In this tutorial, we will learn the basics of performing motion analysis using SolidWorks Motion. Although the tutorial can be completed by anyone with a basic knowledge of SolidWorks parts and assemblies, we have provided enough detail so that students with an understanding of the physics of mechanics will be able to relate the results to those obtained by hand calculations.

We will be looking at three different analyses:

- 1. Rotation of a wheel, in which we will learn how to set up a motion analysis and see the effects of changing the mass moment of inertia on angular acceleration.
- 2. Four-bar linkage, in which we will see how plotting a quantity such as acceleration over a mechanism's full range of motion allows us to identify the extreme values of the quantity.
- 3. Roller on a ramp, in which the effects of friction will be evaluated.
- 1. Rotation of a Wheel

Begin by creating the three part models detailed below, or by downloading the parts from the book's website. The eight-hole pattern on each wheel is added to help visualization of the rotation of the part.

COMPONENTS FOR WHEEL ROTATION MOTION ANALYSIS

WHEEL 1

WHEEL 2

BASE









All dimensions are inches

If you downloaded the part files, the materials for the parts are defined. If you created them yourself, then for each part, define the material by right-clicking "Material" in the FeatureManager and selecting "Edit Material." The Material Editor will appear, as shown here. Select "Alloy Steel" from the list of steels in the SolidWorks materials library. Click Apply and then Close.

To begin, we will analyze a simple model of a wheel subjected to a torque. From Newton's Second Law, we know that the sum of the forces acting on a body equals the mass of the body times the acceleration of the body, or

$$\sum F = ma \tag{1}$$

The above equation applies to bodies undergoing linear acceleration. For rotating bodies, Newton's Second Law can be written as:

$$\sum M = I\alpha \tag{2}$$

Where $\sum M$ is the sum of the moments about an axis passing through the body's center of mass, I is the mass moment of inertia of the body about that axis, and α is the angular acceleration of the body. The moment of inertia about an axis is defined as:

 $I = \int mr^2 dV \tag{3}$

where r is the radial distance from the axis. For simple shapes, the moment of inertia is relatively easy to calculate, as formulas for I of basic shapes are tabulated in many reference books. However, for more complex components, calculation of I can be difficult. SolidWorks allows mass properties, including moments of inertia, to be determined easily.

Open the part "Wheel 1." From the main menu, select Tools: Mass Properties.

The mass properties of the wheel are reported in the pop-up box. For this part, the mass is 15.29 pounds, and the moment of inertia about the z-axis (labeled as "Lzz" in SolidWorks) is 105.36 lb·in². Note that if you centered the part about the origin, then the properties, labeled "Taken at the center of mass and aligned with the output coordinate system" will be identical to those labeled "Principal moments... taken at the center of mass." Note that the units of mass used are actually pounds-mass, that is, a part that weighs one pound has a mass of one pound-mass. When we make our calculations later, we will have to convert our values so that we use units of mass that are consistent with the other units that we are using.





Close the Mass Properties window. Open the part "Wheel 2." From the main menu, select Tools: Mass Properties.

Note that although the mass of 15.45 is almost the same as that of Wheel 1, Wheel 2's moment of inertia is $146.54 \text{ lb} \cdot \text{in}^2$, which is almost 40% greater than that of Wheel 1. The reason for the difference is that more material in Wheel 2 is placed near the outer rim. In the definition of the moment of inertia shown as Eqn. 3, the contribution of each particle of mass on the value of *I* depends on its distance from the axis squared. Therefore, adding mass near the outer rim of the wheel increases its moment of inertia greatly.

Open a new assembly. Insert the component "Base."

Since the first component inserted into an assembly is fixed, it is logical to insert the component representing the stationary component (the "frame" or "ground" component) first.

Insert the part "Wheel 1" into the assembly. Select the Mate Tool. Add a concentric mate between the center hole of the wheel and the hole in the base. Be sure to select the cylindrical faces for the mate and not edges. Add a coincident mate between the back face of the wheel and the front face of the base.

You should now be able to click and drag the wheel, with rotation about the axis of the mated holes the only motion allowed by the mates. The addition of these two mates has added a *revolute joint* to the assembly. A revolute joint is similar to a hinge in that it allows only one degree of freedom.

Click on the "Motion Study 1" tab near the lower left corner, which opens the MotionManager across the lower portion of the screen.





The first two options are always available in SolidWorks. SolidWorks Motion is an add-in program, and must be activated before it can be used.

From the main menu, choose Tools: Add-Ins. In the list of available add-ins, click the check box beside SolidWorks Motion to activate it. Click OK.









Note that the arrow shows that the torque will be applied in the counterclockwise direction relative to the Z-axis (we say that this torque's direction is +Z). The arrows directly below the face selection box can be used to reverse the direction of the torque, if desired.

Scroll down in the Force PropertyManager and set the value to 5 in·lb. Scroll back to the top of the PropertyManager and click the check mark to apply the torque.

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In the MotionManager, click and drag the diamond-shaped icon (termed a "key") from the default five seconds to the desired two seconds (00:00:02). Select the Motion Study Properties Tool, and change the calculation rate from the default of 25 to 100 frames per second. Click the check mark.







Using a larger number of frames per second will result in smoother plots, but will require more calculation time.

Click the Calculator Icon to perform the simulation.



The animation of the simulation can be played back without repeating the calculations by clicking the Play from Start key. The speed of the playback can be controlled from the pull-down menu beside the Play controls.

As noted earlier, SolidWorks Motion provides quantitative analysis results in addition to qualitative animations of motion models. We will create plots of the angular acceleration and angular velocity of the wheel.

Select the Results and Plots Tool. In the PropertyManager, use the pull-down menus to select Displacement/Velocity/Acceleration: Angular Acceleration: Z Component. Click on the front face of the wheel, and click the check mark.



A plot will be created of the angular acceleration versus time. The plot can be dragged around the screen and resized. It can also be edited by rightclicking the plot entity to be modified, similar to the editing of a Microsoft Excel plot.

We see that the acceleration is a constant value, about 1050 degrees per second squared. Since the applied torque is constant, it makes sense that the angular acceleration is also constant. We can check the value with hand calculations. Note that while we

can perform very complex analyses with SolidWorks Motion, checking a model by applying simple loads or motions and checking results by hand is good practice and can prevent many errors.

We earlier found the mass moment of inertia to be 105.36 lb·in². Since the pound is actually a unit of force, not mass, we need to convert weight to mass by dividing by the gravitational acceleration (W = mg). Since we are using inches as our units of length, we will use a value of 386.1 in/s²:

$$I = \frac{105.36 \text{ lb} \cdot \text{in}^2}{386.1 \frac{\text{in}}{\text{s}^2}} = 0.2729 \text{ lb} \cdot \text{in} \cdot \text{s}^2$$
(4)

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Since the torque is equal to the mass moment of inertia times the angular acceleration, we can find the angular acceleration as:

$$\alpha = \frac{T}{I} = \frac{5 \text{ in} \cdot \text{lb}}{0.2729 \text{ lb} \cdot \text{in} \cdot \text{s}^2} = 18.323 \frac{\text{rad}}{\text{s}^2}$$
(5)

Notice that the non-dimensional quantity "radians" appears in our answer. Since we want our answer in terms of degrees, we must make one more conversion:

$$\alpha = 18.323 \ \frac{\text{rad}}{\text{s}^2} \left(\frac{180 \ \text{deg}}{\pi \ \text{rad}} \right) = 1050 \frac{\text{deg}}{\text{s}^2} \tag{6}$$

This value agrees with our SolidWorks Motion result.

Select the Results and Plots Tool. In the PropertyManager, use the pull-down menus to select Displacement/Velocity/Acceleration: Angular Velocity: Z Component. Click on the front face of the wheel, and click the check mark. Resize and move the plot so that both plots can be seen, and format the plot as desired.





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As expected, since the acceleration is constant, the velocity increases linearly. The velocity at the end of two seconds is seen to be about 2100 degrees per second. This result can be verified with a simple hand calculation:

$$\omega = \alpha t = \left(1050 \frac{\text{deg}}{\text{s}^2}\right)(2 \text{ s}) = 2100 \frac{\text{deg}}{\text{s}}$$
(7)

Often, the angular velocity is expressed in revolutions per minute (rpm), commonly denoted by the symbol *N*:

$$N = \left(2100 \frac{\text{deg}}{\text{s}}\right) \left(\frac{1 \text{ rev}}{360 \text{ deg}}\right) \left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 350 \text{ rpm}$$
(8)

We will now experiment with variations of the simulation.

Move the plots out of the way, but do not close them. Click and drag the key at the top of the simulation tree from 2 seconds to four, so that the simulation will now last for four seconds. Place the cursor on the line corresponding to the applied torque (Torque1) at the 2-second mark. (If desired, you can click the + and signs at the right end of the timeline to scale the timeline.) Right-click and select Off.



A new key will be placed at that location. The torque will now be applied for two seconds, but the simulation will continue for the four seconds.

Press the Calculator icon to perform the simulation.

The plots will be automatically updated. Note that the angular acceleration now drops to zero at two seconds, while the angular velocity will be constant after two seconds. Since there is no friction in the model, the wheel will continue to spin at a constant velocity without any torque applied.



In the previous simulations, the torque was applied as a constant value. That means that the change of the acceleration relative to time (commonly referred to as "jerk") is infinite at time = 0 and at time = 2 seconds. A more realistic approximation is to assume that the torque builds up over some period of time, and also ramps down gradually. For example, we will assume that it takes two seconds to reach the full value of torque and two seconds to ramp down.

Right-click the key added to the torque at time = 2 seconds and delete it. Move the key defining the duration of the simulation to six seconds .

Move the time bar back to zero. Right-click on Torque1 and select Edit Feature.

Scroll down in the PropertyManager, and select Segments as the type of Force Function. Enter the three rows as shown here, with Cubic as the Segment Type.

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Two graphs are displayed: the torque as a function of time and the derivative of the torque. You can experiment with different segment types to see how they affect the torque, but the cubic curve will work fine for this example.

Click OK and then the check mark to apply the torque. Calculate the simulation.

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Note that the angular acceleration curve is smooth, and peaks at 1050 deg/s². At the end of the six seconds, the wheel will be turning at about 4,200 deg/s (700 rpm).



Now let's see the effect of replacing the Wheel 1 component with Wheel 2, which has a higher mass moment of inertia. Of course, we could start with a new assembly, but it is easier to replace the component in the existing assembly. This will allow us to retain most of the assembly mates and simulation entities.

Click the model tab at the bottom of the screen. Click on Wheel 1 in the FeatureManager to select it. From the main menu, select File: Replace. Browse to find Wheel 2. In the PropertyManager, click the check mark to accept the replacement of faces in the existing mates with those of the new part. Click the check mark to make the replacement.



Depending on how you modeled the parts, it is possible that errors will be encountered when the program attempts to re-apply the mates. If this happens, close the error messages, delete both mates, and apply new mates manually.

Switch to the Motion Study. Right-click each torque, select Edit Feature, and click on the front face of the wheel to define the direction. In the simulation tree, right-click on each plot, and click on the front face of the wheel to define the component for which velocity/acceleration is to be plotted. Calculate the simulation.



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Note that the maximum angular acceleration is about 755 deg/s², which is significantly less that of the simulation with the earlier wheel. This value can be verified from the ratio:

$$\frac{\alpha_2}{\alpha_1} = \frac{I_1}{I_2} \tag{9}$$

$$\alpha_2 = \frac{I_1}{I_2} \alpha_1 = \frac{105.36}{146.54} \left(1050 \ \frac{\text{deg}}{\text{s}^2} \right) = 755 \ \frac{\text{deg}}{\text{s}^2} \tag{10}$$

2. Four-Bar Linkage

In this exercise, we will model a 4-bar linkage similar to that of Chapter 11 of the text. In the text, we were able to qualitatively simulate the motion of the simulation when driven by a constant-speed motor. In this exercise, we will add a force and also explore more of the quantitative analysis tools available with SolidWorks Motion.

Download or construct the components of the linkage shown on the next page, and assemble them as detailed in Chapter 11 of the text. The material should be Alloy Steel for all of the parts. The Frame link should be placed in the assembly first, so that it is the fixed link.

You should be able to click and drag the Crank link around a full 360 degree rotation.



Note that the Connector link has three holes. The motion of the third hole can follow many paths, depending on the geometry of the links and the position of the hole.

Perpendicular1 🛛 🕐



All dimensions are inches. All parts have thickness of .25

Before beginning the simulation, we will set the links to a precise orientation. This will allow us to compare our results to hand calculations more easily.

Add a perpendicular mate between the two faces shown here.

Expand the Mates group of the FeatureManager, and right-click on the perpendicular mate just added. Select Suppress.



The perpendicular mate aligns the crank link at a precise location. However,

we want the crank to be able to rotate, so we have suppressed the mate. We could have deleted the mate, but if we need to re-align the crank later, we can simply unsuppress the mate rather than recreating it.

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Switch to the Front View. Zoom out so that the view looks similar to the one shown here.

The MotionManager uses the last view/zoom of the model as the starting view for the simulation.



Make sure that the SolidWorks Motion add-in is active. Click the MotionManager tab.

Set the type of analysis to Motion Analysis. Select the Motor icon. In the PropertyManager, set the velocity to 60 rpm. Click on the front face of the Crank to apply the motor, and click the check mark.







The four bar linkage can be designed to produce a variety of motion paths, as illustrated below. C 0 (0We will now add a force to the open hole. Select the Force Tool. In the PropertyManager, the highlighted box prompts you for the location of the force. Click on the edge of the open hole, and the force will be applied at the center of the hole. × orce Type 10) Linear force Force U Torque Moves one or two components as if acted upon by a force. Direction Action only \cap Action & reaction N K 3 "날 🐷 ₿ 00:00:12 V 00:00:10 00:00.08 2 The direction box is now highlighted. Rotate and zoom in so that you can select the top face of the Frame part. The force will be applied normal to this force. As you can see, the force acts upwards. Action & reaction Edge<1>@Connect(3 ₽, Earca calativa to Click the arrows to reverse the direction of Edge<1>@Connecto the force. @Frame-: **Reverse Direction**

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Scroll down in the PropertyManager and set the n force to 20 pounds. Click the check mark to apply	agnitude of the Force Fu	nction		
Run the simulation.				
We will now plot the torque of the motor that is re the 60-rpm motion with the 20-lb load applied.	quired to produce F1 20) lbf		
Select Results and Plots. In the PropertyManager, specify Forces: Applied Torque: Z Component. Click on the RotaryMotor in the MotionManager to select it, and click the check mark in the PropertyManager.	Results and Plots Calculates results and creates graphs. Results Resul	Results It Forces C Component RotaryMotor 1		
Format the resulting plot as desired.				
Note that the applied torque peaks at about 51 in:	b. At t = 0, the torque appears to be	e about -30 in·lb		
Note that the applied torque peaks at about 51 in·l (the negative signs indicates the direction is about View). In order to get a more exact value, we can	b. At $t = 0$, the torque appears to be the -Z axis, or clockwise when view	e about -30 in·lb ed from the Front		
separated values) file that can be read in Word or E	Excel.	' (CUIIIIId-		

Right-click in the graph and choose Export CSV. Save the file to a convenient location, and open it in Excel.



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At time = 0, we see that the motor torque is -29.2 in·lb.

Hand calculations for a *static analysis* of the mechanism are attached, which show a value of -29.4 in·lb.

It is important when comparing these values to recognize the assumptions that are present in the hand calculations:

1. The weights of the members were not included in the forces, and

	A	В	С	D
1	Moment - Z-	RotaryMotor3		
2	Time (sec)	Moment - Z (p	ound_for	ce-inch)
3	0.00	-29.20		
4	0.01	-30.09		
5	0.02	-31.05		
6	0.03	-32.04		
7	0.04	-33.01		
8	0.05	-33.95		
9	0.06	-34.79		
10	0.07	-35.52		
11	0.09	26.09		

2. The accelerations of the members were neglected.

The first assumption is common in machine design, as the weights of the members are usually small in comparison to the applied loads. In civil engineering, this is usually not the case, as the weights of structures such as building and bridges are often greater than the applied forces.

The second assumption will be valid only if the accelerations are relatively low. In our case, the angular velocity of the crank (60 rpm, or one revolution per second) produces accelerations in the members that are small enough to be ignored. Let's add gravity to the simulation to see its effect.

Click on the Gravity icon. In the PropertyManager, select Y as the direction. There will be an arrow pointing down in the lower right corner of the graphics area, showing that the direction is correct. Click the check mark. Run the simulation.





The torque plot is almost unchanged, with the peak

torque increasing by only one in·lb. Therefore, omitting gravity had very little effect on the calculations.

Now we will increase the velocity of the motor to see the effect on the torque.

Drag the key at the end of the top bar in the MotionManager from 1 second to 0.1 second. Use the Zoom In Tool in the lower right corner of the MotionManager to spread out the time line, if desired.



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Drag the slider bar showing the time within the simulation back to zero.

This is an important step before editing existing model items, as changes can be applied at different time steps. Because we want the motor's speed to be changed from the beginning of the simulation, it is important to set the simulation time at zero.

Right-click on the RotaryMotor in the MotionManager. In the PropertyManager, set the speed to 600 rpm. Click the check mark.

Since a full revolution will occur in only 0.1 seconds, we need to increase the frame rate of the simulation to achieve a smooth plot.

Select the Motion Study Properties. In the PropertyManager, set the SolidWorks Motion frame rate to 1000 frames/second. Click the check mark.









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The peak torque has increased from 51 to 180 in·lb, demonstrating that as the speed is increased, the accelerations of the members are the critical factors affecting the torque.

You can verify this conclusion further by suppressing both gravity and the applied 20-lb load and repeating the simulation. The peak torque is decreased only from 180 to 151 in·lb, even with no external loads applied.



To perform hand calculations with the accelerations included, it is necessary to first perform a kinematic analysis to determine the translational and angular accelerations of the members. You can then draw free body diagrams of the three moving members and apply three equations of motion to each:

$$\Sigma F_x = ma_x$$
 $\Sigma F_y = ma_y$ $\Sigma M_z = I_Z \alpha_Z$

The result is nine equations that must be solved simultaneously to find the nine unknown quantities (the applied torque and the two components of force at each of the four pin joints).

The results apply to only a single point in time. This is a major advantage of using a simulation program such as SolidWorks Motion: since it is not evident at what point in the motion that the forces are maximized, our analysis evaluates the forces over the complete range of the mechanism's motion and allows us to identify the critical configuration.

3. Roller on a Ramp

In this exercise, we will add contact between two bodies, and experiment with friction between the bodies. We will begin by creating two new parts – a ramp and a roller (skip these steps if you have downloaded the parts).





Extrude the triangle using the midplane option, with a thickness of 1.2 inches.



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Open a new sketch on the front face of the cylinder. Add and dimension the circles and lines as shown here (the part is shown in wireframe mode for clarity). The two diagonal lines are symmetric about the vertical centerline.



Extrude a cut with the Through All option, with the sketch contours shown selected. If desired, change the color of the cut feature.



Create a circular pattern of the extruded cut features, with five equally-spaced cuts. Save the part with the name "Roller".



Open a new assembly. Insert the ramp first, and place it at the origin of the assembly. Insert the Roller.



Turn off the temporary axis display. Switch to the Motion Study. Select SolidWorks Simulation as the type of analysis. Add gravity in the -y-direction.

Select the Contact Tool. In the PropertyManager, you will be prompted to select the bodies for which contact can occur. Click on each of the two parts. Clear the check boxes labeled "Material" and "Friction." Leave the other properties as their defaults.





We will add friction later, but our initial simulation will be easier to verify without friction. In the Elastic Properties section, note that the default is set as "Impact," with several other properties (stiffness, exponent, etc.) specified. At each time step, the program will check for interference between the selected bodies. If there is interference, then the specified parameters define a non-linear spring that acts to push the bodies apart. Contacts add considerable complexity to a simulation. If the time steps are too large, then the contact may not be recognized and the bodies will be allowed to pass through each other, or a numerical error may result.

Select the Motion Studies Property Tool. Set the frame rate to 500 and check the box labeled "Use Precise Contact." Click the check mark.

For some simulations, it may be necessary to lower the solution tolerance in order to get the simulation to run. For this example, the default tolerance should be fine.

The mates that we added between the parts to precisely locate the roller on the ramp will prevent motion of the roller. Rather than delete these mates, we can suppress them in the MotionManager.

Right-click on each of the mates in the MotionManager and select Suppress. Run the simulation.

You will see that the roller reaches the bottom of the ramp quickly.

Change the duration of the simulation to 0.5 seconds, and run the simulation again. Create a plot of the magnitude of the linear velocity of the roller vs. time.

The roller reaches the bottom of the ramp in about 0.35 seconds, and the velocity at the bottom of the ramp is about 68 in/s. These values agree with those calculated in the attachment at the end of this document.





Now let's add friction.

Move the timeline of the simulation back to zero. Right-click on the contact in the MotionManager tree, and select Edit Feature. Check the "Friction" box, and set the coefficient of friction to 0.25. Calculate the simulation.

The resulting velocity plot shows the velocity at the bottom of the ramp to be about 54 in/s. This value agrees with that of the calculations shown in the attachment.

To confirm that the roller is not slipping, we can trace the position of a single point on the roller.

Select the Results and Plots Tool. Define the plot as Displacement/ Velocity/Acceleration: Trace Path. Click on a point near the outer rim of the roller (not on a face, but on a single point). Click the check mark.



The trace path shows sharp cusps where the point's velocity approaches zero (it will not become exactly zero unless the point is on the outer surface of the roller). For comparison, repeat the analysis with a lower friction coefficient.



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Change the friction coefficient to 0.15 and recalculate the simulation.

This time, the trace paths shows smooth curves when the point is near the ramp's surface, indicating that sliding and rolling are taking place simultaneously.

In the attachment, it is shown that the coefficient of friction require to prevent slipping is about 0.21.



It is interesting to note that the friction coefficient to prevent slipping and the time required to reach the bottom of the ramp are both functions of the ratio of the moment of inertia to the mass of the roller. (A parameter called the *radius of gyration* is defined as the square root of the mass moment of inertia divided by the mass, and is a function only of the part's geometry.) You can confirm this by changing the material of the roller and seeing that the results of the simulation are unchanged. However, if you change the geometry of the roller (the easiest way is by suppressing the cut-out regions), then the results will change.



Note that member CD is a 2-force member, and so the force at the end is aligned along the member's axis.

Apply equilibrium equations:

$$\Sigma M_{\rm B} = (5.714 \text{ in})(CD \sin(75.09^{\circ})) + (1.832 \text{ in})(CD \cos(75.09^{\circ})) - (11.427 \text{ in})(20 \text{ lb}) = 0$$

$$(5.522 \text{ in})CD + (0.4714 \text{ in})CD = 228.5 \text{ in} \cdot \text{lb}$$

$$(5.993 \text{ in})CD = 228.5 \text{ in} \cdot \text{lb}$$

$$CD = \frac{228.5 \text{ in} \cdot \text{lb}}{5.993 \text{ in}} = 38.13 \text{ lb}$$

$$\Sigma F_x = B_x - (38.13 \text{ lb})\cos(75.09^{\circ}) = 0$$

$$B_x = 9.812 \text{ lb}$$



Note that B_x and B_y are shown in opposite directions as in Connector FBD.

Sum moments about A:

$$\Sigma M_A = T + (3 \text{ in})B_x = 0$$

 $T = -(3 \text{ in})(9.812 \text{ lb}) = -29.4 \text{ in} \cdot \text{lb}$



Knowing the distance travelled in the x-direction, and entering the numerical values of g as 386.1 in/s² and of sin β of 0.5 (sin of 30°), we can find the time it takes the block to slide to the bottom:

or

$$12 \text{ in} = \frac{386.1 \text{ in/s}^2}{2} (0.5)t^2$$
$$t = \boxed{0.353 \text{ s}}$$

Substituting this value into Equation 4, we find the velocity at the bottom of the ramp:

$$v_{\rm x} = 386.1 \frac{{\rm in}}{{\rm s}^2} (0.5)(0.353 {\rm s}) = 68.1 \frac{{\rm in}}{{\rm s}}$$

This velocity can also be found by equating the potential energy when the roller is at the top of the ramp (height above the datum equals 6 inches) to the kinetic energy when the roller is at the bottom of the ramp:

$$mgh = \frac{1}{2}mv_{\rm x}^2$$

$$v_{\rm x} = \sqrt{2gh} = \sqrt{2(386.1\frac{{\rm in}}{{\rm s}^2})(6~{\rm in})} = -68.1~\frac{{\rm in}}{{\rm s}}$$

Friction Included:

Free-Body Diagram:



While the roller without friction slides and can be treated as a particle, the roller with friction experiences rigid-body rotation. The equations of equilibrium are:

$$\Sigma F_{x} = W \sin \beta - f = ma_{x}$$
$$\Sigma F_{y} = N - W \cos \beta = 0$$
$$\Sigma M_{c} = f r = I_{o} \alpha$$

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If there is no slipping, then the relative velocity of the roller relative to the ramp is zero at the point where the two bodies are in contact (point O). Since the ramp is stationary, this leads to the observation that the velocity of point O is also zero.

Since point O is the center of rotation of the roller, the tangential acceleration of the center of the roller (a_x) can be written as:

$$a_{\rm x} = r\alpha$$



Substituting this expression into the first equilibrium equation and solving for the friction force,

$$f = W \sin \beta - mr\alpha$$

Substituting this expression into the third equilibrium equation and solving for the angular acceleration α ,

$$(W \sin \beta - mr\alpha) r = I_0 \alpha$$
$$W (\sin \beta) r = I_0 \alpha + mr^2 \alpha$$
$$\alpha = \frac{W (\sin \beta) r}{I_0 + mr^2}$$

The mass and the moment of intertia I_o can be obtained from SolidWorks. For the roller, the values are:

m = 0.017163 lb $I_0 = 0.002515 \text{ lb} \cdot \text{in}^2$

Since pounds are units of weight, not mass, they quantities above must be divided by g to obtain the quantities in consistent units:

$$m = \frac{0.017163 \text{ lb}}{386.1 \text{ in/s}^2} = 4.4453 \text{ X } 10^{-5} \frac{\text{lb} \cdot \text{s}^2}{\text{in}}$$
$$I_0 = \frac{0.002515 \text{ lb} \cdot \text{in}^2}{386.1 \text{ in/s}^2} = 6.5139 \text{ X } 10^{-6} \text{ lb} \cdot \text{in} \cdot \text{s}^2$$

The value of the angular acceleration can now be found:

$$\alpha = \frac{W(\sin\beta)r}{I_0 + mr^2} = \frac{0.017163 \text{ lb}(\sin 30^\circ)(0.5 \text{ in})}{6.5139\text{e} - 6 \text{ lb} \cdot \text{in} \cdot \text{s}^2 + 4.4453\text{e} - 5 \frac{\text{lb} \cdot \text{s}^2}{\text{in}}(0.5 \text{ in})^2} = 243.4 \frac{\text{rad}}{\text{s}^2}$$

Therefore, the linear acceleration in the x-direction is:

$$a_{\rm x} = r\alpha = (0.5 \text{ in})\left(243.4 \frac{\text{rad}}{\text{s}^2}\right) = 121.7 \frac{\text{in}}{\text{s}^2}$$

Integrating to obtain the velocity and position at any time:

$$v_{x} = \int a_{x} dt = a_{x}t + v_{x0} = 121.7 \frac{in}{s^{2}} t$$
$$x = \int (a_{x}t + v_{x0}) dt = \frac{1}{2}a_{x}t^{2} + v_{x0}t + x_{0} = 60.85 \frac{in}{s^{2}} t^{2}$$

For the roller to travel 12 inches in the *x*-direction, the time required is

$$t = \sqrt{\frac{12 \text{ in}}{60.85 \text{ in/s}^2}} = 0.444 \text{ s}$$

And the velocity at the bottom of the ramp is:

$$v_{\rm x} = 121.7 \frac{\rm in}{\rm s^2} \ (0.444 \ \rm s) = 54.0 \ \frac{\rm in}{\rm s}$$

We can also calculate the friction force:

$$f = W \sin \beta - mr\alpha = 0.017163 \text{ lb}(\sin 30^\circ) - \left(4.4453 \text{ e} - 5 \frac{\text{lb} \cdot \text{s}^2}{\text{in}}\right) (0.5 \text{ in}) \left(243.4 \frac{\text{rad}}{\text{s}^2}\right)$$
$$= 0.00318 \text{ lb}$$

From the second equilibrium equation, the normal force is:

$$N = W \cos \beta = 0.017163 \text{ lb}(\cos 30^\circ) = 0.1486 \text{ lb}$$

Since the maximum friction force is the coefficient of friction μ times the normal force, the coefficient of friction must be at least:

$$\mu_{\min} = \frac{0.00318 \text{ lb}}{0.1486 \text{ lb}} = 0.21$$

This is the minimum coefficient of friction required for the roller to roll without slipping.