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A Cobb-Douglas Production Function for Water Desalination

One worldwide area of concern today is the availability of fresh water. Because of this issue, the production of fresh water through the process of desalination of salt water has become a viable area of research. The Middle East is a leader in this form of fresh-water extraction, but desalination is becoming a growing industry along the West Coast of the United States.

Recently three economists used statistical and econometric techniques to estimate the production function for a water desalination plant. The results of the study suggest that the production function is Cobb-Douglas and is given by

$$Q = F^{.6}H^{.4}$$

where Q is cubic meters of desalted water produced per day, F represents factors of production (an aggregation of evaporating pumps, maintenance of those pumps, and labor), and H is the per diem level of heat, which is used in the evaporation process. According to the authors of the study,

> The technical inputs of water desalination can be classified into two groups: those for which the cost per unit of desalted water is increasing when the technical index of the number of effects is increasing, and those for which this cost is decreasing under the same circumstances. This classification permits us to express production of desalted water as a function of two aggregates of inputs, corresponding to the above substitutional groups. Thus a production function is extracted for the general case of full-load annual operation of the desalination plant.

Since the estimated production function is Cobb-Douglas, we can apply our formulas for the marginal products of a Cobb-Douglas production function to obtain an algebraic expression for the marginal product of heat in the production of fresh water,

$$MP_H = .4F^{.6}H^{-.6}$$

and for the marginal product of other factors of production,

$$MP_F = .6F^{-.4}H^{.4}$$

These equations reveal that the production of fresh water obeys the law of diminishing marginal returns.

Source: N. Zagouras, Y. Caouris, and E. Kantsos, "Production and Cost Functions of Water Low-Temperature Solar Desalination," *Applied Economics* 21 (September 1989), pp. 1177–90.