greater convenience in the proportioning of the bolted connections, permissible stresses for bolts are now given in terms applicable to their normal body area, i.e., the area of the unthreaded shank.

The tensile stress permitted for A307 bolts and threaded parts of A36 steel is equivalent to 22 000 psi (pounds per square inch) or 150 MPa (metric) applied at the root area of the threads. See Fig. 29-1-8.

Permissible stresses for rivets are given in terms applicable to the nominal cross-sectional area of the rivet before driving. See Fig. 29-2-1.

| TENSION | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 20 kips/in.² Rivet Size in Inches | | | | | 140 MPa Rivet Size in Millimeters | | | | |
| Rivet Dia. in. (mm) | .50 | .625 | .75 | .875 | 1.00 | 12 | 16 | 20 | 22 | 25 |
| Area in.² (mm²) | .196 | .307 | .442 | .601 | .785 | 113 | 201 | 314 | 380 | 491 |
| Load kips (kN) | 3.93 | 6.14 | 8.84 | 12.03 | 15.71 | 15.82 | 28.14 | 43.96 | 53.2 | 68.74 |

| SHEAR Check Below to Ensure that the Allowable Load Is Not Governed by Bearing | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 15 kips/in.² Rivet Size in Inches | | | | | 100 MPa Rivet Size in Millimeters | | | | |
| Rivet Dia. in. (mm) | .50 | .625 | .75 | .875 | 1.00 | 12 | 16 | 20 | 22 | 25 |
| Single Shear kips (kN) | 2.94 | 4.60 | 6.63 | 9.02 | 11.78 | 11.3 | 20.1 | 31.4 | 38 | 49.1 |
| Double Shear kips (kN) | 5.89 | 9.20 | 13.25 | 18.04 | 23.56 | 22.6 | 40.2 | 62.8 | 76 | 98.2 |

**Fig. 29-2-1** Allowable load in kips per square inch (U.S. Customary) and kilonewtons (metric) for structural steel. (Continued on the next page.)

| BEARING<br>Single and Multiple Shear<br>Check to Ensure that the Allowable Load Is Not Governed by Shear | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Thickness of Material | | 45 kips/in.$^2$ Rivet Size in Inches | | | | | 310 MPa Rivet Size in Millimeters | | | | |
| in. | mm | .50 | .625 | .75 | .875 | 1.00 | 12 | 16 | 20 | 22 | 25 |
| .188 | 5 | 4.22 | 5.27 | 6.33 | 7.38 | | 18.6 | 24.8 | 31 | 34.1 | 38.8 |
| .250 | 6 | 5.62 | 7.03 | 8.44 | 9.84 | 11.25 | 22.3 | 29.8 | 37.2 | 40.9 | 46.5 |
| .312 | 8 | 7.03 | 8.79 | 10.55 | 12.31 | 14.06 | 29.8 | 39.7 | 49.6 | 54.6 | 62 |
| .375 | 10 | | 10.55 | 12.66 | 14.77 | 16.88 | | 49.6 | 62 | 68.2 | 77.5 |
| .500 | 12 | | | | 19.69 | 22.50 | | | | 81.4 | 93 |
| 1.000 | 25 | 22.50 | 28.13 | 33.75 | 39.38 | 45.00 | 3.72 | 4.96 | 6.2 | 6.82 | 7.75 |

For material thickness other than those shown, the bearing value is the value for 1.00 in. (U.S. Customary) or 1 mm (metric) multiplied by the actual thickness. Use values shown in bottom line.

**Fig. 29-2-1** Continued.

The most common methods of bolting or riveting plates together are by lapping or butting the plates, as shown in Fig. 29-2-2. There are many areas where a failure may occur in this type of connection (Fig. 29-2-3). In the lap joint the rivet may shear between the two plates. Since the rivet would shear in only one plate, it is said to be in *single shear*. The area that would shear would be the cross-sectional area of the rivet.

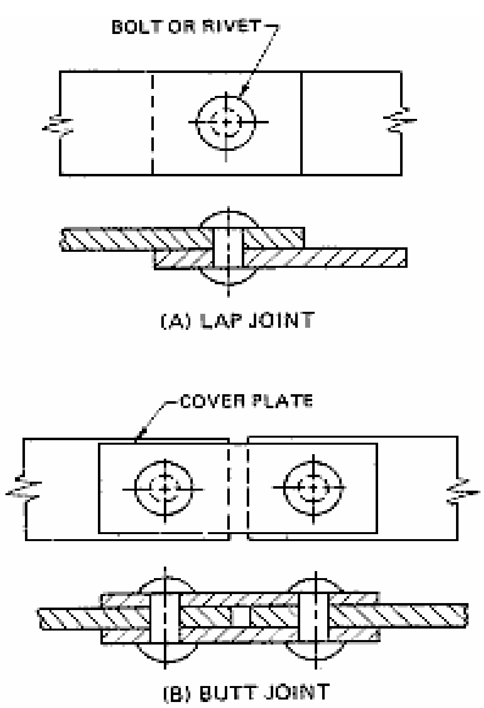



**Fig. 29-2-2** Plate connections.

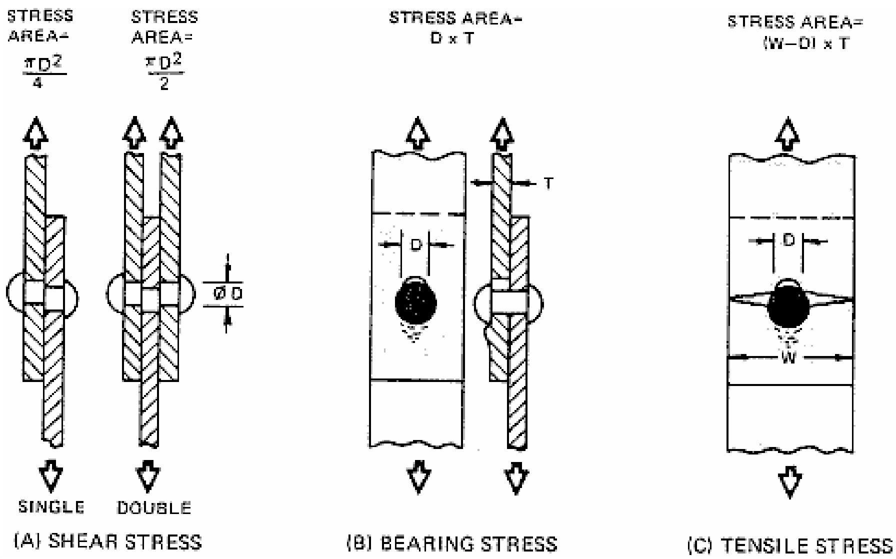



**Fig. 29-2-3** Stress areas in lap and butt joints.

There is a possibility that the plate may fail by tearing away at its weakest point, the section through the rivet or bolt hole. This would be a tension failure. The area that would fail would be the area of the plate at the center of the hole less the area of the hole.

A third type of failure would be for the rivet or bolt to rip through or crush the plate directly beneath it. This is called a *bearing failure,* and the area that would fail in the plate would be equal to the diameter of the fastener times the plate thickness. If more than one bolt or rivet were used, the load would be divided equally on the fasteners. Since only two pieces of metal are joined, they are said to be in *single bearing.*

In the butt joint shown in Fig. 29-2-3, the rivet would have to be sliced into two sections if the joint were to fail by shear. The rivet is said to be in *double shear,* and twice the area of the rivet is used in the shear calculations.

If the joint fails by tension, that is, pulls or tears away, it will do so at its weakest point — the section through the hole. Since the two outside plates are pulling in one direction and the center plate in the other, the smaller of the two areas must be used in calculating the tensile strength of the joint.

In calculating for bearing failure, a greater allowable working stress is permissible for rivets and high-strength bolts over ordinary bolts.

## Rivet Holes

In calculating the stresses in riveted and bolted joints, a distinction must be made between structural joints and joints in boilers, pipes, and tanks. In structural work, the steel members are generally punched and drilled .06 in. (1.5 mm) larger than the rivet in the shop and then taken to the site for assembly. In calculating the tensile stress in the joint, the size of the hole is taken as .12 in. (3 mm) greater than the nominal diameter of the rivet. This is to allow for any unseen damage that may occur around the hole when it is punched and assembled. Areas for shear and bearing are based on the nominal rivet diameter.

In the construction of boilers, where leakage may be a problem, it is essential that the rivet holes line up. These holes are often reamed at assembly. As the finished rivet fills the hole (which is larger than the rivet) completely, the diameter of the hole is used for computing all the stresses.

## Spacing of Rivets or Bolts

The minimum distance between the centers of fastener holes is 3 times the diameter of the fastener, but when possible, the distance shall be not less than shown in Fig. 29-2-4.

| Fastener Diameter | | In Sheared Edge | | In Rolled Edge of Plates | | In Rolled Edge of Structural Shapes | | Minimum Spacing of Rivets or Bolts | |
|---|---|---|---|---|---|---|---|---|---|
| Inches | mm | Inches | mm | Inches | mm | Inches | mm | Inches | mm |
| .50 | 12 | 1.00 | 25 | .90 | 23 | .75 | 20 | 2.00 | 50 |
| .625 | 16 | 1.10 | 28 | 1.00 | 25 | .90 | 23 | 2.25 | 58 |
| .75 | 20 | 1.25 | 32 | 1.10 | 28 | 1.00 | 25 | 2.50 | 65 |
| .875 | 22 | 1.50 | 38 | 1.25 | 32 | 1.10 | 28 | 3.00 | 75 |
| 1.00 | 25 | 1.75 | 45 | 1.50 | 38 | 1.25 | 32 | 3.50 | 90 |
| 1.125 | 30 | 2.00 | 50 | 1.75 | 45 | 1.50 | 38 | 4.00 | 100 |
| 1.25 | 32 | 2.25 | 60 | 2.00 | 50 | 1.75 | 45 | 4.50 | 115 |

**Fig. 29-2-4** Minimum edge distances and spacings for rivets and bolts.

The maximum pitch of rivets, or bolts, in line with the stress of compression members composed of plates and shapes does not exceed 16 times the thickness of the thinnest outside plate or shape or 20 times the thickness of the thinnest enclosed plate or shape, with a maximum of 12 in. (300 mm). When two or more gage center lines are used with rivets and bolts staggered, the maximum pitch of rivets or bolts in the line of stress in each gage line shall not exceed 24 times the thick ness of the thinnest plate or shape, with a maximum of 18 in. (450 mm).

The distance between lines of rivets or bolts measured at right angles to the line of stress shall not exceed 32 times the thickness of the thinnest plate or shape. The minimum distance from the center of any punched hole to any edge shall be that given in Fig. 29-2-4.

**EXAMPLE 1** Lap joint. Two steel bars, .50 x 2.00 in. are lapped and joined by a .75-in. rivet. What is the allowable tensile load that could be applied to the joint? The holes for rivets are to be punched. Plate material is A36 steel.

**Solution:** There are three areas that must be checked:

1. The bars failing under a tensile load at the holes
2. The rivet shearing
3. The bearing on the bars directly below the rivet

As the holes are punched, the diameter of the hole will be taken as .12 in. larger than the rivet diameter for calculating tensile loads.

**Area 1** *Bars failing under a tensile load.*
Area of plate at centerline of hole = .50 x (2.00 - .87) = .565 in.$^2$.
Allowable unit stress = 22 kips/in.$^2$. See Fig. 29-1-8. Therefore:
Allowable load = **S** x *A* = 22 000 x .565 = 12 430 lb

**Area 2** *Rivet shearing.* See Fig. 29-2- 1.
Single shear for ϕ.75 rivet = 6.63 kips or 6 630 lb

**Area 3** *Bearing on plate below rivet.* A ϕ.75 rivet is bearing on .50-in. thick steel.
Allowable load = (33.75 x .50) = 16 875 lb (see note at bottom of bearing table, Fig. 29-2-1).

The weakest area would be the shear on the rivet. Therefore, allowable tensile load that joint could support = 6 630 lb.

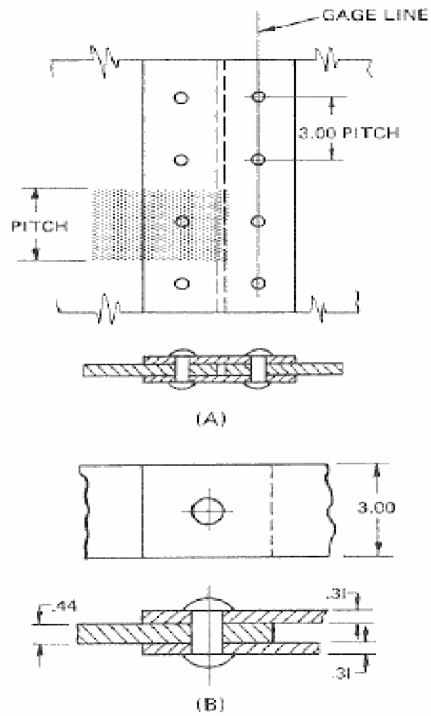



**Fig. 29-2-5** Single-riveted butt joint on a boiler.

**EXAMPLE 2** Single-riveted butt joint (Fig. 29-2-5). A boiler has a single riveted butt joint. The boiler plate is .44 in., and the two cover plates are .31 in. thick. The rivets are ϕ.75 in. and are spaced 3.00 in. apart. Calculate the main stresses that could safely be applied to this joint. Plate material is A36 steel.

**Solution** Since the pitch of the rivets is 3.00 in., it is assumed that the width of the section taken for calculation purposes is 3.00 in. As in Example 1, there are three areas to be checked:

1. The section failing under a tensile load
2. The rivet shearing
3. The bearing on the steel plate below the rivet.

As previously mentioned, for boilerplate construction, the finished rivet is assumed to be the same size as the drilled hole, namely, .81 in.

**Area 1** *Plate failing under a tensile load.* Since the area of the two outside plates is greater than the area of the middle plate, the middle plate will fail first. Diameter of rivet hole = .81 in. Area of middle plate = (3.00 - .81) x .44 = .959 in.$^2$. Allowable unit stress = 22 kips. See Fig. 29-1-8. Therefore

Allowable load = **S** x *A* = 22 000 x .959 = 21 100 lb

**Area 2** *Rivet shear.* Since the rivet would have to shear in two places, it is considered to be in double shear.

Shear area = $\frac{\pi \times 81^2}{4} \times 2 = 1.03$ in.$^2$

Allowable shear stress = 14.5 kips.
Therefore

Allowable load = **S** x *A* = 14 500 x 1.03 = 14 935 lb

**Area 3** *Bearing on plates.* Middle plate (double shear) area = .44 x .31 = .356 in. $^2$. Outside plates (single shear) have an area equal to

Area = 2 x .31 x .31 = .502 in. $^2$

The weaker area would be the middle plate failing under bearing. Allowable unit stress = 45 kips per in.$^2$. Allowable load **S** x *A* = 45 000 x .356 = 16 020 lb.

Therefore, the weakest area of the three areas checked would be the rivet shearing. Allowable load on joint = 14 935 lb.

**EXAMPLE 3** Roof truss (Fig. 29-2-6). A roof truss has loads of 75 and 64 kips acting on the upper and lower chord members. Calculate the number of ϕ.75-in. rivets required to safely carry these loads.

**Solution** Since the .38 in. gusset is enclosed by two .31-in.-thick angles, the rivets are in double shear. In calculating the bearing stress, it will be noted that the two outer angles having a combined thickness of .62 in. (two .31-in.-thick angles) are stronger than the .38-in.-thick gusset. Since the problem is one of determining the number of rivets required to carry the load, it can be assumed that the size of the steel is satisfactory for the applied loads. Refer to Fig. 29-2-1.

1. Number of ϕ.75-in. rivets in double shear required for

Upper chord = 75 + 13.25 = 6 rivets
Lower chord = 64 + 13.25 = 5 rivets

2. Number of ϕ.75-in. rivets bearing on .38-in. plate required for

Upper chord = 75 + 12.66 = 6 rivets
Lower chord = 64 + 12.66 = 5 rivets

Therefore, the minimum allowable rivets required for the upper and lower chords are 6 and 5 rivets, respectively.

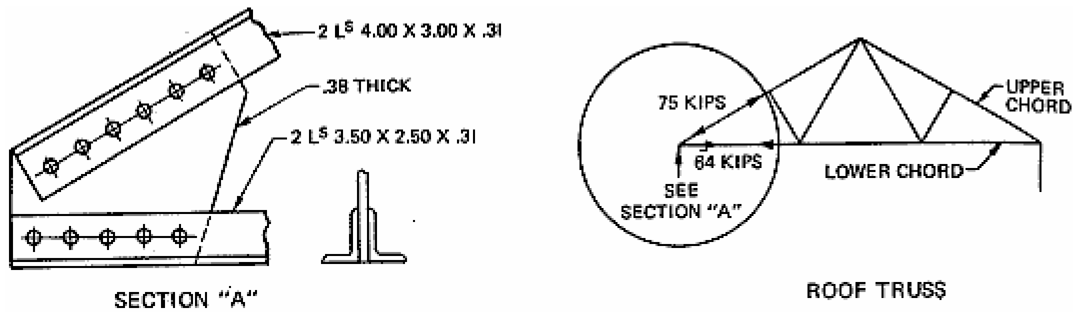



**Fig. 29-2-6** Roof truss.

**EXAMPLE 4** Double-riveted butt joint (Fig. 29-2-7). A boiler has a double-riveted butt joint. The boiler plate is .38 in. thick, and the two cover plates are .25 in. The rivets are ϕ.75 in. A section of the riveted joint is shown. Calculate the main stresses in the joint when the boiler plate is subject to a tensile strength of 6000 psi.

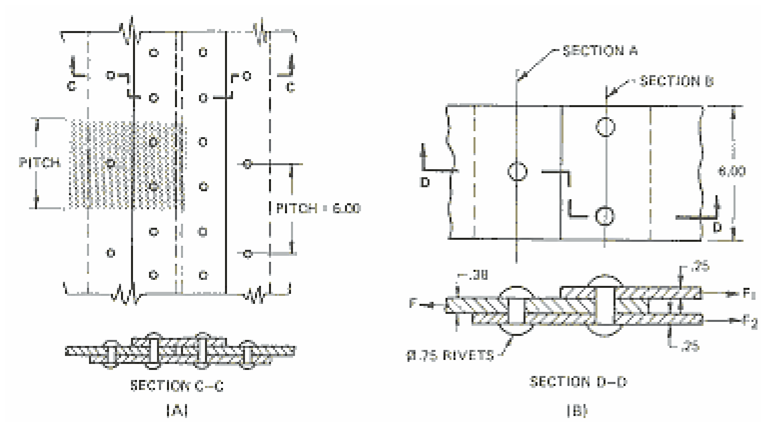



**Fig. 29-2-7** Double-riveted butt joint on a boiler.

**Solution** The length of the repeated section is 6.00 in. Since both sides of the joint are the same, only one side of the joint (shown in Fig. 29-2-7B) is used in computing the stresses. There are two rivets in double shear and one rivet in single shear. The rivets are .75 in. in diameter and are placed in .81-in. drilled holes. As mentioned earlier in boiler work, finished rivets are assumed to be the same size as the drilled holes; thus for calculation purposes the rivets will be .81 in. in diameter. The total force exerted on the repeated section is

**F** = **S** x ***A*** = 6000 x 6.00 x .38 = 13 680 lb

There are two rivets in double shear and one rivet in single shear, comprising shear areas. It will be assumed that each shear area will carry one-fifth of the load. Shear force on each rivet = 13 680 ÷ 5 = 2736 lb. Therefore the unit shear stress on rivets is

$$S = \frac{\mathbf{F}}{A} = \frac{2736}{\left[\left(\pi \times .81^2\right) \div 4\right]} = \frac{2736}{.515} = 5313 \text{ psi}$$

The upper cover plate transmits two-fifths of the load. Therefore $\mathbf{F}_1$ = .4 x 13 680 = 5472 lb.

The lower cover plate transmits three-fifths of the load. Therefore $\mathbf{F}_2$ = .6 x 13 680 = 8208 lb. Stress on boiler plate taken at section *A*

$$S = \frac{\mathbf{F}}{A} = \frac{13\,680}{(6.00 - .81) \times .38} = \frac{13680}{1.97} = 6944 \text{ psi}$$

Since one-fifth of the total load has been transmitted to the lower cover plate at section *A*, the load on the boiler plate at section *B* is .8 x 13 680 = 10 944 lb. Stress on .38-in. boiler plate taken at section *B is*

$$S = \frac{\mathbf{F}}{A} = \frac{10\,944}{(6.00 - 1.62) \times .38} = \frac{10\,944}{1.664} = 6577 \text{ psi}$$

Since the lower cover plate transmits three-fifths of the total load, the largest stress on the two cover plates will occur on the lower cover plate at section *B.* Stress on the bottom cover plate at section *B* is

$$S = \frac{\mathbf{F}_2}{A} = \frac{8208}{(6.00 - 1.62) \times .25} = \frac{8208}{1.095} = 7496 \text{ psi}$$

In calculating the bearing stresses, the bearing area for the rivet in single shear is the rivet diameter times the thickness of the thinner plate connected (the cover plate). The bearing area for the rivet in double shear is the rivet diameter times the thickness of the boiler plate. The rivets in double shear are subjected to twice the bearing load of those in single shear. Bearing stress at a rivet in single shear (section *A*) is

$$S = \frac{\mathbf{F}}{A} = \frac{2736}{.81 \times .25} = \frac{2736}{.20} = 13\,680 \text{ psi}$$

Bearing stress at a rivet in double shear (section *B*) is

$$S = \frac{\mathbf{F}_1}{A} = \frac{5472}{.81 \times .38} = \frac{5472}{.308} = 17\,766 \text{ psi}$$

**Stresses in Thin-Wall Cylinders.** An important application of riveted and welded joints is in the construction of boilers and tanks. The pressure of gases or liquids upon the walls of a tank acts outwardly in all directions and uniformly. Therefore, the cylinder shell on a thin-wall vessel is designed with the assumption that the stress is uniform throughout the wall thickness.

The tensile stress in the ends of the cylinder, caused by the pressure inside, is called *longitudinal stress*, or *tension.* The tensile stress acting in the circumferential direction is called *hoop stress,* or *tension.*

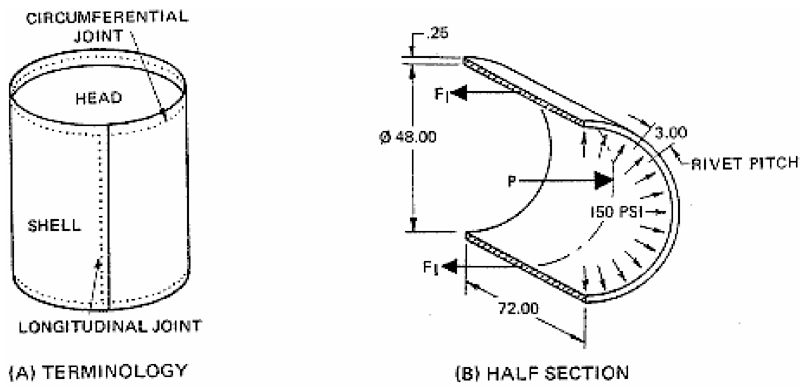



**Fig. 29-2-8** Thin-wall cylinder.

**EXAMPLE 5** A tank of 48.00-in. diameter is made of .25-in. steel plate. The internal pressure is 150 psi. Calculate the size of rivets required if the pitch on the longitudinal and circumferential joints is 3.00 in.

**Solution**

1. Calculate rivet size for longitudinal seam. Figure 29-2-8 shows a half-section of the tank. The internal pressure of 150 psi acts on the shell surface at every point. The total force acting on the half of the tank shown would be equal to the area of the tank taken at its center times the pressure, or (48.00 x 72.00) in.$^2$ x 150 psi = 518 400 lb. The combined equal pressures of $\mathbf{F_1}$ and $\mathbf{F_1}$ acting on the tank wall are equal in magnitude to **P** but act in opposite directions.

Only the pitch distance of 3.00 in. the repeated section, need be used in calculating the size of the rivet along the joint. Therefore:

$$\mathbf{F_1} \text{ for repeated section} = \frac{48.00 \times 3.00 \times 150}{2} = 10\,800 \text{ lb}$$

or load acting on each rivet. As previously mentioned, in boiler construction the diameter of the rivet hole, which is .06 in. larger than the diameter of the rivet, is used in computing all the stresses. Refer to Fig. 29-2-1. The allowable stress in single shear is 15 kips per in$^2$. The chart shows values of 9.02 and 11.78 kips for rivet sizes of $\phi$.875 and $\phi$1.00 in., respectively. Since the finished size of the .875-in. diameter rivet will be. 938 in. in diameter, the allowable load will be computed on the final size. Therefore the allowable load for a .938-in. diameter rivet will be

$$\text{Area x stress} = (\pi \text{ x } .938^2 \div 4) \text{ in.}^2 \text{ x 15 kips} = .69 \text{ in.}^2 \text{ x } 15\,000 = 10\,365 \text{ lb}$$

Since this is less than the load acting on the rivet, the next size larger rivet must be used, and therefore the $\phi$1.00 in. rivet is required. The allowable load in bearing for a $\phi$1.00-in. rivet on .25-in. steel plate is 11 250 lb. Therefore the size of rivet required along the longitudinal seam is $\phi$1.00 in.

**2.** Calculate rivet size for circumferential joint. Number of pitches or repeated sections on circumference equals

$$\frac{\pi \times \text{diameter}}{\text{pitch}} = \frac{\pi \times 48.00}{3.00} = 50.3$$

Use 51 rivets.

Pressure exerted on head of tank = pressure x area = 150 psi x ($\pi$ x $24.00^2$) $\text{in.}^2$ = 271 434 lb. Therefore

$$\text{Load per rivet} = \frac{271\ 434}{51} = 5322 \text{ lb}$$

*Note:* When the pitches on the longitudinal and the circumferential joints are equal, then the load per pitch on the circumferential joint is one-half of the load per pitch on the longitudinal joint.

Refer to Fig. 29-2-1. The allowable load in single shear for a $\phi$.62-in. rivet (use .69 in. for calculations) is 5609 lb and the allowable load in bearing for a $\phi$.62-in. rivet (use .69 in. for calculations) on .25-in. steel is 7762 lb. Therefore the size of rivet required along circumferential joint is $\phi$.62 in.

## BOLTS, SCREWS, AND STUDS

As mentioned at the beginning of this unit, the tensile stress permitted for A307 bolts and threaded parts of A36 steel is equivalent to 22 000 psi applied at the root area of the threads.

**EXAMPLE 6** What force is required to strip the threads on a 1.000-8 UNC regular hex nut and bolt?

**Solution** The sheared area will be equal to the circumference of the root circle multiplied by the height of the nut. Root diameter of 1.000-8 UNC thread = .847 in., circumference = 2.66 in. Height of 1.000 in. regular hex nut (see Appendix) = .875 in. Shear area = 2.66 x .875 = 2.33 $\text{in.}^2$. Ultimate tensile strength of A36 steel = 58 kips (see Fig. 29-1-8).
Therefore

Force required to strip threads = **S** x *A* = 58 000 x 2.33 = 135 140 lb

**EXAMPLE 7** A platform is supported by four A36 steel rods that are suspended from the ceiling. The ends of the rods are threaded, and plate washers and nuts are attached. The platform is to support a load of 16 000 lb and any two of the four rods must be capable of supporting the entire load.

**Solution** The design load for each rod is 16 000 ÷ 2 = 8000 lb. Allowable unit stress (see Fig. 29-1-8) = 22 000 lb per $\text{in.}^2$.

$$A = \frac{P}{S} = \frac{8000}{22\,000} = .364\ \text{in.}^2$$

Note that in example 6 the shear area for a 1.000 in. threaded nut is 2.33 in.$^2$. Therefore, a smaller threaded nut is required. Try a .375 UNC style 1 hex nut.

Area = circumference of root diameter x height of nut = $(\pi \times .312) \times .328 = .321$ in.$^2$. A greater root area is required. A style 2 hex nut that is thicker or a .438 in. nut will be needed. Area of a .375 UNC style 2 hex nut = $(\pi \times .312) \times .406 = .398$ in. $^2$. Therefore, $\phi$.375 rods with UNC threads and style 2 hex nuts meet the design requirements.

As explained in Chap. 8, property class numbers that designate their strength defines metric threaded fasteners. The first number of a two-digit symbol or the first two numbers of a three-digit symbol approximates 1 percent of the minimum tensile stress in megapascals.

The last numeral approximates one-tenth of the ratio expressed as a percentage between minimum yield stress and minimum tensile stress.

**EXAMPLE 8**

What mass can be supported by an M24 × 3 stud, property class 8.8, if a factor of safety of 4 is added to the requirements?

**Solution** Refer to Fig. 29-2-9. Under the 8.8 column, an M24 × 3 thread has a tensile strength (the maximum force permitted) of 293 kN. Adding a factor of safety to this value we find the permissible force is 293 ÷ 4 = 73.25 kN. 1N = 0.102 kg

| Thread | Tensile Stress Area mm² | Thread Stress Area mm² | Tensile Strength (kN) for Property Class | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | 4.6 | 4.8 | 5.8 | 8.8 | 9.8 | 10.9 | 12.9 |
| M10 × 1.5 | 58 | 15.6 | 23.2 | 24.4 | 30.2 | | 52.2 | 60.3 | 70.8 |
| M12 × 1.75 | 84 | 19 | 33.7 | 35.4 | 43.8 | | 75.9 | 87.7 | 103 |
| M14 × 2 | 115 | 22.4 | 46 | 48.3 | 59.8 | | 104 | 120 | 140 |
| M16 × 2 | 157 | 26.1 | 62.8 | 65.9 | 81.6 | 130 | 141 | 163 | 192 |
| M20 × 2.5 | 245 | 33.3 | 98 | | 127 | 203 | | 255 | 299 |
| M24 × 3 | 353 | 40.5 | 141 | | 184 | 293 | | 367 | 431 |
| M30 × 3.5 | 561 | 51.6 | 224 | | | 466 | | 583 | 684 |
| M36 × 4 | 817 | 63.1 | 327 | | | 678 | | 850 | 997 |

**Fig. 29-2-9** Load capacities of threaded fasteners.

Therefore, mass that can be supported = 73 250 x 0.102 = 7472 kg

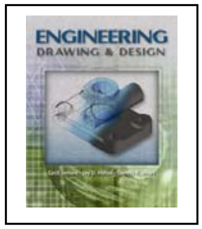
**References and Source Material**

1. American Institute of Steel Construction

**CHAPTER 29**

# Strength of Materials

## UNIT 29-3
## Welded Joints




In addition to riveting, welding is also employed in the joining of structural steel. Fabricated steel construction has also replaced many parts formerly made by casting because of the lower cost and the greater strength at a considerable reduction in size and weight, or mass.

The two types of welds most frequently used are fillet and butt welds. Thus only these types will be covered in this unit.

### FILLET WELDS

The fillet weld is used to join two parts that either overlap or join at an angle, normally perpendicular, to each other. In the calculations of strength of fillet welds, the *effective area* is considered as the *effective length* of weld times the *effective throat thickness,* as illustrated in Fig. 29-3-1.

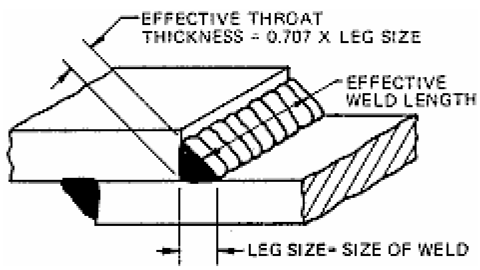



**Fig. 29-3-1** Fillet weld nomenclature.

For example, a .38 in. fillet weld 6.00 in. long (effective length) has an effective area of .38 x .7 x 6.00, or 1.596 in.$^2$.

For purposes of calculating the strength of welds in this unit, the shear stresses shown in Fig. 29-3-2 will be used. The strength of the previous weld would be stress times effective area.

| Electrodes | | | |
|---|---|---|---|
| U.S. Customary | | Metric | |
| E60XX | E70XX | E410XX | E480XX |
| 13 500 lb/in.$^2$ | 15 700 lb/in.$^2$ | 185 MPa | 216 MPa |

**Fig. 29-3-2** Shear stress for electrodes.

With E60xx electrodes the weld strength is

13 500 x 1.596 = 21 546 lb

Using E70xx electrodes, the weld strength is

15 700 x 1.596 = 25 057 lb

Another method of calculating the strength of welds is to multiply the leg size of the weld by effective length. The shear resistance factors (SRF) shown in Fig. 29-3-3 are based on the shear stresses shown in Fig. 29-3-2.

The strength of the previous weld using E70xx electrodes would be 4.2 x 6 = 25.2 kips or 25 200 lb.

The strength value for specified weld sizes is the more convenient method to use.

The following recommendations should be adhered to when welded joints are designed.

- Even-number-mm-size welds, such as shown in Fig. 29-3-3, should be used whenever possible.
- For metric length of welds use lengths evenly divisible by 5, such as 40, 50, 60, etc.
- Fillet welds should be at least .06 in. (2 mm) less than the thickness of the part being welded.
- Welds should be located on both sides of T joints evenly spaced around the line of action of the applied load.

| Fillet Weld Size Inches | Allowable Load Per Inch Length in Kips (Base Metal / Metal Electrodes) | |
|---|---|---|
| | A36[1] E60XX Electrode | A572 E70XX[2] Electrode |
| .18 | 1.8 | 2.1 |
| .25 | 2.4 | 2.8 |
| .31 | 3.0 | 3.5 |
| .38 | 3.6 | 4.2 |
| .44 | 4.2 | 4.9 |
| .50 | 4.8 | 5.6 |
| .62 | 6.0 | 7.0 |
| .75 | 7.2 | 8.4 |

1. Based on shear resistance factor of 0.131 kN per mm of weld for 1 mm of weld length (185 MPa shear stress).
2. Based on shear resistance factor of 0.153 kN per mm of weld for 1 mm of weld length (216 MPa shear stress).

| Fillet Weld Size mm | Allowable Load Per mm Length in kN (Base Metal / Metal Electrodes) | |
|---|---|---|
| | A36[3] E410XX Electrode | A572M[4] E480XX Electrode |
| 4 | 0.5 | 0.6 |
| 6 | 0.8 | 0.9 |
| 8 | 1 | 1.2 |
| 10 | 1.3 | 1.5 |
| 12 | 1.6 | 1.8 |
| 16 | 2.1 | 2.4 |
| 20 | 2.6 | 3 |

3. Based on 600 lb per .062 in. of weld thickness (13 500 lb/in.$^2$ shear stress).
4. Based on 700 lb per .062 in. of weld thickness (15 800 lb/in.$^2$ shear stress).

**Fig. 29-3-3** Strength of fillet welds.

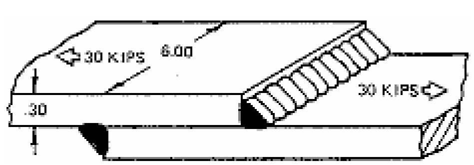



**Fig. 29-3-4** Lap joint.

**EXAMPLE 1** Two .38 x 6.00 in. steel bars are welded with F70 electrodes as shown in Fig. 29-3-4. What size weld is required if a tensile load of 30 kips is applied?

**Solution**

$$\text{Weld size} = \frac{\text{load}}{\text{length} \times \text{SRF}}$$

$$\text{SRF} = \frac{3}{2 \times 6.00} = 2.5$$

Refer to Fig. 29-3-3. A .25 in. fillet weld is required.

**EXAMPLE 2** A .75 x 4.00 in. A572-50 bar is welded to a column. What are the size and length of the fillet welds required if a tensile load of 40 kips is applied?

**Solution** Since two fillet welds will be used, one on each side of the bar, each fillet weld will be designed to resist a force of 40 kips ÷ 2 = 20 kips.

With a weld running the entire length of 4.00 in., a weld having the strength of 20 ÷ 4 or 5 kips per inch of length is required.

Refer to Fig. 29-3-3: a .50 in. fillet weld is selected. With a .62 in. weld, a weld length of 20 ÷ 7.0 = 2.85 in. (use 3.00 in.) would be required.

**EXAMPLE 3** A 250 x 10 mm. A572M-380 steel plate is connected by a pair of fillet welds to the bottom flange of a beam, as shown in Fig. 29-3-5. The plate is subjected to a tensile load of 450 kN. What are the minimum size and length of weld recommended for this connection, if the maximum stress on the plate is 220 MPa?

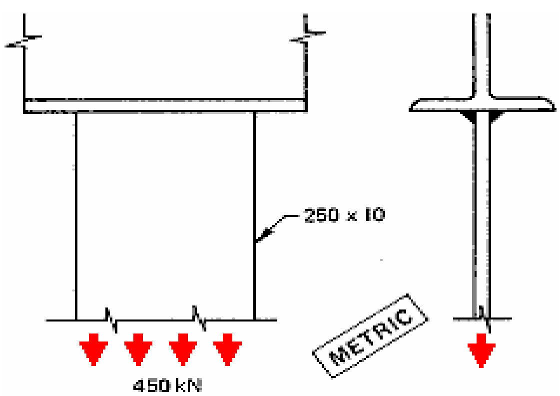



**Fig. 29-3-5** Bar fillet welded both sides.

**Solution** Before the weld size is chosen, the minimum plate area at the weld should be established. The minimum plate area, for calculating purposes, at the welded area (see Fig. 29-3-6) is width times plate thickness.

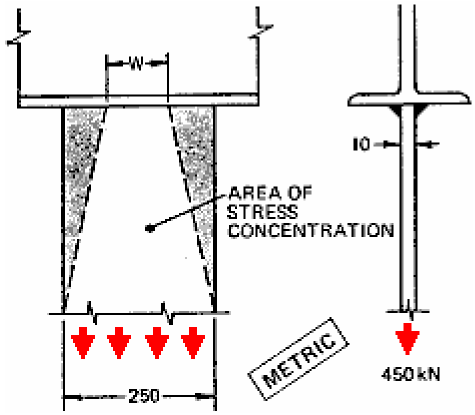



**Fig. 29-3-6** Plate area at weld.

Load = stress x area.

Therefore

$$\text{Minimum width of plate} = \frac{\text{load}}{\text{stress} \times \text{plate thickness in meters}}$$
$$= \frac{450\,000}{(220 \times 10^{6})\,\text{Pa} \times 0.01\,\text{m}}$$
$$= 0.2045\,\text{m or } 204.5\,\text{mm}$$

Thus, the plate width of 250 mm is acceptable. If the minimum weld length is 210 mm per side (min. plate width), the load per min of weld is 450 ÷ (2 x 210) = 1.07 kN.

Refer to Fig. 29-3-3. An 8-mm fillet weld is required. If the fillet weld were to run the entire length of the plate, then the load per mm of weld = 450 + (2 x 250) = 0.9 kN. This would permit a 6-mm weld to be used, which is more economical.

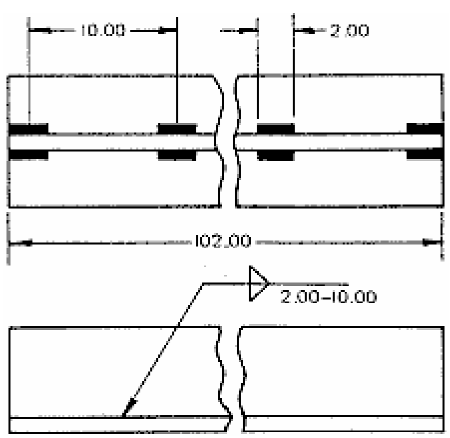



**Fig. 29-3-7** Intermittent weld.

## Intermittent Fillet Welds

Intermittent fillet welds may be used to transfer calculated stress across a joint when the strength required is less than that developed by a continuous fillet weld of the smallest permitted size, and to join components of built-up members. The effective length of any segment of intermittent fillet welding should be not less than 4 times the weld size, with a minimum of 1.50 in. (40 mm).

**EXAMPLE 4** Calculate the size of the intermittent weld shown in Fig. 29-3-7 to safely carry a tensile load of 180 kips. Use F70xx electrodes.

**Solution**

Total length of welds = 11 x 2.00 x 2 = 44.00 in.

$$\text{Load per inch length of weld} = \frac{180}{44} = 4.09 \text{ kips.}$$

Refer to Fig. 29-3-3. Size of fillet weld required is .38 in.

## Fillet Welds for Angle Iron

When tension or compression members are connected by two side fillet welds as shown in the previous examples, the weld should be placed in the same line of action as the force being transmitted by the weld. For members having symmetrical cross sections, the length of weld on each side of the member should be equal. For members having unsymmetrical cross sections, as shown in Fig. 29-3-8 where an angle iron is welded to a steel plate, the lengths of welds are so proportioned that the line of action of the force transmitted by the weld will be along the axes of the two members. This is accomplished by assuming that the line of action on the angle member is on the *centroidal axis* (center of gravity) and by making the lengths of welds such that $L_1 \times A = L_2 \times B$.

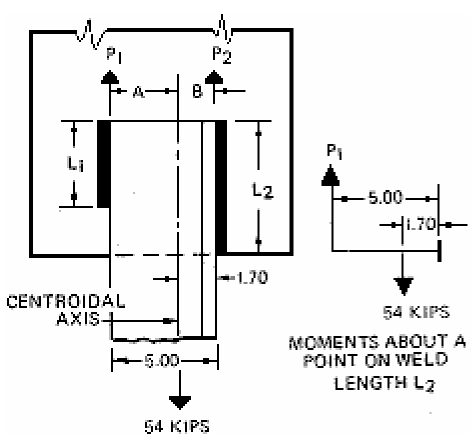



**Fig. 29-3-8** Fillet weld on both sides of angle iron.

**EXAMPLE 5** A 5.00 x 3.00 x .38 in. A572-45 angle welded to a steel plate transmits a load of 54 kips. Calculate the length of welds on each side of the angle so that the load acts along the centroidal axis of the angle. See Fig. 29-3-8.

**Solution** The maximum fillet weld for .38-in.-thick material is .31 in. The allowable load per inch length of a .31-in. weld is 3.5 kips. See Fig. 29-3-3. Therefore:

$$\text{Minimum permissible length of weld} = \frac{54}{3.5} = 15.43 \text{ in.} (\text{use } 15.50 \text{ in.})$$

Consider the 54 kip load being transferred to the plate by loads $P_1$ and $P_2$ where $P_1 = 3.5 \times L_1$; $P_2 = 3.5 \times L_2$; and $P_1$ and $P_2$ both equal 54 kips. Taking moments about a point on length $L_2$ (refer to Fig. 29-4-3 for calculation of moments), we have

$$\begin{aligned} P_1 \times 5.00 &= 54 \times 1.70 \\ P_1 &= 54 \times 1.70 \div 5.00 \\ &= 18.3 = 3.5 \times L_1 \\ L_1 &= 5.23 \text{ in.} (\text{use } 5.25 \text{ in.}) \end{aligned}$$

Therefore

$$\begin{aligned} L_1 &= 5.23 \text{ in.} (\text{use } 5.25 \text{ in.}) \\ L_1 + L_2 &= 15.50 \text{ in.} \\ L_2 &= 15.50 - 5.25 = 10.25 \text{ in.} \end{aligned}$$

Therefore weld lengths of 5.25 ($L_1$) and 10.25 in. ($L_2$) are selected.

**EXAMPLE 6** Use the same members and load as in the previous example except that the fillet weld is welded on three sides, as shown in Fig. 29-3-9.

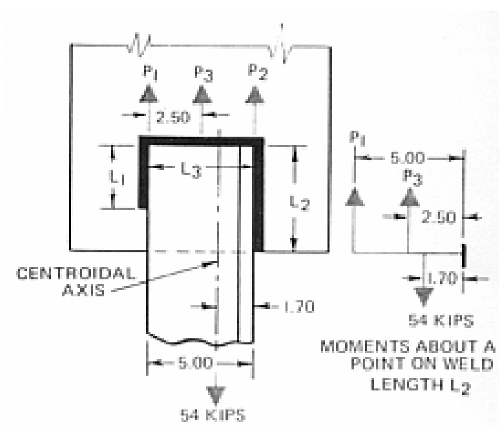



**Fig. 29-3-9** Fillet weld on sides and end of angle iron.

**Solution** The design calls for 15.50 in. of weld to be used; 5.00 in. of the weld lies along the back of the angle so that the remaining 10.25 in. of weld is equal to the combined length of welds $L_1$ and $L_2$.

The 15.50-in. of weld should be located so that the line of action of the force transmitted by the weld is along the centroidal axis of the angle.

For calculation purposes, assume there are three welds, $P_1$, $P_2$, and $P_3$ whose combined loads equal 54 kips, and the allowable load per inch length of a .06 in. weld is 3.5 kips. $P_1$ lies along distance $L_1$ and is equal to $3.5L_2$ kips. $P_2$ lies along distance $L_2$ and is equal to $3.5L_2$ kips. $P_3$ lies midway along the 5.00-in. width, or 2.50 in. from $P_1$, and is equal to $5.00 \times 3.5 = 17.5$ kips. At a point on line $L_2$

Clockwise moments

$= (P_1 \times 5.00) \div (P_3 \times 2.50)$

$= (L_1 \times 3.5 \times 5.00) + (17.5 \times 2.50)$

$= 17.5L_1 + 43.75$ in. kips

Counterclockwise moments

$= 1.70 \times 54 = 91.8$ in. kips

Clockwise moments = counterclockwise moments

$17.5\,L_1 + 43.75$ in. kips = 91.8 in. kips

$17.5\,L_1 = 48.05$ in. kips

$L_1 = 2.75$ in.

$L_1 + L_2 = 10.50$ in.

$L_2 = 10.50 - 2.75$

$L_2 = 7.75$ in.

Therefore weld lengths of 2.75-in. ($L_1$) and 7.75 in. ($L_2$) are used on the sides of the angle.

## BUTT WELDS

The butt weld is used to join two pieces of metal that lie on the same plane. In the calculations of strength of butt joints, the effective area of butt welds shall be considered as the effective length of weld times the effective throat thickness. The effective throat thickness depends on the metal thickness, the gap between the adjoining parts, the type of butt weld, and whether the weld is on one or both sides. See Fig. 29-3-10.

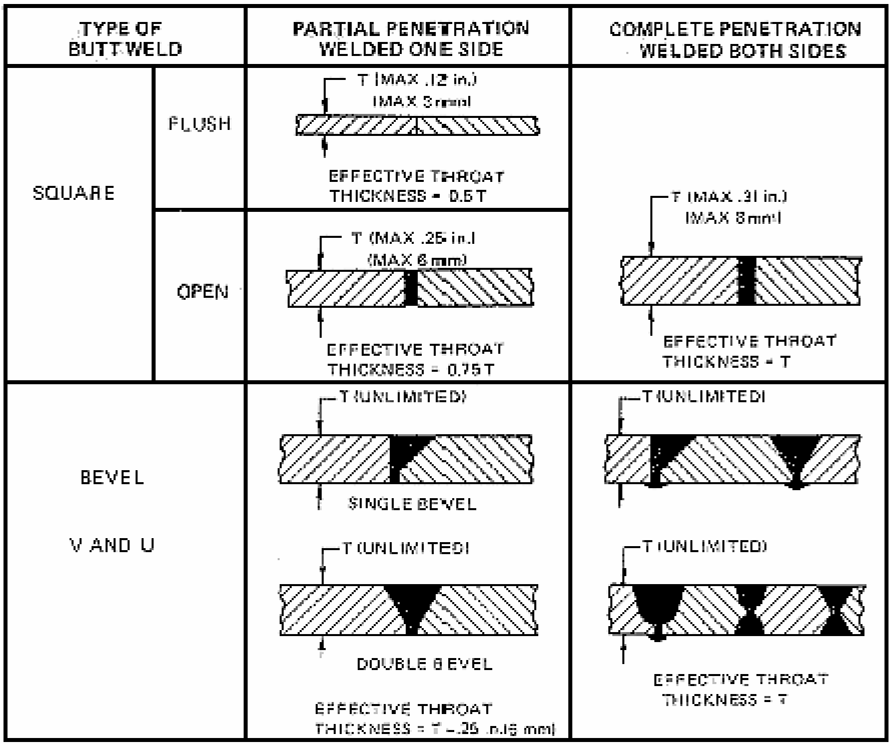



**Fig. 29-3-10** Strength of butt welds.

**EXAMPLE 7** An open-square butt weld, welded one side, is used to join two A36 steel plates .25 x 10.00 in. Compute the safe tensile load that can be applied to the joint.
**Solution** Refer to Fig. 29-3-10. The effective throat thickness for an open-square butt weld = 0.75*T* = 0.75 x .25 = .188 in. Next (refer to Fig. 29-1-8), the allowable unit tensile stress for A36 steel is 22 kips. Therefore:

Safe load = area x unit stress
= (.188 x 10.00) x 22 000
= 141.36 kips

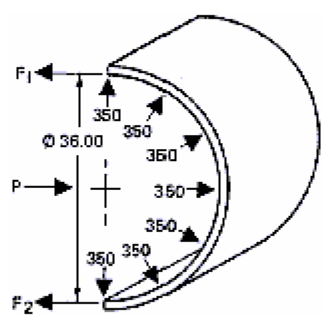



**Fig. 29-3-11** Welded boiler section.

**EXAMPLE 8** A ϕ36 in. boiler, made of A36 steel, has to withstand a steam pressure of 350 psi. A single-V butt joint, welded one side, is to be used. What is the thickness of boiler plate required?

**Solution** Figure 29-3-11 shows a half section of the boiler. The total force $P$ acting on the cylinder is the resultant pressure of the internal pressure of 350 psi acting in all directions on the cylinder wall. It will be assumed that for thin-walled cylinders the resultant force $P$ will equal the diameter of the cylinder in inches, times the length of the cylinder in inches, times the pressure acting within the cylinder.

The total force $P$ is resisted by two equal forces $F_1$ and $F_2$. Taking a section of the tank 1 in. in length and calculating the forces $P$, $F_1$, and $F_2$, we have

$P = \mathbf{S} \times A$
$= 350 \times 1.00 \times 36.00$
$= 12\,600$ lb
$F_1 = 12\,600 \div 2 = 6300$ lb

The single-V butt weld will have to withstand a force of 6300 lb for every inch of weld. Allowable unit stress for A36 steel is 22 000 psi. Weld stress equals plate stress. Therefore

Effective throat thickness of weld
$= F_1 \div (S \times$ length of section)
$= 6300 \div (22\,000 \times 1.00)$
$= .28$ in.

Refer to Fig. 29-3-10; note that the effective throat thickness of a single-V butt weld, welded one side, under tension is equal to $T$ - .25 in., where $T$ is equal to the thickness of plate. Therefore, minimum plate thickness must be .25 + .28 = .53 in. to safely carry the load. With a welded-both-sides joint, the plate thickness could be reduced to .28 in., which would be a considerable saving.

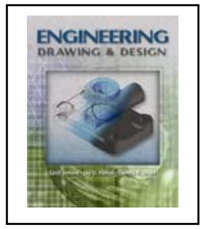
**Reference and Source Material**

1. American Institute of Steel Construction.

**CHAPTER 29**

# Strength of Materials

**UNIT 29-4**
**Beams**




A *beam* is a structural member or machine part which supports transverse (i.e., perpendicular) loads and reactions. Most beams are placed in a horizontal position with vertical forces acting on them. Examples are floor and ceiling joists, lintels, and floor beams. This unit covers the design of simple beams only where buckling and twisting are not factors and where the beams are of uniform size and shape for the entire length. Forces acting on the beams are assumed to be in the same plane.

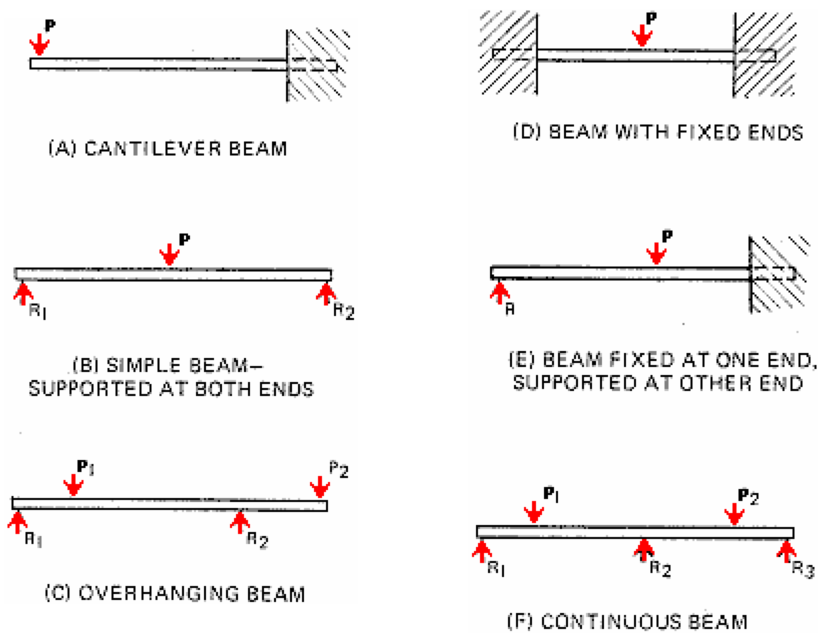



**Fig. 29-4-1** Common types of beams.

### Types of Beams

Beams are classified according to the manner in which they are supported. Some of the more common types of beams are shown in Fig. 29-4-1. They are

1. Cantilever beam: a beam that has one fixed end
2. Simple beam: a beam that is supported at each end
3. Overhanging beam: a beam that has one or both ends projecting beyond its supports
4. Beams with both ends fixed
5. Beams fixed at one end and supported at the other end
6. Continuous beam: a beam supported at more than two points

## KINDS OF LOADS

Two types of loads commonly occur on beams: *concentrated* and *uniformly distributed* loads. A concentrated load extends over a short length of the beam and for calculation purposes is considered as acting at one point. It is usually represented by a line with an arrow indicating its direction of force and the letter *P* as shown in Fig. 29-4-2A. Concentrated loads are generally expressed in pounds or kips (U.S. Customary), or kilonewtons or meganewtons (metric). One kip equals 1000 pounds. A uniformly distributed load is one in which the load is distributed uniformly over a given length or over the entire length of the beam. The weight or mass of the beam is an example of a uniformly distributed load. This type of load is generally expressed in pounds or kips per foot (U.S. Customary), or newtons per meter or kilonewtons per meter (metric). The load is represented in the figure by a rectangular block resting on the beam, as shown in Fig. 29-4-2B.

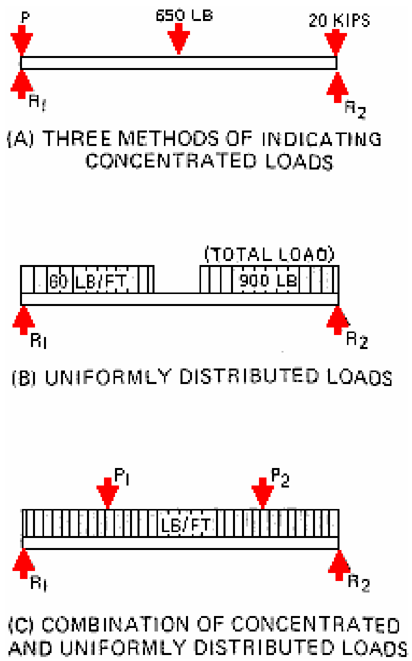



**Fig. 29-4-2** Representation on beam drawings of loads.

The upward forces, or supports, that hold the beam in a state of equilibrium are called the *reactions* and are designated by the letters $R_1$ (left side) and $R_2$ (right side). The sum of the reactions $R_1 + R_2$, known as the *forces acting upward,* are equal and opposite to the downward forces or loads.

## MOMENTS

When a force acts upon an object at a distance from the object, as through a beam, the force is called a *moment.* See Fig. 29-4-3. A moment is the tendency of a force to cause rotation about a given point or axis. The magnitude of a moment is equal to the magnitude of the force times the perpendicular distance to the point. Since the force is measured in pounds and the distance in feet or inches, moments are measured in foot-pounds (ft-lb) or inch-pounds (in.-lb). In the metric system the moments are measured in newton-meters (N-m).

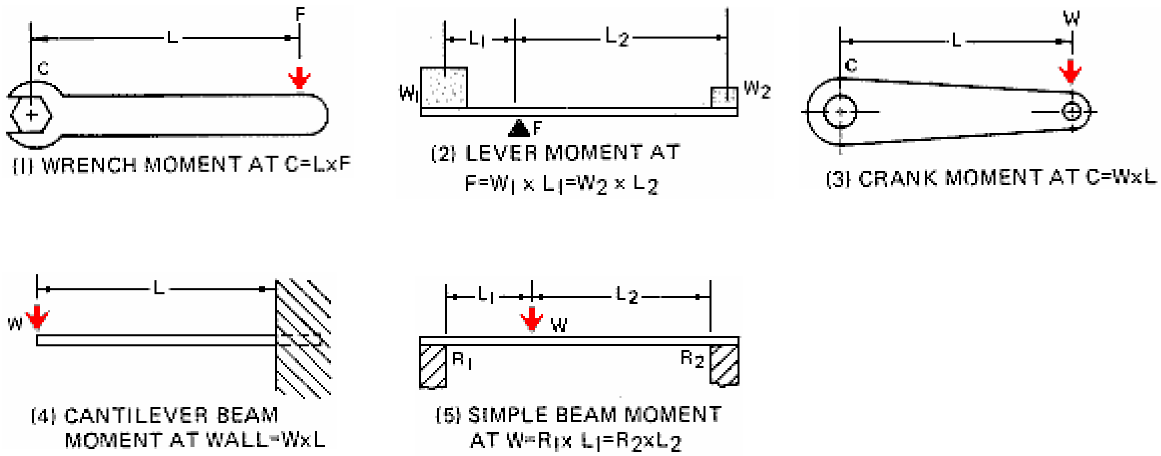



**Fig. 29-4-3** Application of moments.

If a number of forces acting on a point are in equilibrium, the sum of the moments of all the forces about that point is zero. Therefore, the sum of the moments of all forces that tend to produce clockwise moments about a given point is equal and opposite to the sum of all the forces that tend to produce counterclockwise moments at that given point. This *law of equilibrium is* very helpful in solving beam reaction.

**EXAMPLE 1** A force of 20 lb is applied at the end of a wrench 12 in. from the center of the bolt that is being held by the wrench. Calculate the moment.

**Solution** The moment may be found by multiplying the force times the distance: 20 x 12 = 240 in-lb.

**EXAMPLE 2** A cantilever beam supports a concentrated load of 500 lb located 12 ft from the support. Neglecting the mass of the beam, calculate the moment at the wall.

**Solution** The moment taken at the wall or support may be found by multiplying the force times the distance: 500 x 12 = 600 ft-lb.

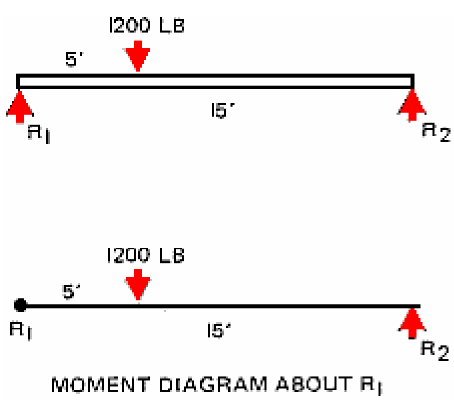



**Fig. 29-4-4** Simple beam with concentrated load.

**EXAMPLE 3** A beam 15 ft long has a concentrated load of 1200 lb acting 5 ft away from the left reaction. Neglecting the mass of the beam, calculate the reaction forces. See Fig. 29-4-4.

**Solution** Taking moments about reactions $R_1$ and $R_2$ we have

Clockwise moments using pounds and feet = 5 x 1200 = 6000 ft-lb.

Counterclockwise moments = 15 x $R_2$

Clockwise moments = Counterclockwise moments

6000 ft-lb = 15 x $R_2$

Therefore

$$R_2 = \frac{6000}{15} = 400 \text{ lb}$$

$$R_1 + R_2 = 1200 \text{ lb}$$

Thus

$$R_1 = 1200 - 400 = 800 \text{ lb}$$

**EXAMPLE 4**

Use the same data given in Example 3, but include the force of gravity acting on the beam, which is 40 lb/ft. See Fig. 29-4-5.

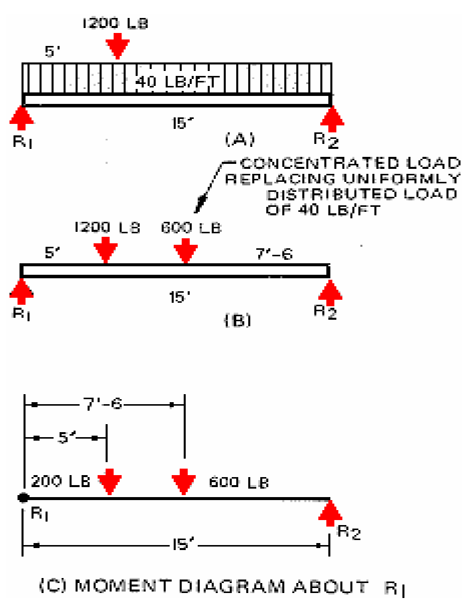



**Fig. 29-4-5** Simple beam with uniformly distributed and concentrated loads.

**Solution** Since the force of gravity acting on the beam is uniformly distributed, a concentrated load of 600 lb located midway on the beam, as shown in Fig. 29-4-5B, would have the same effect on the reactions $R_1$ and $R_2$.

Therefore by substituting the 600-lb concentrated load 7'-6 from the reactions for the uniformly distributed load, the reaction forces can now be found.

Taking moments about reaction $R_1$, (Fig. 29-4-5C), we have

Clockwise moments using pounds and feet

$$= (1200 \times 5) + (600 \times 7.5)$$
$$= 6000 + 4500$$
$$= 10\ 500 \text{ ft-lb}$$

Clockwise moments = Counterclockwise moments

10 500 ft-lb = 15 x $R_2$

Therefore:

$$R_2 = 10\ 500 \div 15 = 700 \text{ lb}$$

$R_1 + R_2 = 1800$ 1b

Thus

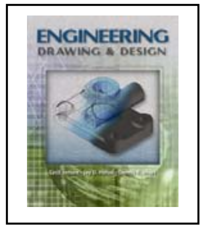
$$R_1 = 1800 - 700 = 1100 \text{ lb}$$

**CHAPTER 29**

## Strength of Materials

## UNIT 29-5

### Shear Diagrams




When a beam supports a load, there is a tendency for the beam to fail by shear. In design work, it is essential to know what shear force a beam must resist at any section. The *vertical shear force* at a section of a beam is the algebraic sum of all the external forces acting on either side of the section. This can be further simplified by stating that the vertical shear at any section is equal to the product of the reaction minus the loads. The *section* is the name given to the cross section of the beam where the calculations are made. For simplification, only the forces acting to the left of the section will be calculated in this unit.

Shear is designated as either *positive shear* or *negative shear*. When the sum of the vertical forces to the left of the section is upward, the shear is positive. When the sum of the vertical forces to the left of the section is downward, the shear is negative. See Fig. 29-5-1.

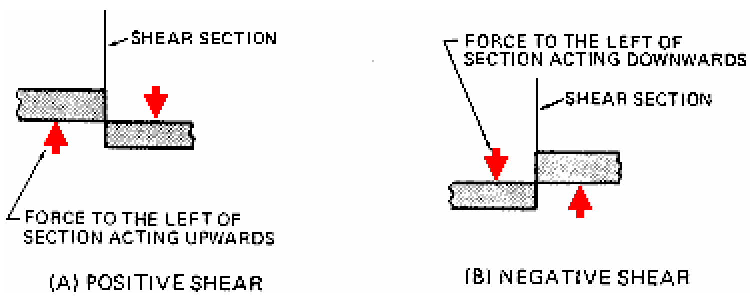



**Fig. 29-5-1** Designation of positive and negative shear.

This information is represented in a shear force diagram that is normally drawn below the loading diagram of the beam. A horizontal zero base line is drawn to the same horizontal scale as the loading diagram, and the positive shear is shown above this line while the negative shear is drawn below it. The magnitude of the shear at each section is shown by vertical lines drawn to a convenient scale.

In order to identify the section at which the shear is taken, a symbol (the letter $V$ followed by a number) is used. The letter $V$ refers to the magnitude of the vertical shear, and the number refers to the horizontal distance from the left end of the beam. Thus $V_4$ refers to the shear force taken at a section 4 ft away from the left reaction ($R_1$) of a simple beam, or 4 ft away from the free end of a cantilever beam. The mass of the beam will not be considered unless specified in the examples or problems.

## CANTILEVER BEAMS

Cantilever beams should be drawn with the support shown at the RH side.

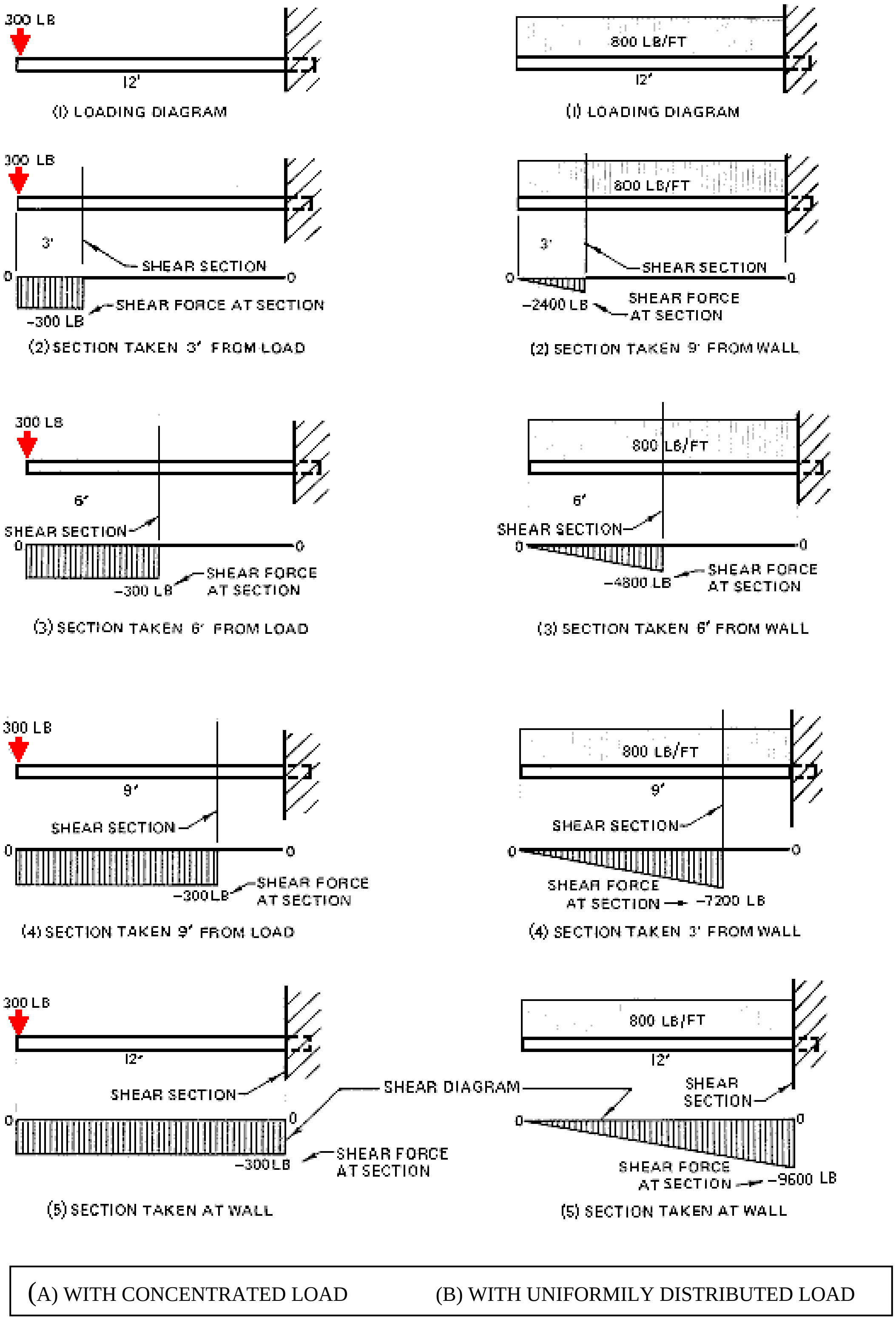



**Fig. 29-5-2** Construction of shear diagram for cantilever beams.

**EXAMPLE 1** Figure 29-5-2A represents a cantilever beam with a concentrated load at the free end. Construct the shear diagram.

**Solution** Taking sections at various points along the beam and calculating *V*, the vertical shear to the left of the section, we have

$V_0$ = 0 – 300 = -300 lb
$V_3$ = 0 – 300 = -300 lb
$V_6$ = 0 – 300 = -300 lb
$V_9$ = 0 – 300 = -300 lb
$V_{12}$ = 0 – 300 = -300 lb

Since there is no reaction to the left of the section, the shear values are all negative and are drawn below the base line.

**EXAMPLE 2** Figure 29-5-2B illustrates a cantilever beam with a uniformly distributed load. Construct the shear diagram.
**Solution** Taking sections at various points along the beam, starting at the free end, we have

$V_0$ = 0 - 0 = 0 lb
$V_3$ = 0 - (800 x 3) = -2400 lb
$V_6$ = 0 - (800 x 6) = - 4800 lb
$V_9$ = 0 - (800 x 9) = -7200 lb
$V_{12}$ = 0 - (800 x 12) = -9600 lb

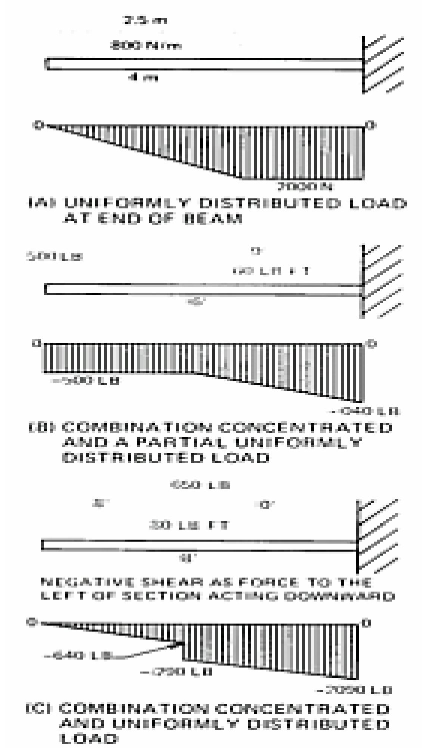



**Fig. 29-5-3** Shear diagrams for cantilever beams.

**EXAMPLE 3** Figure 29-5-3A illustrates a cantilever beam with a uniformly distributed load at the end of the beam. Construct the shear diagram.

**Solution** Taking sections at various points along the beam, starting at the free end, we have

$V_0 = 0 - 0 = 0$ N
$V_1 = 0 - 800 = -800$ N
$V_2 = 0 - 800 \times 2 = -1600$ N
$V_{2,5} = 0 - 800 \times 2.5 = -2000$ N
$V_3 = 0 - 800 \times 2.5 = -2000$ N
$V_4 = 0 - 800 \times 2.5 = -2000$ N

**EXAMPLE 4** Figure 29-5-3B shows a cantilever beam with a concentrated load at the free end of the beam and a uniformly distributed load at the fixed end. Construct the shear diagram.

**Solution** Taking sections at various points along the beam, starting at the free end, we have

$V_0 = 0 - 500 = - 500$ lb
$V_4 = 0 - 500 = - 500$ lb
$V_8 = 0 - 500 - (1 \times 60) = -560$ lb
$V_{12} = 0 - 500 - (5 \times 60) = -800$ lb
$V_{16} = 0 - 500 - (9 \times 60) = -1040$ lb

**EXAMPLE 5** Figure 29-5-3C shows a cantilever beam with a uniformly distributed load and a concentrated load acting in the middle section of the beam. Construct the shear diagram.

**Solution** Taking sections at various points along the beam, starting at the free end, we have

$V_0$ = 0 - 0 = 0 lb
$V_4$ = 0 - (4 x 80) = - 320 lb
$V_8$ = 0 - (8 x 80) = - 1290 lb
$V_{12}$ = 0 (12 x 80) = - 1610 lb
$V_{18}$ = 0 (18 x 80) = - 2090 lb

## SIMPLE BEAMS

In constructing the shear diagram for a simple beam, the magnitude of the reactions must be calculated first. The shear diagram is constructed in the same manner as for a cantilever beam. For calculation purposes $V_0$ will be considered as the section where the beam leaves the reaction $R_1$, and the shear section taken at $R_2$ will be considered as the section where the beam leaves the reaction $R_2$.

**EXAMPLE 6** Figure 29-5-4A illustrates a simple beam with a concentrated load of 800 lb at the center of the span. Construct the shear diagram.

**Solution** From the figure it is apparent that the reactions are 400 lb each, because of the symmetrical loading. As previously mentioned, only the forces acting to the left of the section will be used for calculating the shear diagram. At the left end of the beam, the only force acting on the beam is the upward force of the reaction $R_1$. Therefore the shear force at $V_0$ = 1400 - 0 = + 400 lb. Taking sections along the beam, we find that

$V_4$ = 400 - 0 = + 400 1b
$V_7$ = 400 - 0 = + 400 lb

Up to and including the section just to the left of the center of the beam, no new forces are encountered; therefore from $V_0$ to $V_{7.99}$ the shear force is +400 lb. At the center of the beam the downward force of 800 lb occurs; thus

$V_8$ = 400 - 800 = - 400 lb

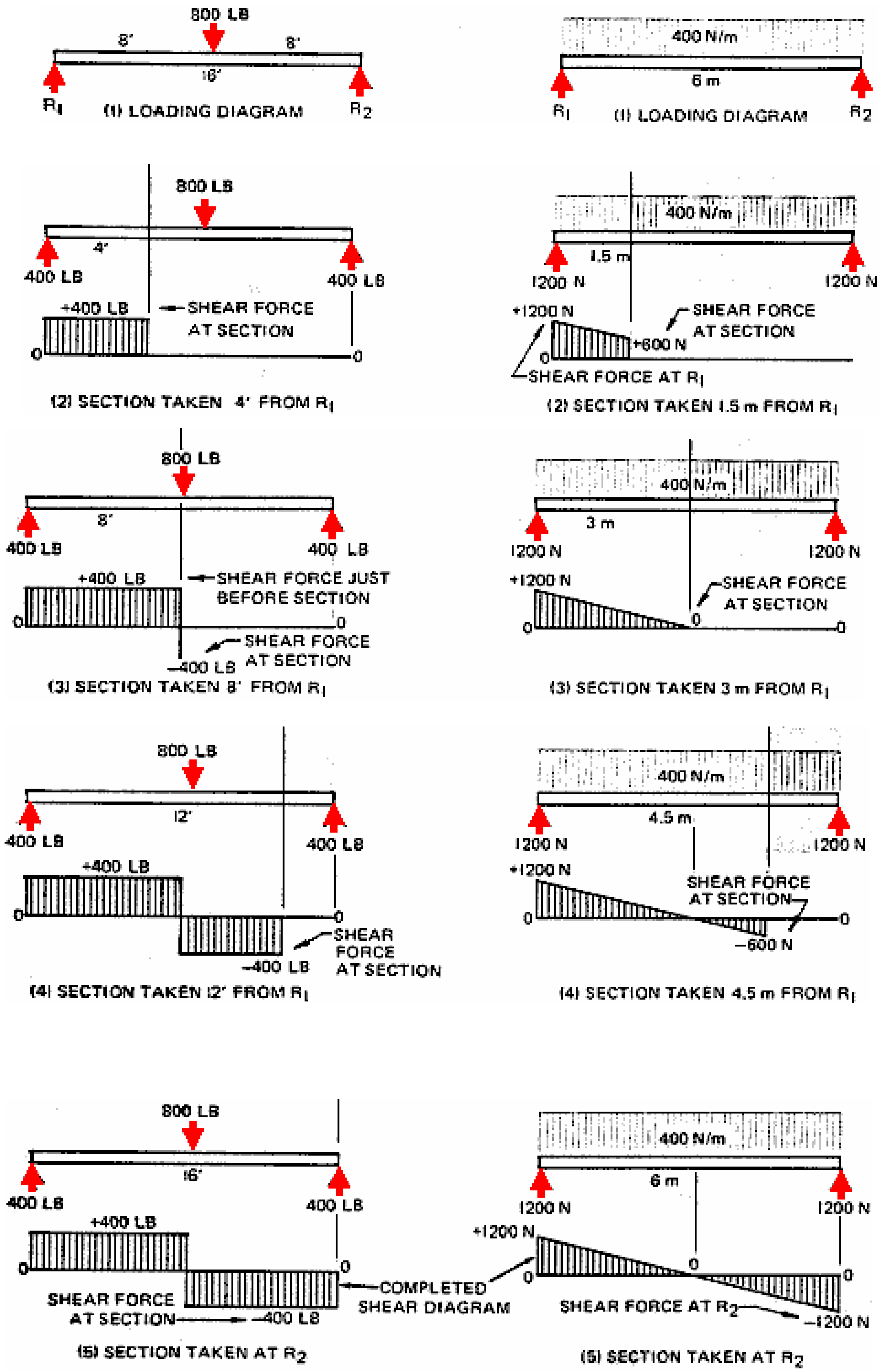
Since no new loads are encountered for the remainder of the beam unit $R_2$ is reached,

$V_{12}$ = 400 - 800 = -400 lb

$V_{14}$ = 400 - 800 = -400 lb

Just before $R_2$ is reached, the shear force from $V_8$ to $V_{16}$ is - 400 lb.

Note that the shear diagram passes through zero, from + 400 lb to - 400 lb at the 800-lb load.




| (A) WITH CONCENTRATED LOAD | (B) WITH UNIFORMILY DISTRIBUTED LOAD |
|---|---|

**Fig. 29-5-4** Construction of shear diagram for simple beams.

**EXAMPLE 7** Figure 29-5-4B illustrates a simple beam with a uniformly distributed load. Construct the shear diagram.
**Solution** Taking sections at intervals along the beam, we have

$V_0 = 1200 - 0 = +1200$ N
$V_{1.5} = 1200 – (1.5 \times 400) = +600$ N
$V_3 = 1200 – (3 \times 400) = +0$ N
$V_{4.5} = 1200 – (4.5 \times 400) = -600$ N
$V_6 = 1200 – (6 \times 400) = -1200$ N

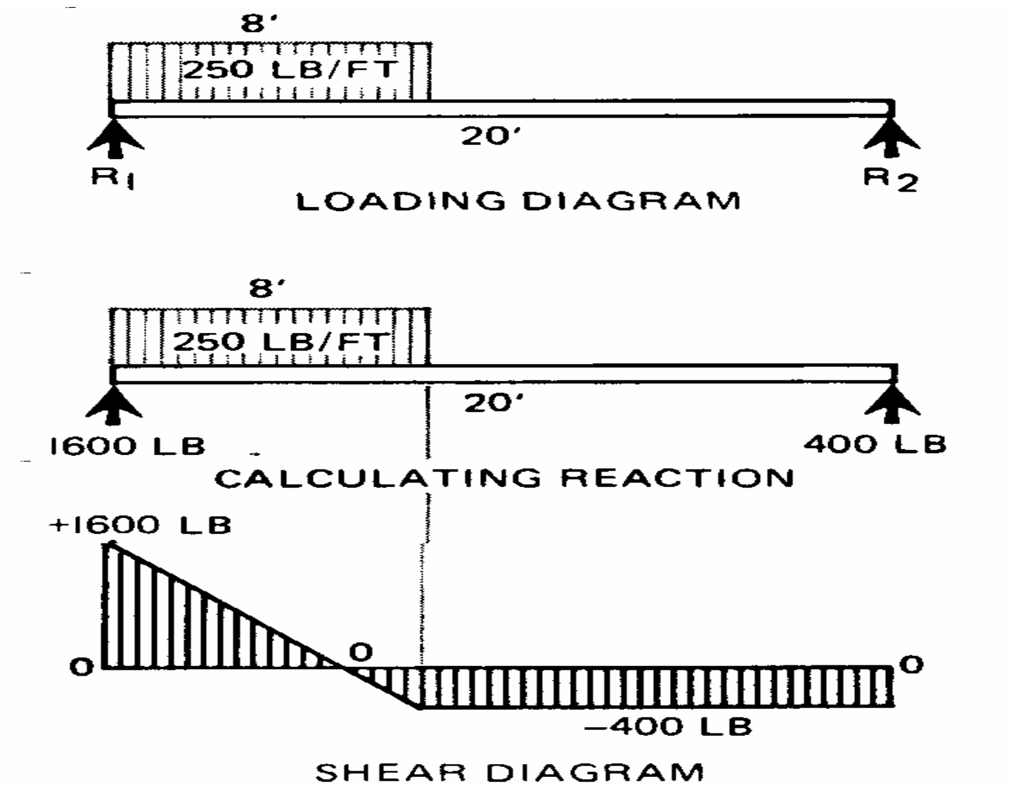



**Fig. 29-5-5** Simple beam with partial, uniformly distributed load.

**EXAMPLE 8** Figure 29-5-5 illustrates a simple beam with a partial, uniformly distributed load starting at reaction $R_1$. Construct the shear diagram.

**Solution** The values of reactions $R_1$ and $R_2$ must first be found. For calculation purposes, a concentrated load of $250 \times 8$, or 2000 lb, acting at the center of the uniformly distributed load will be used in place of the uniformly distributed load. Taking moments about $R_1$, we have:

Clockwise moments = $4 \times (8 \times 250) = 8000$ ft-lb

Counterclockwise moments = $20 \times R_2$
$R_2 = 8000 \div 20 = 400$ lb

$R_1 = 2000 – 400 = 1600$ lb

Taking sections at intervals along the beam starting at reaction $R_1$, we have:

$V_0 = 1600 - 0 = + 1600$ lb
$V_4 = 1600 - (4 \times 250) = + 600$ lb
$V_8 = 1600 - (8 \times 250) = - 400$ lb
$V_{12} = 1600 - (8 \times 250) = - 400$ lb
$V_{20} = 1600 - (8 \times 250) = - 400$ lb

From the shear diagram it can be seen that the shear passes from positive shear to negative shear between $V_4$ and $V_8$. How to locate the position of zero shear will be discussed in Unit 29-6.

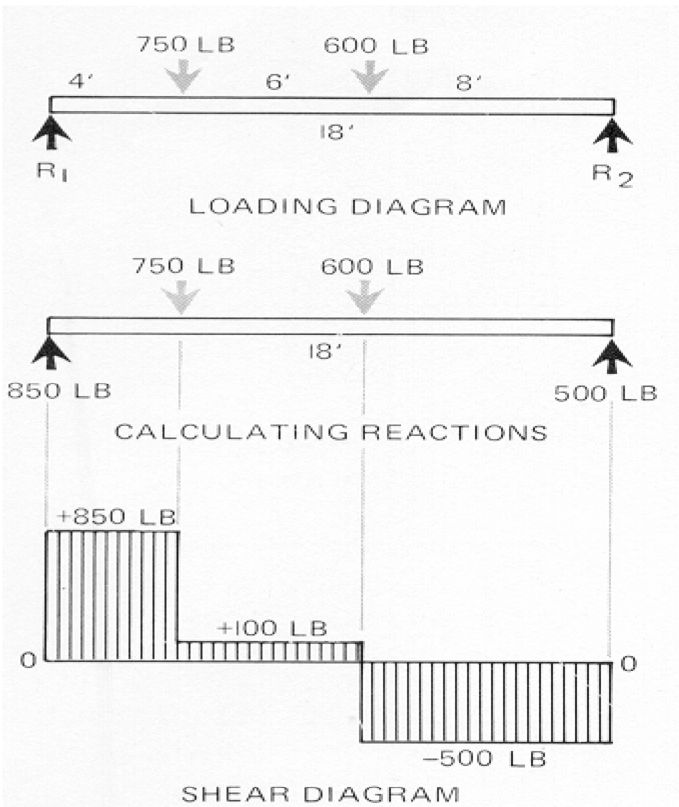



**Fig. 29-5-6** Simple beam with two concentrated loads.

**EXAMPLE 9** Figure 29-5-6 illustrates a simple beam with two concentrated loads. Construct the shear diagram.
**Solution** The reactions must first be calculated. Taking moments about reaction $R_1$, we have

Clockwise moments
= (4 x 750) + (10 x 600)
= 3000 + 6000 = 9000 ft-lb

Counterclockwise moments = 18 x $R_2$

Clockwise moments = Counterclockwise moments

18 x $R_2$ = 9000 ft-lb

$$R_2 = \frac{9000}{18} = 500\,\text{lb}$$

$R_1$ + $R_2$ = 750 + 600 = 1350 lb

Therefore

$R_1$ = 1350 - 500 = 850 1b

Taking sections at intervals along the beam, we have

$V_0$ = 850 - 0 = + 850 lb
$V_4$ =850-750= + l00 lb
$V_{10}$ = 850 - 750 - 600 = - 500 lb
$V_{18}$ = 850 - 750 - 600 = - 500 lb

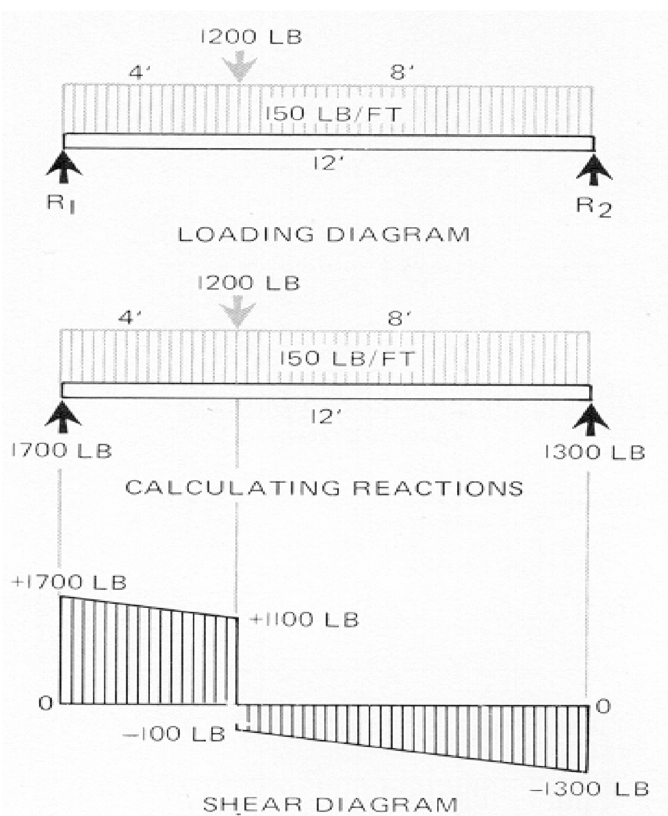



**Fig. 29-5-7** Simple beam with uniformly distributed load and concentrated load.

**EXAMPLE 10** Figure 29-5-7 illustrates a simple beam with a uniformly distributed load and a concentrated load acting on it. Construct the shear diagram.

**Solution** The reactions must be calculated first. For calculation purposes, a concentrated load of 12 x 150, or 1800 lb, acting at the center of the beam, will be used in place of the uniformly distributed load. Taking moments about $R_1$, we have

Clockwise moments
= (4 x 1200) + (6 x 1800)
= 4800 + 10 800 = 15 600 ft-lb

Clockwise moments = Counterclockwise moments
$12 \times R_2 = 15\,600$ ft-lb
$R_2 = 15\,600 \div 12 = 1300$ lb
$R_1 + R_2 = 1200 + 1800 = 3000$ lb

Therefore

$R_1 = 3000 - R_2 = 3000 - 1300 = 1700$ lb.

Taking sections at intervals along the beam gives us

$V_0 = 1700 - 0 = +17001b$
$V_2 = 1700 - (2 \times 150) = + 1400$ lb
$V_4 = 1700 - (4 \times 150) – 1200 = - 100$ lb
$V_8 = 1700 - (8 \times 150) – 1200 = - 700$ lb
$V_{10} = 1700 - (10 \times 150) - 1200 = - 1000$ lb
$V_{12} = 1700 - (12 \times 150) - 1200 = - 1300$ lb

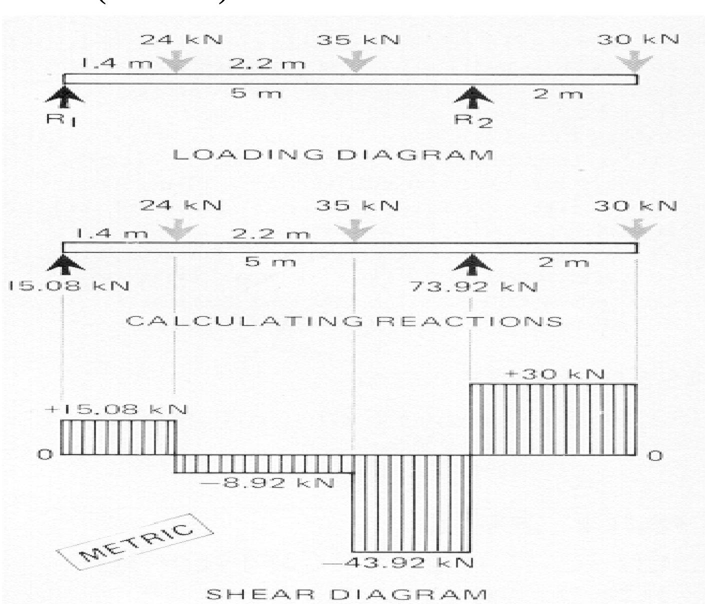



**Fig. 29-5-8** 0verhanging beam.

**EXAMPLE 11** Figure 29-5-8 illustrates an overhanging beam. Construct the shear diagram.
**Solution** First the reactions must be calculated. Taking moments about $R_1$, we have

Clockwise moments

$= (1.4 \times 24) + (3.6 \times 35) + (7 \times 30)$

$= 33.6 + 126 + 210$

$= 369.6$ kN-m

Counterclockwise moments = $5 \times R_2$

Clockwise moments = Counterclockwise moments

$5 \times R_2 = 369.6$ kN-m

$R_2 = 369.6 \div 5 = 73.92$ kN

$R_1 + R_2 = 24 + 35 + 30 = 89$ kN

Therefore

$R_1 = 89 - 73.92 = 15.08$ kN

Taking sections at intervals along the beam gives us

$V_0 = 15.08 - 0 = +15.08$ kN

$V_{1,4} = 15.08 - 24 = -8.92$ kN

$V_3 = 15.08 – 24 = -8.92$ kN

$V_{3,6} = 15.08 - 24 - 35 = -43.92$ kN

$V_5 = 15.08 - 24 - 35 + 73.92 = +30$ kN

$V_7 = 15.08 - 24 - 35 + 73.92 = +30$ kN

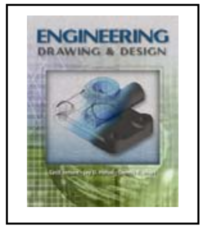
## Conclusion

From the examples given, the following conclusions can be drawn for shear diagrams.

- Where there are concentrated loads, the shear lines are straight horizontal lines changing in value at the loads.
- Where there are uniformly distributed loads, the shear lines are straight inclined lines, the slope of the line being proportional to the load.
- At each concentrated load, including reactions, the shear line rises or drops vertically by an amount equal to the load at that section.

**CHAPTER 29**

# Strength of Materials




## UNIT 29-6
## Bending Moment Diagrams

As previously mentioned, when a load acts on a beam, the force tends to shear the beam. In addition to producing this shearing action, the load tends to *deflect* or *bend* the beam.

To determine this deflection, which varies along the beam, the bending stresses must be calculated. Just as the shear diagram shows the shear at any section along the beam, a bending moment diagram is similarly constructed to show the bending moment at any point along the beam and also to indicate where the maximum bending occurs.

The *bending moment* at any section along the beam is equal to the sum of all the moments of the forces acting to the right or left of the beam. In drawing bending moment diagrams, the following points should be noted:

1. Forces are taken to the left of the section.
2. Upward moments are considered positive and are shown above the base line on the bending moment diagram.
3. Downward moments are considered negative and are shown below the base line.
4. The bending moment diagram is drawn directly below the shear diagram and to the same scale.
5. Shear is equal to reaction minus loads.
6. Bending moments are equal to reaction moments minus load moments.
7. In calculating the bending moments at any given section along the beam, the capital letter $M$ is used to designate the bending moments. It is followed by a subscript which indicates the distance from the LH end of the beam. Thus, $M_4$ indicates the bending moments 4 ft from the LH end of the beam.

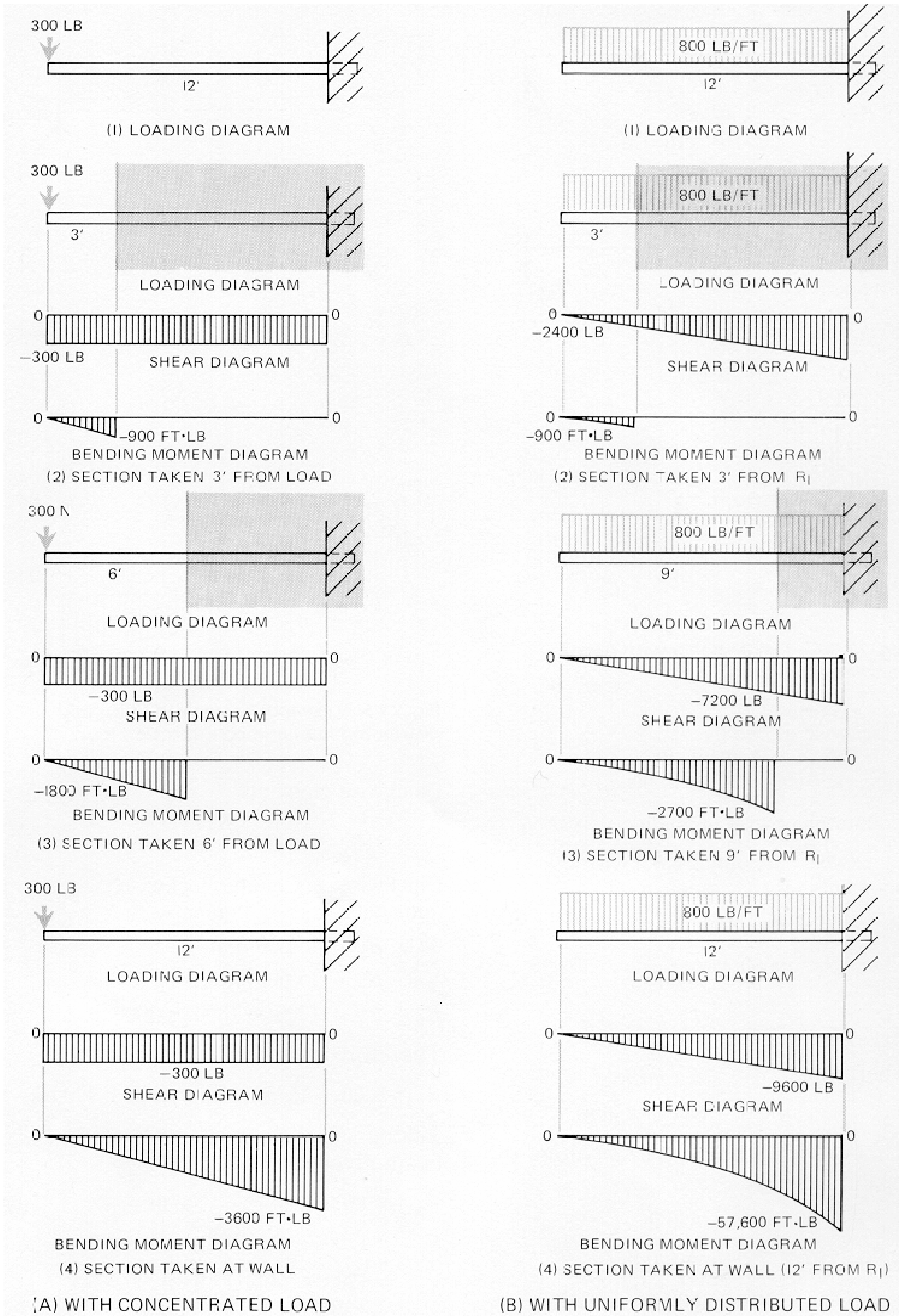



**Fig. 29-6-1** Construction of bending moment diagram for cantilever beams.

**EXAMPLE 1** Figure 29-6-1A shows a cantilever beam with a concentrated load applied to the free end. Figure 29-5-2A shows the shear diagram development for this beam. Construct the bending moment diagram.

**Solution** Taking moments at intervals along the beam starting at the LH end, we find the moments will be negative as the force is acting downward. Thus we have

$M_0$ = - 300 x 0 = 0
$M_3$ = - 300 x 3 = - 900 ft·lb
$M_6$ = - 300 x 6 = - 1800 ft·lb
$M_9$ = - 300 x 9 = - 2700 ft·lb
$M_{12}$ = - 300 x 12 = - 3600 ft·lb

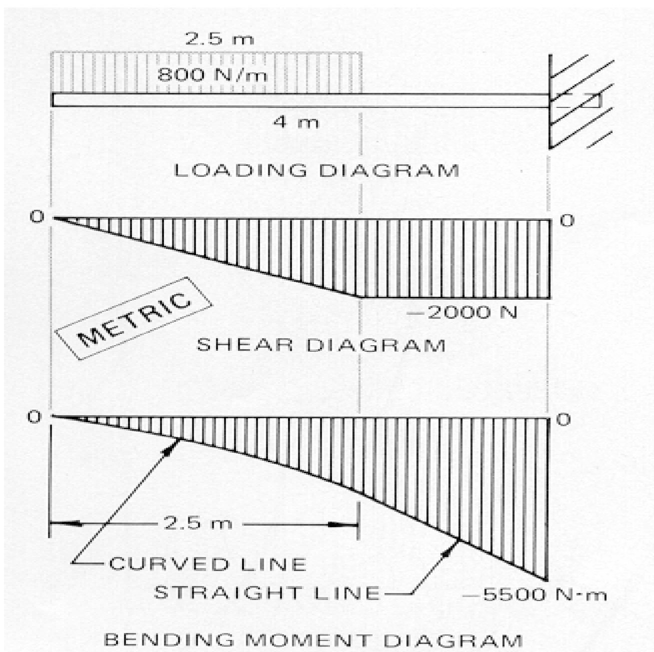



**Fig. 29-6-2** Cantilever beam with uniformly distributed load at end of beam.

**EXAMPLE 2** Figure. 29-6-1B shows a cantilever beam with a uniformly distributed load. Figure 29-5-2B shows the shear diagram development for this beam. Construct the bending moment diagram.

**Solution** The bending moment at $M_0$ is zero. The 3-ft section to the right of $R_1$ weighs 3 x 800 or 2400 lb. The force of any uniform load can be considered as acting at its center of gravity. Thus the 2400 lb load can be considered as acting 1.5 ft away from $R_1$. Therefore, we have

$M_3$ = - (800 x 3) x 1.5 = -3600 ft·lb
$M_6$ = - (800 x 6) x 3 = -14 400 ft·lb
$M_9$ = - (800 x 9) x 4.5 = -32 400 ft·lb
$M_{12}$ = - (800 x 12) x 6 = -57 600 ft·lb

**EXAMPLE 3** Figure 29-6-2 shows a cantilever beam with a uniformly distributed load at the end of the beam. The shear diagram development was explained in Unit 29-5, Example 3, Fig. 29-5-3A. Construct the bending moment diagram.
**Solution** Taking moments at intervals along the beam starting from the LH end, we have

$M_0 = 0$
$M_1 = - 800 \times 1 \times 0.5 = - 400$ N·m
$M_2 = - 800 \times 2 \times 1 = - 1600$ N·m
$M_{2.5} = - 800 \times 2.5 \times 1.25 = - 2500$ N·m
$M_3 = - 800 \times 2.5 \times (3 – 1.25) = -3500$ N·m

Note that at $M_3$, the distance from the section to the center of gravity of the load is 3 - 1.25 = 1.75 m, since the center of gravity is 1.25 m from the LH end.

$M_4 = -800 \times 2.5 \times (4 - 1.25) = - 5500$ N·m

Note that from $M_{2.5}$ to $M_{4,}$ the line on the bending moment diagram is straight.

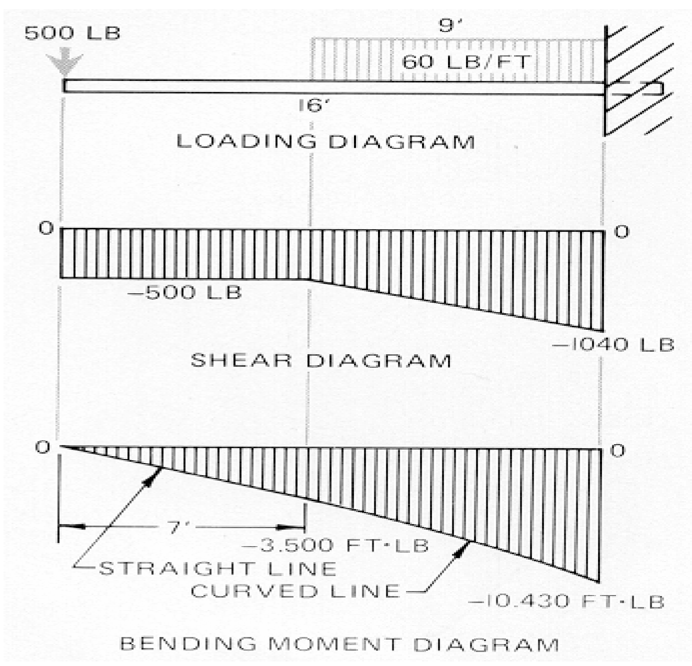



**Fig. 29-6-3** Cantilever beam with concentrated load and partial, uniformly distributed loads.

**EXAMPLE 4** Figure 29-6-3 shows a cantilever beam with a combination concentrated and a partial, uniformly distributed load. The shear diagram development was explained in Unit 29-5, Example 4, Fig. 29-5-3B. Construct the bending moment diagram.

**Solution** The bending moments from $M_0$ to $M_{16}$ are calculated in the same manner as in Example 1.

$M_0$ = - 500 x 0 = 0
$M_1$ = - 500 x 1 = - 500 ft·lb
$M_4$ = - 500 x 4 = - 2000 ft·lb
$M_7$ = - 500 x 7 = - 3500 ft·lb
$M_9$ = - (500 x 9) - (60 x 2 x 1) = -4620 ft·lb
$M_{12}$ = - (500 x 12) - (60 x 5 x 2.5) = -6750 ft·lb
$M_{16}$ = - (500 x 16) - (60 x 9 x 4.5) = -10 430 ft·lb

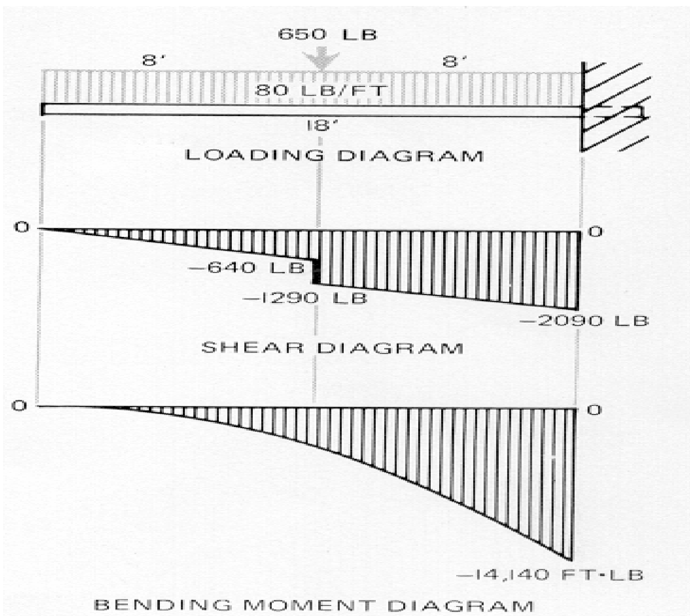



**Fig. 29-6-4** Cantilever beam with concentrated and uniformly distributed loads.

**EXAMPLE 5** Figure 29-6-4 shows a cantilever beam with a concentrated and uniformly distributed load. The shear diagram development was explained in Unit 29-5, Example 5, Fig. 29-5-3C. Construct the bending moment diagram.

**Solution** The bending moments from $M_0$ to $M_8$ are calculated in the same manner as in Example 2.

$M_0$ = 0
$M_2$ = - 80 x 2 x I = - 160 ft·lb
$M_4$ = - 80 x 4 x 2 = - 640 ft·lb
$M_8$ = - (80 x 8 x 4) - (650 x 0) = - 2560 ft·lb
$M_{10}$ = - (80 x 10 x 5) - (650 x 2) = - 5300 ft·lb
$M_{12}$ = - (80 x 12 x 6) - (650 x 4) = - 8360 ft·lb
$M_{14}$ = - (80 x 14 x 7) - (650 x 6) = - 11 740 ft·lb
$M_{18}$ = - (80 x 16 x 8) - (650 x 6) = - 14 140 ft·lb

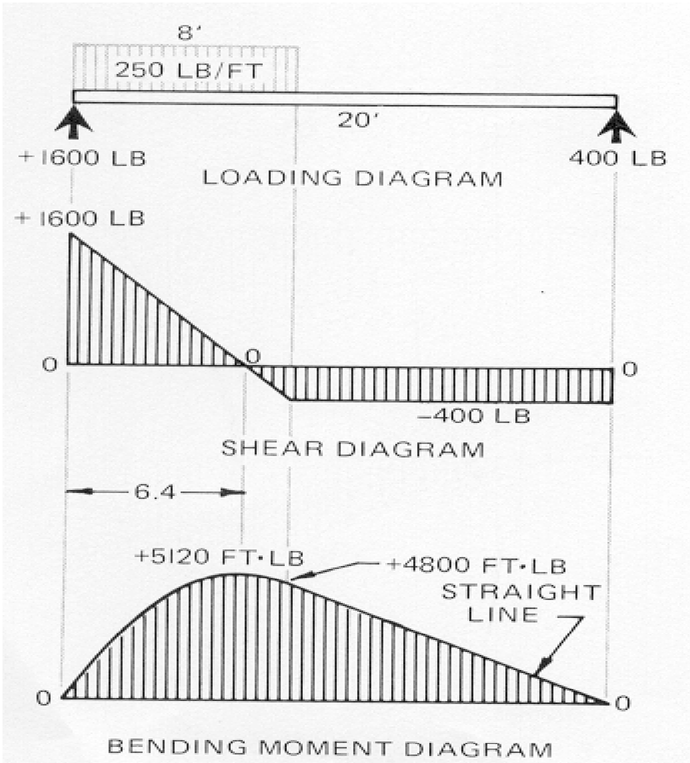



**Fig. 29-6-5** Simple beam with partial, uniformly distributed load.

**EXAMPLE 6** Figure 29-6-5 shows a simple beam with a partial, uniformly distributed load. The development of the shear diagram was explained in Unit 29-5, Example 8, Fig. 29-5-5. Construct the bending moment diagram.

**Solution** Taking moments at intervals along the beam, starting at reaction $R_1$, gives us

$M_0 = +1600 \times 0 = 0$

$M_4 = +1600 \times 4 - 250 \times 4 \times 2 = +3400$ ft·lb

$M_6 = +1600 \times 6 - 250 \times 6 \times 3 = +5100$ ft·lb

$M_8 = +1600 \times 8 - 250 \times 8 \times 4 = +4800$ ft·lb

From the bending moment calculations it can be seen that the maximum bending moment occurs somewhere between $M_6$ and $M_8$ where the zero shear takes place.

The distance between $R_1$ and zero shear can be found as follows. Let the distance from $R_1$ to the point at zero shear be $X$. Thus, we have

$V_X = +1600 - 250 \times X = 0$

$X = 1600 \div 250 = 6.4$ ft

Maximum bending moment occurs at $M_{6.4}$

$M_{6.4} = +(1600 \times 6.4) - (250 \times 6.4 \times 3.2) = +5120$ ft·lb

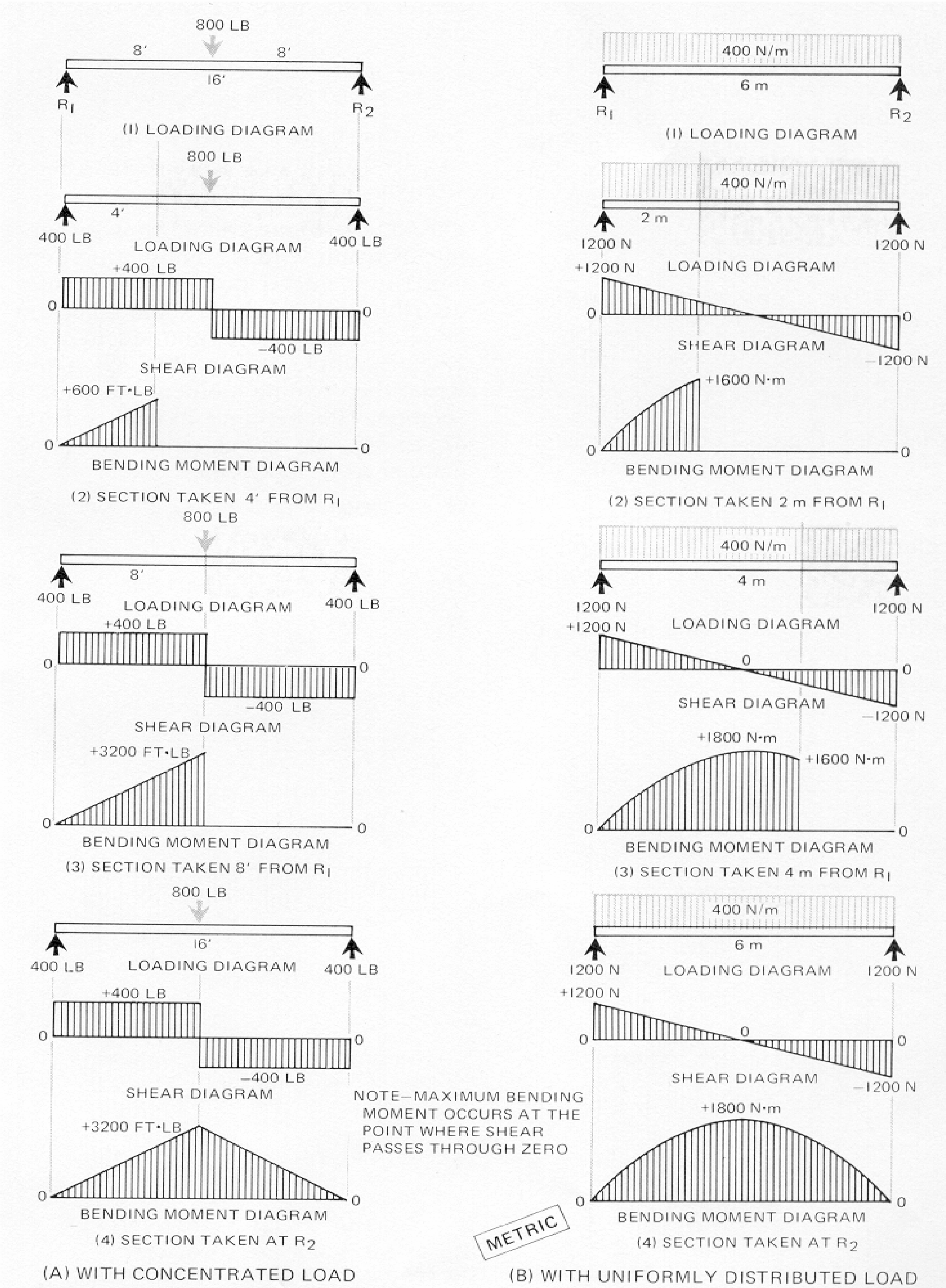



**Fig. 29-6-6** Construction of bending moment diagram for simple beam.

For calculating the moments for the remaining sections, the uniformly distributed load will be considered as a 2000 lb concentrated load acting 4 ft from $R_1$.

$M_{12}$ = + (1600 × 12) - (2000 × 8) = + 3200 ft·lb
$M_{16}$ = + (1600 × 16) – (2000 × 12) = + 1600 ft·lb
$M_{20}$ = + (1600 × 20) – (2000 × 16) = 0

**EXAMPLE 7** Figure 29-6-6A shows a simple beam with a concentrated load acting in the center of the beam. The development of the shear diagram was explained in Unit 29-5, Example 6, Fig. 29-54A. Construct the bending moment diagram.
**Solution** Taking moments at intervals along the beam, starting at reaction $R_1$, which is acting upward, we have

$M_0$ = +400 x 0 - 0
$M_2$ = + 400 x 2 = + 800 ft·lb
$M_4$ = + 400 x 4 = + 1600 ft·lb
$M_8$ = + 400 x 8 - 800 x 0 = + 3200 ft·lb
$M_{12}$ = + 400 x 12 - 800 x 4 = + 1600 ft·lb
$M_{14}$ = +400 x 14 - 800 x 6 = +800 ft·lb
$M_{16}$ = + 400 x 16 - 800 x 8 = 0

*Note:* The maximum bending moment occurs at the point where shear passes through zero.

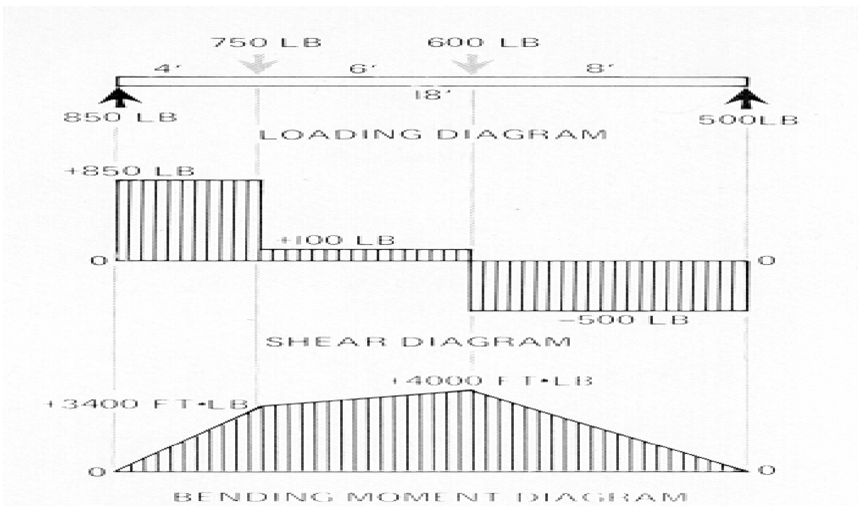
**EXAMPLE 8** Fig. 29-6-6B shows a simple beam with a uniformly distributed load. The development of the shear diagram was explained in Unit 29-5, Example 7, Fig. 29-5-4B. Construct the bending moment diagram.

**Solution** The bending moment at $M_0$ is zero. The 1-m section to the right of $R_1$ creates a force of 400 N. The force of any uniform load can be considered as acting at its center of gravity. Thus, the 400-N load can be considered as acting 0.5 m away from $R_1$. Therefore, we have

$M_1$ = + 1200 x 1 - 400 x 1 x 0. 5 = +1000 N·m
$M_2$= + 1200 x 2 - 400 x 2 x 1 = + 1600 N·m
$M_3$ = + 1200 x 3 - 400 x 3 x 1. 5 = + 1800 N·m
$M_4$ = + 1200 x 4 - 400 x 4 x 2 = + 1600 N·m
$M_5$ = + 1200 x 5 - 400 x 5 x 2.5 = + 1000 N·m
$M_6$ = + 1200 x 6 - 400 x 6 x 3 = 0

Note that the maximum bending moment of + 1800 N·m occurs at zero shear.

**Fig. 29-6-7** Simple beam with two concentrated loads.




**EXAMPLE 9** Figure 29-6-7 shows a simple beam with two concentrated loads. The development of the shear diagram was explained in Unit 29-5, Example 9, Fig. 29-5-6. Construct the bending moment diagram.

**Solution** Taking moments at intervals along the beam, starting at reaction $R_1$, gives us

$M_0 = +850 \times 0 = 0$

$M_2 = +850 \times 2 = +1700$ ft·lb

$M_4 = +850 \times 4 - 750 \times 0 = +3400$ ft·lb

$M_5 = +(850 \times 5) - (750 \times 1) = +3500$ ft·lb

$M_8 = +(850 \times 8) - (750 \times 4) = +3800$ ft·lb

$M_{10} = +(850 \times 10) - (750 \times 6) - (600 \text{ x } 0) = +4000$ ft·lb

$M_{12} = +(850 \times 12) - (750 \times 8) - (600 \text{ x } 2) = +3000$ ft·lb

$M_{14} = +(850 \times 14) - (750 \times 10) - (600 \text{ x } 4) = +2000$ ft·lb

$M_{18} = +(850 \times 18) - (750 \times 14) - (600 \text{ x } 8) = 0$

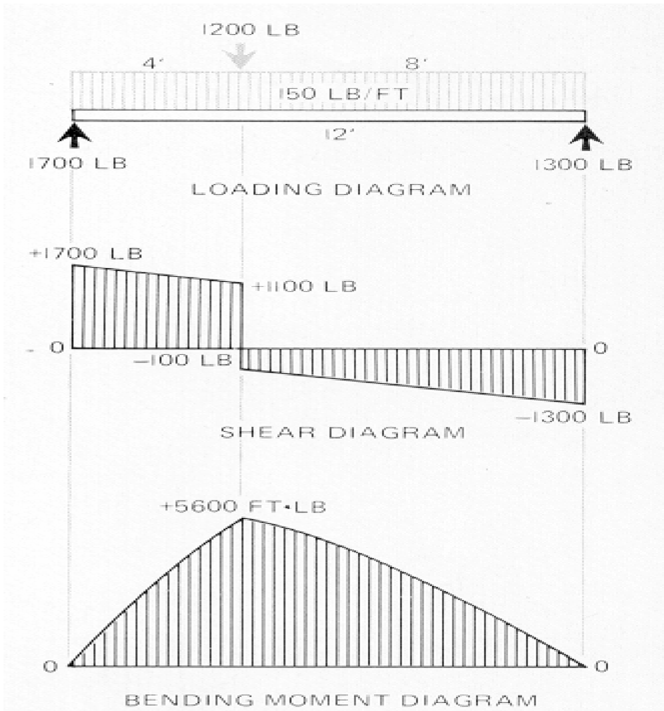



**Fig. 29-6-8** Simple beam with uniformly distributed and concentrated loads.

**EXAMPLE 10** Figure 29-6-8 shows a simple beam with a uniformly distributed load and a concentrated load. The development of the shear diagram was explained in Unit 29-5, Example 10, Fig. 29-5-7. Construct the bending moment diagram.

**Solution** Taking moments at intervals along the beam, starting at reaction $R_{1,}$ we have

$M_0 = + 1700 \times 0 = 0$
$M_2 = + (1700 \times 2) - (150 \times 2) \times 1 = + 3\,100$ ft-lb
$M_4 = + (1700 \times 4) - (150 \times 4 \times 2) - (1200 \times 0) = + 5600$ ft-b
$M_6 = + (1700 \times 6) - (150 \times 6 \times 3) - (1200 \times 2) = + 5100$ ft-lb
$M_8 = + (1700 \times 8) - (150 \times 8 \times 4) - (1200 \times 4) = + 4000$ ft-lb
$M_{10} = + (1700 \times 10) - (150 \times 10 \times 5) - (1200 \times 6) = + 2300$ ft-lb
$M_{12} = + (1700 \times 10) - (150 \times 12 \times 6) - (1200 \times 8) = 0$

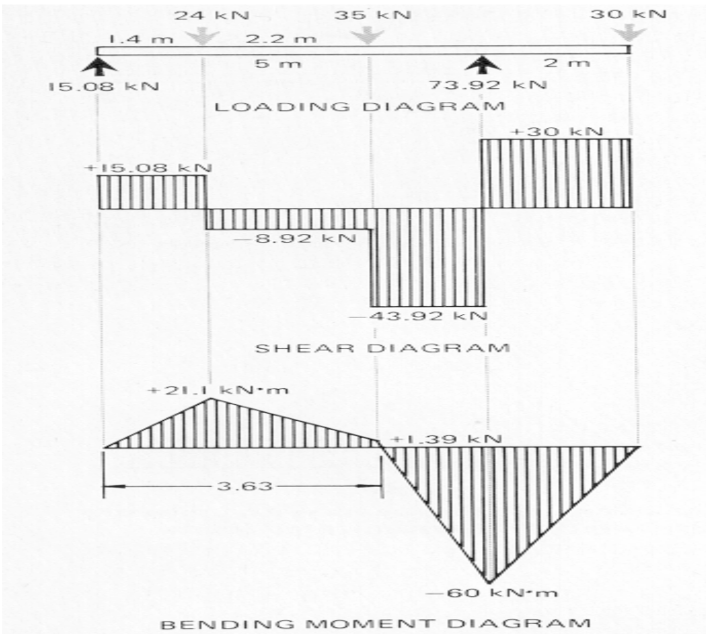



**Fig. 29-6-9** Overhanging beam.

**EXAMPLE 11** Figure 29-6-9 shows an overhanging beam with three concentrated loads. The development of the shear diagram was explained in Unit 29-5, Example 11, Fig. 29-5-8. Construct the bending moment diagram.

**Solution** Taking moments at intervals along the beam, starting at reaction $R_1$, we have

$M_0 = +\ 15.08 \times 0 = 0$
$M_1 = +\ 15.08 \times 1 = +\ 15.08$ kN·m
$M_{1.4} = +\ (15.08 \times 1.4) - (24 \text{ x } 0) = +\ 21.1$ kN.m
$M_2 = +\ (15.08 \times 2) - (24 \times 0.6) = +\ 16.12$ kN·m
$M_3 = +\ (15.08 \times 3) - (24 \times 1.6) = +\ 6.84$ kN-rn
$M_{3.6} = +\ (15.08 \times 3.6) - (24 \times 2.2) - (35 \times 0) = +\ 1.39$ kN·m
$M_4 = +\ (15.08 \times 4) - (24 \times 2.6) - (35 \times 0.4) = -\ 16.08$ kN·m
$M_5 = +\ (15.08 \times 5) - (24 \times 3.6) - (35 \times 1.4) + (73.92 \times 0) = -60$ kN·m
$M_6 = +\ (15.08 \times 6) - (24 \times 4.6) - (35 \times 2.4) + (73.92 \times 1 = -\ 30$ kN·m
$M_7 = +\ (15.08 \times 7) - (24 \times 5.6) - (35 \times 3.4) + (73.92 \times 2) = 0$

From the bending moment diagram it can be seen that zero bending moment occurs to the right of the 35-kN load. Its exact location can be found as follows.
Let the distance between $R_1$ and the point where zero takes place be $X$. Then
$M_X = 0$
$M_X = +\ (15.08 \text{ x } X) - 24(X - 1.4) - 35\ (X - 3.6) = 0$
$M_X = +\ 15.08X - 24X + 33.6 - 35X + 126$
$43.92X = 159.6$
$X = 3.63$ mm

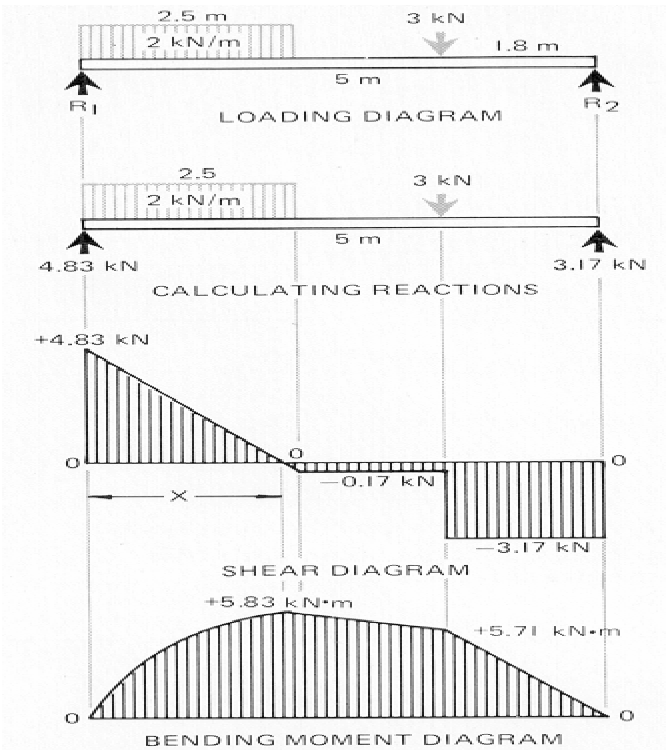



**Fig. 29-6-10** Simple beam with a partial, uniformly distributed load and a concentrated load.

**EXAMPLE 12** Figure 29-6-10 shows a simple beam with a partial, uniformly distributed load and a concentrated load. Find the position and magnitude of the maximum bending moment.

**Solution** Reactions $R_1$ and $R_2$ must be calculated first. Taking moments about $R_1$, we have

Clockwise moments

$= 1.25 \times 2 \times 2.5 + 3.2 \times 3$

$= 15.85$ kN·m

Counterclockwise moments = $5 \times R_2$

$R_2 = 15.85 \div 5 = 3.17$ kN

$R_1 = (2.5 \times 2) + 3 - 3.17 = 4.83$ kN

Next construct the shear diagram, taking sections at intervals along the beam, starting at reaction $R_1$:

$V_0 = 4.83 – 0 = +4.83$ kN

$V_1 = 4.83 - 1 \times 2 = +2.83$ kN

$V_2$ = 4.83 – 2 x 2 = +0.83 kN
$V_{2.5}$ = 4.83 - 2.5 x 2 = - 0.17 kN
$V_{3.2}$ = 4.83 - 2.5 x 2 - 3 = - 3.17 kN
$V_5$ = 4.83 – 2.5 x 2 - 3 = - 3.17 kN

From the shear diagram, it is noted that the section having zero shear lies somewhere between $R_1$ and the end of the 2.5-m uniformly distributed load. Its exact position, $X$ distance from $R_1$, may be found by taking the shear at $V_X$, which is zero.

Thus, $V_X = 4.83 - X \times 2 = 0$. Therefore, $X = 4.83 \div 2 = 2.415$m.

The maximum bending moment will occur where the shear passes through zero or 2.415 m from $R_1$. Thus the maximum bending moment is

$$M_{2.415} = (4.83 \times 2.415) - (2 \times 2.415 \times 1.2075) = 5.83 \text{ kN·m}$$

**EXAMPLE 13** A simple beam 8 m long carries a 3600-N concentrated load 2 m from the left abutment. Calculate the maximum bending moment and shear.

**Solution** The maximum bending moment for a simple beam with a concentrated load at any point (see Fig. 29-6-11) is

$$\frac{FAB}{L} = \frac{3600 \times 2 \times 6}{8} = 5400 \text{ N} \cdot \text{m}$$

Maximum shear =

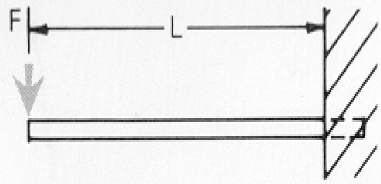
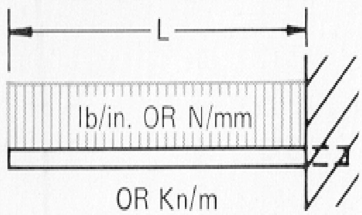
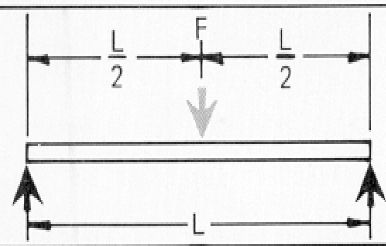
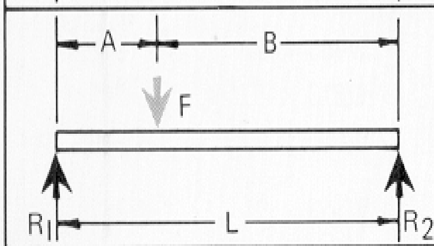
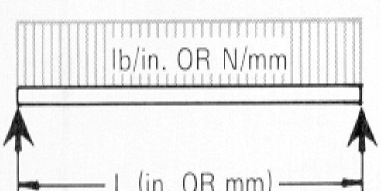
$$\frac{FB}{L} = \frac{3600 \times 6}{8} = 2700 \text{ N}$$

## Conclusion

From the examples given, the following conclusions can be drawn from bending moment diagrams.

1. Where there are no loads on a part of a beam, the bending moment line is a straight, sloping line.
2. Where there is a uniformly distributed load, the bending moment line is a curve.
3. The maximum bending moment occurs at a section on the beam at which the shear passes through zero.

Standard beam formulas are shown in Fig. 29-6-11.

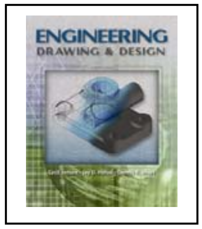
| BEAM AND LOADING<br>L IN METERS | MAXIMUM BENDING MOMENTS | MAXIMUM SHEAR | MAXIMUM DEFLECTION |
|---|---|---|---|
|  | $FL$ | $F$ | $\frac{FL^3}{3EI}$ |
|  | $\frac{NL^2}{2}$ | $NL$ | $\frac{NL^4}{8EI}$ |
|  | $\frac{FL}{4}$ | $\frac{F}{2}$ | $\frac{FL^3}{48EI}$ |
|  | $\frac{FAB}{L}$ | WHEN B IS GREATER THAN A $\frac{FB}{L}$ | $\frac{FA^2B^2}{3EIL}$ |
|  | $\frac{NL^2}{8}$ | $\frac{NL}{2}$ | $\frac{5NL^3}{384EI}$ |

E = MODULUS OF ELASTICITY I = MOMENT OF INERTIA

**Fig. 29-6-11** Maximum bending moments, shear, and deflection for commonly occurring loads on beams.

CHAPTER 29

# Strength of Materials




## UNIT 29-7
## Beam Design

It has been found from experience that beams normally fail at the section where the bending moment is maximum, rather than by shearing at the supports. Therefore, in beam design, it is customary first to select a suitable beam size to withstand the bending forces and then to check it for shear and deflection. The ability of a beam to resist bending depends on such factors as the material used, the shape of its cross section, and the way the cross section is turned with respect to the load. To illustrate this last point, one may bend a flat steel rule across its thin axis; but if the steel rule is set on its edge, then it is virtually impossible to bend the rule in the direction of its width. This resistance to bending can be measured in terms of a quantity called the *section* modulus of the section concerned. The theory and the mathematics behind the development of the section modulus of beams and shapes will not be covered in this text.

Thus the ability of any beam to resist bending is directly related to its section modulus, which is expressed in cubic inches (U.S. Customary) or cubic millimeters (metric) and is denoted $Z$ in calculations. The bending stress $S$, the bending moment $M_0$, and the section modulus are related by the formula $M_0 = Z \times S \div 10^6$ in which the quantities are inch-pounds, cubic inches, and pounds (U.S. Customary) and newtonmillimeters, cubic millimeters, and pascals (metric), respectively. The stress in pascals is divided by $10^6$ to obtain the stress per square millimeter. The section modulus for certain regular sections can be found from the formulas given in Fig. 29-7-1. The values of $Z$ for structural-steel shapes and many common circular and rectangular sizes are tabulated in most engineers' handbooks.

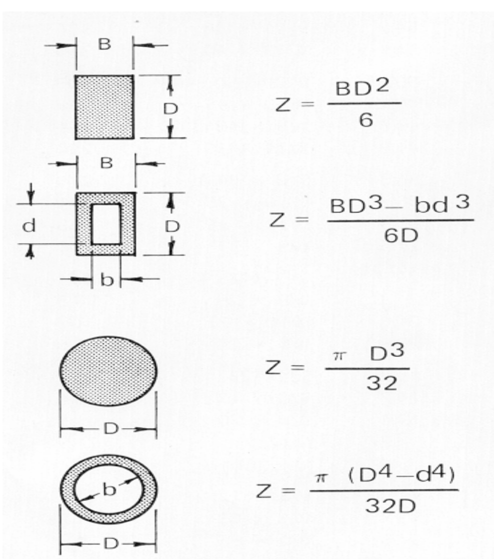



**Fig. 29-7-1** Formulas for section moduli for common shapes.

The letter *S* is frequently used in textbooks to designate section modulus. However, to avoid confusion with the letter *S* for stress, the letter *Z* will be used to designate section modulus throughout this chapter.

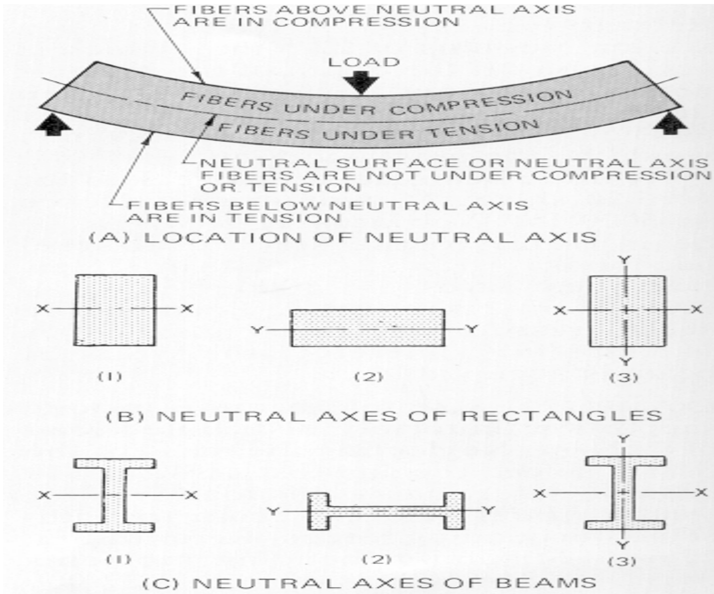



**Fig. 29-7-2** Neutral axis.

Structural shapes may be placed in two general positions, as shown in Fig. 29-7-2. Since the resistance to bending will depend on the position of the beam with regard to its neutral axis, two section modulus values are generally shown in engineering tables. One value is used when the beam is in the upright position, as shown in Fig. 29-7-2C(l) where the *X-X* axis is the neutral axis; the other is used when the beam is in the flat position, as shown in Fig. 29-7-2C(2), where the *Y- Y* axis is the neutral axis. The *neutral axis is* defined as the axis that passes through the centroid of the cross-sectional area.

The majority of engineering handbooks show only one illustration of the structural shape with both the *X-X* and *Y- Y* axes shown as illustrated in Fig. 29-7-2C(3).

## SHEARING STRESSES IN BEAMS

In designing beams for vertical shear, it is customary to consider only the full height of the webs of S, C, WT, and WWT beams to carry the full load; the flanges are not considered.

**EXAMPLE I** A cantilever beam 10 ft long supports a 5000-lb load at the end of the beam. What size A36 beam is required?

**Solution** First select a beam to withstand the bending forces. Refer to Fig. 29-6-11; maximum bending moments = $FL$ = 5000 x 10 x 12 = 600 000 in-lb. The allowable bend stress for A36 steel (see Fig. 29-1-8) is 24 kips/in.$^2$.

Section modulus required = $Z = M \div S$ = 600 000 $\div$ 24 000 = 25 in.$^3$

Refer to Fig. 29-7-3. A W8 x 31 has a section modulus of 27.5 in.$^3$, which is acceptable. If the depth of the beam is not an important design factor, then the W10 X 29 beam, which is lighter and has a section modulus of 30.9 in.$^3$, would be the most economical.

Next the beam must be checked for vertical shear. Maximum shear force = 5000 lb. Web area of a W8 x 31 beam (refer to the structural- steel handbook) = 8 x .31 = 2.48 in$^2$. Vertical shear stress = 5000 ÷ 2.48 = 2016 lb or 2 kips/in.$^2$. Permissible shear stress for steel (see Fig. 29-1-8) is 14.5 kips/in.$^2$. Therefore the W8 x 31 beam is acceptable.

**EXAMPLE 2** A simple beam 6 m long supports a uniformly distributed load of 6 kN/m. Neglecting the mass of the beam, select the lightest A572M-310 beam to safely carry this load.

**Solution** First select a beam to withstand the bending forces. Refer to Fig. 29-6-11. Maximum bending moments = $NL^2 \div 8$ = (6000 x $6^2$) ÷ 8 = 27 000 N-m, or 27 x $10^6$ N-mm. The allowable bending stress for A572M-310 steel (see Fig. 29-1-8) is 205 MPa. Section modulus required = $Z = M \div (S \div 10^6)$ = 27 x $10^6$ x $10^6$ $\div$ (205 x $10^6$) = 131 700 mm$^3$.

Referring to Fig. 29-7-3, we find that a W150 x 18 beam has a section modulus of 136 000 mm$^3$, which is acceptable.

Next the beam must be checked for vertical shear.

Maximum shear force (Fig. 29-6-11)

$$\frac{NL}{2} = \frac{6\,\text{kN} \times 6}{2} = 18\,\text{kN}$$

Web area of a W150 x 18 beam (refer to the structural- steel handbook) = 153 x 6 = 918 mm$^2$. Average vertical shear stress = 18 000 + 0.000 918 = 19.6 MPa. The permissible vertical shear stress for A572M-310 steel (see Fig. 29-1-8) is 125 Mpa. Therefore the W150 x 18 beam is acceptable.

| U.S. CUSTOMARY | | | METRIC* | | |
|---|---|---|---|---|---|
| Section Modulus In.$^3$ | Shape | Moment of Inertia I In.$^{4**}$ | Section Modulus $10^3$mm$^3$ | Shape | Moment of Inertia = I $10^6$mm$^{4**}$ |
| 157 | W18 x 85 | 1440 | 3050 | W460 x 128 | 637 |
| 151 | W16 x 88 | 1220 | 2850 | W410 x 132 | 538 |
| 151 | W21 x 73 | 1600 | 2830 | W530 x 109 | 667 |
| 142 | W18 x 77 | 1290 | 2670 | W460 x 113 | 556 |
| 140 | W21 x 68 | 1480 | 2620 | W530 x 101 | 617 |
| 131 | W14 x 84 | 928 | 2560 | W360 x 134 | 415 |
| 125 | W12 x 92 | 789 | 2420 | W310 x 143 | 348 |
| 121 | W14 x 78 | 851 | 2270 | W360 x 122 | 365 |
| 118 | W18 x 64 | 1050 | 2180 | W460 x 97 | 445 |
| 116 | W16 x 71 | 941 | 2160 | W310 x 129 | 308 |
| 116 | W12 x 85 | 723 | 2130 | W410 x 100 | 398 |
| 112 | W14 x 74 | 797 | 2060 | W360 x 110 | 331 |
| 107 | W12 x 79 | 663 | 1950 | W310 x 118 | 275 |
| 105 | W16 x 64 | 836 | 1830 | W460 x 82 | 370 |
| 97.5 | W12 x 72 | 597 | 1770 | W310 x 107 | 248 |
| 94.3 | W16 x 58 | 748 | 1730 | W410 X 85 | 315 |
| 92.2 | W14 x 61 | 641 | 1680 | W360 x 91 | 267 |
| 88 | W12 x 65 | 533 | 1590 | W310 x 97 | 222 |
| 80.9 | W16 x 50 | 657 | 1510 | W41 0 x 74 | 275 |
| 73.6 | W10 x 66 | 382 | 1400 | W250 x 101 | 164 |
| 70.6 | W12 x 53 | 426 | 1280 | W310 x 79 | 177 |
| 64.8 | W12 x 50 | 395 | 1190 | W310 x 74 | 165 |
| 64.8 | S15 x 50 | 486 | 1140 | S380 x 64 | 187 |
| 62.7 | W14 x 43 | 429 | 1140 | W360 x 64 | 178 |
| 60.5 | W10 x 54 | 306 | 1140 | S380 x 64 | 187 |
| 59.6 | S15 x 42.9 | 447 | 1090 | W250 x 80 | 126 |
| 54.7 | W14 x 38 | 386 | 1010 | W360 x 57 | 161 |
| 51.9 | W12 x 40 | 310 | 1010 | S310 x 74 | 128 |
| 50.8 | S12 x 50 | 305 | 941 | W310 x 60 | 129 |
| 49.2 | WIO x 45 | 249 | 901 | W250 x 67 | 104 |
| 43.3 | W8 x 48 | 184 | 803 | W200 x 71 | 76.6 |
| 41.8 | W14 x 30 | 290 | 779 | W360 x 45 | 122 |
| 36.3 | S12 x 31.8 | 218 | 690 | S310 x 47 | 91.1 |
| 35.1 | W10 x 33 | 171 | 633 | W250 x 49 | 70.6 |
| 30.9 | W10 x 29 | 158 | 602 | W250 x 45 | 71.1 |
| 27.5 | W8 x 31 | 110 | 496 | W200 x 46 | 45.5 |
| 24.8 | SIO x 25.4 | 124 | 465 | S250 x 38 | 51.4 |
| 24.3 | W8 x 28 | 97.8 | 446 | W200 x 42 | 40.9 |
| 21.6 | WIO x 21 | 107 | 424 | W250 x 33 | 48.9 |
| 20.8 | W8 X 24 | 82.5 | 380 | W200 x 36 | 34.4 |
| 16.2 | S8 x 23 | 64.9 | 316 | S200 x 34 | 27.0 |
| 14.2 | W8 x 17 | 56.6 | 279 | W200 x 27 | 25.8 |
| 13.4 | W6 x 20 | 41.5 | 244 | W150 x 30 | 17.2 |
| 12 | M12 x 11.8 | 71.9 | 232 | M3 10 x 17.6 | 29.7 |
| 10.1 | W6 x 16 | 31.7 | 192 | W150 x 24 | 13.4 |
| 7.23 | W6 x 12 | 21.7 | 136 | W150 X 18 | 9.16 |
| 5.43 | W4 x 13 | 11.3 | 103 | W100 X 19 | 4.76 |
| 5.08 | W6 x 85 | 14.8 | 103 | W150 X 14 | 6.87 |

*Soft Converted. *Taken at X-X axis.

**Fig. 29-7-3** Section modulus and moment of Inertia for shapes used as beams.

**EXAMPLE 3** A simple beam 16 ft long supports a 30 000 lb concentrated load 4 ft from the left abutment. What size A36 beam is required?

**Solution** First select a beam to withstand the bending forces. Refer to Fig. 29-6-11. Maximum bending moments = $FAB \div L$=30 000 x 4 x 12 ÷ 16 = 90 000 ft-lb, or 1 080 000 in-lb. Allowable bending stress for A36 steel (see Fig. 29-1-8) is 24 kips/in.$^2$.

Section modulus required = $Z$

$$= M \div S$$
$$= 1080\,000 + 24\,000$$
$$= 45 \text{ in.}^3$$

Referring to Fig. 29-7-3, we find that a W10 x 45 beam has a section modulus of 49.2.

Next the beam must be checked for vertical shear.

Maximum shear force

$$= \frac{30\,000 \times 12}{16} = 22\,500 \text{ lb}$$

Web area of a W10 x 45 beam (refer to the structural-steel handbook) = 10.12 x .38 = 3.85 in.$^2$.

Average vertical shear stress

$$= \frac{22\,500}{3.85} = 5844 \text{ lb}$$

The permissible vertical shear stress for A36 steel (see Fig. 29-1-8) is 14.5 kips/in.$^2$. Therefore the W10 X 45 beam is acceptable.

**EXAMPLE 4** A floor 5 m wide has a uniformly distributed load of 3000 N-m. The floor joists are 38 mm wide and are spaced 400 mm center-to-center. If the allowable bending stress is not to exceed 9600 kPa, what depth of floor joists must be used?

**Solution** On each floor joist, the uniformly distributed load is 3000 x (400 ÷1000) = 1200 N-m. For a simple beam with a uniformly distributed load, the maximum bending moment = $FL^2 \div 8$ = (1200 x 5 x 5) ÷ 8 = 3750 N-m, or 375 x $10^4$ N-mm. Allowable bending stress = 10 MPa. Therefore

Section modulus required = $Z$

$$= M \div \frac{S}{10^6}$$
$$= 375 \times 10^4 \times \frac{10^6}{10 \times 10^6}$$
$$= 375\,000 \text{ mm}^3$$

The section modulus for the joist = $bd^2 \div 6$ where *b* = width = 38 mm and *d* = depth, which is unknown. Therefore:

$$d = \sqrt{\frac{375\,000 \times 6}{38}} = 243\,\text{mm}$$

Since standard joist sizes are 38 x 184, 38 x 235, and 38 x 286, the joist size of 38 x 286 is selected.

## DEFLECTION OF BEAMS

The vertical distance a horizontally placed beam moves when it bends under an applied load is called *deflection.* Since deflection may cause cracking in plastered ceilings or buckling of floors, the limitations placed on the deflection of the beam may be the governing factor in its selection. In building construction the maximum deflection of beams is limited to 1/360 of the span, or, for cantilever beams, 1/180 of the span. After the beam is selected to withstand the bending and shearing stresses, it must then be checked for deflection. The theory and the mathematics behind the development of beam defection will not be covered in this text. The formulas using the *double-integration method* for finding the deflection of simple beams are shown in Fig. 29-6-11.

**EXAMPLE 5** A W8 x 28 cantilever beam 8 ft long has a concentrated load of 5000 lb at its free end. Is the deflection excessive?

**Solution** Refer to Fig. 29-6-11. The maximum deflection for a cantilever beam having a concentrated load at its free end is $FL^3 \div 3EI$, where *F* = 5000 lb, *L* = 96 in., $E = 29 \times 10^6$ lb/in$^2$ (see Fig. 29-1-3), and *I* = 97.8 in.$^4$ (Fig. 29-7-3). Therefore

Maximum deflection

$$= \frac{500 \times 96^3}{3 \times 97.8 \times 29 \times 10^6} = .52 \text{ in.}$$

Allowable deflection = 1/180 of the span (for cantilever beams) = 96 ÷180 = .53 in. Therefore, since the maximum deflection is less than the allowable, the W8 x 28 beam is acceptable.

**EXAMPLE 6** A W310 x 60 simple beam has a concentrated load of 27 kN acting at the center of the beam. The beam span is 6 m. Check for deflection.

**Solution** Refer to Fig. 29-6-11. The maximum deflection for a simple beam with a concentrated load is $FL^3 \div 48EI$ where *F* = 27 kN, *L* = 6000 mm, *E* = 200 000 MPa (Fig. 29-1-3), and $I = 129 \times 10^6$ mm$^4$ (Fig. 29-7-3). Therefore

Maximum deflection

$$= \frac{27\,000 \times 6000^3}{48 \times 200\,000 \times 129 \times 10^6} = 4.7 \text{ mm}$$

Allowable deflection = span ÷ 360 = 6000 360 = 16.7 mm. Since the maximum deflection is less than the allowable, the W310 x 60 beam is acceptable.