Chapter 7

Limits to growth?

The Solow model with scarce natural resources

So far our presentation of growth theory has been a bit like 'playing Hamlet without the Prince of Denmark'. The theory of economic growth was initiated by the great classical economists, Adam Smith, Thomas Malthus and David Ricardo. They thought that the presence of land as a fixed factor in production implied a severe constraint on the economy's long-term growth potential. In their time agriculture accounted for a much greater share in total production than today, so it is no wonder that the economists of the time considered land to be essential. However, today land is still in fixed supply and remains an important input to aggregate production. Not only is agriculture still of importance, but any kind of production requires at least some space and hence land. Moreover, if we interpret the services of 'land' in a broad sense to include all the life support services of the natural environment – such as its ability to generate clean air and water and to absorb the waste products of human activity – it should be clear that 'land' is vital for economic activity.

The analysis of the classical economists taught them that the presence of an irreplacable factor in fixed supply would imply a tendency towards long-run decline in income per capita and eventually stagnation at a low income level. In modern terms their argument was the following.

Assume there is some population growth and disregard technological progress for a moment. Assume further that the economy manages to build up capital at the same speed as the population increases. Note that this is exactly what we found in the basic Solow model's steady state: capital and labour were growing at the same rate (the capital–labour ratio was constant). Because of *constant returns to capital and labour*, this implied that output was also growing at this rate, leaving output per worker constant (possibly at a high level).

Assume now, realistically, that there are three inputs in the aggregate production function: capital, labour and land. Suppose further that land is in fixed supply. From the replication argument this production function should have constant returns to capital, labour *and* land, implying *diminishing returns to the combination of capital and labour*. Now, as the inputs of capital and labour increase proportionally, as we assume they do, while land stays fixed, output will grow less than proportionally to labour and capital, and hence output per worker will decline in the long run. In a nutshell, this is the classical argument why income per capita has to decline in the long run as a consequence of population growth. The ultimate root of this decline is the diminishing returns to capital and labour arising from the presence of a fixed factor, land.

This is not even the end of the story according to the classical economists. As income per capita falls, savings per capita would also fall, so capital would not really be able to increase at the same speed as labour in the long run. This would imply an even faster economic decline. Furthermore, with decreasing incomes, population growth would also eventually be brought to a halt. Fertility might stay unchanged, but because of the miserable living conditions of workers, mortality would be so high that the labour force would stay constant. This would eliminate population growth, the original source of declining incomes, but in the 'end state' income per capita would have stagnated at a subsistence level. In the words of Malthus:

> Population, when unchecked, increases in a geometrical ratio. Subsistence increases only in an arithmetical ratio. A slight acquaintance with numbers will show the immensity of the first power in comparison of the second ... This implies a strong and constantly operating check on population from the difficulty of subsistence ... its effects are ... Among mankind, misery and vice. (from Chapter I of Thomas Malthus' famous book, *An Essay on Population*, 1798).

The labelling of economics as the 'dismal science' arose from this idea. The economic history of the last 200 years suggests that the pessimism of the classical economists was unwarranted. Income per capita has been growing at rates around 2 per cent per year over very long periods in many Western countries. This does not mean that the classical economists were wrong in their reasoning, only that there were perhaps some other factors to which they did not pay enough attention. In fact, the modelling of this chapter will show that the classical economists were in some sense right: land *does* imply a growth drag such that population growth leads to negative growth in income per capita *in the absence of technological progress*.

The possibilities of prolonged technological progress and birth control was not emphasized by the classical economists. As we saw in Chapter 5, and in Chapter 6 as well, technological growth is a source of long-run growth in income per capita. Fixed natural resources and technological progress seem to be two countervailing influences on long-run growth. Which one will be the strongest in the long run is a crucial issue and a central theme in this chapter.

We will study a Solow growth model that also includes a fixed factor, 'land', as an input in the aggregate production function. This will enable us to take a stand on the essential issue whether fixed land or technical progress has the strongest influence on long-run growth. The model we consider comes from an article by the economist William D. Nordhaus,[1] taking up a discussion about the limits to growth provoked by the pessimistic views expressed in some famous books from 1972 and 1992 known as the 'Limits to Growth' reports.[2]

Like Nordhaus, we will go one step further and also present a model with exhaustible natural resources. If a fixed factor such as land, the supply of which stays unchanged as it is

1. William D. Nordhaus, 'Lethal Model 2: The Limits to Growth Revisited', *Brookings Papers on Economic Activity*, **2**, 1992, pp. 1–59.
2. The most well-known of these reports was *The Limits to Growth* published in 1972 by a group of scientists gathered in the Club of Rome. In neo-Malthusian fashion, this book predicted that economic growth and population growth would be brought to an abrupt halt some time in the present century, either as a result of acute shortages of food or raw materials, or as a consequence of an ecological breakdown arising from man-made pollution.

used in production, can imply a drag on growth, one should expect that exhaustible resources such as oil and gas, which disappear as they are used, can imply even more of a growth drag. Our model will show that this intuition is indeed correct. We will again use the model to address the essential question: is the growth drag so strong that it will ultimately bring growth in income per capita to a halt despite continued technological progress?

7.1 The Solow model with land

The production function of the representative, profit-maximizing firm will now include three inputs: capital, labour and land. The consumers own the land (as well as the capital and labour), and the amount of land is fixed and does not change with its use in production. Consumers sell the services of capital, labour and land to the firm in competitive markets where the real factor prices are denoted by r_t, w_t and v_t, respectively. The consumers as a group (or consumers and government together) save a fraction s of total income which is the sum of interest earned on capital, wage income and rent from land. We should note that although the fixed factor is referred to as 'land' for convenience, its services could be interpreted broadly to include all the life support services of the natural environment.

The model

Equation (1) is a Cobb–Douglas aggregate production function with land and exogenous, constant technical coefficients α, β and κ:

$$Y_t = K_t^{\alpha}(A_tL_t)^{\beta}X^{\kappa}, \qquad \alpha > 0, \beta > 0, \kappa > 0, \qquad \alpha + \beta + \kappa = 1. \tag{1}$$

The input X of land does not carry a time subscript since land is in fixed supply. The competitive market for the services of land implies, by adjustment of v_t, that all land supplied is also demanded. The production function exhibits constant returns to the three inputs – capital, labour and land – as it should according to the replication argument. Hence there will be diminishing returns to any single factor, and also to the combination of any two factors. In particular, a doubling of both K_t and L_t will imply that output, Y_t, is multiplied by $2^{\alpha+\beta}$, which is less than two, since $\alpha + \beta$ (= $1 - \kappa$) is less than one. The degree to which $\alpha + \beta$ is below one, or the size of κ, measures *how much* the returns to capital and labour diminish.

Equation (2) is the capital accumulation equation, assuming given rates of gross investment in physical capital (now disregarding human capital) and of depreciation:

$$K_{t+1} = sY_t + (1 - \delta)K_t, \qquad 0 < s < 1, \qquad 0 < \delta < 1. \tag{2}$$

Finally, Eqs (3) and (4) assume given growth rates for the labour force, L_t, and for the labour-augmenting efficiency factor, A_t, respectively. Thus n is the growth rate of the labour force, and g reflects the rate of labour-augmenting technological progress. We assume that both of these rates are at least zero:

$$L_{t+1} = (1 + n)L_t, \qquad n \geqslant 0, \tag{3}$$

$$A_{t+1} = (1 + g)A_t, \qquad g \geqslant 0. \tag{4}$$

In each period the values of the state variables, K_t, L_t and A_t, are predetermined, so for given initial values the four equations above will determine the full time sequence for the state variables and for Y_t.

The competitive factor markets imply that, in each period, each factor will earn its marginal product which depends on the predetermined levels of K_t, L_t, A_t and X:[3]

$$r_t = \alpha\left(\frac{K_t}{A_tL_t}\right)^{\alpha-1}\left(\frac{X}{A_tL_t}\right)^{\kappa}, \tag{5}$$

$$w_t = \beta\left(\frac{K_t}{A_tL_t}\right)^{\alpha}\left(\frac{X}{A_tL_t}\right)^{\kappa}A_t, \tag{6}$$

$$v_t = \kappa\left(\frac{K_t}{A_tL_t}\right)^{\alpha}\left(\frac{X}{A_tL_t}\right)^{\kappa-1}. \tag{7}$$

These three latter equations complete the description of the model, and you can easily verify that in the special case of $\kappa = 0$, where land is of no importance, the model boils down to the Solow model of Chapter 5. You can also easily show that the income *shares* of capital, labour and land (r_tK_t/Y_t, etc.) are α, β and κ, respectively, so the model is again one of constant income shares.

Some production function arithmetic

As with the other Solow models we have analysed, let us first see what can be said about the sources of economic growth from the production function alone. Defining output per worker as $y_t \equiv Y_t/L_t$, capital per worker as $k_t \equiv K_t/L_t$, and land per worker as $x_t \equiv X/L_t$, and dividing on both sides of (1) by L_t, one gets:

$$y_t = k_t^{\alpha}A_t^{\beta}x_t^{\kappa}, \tag{8}$$

and hence, by taking logs and time differences, the approximate growth rates $g_t^y \equiv \ln y_t - \ln y_{t-1}$, etc., must fulfil:

$$g_t^y = \alpha g_t^k + \beta g_t^A - \kappa g_t^n \cong \alpha g_t^k + \beta g - \kappa n. \tag{9}$$

Equation (8) shows that an increase in income per worker can now be obtained (only) by an increase in capital per worker, by better technology, or by an increase in land per worker. With our assumptions the latter effect can only give a decline in growth since the supply of land is constant and we have assumed that population growth is at least zero. In fact, Eq. (9) shows that the approximate growth rate of output per worker, g_t^y, is (approximately) the weighted average of g_t^k, g and $-n$, with weights α, β and κ.

The dynamics of the Solow growth models we have considered earlier were such that the economy converged towards a steady state in which the capital–output ratio was constant. Let us assume for a moment that the dynamics of the Solow model with land has the same implication. If K_t/Y_t converges towards a constant steady state level, so must

3. In the introduction and in this section we have opted for a broad interpretation of 'land' to include nature's ability to absorb waste, etc. In connection with a market for the services of land a more literal interpretation is appropriate since nature's ability to absorb waste is typically not owned or marketed by private people. Note, however, that the dynamics of Eqs (1)–(4) are independent of the real factor prices, so as long as we include only Eqs (1)–(4) in our model, the broader interpretation of X remains valid.

k_t/y_t, and then the steady state growth rates of capital per worker and of output per worker must be identical. In other words, $g_t^k = g_t^y$. Inserting this equality into (9) above and solving for g_t^y shows that g_t^y converges towards a constant g^y fulfilling:

$$g^y \cong \frac{\beta g - \kappa n}{1 - \alpha} = \frac{\beta g - \kappa n}{\beta + \kappa}. \tag{10}$$

We have already arrived at an expression for the steady state growth rate in output per worker. It is not entirely correct to say that this was derived only from 'production function arithmetic'. It was derived from the production function *and* an assumption of a constant capital–output ratio in steady state (and constant growth rates for technology and the labour force). For a proper analysis we should now demonstrate convergence of the capital–output ratio to a constant steady state level. We will do exactly that in the next subsection. Having established a firm foundation for our expression, we will discuss it at length.

Convergence to steady state

You may have the idea that we should (as usual) analyse the model in terms of technology adjusted variables, $\tilde{y}_t \equiv y_t/A_t$, $\tilde{k}_t \equiv k_t/A_t$ and $\tilde{x}_t \equiv x_t/A_t$. However, since $\tilde{x}_t = X/(A_t L_t)$, it cannot possibly converge towards a constant value, except in the case $g = n = 0$, which does not interest us. We are looking for variables that could converge towards constant steady state values. If the assumption we built on in the previous subsection is correct, the capital–output ratio, $z_t \equiv K_t/Y_t = k_t/y_t$, is one such variable. We will therefore analyse the dynamics of our model directly in terms of z_t.

From the per capita production function, $y_t = k_t^\alpha A_t^\beta x_t^\kappa$, we get $z_t = k_t/y_t = k_t/(k_t^\alpha A_t^\beta x_t^\kappa)$, or:

$$z_t = k_t^{1-\alpha} A_t^{-\beta} x_t^{-\kappa}. \tag{11}$$

Leading this expression for z_t one period and inserting appropriate model equations gives:

$$\begin{aligned}
z_{t+1} = k_{t+1}^{1-\alpha} A_{t+1}^{-\beta} x_{t+1}^{-\kappa} &= \left(\frac{K_{t+1}}{L_{t+1}}\right)^{1-\alpha} A_{t+1}^{-\beta} \left(\frac{X}{L_{t+1}}\right)^{-\kappa} \\
&= \left(\frac{sY_t + (1-\delta)K_t}{(1+n)L_t}\right)^{1-\alpha} (1+g)^{-\beta} A_t^{-\beta} \left(\frac{X}{(1+n)L_t}\right)^{-\kappa} \\
&= \left(\frac{1}{(1+n)(1+g)}\right)^{\beta} (sy_t + (1-\delta)k_t)^{1-\alpha} A_t^{-\beta} x_t^{-\kappa} \\
&= \left(\frac{1}{(1+n)(1+g)}\right)^{\beta} \left(s\frac{y_t}{k_t} + (1-\delta)\right)^{1-\alpha} k_t^{1-\alpha} A_t^{-\beta} x_t^{-\kappa} \\
&= \left(\frac{1}{(1+n)(1+g)}\right)^{\beta} \left(\frac{s}{z_t} + (1-\delta)\right)^{1-\alpha} z_t,
\end{aligned}$$

where for the last equality we used the expression in (11) for z_t, and where we have exploited the fact that $\alpha + \beta + \kappa = 1$. We have thus arrived at a transition equation for z_t, most conveniently written as:

$$z_{t+1} = \left(\frac{1}{(1+n)(1+g)}\right)^{\beta} [(s + (1-\delta)z_t)]^{1-\alpha} z_t^{\alpha}. \tag{12}$$

To show that this transition equation implies convergence of the capital–output ratio, z_t, to a particular and constant steady state level, z^*, from any strictly positive initial level, z_0, it suffices to demonstrate the following four properties of the transition curve:

1. It passes through (0, 0). This follows directly from (12) setting $z_t = 0$.
2. It is everywhere strictly increasing. This also follows directly from inspection of (12).
3. It has exactly one strictly positive intersection with the 45° line. We show this by solving the transition equation (12) for $z_{t+1} = z_t = z$. This gives:

$$z^{1-\alpha} = \left(\frac{1}{(1+n)(1+g)}\right)^{\beta} [(s + (1-\delta)z)]^{1-\alpha} \Leftrightarrow$$

$$z = z^* \equiv \frac{s}{[(1+n)(1+g)]^{\beta/(\beta+\kappa)} - (1-\delta)} > 0, \tag{13}$$

where we have used $1 - \alpha = \beta + \kappa$, and the inequality follows from $n \geqslant 0$, $g \geqslant 0$, and $\delta < 1$. We have thus found a unique value, z^*, for the intersection with the 45° line.

4. It has a slope at (0, 0) which is strictly larger than one, the slope of the 45° line.[4] By differentiation of (12) with respect to z_t we get:

$$\frac{dz_{t+1}}{dz_t} = \left(\frac{1}{(1+n)(1+g)}\right)^{\beta}$$
$$\times\ [(1-\alpha)(1-\delta)[(s+(1-\delta)z_t)]^{-\alpha} z_t^{\alpha} + [(s+(1-\delta)z_t)]^{1-\alpha} \alpha z_t^{\alpha-1}].$$

This goes to infinity as z_t goes to zero because of the presence of $z_t^{\alpha-1}$, where $\alpha - 1 < 0$.

One can now sketch the transition diagram for z_t as it must look given the four properties, and convince oneself by 'staircase iteration' that z_t indeed converges to z^* in the long run from any initial $z_0 > 0$ (do this).

In the particular case of $\kappa = 0$, the steady state capital–output ratio z^* becomes $s/(n + g + \delta + ng)$, the same as that in the Solow model (just compute k_t^*/y_t^* from the appropriate equations in Section 3 of Chapter 5).

The steady state growth rate

Summarizing, we have established that the capital–output ratio $z_t = k_t/y_t$ converges towards the constant steady state level z^*. This means that the growth rates of k_t and y_t must converge towards the same rate. Above we found the expression (10) for the common value of the approximate growth rates. This value was:

$$g^y \cong \frac{\beta}{\beta+\kappa} g - \frac{\kappa}{\beta+\kappa} n. \tag{14}$$

An exercise at the end of this chapter will ask you to find the exact growth rate of y_t in steady state and show that this is indeed close to the expression in (14). The same exercise will ask you to verify that the steady state of the Solow model with land meets the requirements for balanced growth formulated in Chapter 2.

4. We could alternatively show that the slope at z^* is smaller than one (do that and explain why this is also sufficient), but here the procedure suggested is the simplest.

Since total output is $Y_t = y_t L_t$, and total capital is $K_t = k_t L_t$, the approximate growth rates of Y_t and K_t in steady state are both close to $g^y + n$, or:

$$g^Y = g^K \cong \frac{\beta}{\beta + \kappa}(g + n). \tag{15}$$

It is illuminating to compare the steady state growth rate g^y in output per worker that we have arrived at here to that of the real Solow model, which was equal to the rate g of labour-augmenting technological progress. In the particular case of $\kappa = 0$, the growth rate g^y in (14) also takes the value g, which should be no surprise. However, when land has some importance in production so that $\kappa > 0$, the g^y in (14) is smaller than g (whenever $g > 0$ or $n > 0$), and the more important land is (the larger κ is), the more g^y will fall below g.

In the 'classical case' where there is no technological progress, $g = 0$, but some population growth, $n > 0$, economic growth will be negative according to (14). The growing input of labour will inevitably press on the limited amount of land. As land becomes more and more scarce relative to labour (and capital), diminishing returns will imply that income per worker will decline. Note from (15) that the input of capital will not be growing at the same rate as labour, but at a somewhat lower rate. Again this reflects the diminishing returns to capital and labour which imply that income per worker, and hence savings per worker, cannot keep pace with the labour force. As you will be asked to show in an exercise, the real wage will fall by the same rate as output per worker, while the real rent on land will be increasing at the same rate as total output, Y_t. The classical economists thought that decreasing wages would ultimately bring population growth to a stop. When that happened, wages would be at a subsistence level while the rent on land would be at a maximum. Working people would be poor while the landlords would live rich and idle lives, an unfortunate state of affairs much emphasized by the classical economists.

If there is no population growth, $n = 0$, but there is some technological growth, $g > 0$, (the 'modern case'), then $0 < g^y < g$. There will be growth in income per worker, but the growth rate will be smaller than the rate of labour-augmenting technological progress. Technological progress means that the *effective* labour input, $A_t L_t$, increases at rate g, while capital, K_t, will increase at a somewhat lower rate than g (see again (15)). The increased amounts of effective labour and capital will press on the fixed amount of land, implying, through diminishing returns to capital and effective labour, that income per worker cannot keep pace with labour-augmenting technological progress.

We can conclude that the classics were right on their own assumptions and perhaps more right than they thought. Not only does land imply a drag on growth in the presence of population growth, it also implies a drag on growth in the sense that technological progress is less effective in creating economic growth than it would have been if land had no importance for aggregate production.

Technological progress *does* have a positive influence on growth in income per capita according to (14), but will it, realistically, be enough to outweigh the negative influence from population growth? The condition for strictly positive growth in GDP per worker is $\beta g > \kappa n$. Is this likely to be fulfilled? A lower bound for labour's share, β, would be approximately 0.6, which would leave 0.4 to the sum of capital's share α and land's share κ. At the very most, κ could thus be close to 0.4. So, in an absolutely worst case, the condition for positive economic growth would be $g > (\frac{2}{3})n$. With an annual population growth rate of 1 per cent, which is between plausible and high by Western standards, an annual growth rate of labour-augmenting technological progress of $\frac{2}{3}$ per cent would suffice for positive long-run growth in income per worker according to our model, and $\frac{2}{3}$ per cent is well below empirical estimates of long-run annual growth rates for labour-augmenting technological progress in Western countries.

We have just derived a worst case condition for long-run economic growth. If we consider more realistic parameter values such as $\beta = 0.6$ and $\alpha = \kappa = 0.2$,[5] we get $g^y = (\frac{3}{4})g - (\frac{1}{4})n$. Hence, if the annual rates are, say, $g = 2$ per cent and $n = 0.5$ per cent, which could be typical for a Western economy, then g^y would be 1.375 per cent per year, which is a decent long-run growth rate and certainly far from zero. The fact that GDP per capita has grown at annual rates of around 2 per cent in many Western countries may suggest that technological growth rates (our Model's g) have been somewhat above 2 per cent per year. Note, on the other hand, that if a country has little control over population growth and experiences annual values for n of around 3 per cent (which is the case in some developing countries, see Table A), then the drag on growth caused by population growth could be as large as $\frac{3}{4}$ of a percentage point, and rates of technological growth, g, above 1 per cent would be needed just to overcome the negative influence of population growth.

Again, our analysis suggests that the classics were, in principle, right. Two hundred years of economic history in the West have shown that they were not sufficiently aware of the possibilities for long-run technological progress and for birth control not caused by 'misery and vice'. However, the calculations above show that the classical pessimism may be sadly relevant for many developing countries that have not got population growth under control.

The steady state balanced growth path

One may get the impression that to have much land is bad. Land implies a drag on growth (compared to a fictitious situation where land is not important) and more so the more important land is. Isn't there anything good about an abundance of land? A glance at the production function will reveal that there is. For given inputs of other factors, a larger amount of land, X, will imply higher output and average income. The amount of land does not affect the *growth rate* in output per worker along the steady state growth path, but it does affect the *level* of this path. We can see this as follows.

Start again from the per capita production function, $y_t = k_t^\alpha A_t^\beta x_t^\kappa$, as stated in (8). To get the capital–output ratio on the right-hand side, divide both sides by y_t^α, thereby getting $y_t^{1-\alpha} = z_t^\alpha A_t^\beta (X/L_t)^\kappa$. In steady state the capital–output ratio, z_t, is constant and always equal to the z^* of (13), so the steady state growth path for y_t is (remembering that $1 - \alpha = \beta + \kappa$):

$$y_t^* = (z^*)^{\alpha/(\beta+\kappa)} A_t^{\beta/(\beta+\kappa)} \left(\frac{X}{L_t}\right)^{\kappa/(\beta+\kappa)}. \tag{16}$$

5. In the article by Nordhaus cited earlier in this chapter (see footnote 1), a labour share of 0.6, and a capital share of 0.2 are considered realistic. This leaves another 0.2 for land's share.

If we want to trace the growth path back to parameters (including those entering z^*) and initial values, we can insert $A_t = A_0(1+g)^t$ and $L_t = L_0(1+n)^t$ to get:

$$y_t^* = (z^*)^{\alpha/(\beta+\kappa)} A_0^{\beta/(\beta+\kappa)} \left(\frac{X}{L_0}\right)^{\kappa/(\beta+\kappa)} (1+g)^{\beta/(\beta+\kappa)t}(1+n)^{-\kappa/(\beta+\kappa)t}. \tag{17}$$

Note that in line with our discussion in the previous subsection, the growth rate along this path is affected by both g and n. In fact, it appears that the growth rate must be the g^y of (14). Taking g and n as given, the growth path lies higher the larger X/L_0 is, that is, the more land there is per worker in some base year. Furthermore (16) shows that in any given year steady state income per worker, y_t^*, depends positively on X/L_t.

We can test these predictions empirically using data for the value of natural resources, among them 'agricultural land', from the World Bank (see the legend of Fig. 7.1 for the exact source) in association with data from the Penn World Table (PWT). The data on natural resources only appear for 1994, so we will have to use these. Taking logs on both sides of (16) gives:

$$\ln y_t^* = \frac{\beta}{\beta+\kappa}\ln A_t + \frac{\alpha}{\beta+\kappa}\ln z^* + \frac{\kappa}{\beta+\kappa}\ln\left(\frac{X}{L_t}\right).$$

Here we should insert the expression for z^* from (13), but taking the log of z^* does not give a handy expression because of the exponent in the denominator of z^*. Since β is realistically at least 0.6, and κ is relatively small, 0.2 say, we may crudely approximate the exponent $\beta/(\beta+\kappa)$ to be 1, in which case $\ln z^* \cong \ln s - \ln(n+g+\delta)$, here also using $ng \cong 0$. Hence from the above formula for $\ln y_t^*$:

$$\ln y_t^* \cong \frac{\beta}{\beta+\kappa}\ln A_t + \frac{\alpha}{\beta+\kappa}[\ln s - \ln(n+g+\delta)] + \frac{\kappa}{\beta+\kappa}\ln\left(\frac{X}{L_t}\right). \tag{18}$$

We have arrived (more or less) at the steady state equation for the general Solow model corrected for the presence of land (setting $\kappa = 0$ in (18) gives the steady state equation of Chapter 5 with $ng = 0$). With our usual short-cuts, that is, assuming that the basic parameters α, β and κ as well as A_t are the same in all countries, (18) suggests the following regression equation across countries i:

$$\ln y_{00}^i = \gamma_0 + \gamma_1[\ln s^i - \ln(n^i + 0.075)] + \gamma_2 \ln\left(\frac{X^i}{L_{94}^i}\right).$$

Here X^i is the 1994 value of 'agricultural land' for country i according to the data from the World Bank, L_{94}^i is the labour force in 1994 from the PWT, and, as usual, y_{00}^i is the GDP per worker in year 2000, while s^i and n^i are annual averages over the period 1960 to 2000. Strictly speaking, (18) suggests using the same year for GDP per worker and for land per worker, but 1994 is so close to 2000 that our inaccuracy should not be serious. An estimation based on 73 countries with at least grade C for data quality in PWT and for which data are available from the PWT and the World Bank gives:

$$\ln y_{00}^i = 2.71 + \underset{(\text{se}=0.14)}{1.35}[\ln s^i - \ln(n^i + 0.075)] + \underset{(\text{se}=0.11)}{0.38}\ln(X^i/L_{94}^i), \text{ adj. } R^2 = 0.61. \tag{19}$$

There seems to be a positive and significant influence from land per worker on GDP per worker, and the influence from $\ln s - \ln(n+g+\delta)$ is also estimated to have the 'correct'

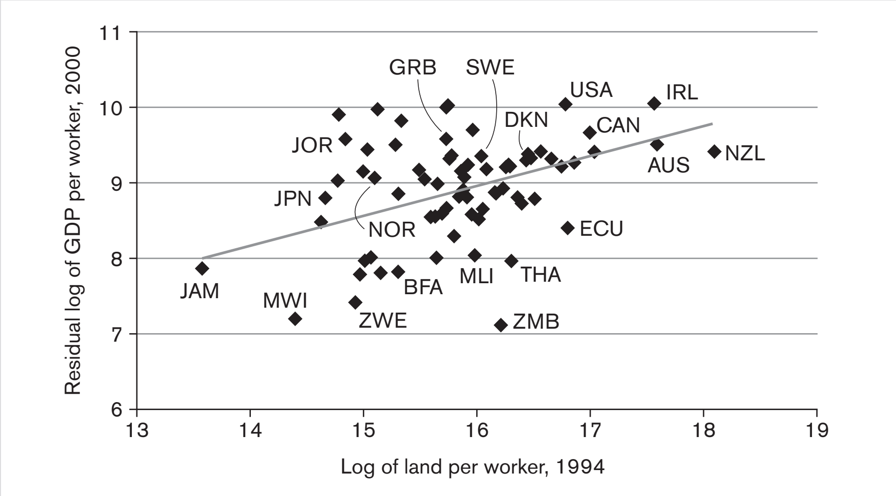

Figure 7.1: Residual log of GDP per worker against log of the value of land per worker, 73 countries
Source: Penn World Table 6.1. Estimates of land from World Bank: Wealth Estimates, 1994.

sign. An exercise will ask you if the estimates of γ_1 and γ_2 seem reasonable in view of plausible values for the parameters α, β and κ.

Figure 7.1 gives an illustration. It plots the 'residual value' of the log of GDP per worker, $\ln y^i_{00} - 1.35[\ln s^i - \ln(n^i + 0.075)]$, that is, the part not explained by the structural parameters s^i and n^i, against land per worker, $\ln(X^i/L^i_{94})$, across countries. The figure gives an indication of a relatively tight positive relationship.

7.2 The Solow model with oil

Land is not the only natural resource of importance for aggregate production. Non-renewable resources such as oil, gas, coal, metals, etc., are also important. Unlike land, the amount of which stays constant when used in production, non-renewable resources are depleted as they are used. Intuitively this should imply even more of a drag on growth than does land. We will investigate this intuition by analysing a Solow model with 'oil', using the term 'oil' generically for exhaustible resources. The model will abstract from the presence of land, but in the next section we will consider a model with both land and oil.

The underlying micro world one should now have in mind is like the one for the Solow model with land, with one exception: the natural resource owned by consumers is no longer a constant amount of land but rather, at the beginning of any period t, the total *remaining* stock, R_t, of an exhaustible resource, oil. The part of this stock that is used as energy input during period t will be denoted by E_t. An important resource depletion equation will state that the stock of oil will be reduced from one period to the next by exactly the amount used in production in the first of the periods.

The model

The production function of the representative firm is now:

$$Y_t = K_t^{\alpha}(A_t L_t)^{\beta} E_t^{\varepsilon}, \qquad \alpha > 0, \beta > 0, \varepsilon > 0, \qquad \alpha + \beta + \varepsilon = 1. \tag{20}$$

In line with the replication argument, this production function has constant returns to scale with respect to the three inputs, K_t, L_t and E_t, that can be varied at the firm level within a period. The equations describing capital accumulation, population growth and technological progress are as before:

$$K_{t+1} = sY_t + (1-\delta)K_t, \qquad 0 < s < 1, \qquad 0 < \delta < 1, \tag{21}$$

$$L_{t+1} = (1+n)L_t, \qquad n \geqslant 0, \tag{22}$$

$$A_{t+1} = (1+g)A_t, \qquad g \geqslant 0. \tag{23}$$

The depletion of the natural resource is described by:

$$R_{t+1} = R_t - E_t. \tag{24}$$

This leaves us with the question: how much of the remaining stock, R_t, of the exhaustible resource will be used in production as the input, E_t, in each period t?

For a proper micro-founded answer (as given for the inputs of labour and capital) we should assume a market for oil, let us say, for simplicity, a competitive one with a real price, u_t, per unit of oil. The demand in this market would come from the representative firm and would in any period obey a usual marginal product (of energy E_t) equal to real price condition. The supply would, under the not quite realistic assumption of a perfectly competitive market, come from many small suppliers (possibly 'represented' by one) who would in total own R_t, and each of whom would have negligible influence on the current and future prices of oil and on the total future stock of oil. A supplier would be faced with a basic trade-off. Since the stock of oil is gradually used up, energy prices would and should be expected to increase over time, which speaks for not selling too much off today, but for saving it for later use. The possibility of placing a revenue from the sales of oil in interest-bearing capital speaks, on the other hand, for selling off more now.

The interaction between demand and supply involving current and (correctly foreseen) future prices would solve the full problem of how a given remaining stock of the exhaustible resource would be allocated over time. This problem is complex and we will not present a solution here. Instead we will postulate that the counteracting forces described can result in the 'rule' that in each period a certain and constant fraction s_E of the remaining stock is used in production, that is:

$$E_t = s_E R_t, \qquad 0 < s_E < 1. \tag{25}$$

Alternatively, we can view our analysis as simply exploring the *consequences* of a constant extraction rate, s_E, without bothering about the micro foundations for such a rule.

In (25) we have assumed $s_E < 1$, which is, of course, required. In Nordhaus' article an annual value for the extraction rate s_E of 0.005 is considered reasonable, corresponding to one half of a per cent of the remaining stock of exhaustible resources being used each year. At any rate, an assumption of $s_E < \delta$ is plausible as annual depreciation rates for aggregate capital are usually estimated to at least 0.05. We will therefore make this assumption, which will be of relevance for what you are asked to demonstrate in Exercise 5 at the end of the chapter.

Our model consists of the six equations above. It is easy to see that for given initial values of the state variables, K_t, L_t, A_t and R_t, the six equations determine the full evolution of all the endogenous variables (convince yourself of this). Note that the two equations (24) and (25) imply that $R_{t+1} = (1 - s_E)R_t$. Given our resource extraction rule, the growth rate of the stock of the resource is constant and equal to $-s_E$. This means that just like technology, A_t, and population, L_t, the stock of resources, R_t, lives a dynamic life of its own and evolves over time as $R_t = (1 - s_E)^t R_0$ from an initial value R_0.

Having assumed a specific rule for the use of the exhaustible resource, it is easy to complete the model with equations stating the (rental) prices of the factors, r_t, w_t and u_t, respectively, in terms of the state variables. An exercise will ask you to do so and to think about how the factor rewards evolve over time. It should be no surprise that the income shares of capital and labour will be constant, α and β, respectively. Similarly, energy's share, $u_t E_t/Y_t = u_t s_E R_t/Y_t$, remains constant at ε in all periods as the stock of oil is run down.

The steady state growth rate

Inserting $E_t = s_E R_t$ into the production function gives: $Y_t = K_t^\alpha (A_t L_t)^\beta (s_E R_t)^\varepsilon$. Dividing both sides by L_t gives:

$$y_t = k_t^\alpha A_t^\beta \left(\frac{s_E R_t}{L_t}\right)^\varepsilon = s_E^\varepsilon k_t^\alpha A_t^\beta R_t^\varepsilon L_t^{-\varepsilon}. \tag{26}$$

From the latter equality, taking logs and time differences, one gets in the now familiar notation:

$$g_t^y = \alpha g_t^k + \beta g_t^A + \varepsilon g_t^R - \varepsilon g_t^L \cong \alpha g_t^k + \beta g - \varepsilon s_E - \varepsilon n, \tag{27}$$

where it was used that g_t^R is approximately equal to $-s_E$. Assuming again that the dynamics of our full model are such that the capital–output ratio, k_t/y_t, converges towards a constant steady state level, the growth rates of output and capital per worker must converge towards the same rate. Setting $g_t^y = g_t^k$ in (27) and solving shows that both rates must converge towards a common constant level, approximately given by:

$$g^y \cong \frac{\beta}{\beta + \varepsilon} g - \frac{\varepsilon}{\beta + \varepsilon} n - \frac{\varepsilon}{\beta + \varepsilon} s_E. \tag{28}$$

We should demonstrate (rather than assume) that the dynamics of our model imply convergence towards a steady state with a constant capital–output ratio. An exercise will ask you to do this yourself. It is done in much the same way as for the Solow model with land.

The growth rate, g^y, of (28) is in two respects just like the one obtained in (14) for the Solow model with land. First, a given rate, g, of labour-augmenting technological progress is less effective in creating economic growth than in the absence of natural resources, i.e., $\beta/(\beta + \varepsilon)$ is less than one. Second, population growth implies a drag on economic growth of the size $-[\varepsilon/(\beta+\varepsilon)]n$. These two features occur for the same reasons as in the model with land. Increasing amounts of effective labour in association with increasing amounts of capital will press on the limited amount of the natural resource and therefore, through diminishing returns, imply a slower (and possibly negative) growth in income per worker than with no natural resource. This time, however, the natural resource is in even more limited supply as it disappears gradually through its use in production. This means that over time, the diminishing returns to capital and effective labour arising from the scarcity of the natural resource become more and more severe, implying even more of a drag on growth than in the case of a fixed factor. The last term, $-[\varepsilon/(\beta+\varepsilon)]s_E$, in (28) is explained by this increased scarcity of natural resources. Clearly, the larger is the extraction rate, s_E, the faster the exhaustible resource will be depleted, and the faster the negative influence from diminishing returns to the other factors will grow.

We could proceed as in the model with land, trying to make a numerical evaluation of g^y assuming realistic values for α, β, ε and s_E, etc. You should do such numerical exercises around (28) yourself. However, doing so would result in an 'error' that tends to overestimate the negative influence on growth of land and exhaustible resources. The reason is as follows. First we considered a model with land, but no exhaustible resource. We assumed plausible shares of capital and labour ($\alpha = 0.2$ and $\beta = 0.6$) and then gave the remaining share 0.2 to the remaining factor, land. Now we consider a model with an exhaustible resource but without land, and we are again about to give all of the remaining share, the 0.2, to this last factor. In the real world there are both fixed natural resources and exhaustible ones and with constant returns to all factors, we cannot give all of the income share that is left after capital and labour's shares to each of them. In other words, for a really meaningful numerical exercise there is no way around the job of setting up a model with both land and exhaustible resources.

This is exactly what we do in the next section. You can view the model of this section as a building block on the way, but a very important one, since it demonstrates the *isolated effects* of the presence of an exhaustible natural resource.

The steady state balanced growth path

Consider again (26). Dividing on both sides of the first equality by y_t^α one gets, as in the model with land, the capital–output ratio $z_t = k_t/y_t$ on the right-hand side: $y_t^{1-\alpha} = z_t^\alpha A_t^\beta s_E^\varepsilon (R_t/L_t)^\varepsilon$. An exercise will ask you to show that in this model z_t also converges towards a constant steady state level, z^*, and to find the appropriate expression for z^*. Inserting z^* for z_t, raising to the power of $1/(1-\alpha)$ on both sides, and using that $1-\alpha = \beta+\varepsilon$, gives the steady state balanced growth path for y_t:

$$y_t^* = (z^*)^{\alpha/(\beta+\varepsilon)} A_t^{\beta/(\beta+\varepsilon)} s_E^{\varepsilon/(\beta+\varepsilon)} \left(\frac{R_t}{L_t}\right)^{\varepsilon/(\beta+\varepsilon)}. \tag{29}$$

If one wants to trace this back to parameters and initial values, one can insert $A_t = (1+g)^t A_0$, $L_t = (1+n)^t L_0$, $R_t = (1-s_E)^t R_0$, and of course the correct expression for z^*. In any case, the growth path will be higher the higher R_0/L_0 is, given the rates n, s_E, etc. One can also see directly from (29) that y_t^* is positively affected by R_t/L_t. Hence, according to our model, it is good to have an abundance of exhaustible resources, in a base year or left over in the current year, other circumstances being equal.[6]

As stated, finding the right expression for z^* is left to you as an exercise, but it should be no surprise that, as in the model with land, $\ln z^* \cong \ln s - \ln(n + g + \delta)$ should hold as a crude approximation. Hence, taking logs on both sides in (29), the steady state balanced growth path can be expressed as:

$$\ln y_t^* \cong \frac{\beta}{\beta+\varepsilon}\ln A_t + \frac{\varepsilon}{\beta+\varepsilon}\ln s_E + \frac{\alpha}{\beta+\varepsilon}[\ln s - \ln(n+g+\delta)] + \frac{\varepsilon}{\beta+\varepsilon}\ln\left(\frac{R_t}{L_t}\right). \qquad (30)$$

With the usual short-cuts (assuming that the basic parameters α, β and ε as well as both A_t and s_E are the same in all countries), (30) suggests a regression equation across countries i:

$$\ln y_{00}^i = \gamma_0 + \gamma_1[\ln s^i - \ln(n^i + 0.075)] + \gamma_3 \ln\left(\frac{R_{94}^i}{L_{94}^i}\right). \qquad (31)$$

For R_{94}^i we use World Bank data for the value of 'subsoil assets' (including oil, gas, coal, metals, etc.). Again, the error from mixing year 2000 data for GDP per worker with 1994 data for 'oil' per worker should be small. An estimation of (31) based on 60 countries gives:

$$\ln y_{00}^i = 6.83 + \underset{(\text{se}=0.16)}{1.26}[\ln s^i - \ln(n^i + 0.075)] + \underset{(\text{se}=0.03)}{0.14}\ln(R_{94}^i/L_{94}^i), \text{ adj. } R^2 = 0.61.$$

Figure 7.2 illustrates by plotting the 'residual log of GDP per worker' (that part of GDP which is not explained by the structural parameters s^i and n^i) in 2000 against the log of the value of subsoil assets per worker in 1994.

The conclusion from this empirical investigation of the *partial* influence of 'oil' per worker on income per worker is that indeed there seems to be a positive and significant influence in accordance with our model.

7.3 The Solow model with both land and oil

The model

The aggregate production function with both a factor in fixed supply, land, and an exhaustible resource such as oil, is (in the obvious notation):

$$Y_t = K_t^\alpha (A_t L_t)^\beta X^\kappa E_t^\varepsilon, \qquad \alpha > 0, \beta > 0, \kappa > 0, \varepsilon > 0, \qquad \alpha + \beta + \kappa + \varepsilon = 1.$$

There are constant returns to the four factors, capital, labour, land and energy. Hence the

6. Note that this is a statement about the influence of natural resources on the *level* of income, and not about the impact on the *growth rate*. Referring to the latter impact, some economists have spoken of 'the curse of natural resources'. By this they mean that in many resource-abundant countries, opposing interest groups have often been in political and even armed conflict to secure the rents from the exploitation of the resources for themselves. There is also a danger that resource abundance in a country will reduce the perceived need to invest in growth-enhancing activities such as education, research and development, etc. For whatever reason, it is a fact that many resource-rich countries have had a poor historical growth performance. Spectacular examples of this were given in Fig. 2.1 which showed that oil-rich countries like Nigeria and Venezuela have experienced *negative* growth in income per worker since 1960.

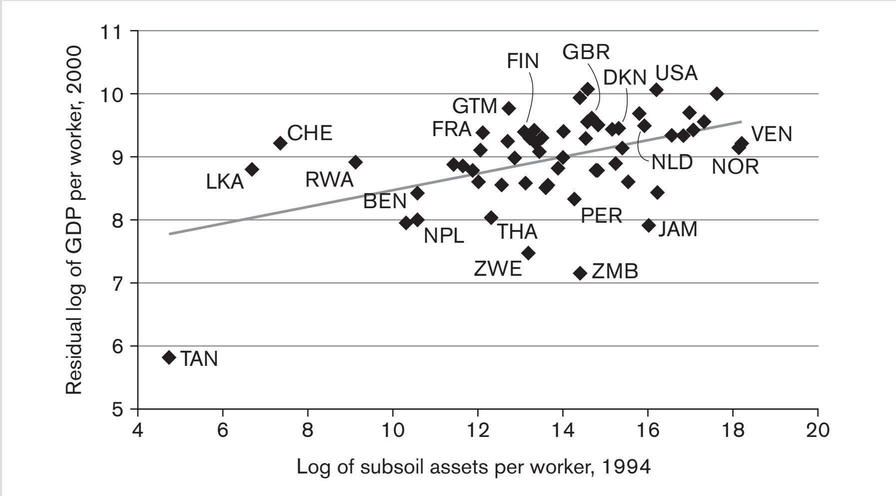

Figure 7.2: Residual log of GDP per worker against log of the value of subsoil assets per worker, 60 countries

Source: Penn World Table 6.1. Estimates of subsoil assets from World Bank: Wealth Estimates, 1994.

sum of the income shares of the two latter factors, $\kappa + \varepsilon$, must be what is left over after capital and labour's shares. The remaining equations of the model are again (21)–(25) above, where we continue to assume $s_E < \delta$. One can complete the model with equations for the factor reward rates, r_t, w_t, v_t and u_t, from which constant income shares would follow (show this).

The steady state growth rate

Inserting into the production function that $E_t = s_E R_t$ and dividing both sides by L_t now gives:

$$y_t = k_t^\alpha A_t^\beta \left(\frac{X}{L_t}\right)^\kappa \left(\frac{s_E R_t}{L_t}\right)^\varepsilon = s_E^\varepsilon X^\kappa k_t^\alpha A_t^\beta R_t^\varepsilon L_t^{-\kappa-\varepsilon}. \tag{32}$$

From the latter equality we get, taking log differences and in the usual notation:

$$g_t^y = \alpha g_t^k + \beta g_t^A + \varepsilon g_t^R - (\kappa + \varepsilon) g_t^L \cong \alpha g_t^k + \beta g - \varepsilon s_E - (\kappa + \varepsilon) n. \tag{33}$$

In the long run the capital–output ratio converges towards a constant steady state level, as you are again asked to show in an exercise. Therefore, g_t^y and g_t^k converge towards the same rate. Setting $g_t^y = g_t^k$ in (33), remembering that $1 - \alpha = \beta + \kappa + \varepsilon$, and calling the constant solution g^y gives:

$$g^y = \frac{\beta}{\beta + \kappa + \varepsilon} g - \frac{\kappa + \varepsilon}{\beta + \kappa + \varepsilon} n - \frac{\varepsilon}{\beta + \kappa + \varepsilon} s_E. \tag{34}$$

This formula is, in a sense, a perfect combination of the two g^y formulae from the model with only land and that with only oil. The explanations of the formula, in terms of the diminishing returns arising from the presence of land and the increasingly severe diminishing returns arising from the presence of oil, are also the same. There is an important difference to the model with just oil with respect to the negative influence of s_E. Taking capital's and labour's shares as given, e.g. $\alpha = 0.2$ and $\beta = 0.6$, one would have to set $\varepsilon = 0.2$ in the model with only oil, arriving, from (28), at a coefficient on s_E of $\frac{1}{4}$. In the present model κ and ε should share the remaining 0.2, e.g. $\kappa = \varepsilon = 0.1$, in which case the coefficient on s_E becomes $\frac{1}{8}$. With both land and oil in the model, the negative influence on growth from the extraction rate, s_E, is potentially much smaller.

Indeed, Nordhaus considers the values $\alpha = 0.2$, $\beta = 0.6$ and $\kappa = \varepsilon = 0.1$ to be realistic. With these values (34) becomes

$$g^y = 0.75g - 0.25n - 0.125s_E. \tag{35}$$

If we set $n = 0.01$ (which is a plausible *upper* estimate for developed countries, many of which are considerably below) and, taking again a plausible value from Nordhaus's article, set $s_E = 0.005$, we get for the long-run growth rate of income per worker:

$$g^y = 0.75g - 0.0025 - 0.000625 \cong 0.75g - 0.003. \tag{36}$$

This equation can be said to involve two growth drags. The most serious one seems to be that (labour-augmenting) technological growth is only three quarters as effective in creating economic growth as it would be in the (fictitious) absence of natural resources. However, a rate of technological progress of, e.g., 2.4 per cent per year will still contribute 1.8 percentage points to growth in GDP per capita. The other one, arising from the combined effect of increasing population and exhaustion of the limited resources, appears as a further drag (on the 'reduced' growth rate $0.75g$) of at most 0.3 percentage points.

Clearly, *none of the drags on growth arising from natural resources seem to be of a growth-preventing magnitude as long as there is a reasonable amount of technological progress and population growth is under control* as typically seen in the most developed part of the world. This is indeed a major conclusion from our analysis.

One should again bear in mind that a population growth rate of 3 per cent, as seen in some developing countries, would imply that the second of the above growth drags would be around 0.8 percentage points, so a g of around 1 per cent would be needed just to overcome the negative influence of population growth. *If population growth is not under control, the drag on growth arising as the number of people presses on the limited resources is of a serious, possibly growth-preventing magnitude.*

Equation (35) above says that a one percentage point *change* in the population growth rate gives a quarter of a percentage point *change* in the growth rate of GDP per capita, other things being equal. This is an effect completely different from the level effect we have discussed before. It is not (just) the *level* of the long-run growth path for GDP per worker that is negatively affected by population growth, it is also the *growth rate* along this path. Furthermore, it is an effect of potentially serious magnitude. For instance, *the observed differences in annual population growth rates between developed and developing countries of up to three percentage points would, according to our model, imply differences in annual growth rates of GDP per worker of 0.75 percentage points*. This is a big difference taking long-run compound interest effects into account.

Our model's steady state prediction of how economic growth should depend on population growth can be confronted with the usual type of cross-country data. One can simply plot long-run growth rates in GDP per worker against long-run population growth rates as in Fig. 7.3. Details of the estimation of the straight line are given in the figure's legend.

The figure and the estimation point to a relationship that is perhaps not too tight, but significantly negative. Note that the estimated slope is around −0.5, with a standard error of 0.16. Thus the estimated slope is within two standard deviations of the theoretical value of −0.25 predicted by our model (see the coefficient on n in (35)). Hence the direction as well as the magnitude of the influence of population growth on economic growth in our model's steady state are not contradicted by the data.

We should remind you once again of the warning concerning reversed causality that is very often important for the interpretation of empirical relationships in economics. Does Fig. 7.3 really illustrate a negative influence of population growth on economic growth across countries, or is it rather the other way around that a better growth performance helps to get population growth under control? One simply cannot tell from the empirical evidence presented here. Our model provides theoretical reasons why the causality should run from population growth to economic growth, but there may well be other mechanisms which tend to generate a feedback from higher income growth to lower population growth. Indeed, the existence of such mechanisms might help to explain why our estimated correlation between population growth and income growth is somewhat higher than suggested by our theoretical model.

The steady state balanced growth path

If we divide both sides of the first equality of (32) by y_t^{α}, again to get the capital–output ratio on the right-hand side, we obtain: $y_t^{1-\alpha} = z_t^{\alpha} A_t^{\beta} (X/L_t)^{\kappa} s_E^{\varepsilon} (R_t/L_t)^{\varepsilon}$, where $z_t \equiv K_t/Y_t$. In

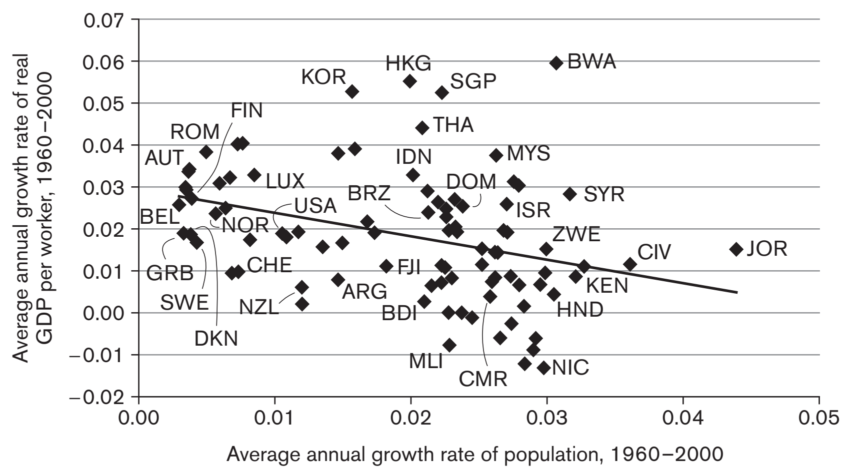

Figure 7.3: Average annual growth rate of real GDP per worker against average annual growth rate of population, 90 countries

Note: The estimated line is $y = 0.0294 - 0.5584x$. Some details for the statistically proficient: $R^2 = 0.12$ and $t = -3.5$ (standard error on estimated coefficient is 0.16).

Source: Penn World Table 6.1.

steady state z_t has reached a constant value, z^*. Hence the steady state balanced growth path for y_t is:

$$y_t^* = (z^*)^{\alpha/(\beta+\kappa+\varepsilon)} A_t^{\beta/(\beta+\kappa+\varepsilon)} \left(\frac{X}{L_t}\right)^{\kappa/(\beta+\kappa+\varepsilon)} s_E^{\varepsilon/(\beta+\kappa+\varepsilon)} \left(\frac{R_t}{L_t}\right)^{\varepsilon/(\beta+\kappa+\varepsilon)}.$$

Getting an expression for z^* requires a dynamic analysis like that conducted for the model with only land, but it should be no surprise that again $\ln z^* \cong \ln s - \ln(n+g+\delta)$ can be used as a crude approximation (an exercise will ask you to demonstrate convergence and find z^*). Hence, by taking logs on both sides of the equation above, the steady state balanced growth path can be expressed as:

$$\ln y_t^* \cong \frac{\beta}{\beta+\kappa+\varepsilon} \ln A_t + \frac{\varepsilon}{\beta+\kappa+\varepsilon} \ln s_E + \frac{\alpha}{\beta+\kappa+\varepsilon} [\ln s - \ln(n+g+\delta)] + \frac{\kappa}{\beta+\kappa+\varepsilon} \ln\left(\frac{X}{L_t}\right) + \frac{\varepsilon}{\beta+\kappa+\varepsilon} \ln\left(\frac{R_t}{L_t}\right). \quad (37)$$

This suggests a regression equation:

$$\ln y_{00}^i = \gamma_0 + \gamma_1[\ln s^i - \ln(n^i + 0.075)] + \gamma_2 \ln\left(\frac{X^i}{L_{94}^i}\right) + \gamma_3 \ln\left(\frac{R_{94}^i}{L_{94}^i}\right), \quad (38)$$

across countries. Once again we take X^i to be the 1994 value of 'agricultural land' for country i, while R_{94}^i is the value of 'subsoil assets' in 1994, according to the data from the World Bank. An estimation based on 60 countries gives:

$$\ln y_{00}^i = 3.48 + \underset{(\text{se}=0.16)}{1.23} [\ln s^i - \ln(n^i + 0.075)] + \underset{(\text{se}=0.11)}{0.25} \ln\left(\frac{X^i}{L_{94}^i}\right) + \underset{(\text{se}=0.04)}{0.12} \ln\left(\frac{R_{94}^i}{L_{94}^i}\right), \text{ adj. } R^2 = 0.63. \quad (39)$$

One should note that for both of the types of natural resources considered, the influence is in the expected direction and statistically significant.

7.4 Unlimited substitution? An important issue of concern

This chapter deals with the very big issue *whether sustained growth in per capita income can essentially go on for ever despite the limitations given by the natural environment*. Our main conclusions are repeated in the following double statement:

- *If underlying technological progress and population growth rates can be held at levels that have typically been seen over long (but recent) periods in Western countries, permanent economic growth seems to be sustainable, that is, not in conflict with nature's finiteness.*
- *If population growth rates are at the highest levels seen in developing countries, the limited natural resources imply a serious drag on growth that may eliminate most or all of the positive influence of technological progress on income per capita.*

The second of these statements means that for the poorest parts of the world the classical ('dismal science') views are still sadly relevant. Nevertheless the overall conclusion with respect to the possibility of sustainable growth is optimistic because of the first of the statements above.

The mainly optimistic conclusion is, of course, reached on the premises of the models we have considered. It is now time to face one critical modelling assumption explicitly: our Cobb–Douglas production functions assume that there are no limits to the technological possibilities of substituting capital and technologically augmented labour for scarce natural resources. Indeed, we implicitly assumed that production can continue to grow even as the input of natural resources becomes infinitely small relative to other inputs. Much of the public debate on the possible limits to growth is really about the validity of this assumption, which is clearly not an innocent one.

Influenced by the economic history of Western countries over the last two centuries, most economists tend to be technological optimists, believing that the substitution possibilities in production are essentially unlimited in the long run. They point out that whenever a particular natural resource becomes scarce, its relative price will tend to go up, providing strong incentives for the development of alternative production techniques and consumption patterns which rely less on the scarce factor. According to this view the modelling assumption of unlimited substitution should not be taken literally, but should be seen as representing the described incentive effect. In other words, when the world has only one ton of copper left it is not that production will literally still use small amounts of the remaining copper in association with extremely sophisticated labour in accordance with an old production function. Rather, copper will have been replaced by a substituting product invented, as copper, because of its scarcity, became extremely expensive and the economic gains from developing a substituting product became very large. With just one or two types of natural resources in our production function we cannot describe this process explicitly, but the assumption of unlimited substitution can be seen as representing it.

The same (optimistic) economists also argue that the deterioration of the natural environment caused by the polluting activities of firms and consumers can be held in check by intelligent use of 'green' taxes and other economic instruments in environmental policy.

On the other hand, many natural scientists and environmentalists argue that there are in fact limits to the possibilities of substituting other factors for certain essential raw materials and life support services offered by the environment. The more moderate critics doubt that the market mechanisms will always ensure the development of new substitute techniques in time to prevent serious disruptions stemming from environmental degradation.

Passing a fully qualified judgement on these fundamental issues requires insight into the natural sciences as well as into economics. With this chapter we do not pretend to have given a definitive answer to the question: 'Are there limits to growth?' However, we hope to have shed some light on the basic assumptions one needs to make to warrant either growth optimism or growth pessimism.

7.5 Summary

1. Land and natural resources are important inputs into production. This is particularly the case if 'land' is interpreted in a broad sense to include all the life support services of the natural environment. This chapter has developed an extended Solow model where capital and labour are combined with natural resources in production. By the replication argument, an increase in all factor inputs yields constant returns to scale, but the need for natural resources implies diminishing returns to the combination of labour and capital. The crucial issue is whether technological progress is sufficient to generate perpetual economic growth despite the scarcity of natural resources.

2. In a three-factor Solow model where labour and capital are combined with a *fixed supply of land*, technological progress serves in part to offset the negative influence of increasing land scarcity on income per worker. Hence the growth rate of per capita income is lower than the rate of labour-augmenting technological progress even if there is no population growth. Positive population growth implies a further drag on growth in per capita income when land is in fixed supply and could even bring growth to a halt, as predicted by the classical economists. However, for parameter values characteristic of developed countries, a fixed supply of natural resources would not realistically prevent sustained positive growth in income per capita.

3. The Solow model with a fixed supply of land predicts that a country with a higher initial stock of land per worker will have a higher level of GDP per worker, all other things equal. The international empirical evidence is consistent with this prediction.

4. In a three-factor Solow model where labour and capital are combined with an *exhaustible* natural resource ('oil'), the gradual depletion of the natural resource creates even stronger diminishing returns to capital and labour than in the model with a fixed stock of land. Hence a higher rate of technical progress is required to outweigh the negative effect on growth arising from population growth and increasing natural resource scarcity.

5. The Solow model with oil implies that a larger initial stock of the exhaustible resource per worker should, *ceteris paribus*, yield a higher GDP per worker. Confirming this, the international evidence shows a positive relationship between the stock of subsoil assets and the level of real GDP per worker.

6. We also developed a four-factor Solow model where production uses a fixed stock of land as well as inputs of an exhaustible natural resource in combination with capital and labour. In such an economy positive economic growth is sustainable for parameter values typical for Western countries, but may not be attainable in poor developing countries which have not managed to bring population growth under control.

7. The Solow model with land and oil predicts that the average rate of growth in GDP per worker will be lower the higher the rate of population growth, other things equal. The international cross-country evidence does indeed show a negative relationship between population growth and economic growth, and the quantitative relationship is roughly in line with the prediction of the Solow model. However, causality may not only run from population growth to economic growth, but also in the opposite direction.

8. A crucial assumption in the Solow model with scarce natural resources is that there are unlimited technological possibilities of substituting capital and (educated) labour for increasingly scarce natural resource inputs. This assumption should not be taken literally, but should be seen as reflecting a belief that growing scarcity of a particular natural resource will generate an economic incentive to develop new substitute inputs. Much of the debate about the possible limits to growth is really about the validity of the assumption of unlimited substitution possibilities. Resolving this issue requires insight into the natural sciences as well as into economics.

7.6 Exercises

Exercise 1. Exact growth rates, and balanced growth, in the steady state of the Solow model with land

1. Show that in the Solow model with land the exact (in contrast to the approximate), common growth rate g^{ye} of output per worker and of capital per worker in steady state is:

$$g^{ye} = (1+g)^{\beta/(\beta+\kappa)}\left(\frac{1}{1+n}\right)^{\kappa/(\beta+\kappa)} - 1.$$

(Hint: Start again from the per capita production function (8), this time do not take logs, but write (8) also for period $t-1$, divide one by the other and proceed in terms of growth *factors*.) Show that g^{ye} is approximately equal to the g^y of (14) in Section 1. (Hint: Use repeatedly the approximation, $a - 1 \cong \ln a$.)

2. Show that the steady state of the Solow model with land is in accordance with balanced growth, not only in the respect that the capital–output ratio is constant, but also in the following respects: the real wage rate, w_t, grows by the same rate as both output per worker and capital per worker, and the real interest rate, r_t, is constant. Show also that the real rate of rent on land, v_t, grows at the same rate as total output (not per worker), and that whenever $n > 0$, the real rent on land, v_t, grows faster than the real wage rate.

3. Assume that $n > 0$, but $g = 0$, more or less as considered by the classical economists. Describe how the real rates, r_t, w_t and v_t, evolve over time. If there is a fixed number of landlords owning the land, how will their life conditions evolve compared to the conditions of the workers?

Exercise 2. The Solow model with land in continuous time

Discrete time may be easier to grasp, but analysis in continuous time often runs more elegantly. The latter may be particularly true for the models considered in this chapter. Consider the following continuous time version of the Solow model with land (in obvious notation):

$$Y = K^{\alpha}(AL)^{\beta}X^{\kappa}, \qquad \alpha, \beta, \kappa > 0 \qquad \alpha + \beta + \kappa = 1,$$

$$\dot{K} = sY - \delta K, \qquad 0 < s, \delta < 1,$$

$$\frac{\dot{L}}{L} = n, \qquad n \geqslant 0$$

$$\frac{\dot{A}}{A} = g, \qquad g \geqslant 0.$$

1. Show that the per capita production function and the capital–output ratio (still in obvious notation, e.g. $x \equiv X/L$) are, respectively:

$$y = k^{\alpha} A^{\beta} x^{\kappa},$$
$$z = k^{1-\alpha} A^{-\beta} x^{-\kappa}.$$

2. Show that the law of motion for z following from the model is the following *linear* differential equation in z:

$$\dot{z} = (\beta + \kappa)s - \lambda z,$$

where $\lambda \equiv (\beta + \kappa)\delta + \beta(n + g)$. (Hint: Start from the expression for z you found in **1**. Take logs and differentiate with respect to time. In the equation you arrive at, use, e.g., that $\dot{k}/k = \dot{K}/K - n$, etc. Remember that $1 - \alpha = \beta + \kappa$).

3. Show that the steady state value, z^*, for the capital–output ratio is:

$$z^* = \frac{s}{\frac{\beta}{(\beta + \kappa)}(n + g) + \delta},$$

and show that the above differential equation implies convergence of z to z^*. (For the latter you can just draw $\dot{z}$ as a function of z or you can solve the differential equation as in Exercise 3 of Chapter 3.)

4. Show that the growth rate of y in steady state is *exactly*:

$$\frac{\beta g - \kappa n}{\beta + \kappa},$$

and compare this to the approximate growth rate, g^y, found in Section 1 of this chapter.

Exercise 3. Golden rule in the Solow model with land

For the Solow model with land, find the value of the savings/investment rate, s, that will imply the highest possible position for the growth path of consumption per worker in steady state. Compare your result to the golden rule of the Solow model of Chapter 5, and explain the similarity. Why is the result nevertheless different here? From reasonable parameter values considered in this chapter, of what magnitude is the golden rule savings rate you have just found? Compare with empirical investment rates in typical developed countries (from Table A).

Exercise 4. Are the estimated coefficients in the Solow model with land reasonable?

Consider the estimation in (19). Are the estimated values of γ_1 and γ_2 reasonable given (18) and plausible parameter values? Why is γ_1 estimated to be smaller than the γ in Chapter 5? Does the fact that it is smaller tend to make it more reasonable? Is it enough? From (18) and (19) one can also find implied values of α, β and κ. Find these and judge their reasonableness. A major point in Chapter 6 was that by introducing human capital into the Solow model, the estimated coefficients in the model's steady state equation became much more reasonable. Does the introduction of land imply a similar improvement?

Exercise 5. Convergence to steady state in the Solow model with oil

For the Solow model with oil, set up the transition equation for the capital–output ratio, $z_t \equiv K_t/Y_t = k_t/y_t$ in usual notation, and show that this transition equation implies convergence of z_t towards a constant level z^*. State the expression for z^*. (Hint: You can go through a number of steps which are counterparts of the steps we went through for the model with just land in Section 1, in order to establish convergence to z^* and to find the value for z^*). Show that the approximation we used in Section 2, $\ln z^* \cong \ln s - \ln(n + g + \delta)$, should not be far-fetched. (Note that for the latter you need one more approximation than we used in Section 1, not only $\beta/(\beta + \varepsilon) \cong 1$, but also $\varepsilon/(\beta + \varepsilon) \cong 0$ and/or $1 - s_E \cong 1$ should be used).

Exercise 6. Golden rule in the Solow model with oil

State the steady state growth path for consumption per worker of the Solow model with oil. Does more oil per worker, in a base year or left over in the current year, given all other parameters, etc., imply a higher or lower position of this path? What is the golden rule value for the savings/investment rate, s, in this model? Comment.

Exercise 7. Balanced growth and the behaviour of the factor compensations in the steady state of the Solow model with oil

1. Consider the Solow model with oil. State expressions for the factor reward rates, r_t, w_t and u_t, as depending, in any period, on the state variables K_t, L_t, A_t and R_t. Show that the income shares are constant, in particular that the share of energy (or oil), $u_t E_t/Y_t = u_t s_E R_t/Y_t$, is equal to ε in all periods.

2. Consider now only the steady state of the same model. Show that the approximate growth rates of Y_t and K_t fulfil:

$$g^Y = g^K \cong \frac{\beta}{\beta + \varepsilon}(g + n) - \frac{\varepsilon}{\beta + \varepsilon} s_E.$$

Show that there is balanced growth in steady state (also) in the respects mentioned in Exercise 1, Question 2.

3. Find expressions for the approximate growth rates, g^r, g^w and g^u, of the real interest rate, r_t, the real wage rate, w_t, and the real oil price, u_t, respectively, in steady state. How does u_t evolve compared to w_t?

Exercise 8. Convergence to steady state, and the steady state growth path, in the Solow model with both land and oil

For the Solow model with both land and oil, do what you were asked to do for the Solow model with just oil in Exercise 5 above. In particular, you are asked to justify equation (37) of Section 3, in the sense that the steady state growth path for y_t (involving the steady state value z^* for the capital–output ratio) with appropriate approximations leads to (37).

Exercise 9. Further numerical evaluation of g^y in the model with both land and oil

Consider equation (35), which assumes $\alpha = 0.2$, $\beta = 0.6$, $\kappa = \varepsilon = 0.1$, and (36), which assumes further $n = 0.01$ and $s_E = 0.005$. In the text we reminded you that a considerably higher population growth rate, e.g. $n = 0.03$, would change the conclusion that the growth drag from population growth is of a modest size. Is something similar true for the growth drag from depletion of the exhaustible resource? What would this drag be if, e.g., $s_E = 0.015$ were more realistic? Why is the trebling of the relevant rate less important for s_E than for n?

Exercise 10. Are the estimated coefficients in the Solow model with land and oil reasonable?

Consider equation (37) and the estimation in (38) based on it. Are the estimated values of γ_1, γ_2 and γ_3 reasonable for plausible parameter values? Back out implied values for α, β, κ and ε from the estimation and judge their reasonableness. As in Exercise 3 above, discuss if the introduction of both land and oil can substitute for the introduction of human capital.

Exercise 11. The Solow model with natural resources and human capital

In some exercises above you may have found that our parameter estimates based on the Solow models with natural resources are not too plausible. It is an obvious idea to build in human capital as well to see if this will make parameter estimates more reasonable. A full analysis of such a model is beyond the scope of this book, but finding the balanced growth rate should not be too complicated.

In a Solow model with both natural resources and human capital the production function would be:

$$Y_t = K_t^{\alpha} H_t^{\varphi} (A_t L_t)^{\beta} X^{\kappa} E_t^{\varepsilon},$$

in an obvious combination of the notations of this chapter and the previous one. Assume that A_t grows at constant rate g, L_t at constant rate n, and that the exhaustion rate is a constant, s_E.

1. With usual notation like $y_t \equiv Y_t/L_t$, $k_t \equiv K_t/L_t$, $h_t \equiv H_t/L_t$, $x_t \equiv X/L_t$, $e_t \equiv E_t/L_t$, and $g_t^y \equiv \ln y_t - \ln y_{t-1}$, etc., show that the per capita production function is:

$$y_t = k_t^{\alpha} h_t^{\varphi} A_t^{\beta} x_t^{\kappa} e_t^{\varepsilon},$$

and that:

$$g_t^y \cong \alpha g_t^k + \varphi g_t^h + \beta g - (\kappa + \varepsilon) n - \varepsilon s_E.$$

2. Define the physical capital–output ratio, $z_t \equiv K_t/Y_t$, as well as the human, $q_t \equiv H_t/Y_t$. Show that along a balanced growth path where both z_t and q_t are constant, the approximate growth rate of income per worker is:

$$g^y \cong \frac{\beta}{\beta + \kappa + \varepsilon} g - \frac{\kappa + \varepsilon}{\beta + \kappa + \varepsilon} n - \frac{\varepsilon}{\beta + \kappa + \varepsilon} s_E.$$

(Hint: $1 - \alpha - \varphi = \beta + \kappa + \varepsilon$.) Compare this expression for g^y with the corresponding one, (34), from the model without human capital. Although the expressions are algebraically identical there is a difference. Why?

3. Assume that labour's share, $\beta + \varphi$, is 0.6 with raw labour and human capital having equal shares β and φ, respectively, and that capital's share is 0.2, while land's and oil's shares are both 0.1 (as we assumed in this chapter). Rewrite the formula for g^y using these values and comment with respect to growth drags. (Insert, for instance, also $s_E = 0.005$ and consider values of n typical for developed and developing countries, respectively.) In particular, how is the coefficient on n affected by also having human capital in the model? In view of what we found in Fig. 7.3, has the model become more realistic by introducing human capital?