CHAPTER 19

Measures of association

Chapter contents

Learning objectives

When you have read this chapter, you should understand:

- how correlation analysis may be applied to study relationships between two or more variables
- the uses, requirements and interpretation of correlation coefficients
- how predictions are made with regression analysis
- how to test regression models for linearity and assess their fit
- the non-parametric measures of association and the alternatives they offer when key assumptions and requirements for parametric techniques cannot be met.

19.1 Introduction

In the previous chapter, we emphasized testing hypotheses of difference. However, management questions in business frequently revolve around the study of relationships between two or more variables. In such cases, a relational hypothesis is necessary. In the research question, 'Do homogeneous management teams perform better than heterogeneous teams?' the nature of the relationship between the two variables ('team homogeneity' and 'performance') is not specified. The implication, nonetheless, is that one variable is responsible for the other. A correct relational hypothesis for this question would state that the variables occur together in some specified manner without implying that one causes the other.

Various objectives are served with correlation analysis. The strength, direction, shape and other features of the relationship may be discovered. Or tactical and strategic questions may be answered by predicting the values of one variable from those of another. Some typical management questions are as follows:

- In the mail-order business, excessive catalogue costs quickly squeeze margins. Many mailings fail to reach receptive or active buyers. What is the relationship between various categories of mailings that delete inactive customers and the improvement in profit margins?
- Medium-sized companies often have difficulty attracting the best business students, and when they are successful, they have trouble retaining them. What is the relationship between the ranking of candidates based on executive interviews and the ranking obtained from testing and assessment?
- Retained cash flow, undistributed profits plus depreciation, is a critical source of funding for equipment investment. During a period of decline, capital spending suffers. What is the relationship between retained cash flow and equipment investment over the last year? Between cash flow and dividend growth?
- Aggressive US high-technology companies have invested heavily in the European chip market, and their sales have grown 20 per cent over the three largest European firms. Can we predict next year's sales based on present investment?

***Exhibit 19.1** Commonly used measures of association.*

Measurement	Coefficient	Comments and uses
Interval and ratio	Pearson (product moment) correlation coefficient	For continuous linearly related variables
	Correlation ratio (eta)	For non-linear data or relating a main effect to a continuous dependent variable
	Biserial	One continuous and one dichotomous variable with an underlying normal distribution
	Partial correlation	Three variables; relating two with the third's effect taken out
	Multiple correlation	Three variables; relating one variable with two others
	Bivariate linear regression	Predicting one variable from another's scores
Ordinal	Gamma	Based on concordant–discordant pairs: $(P - Q)$; proportional reduction in error (PRE) interpretation
	Kendall's tau *b*	$P - Q$ based; adjustment for tied ranks
	Kendall's tau *c*	$P - Q$ based; adjustment for table dimensions
	Somers's *d*	$P - Q$ based; asymmetrical extension of gamma
	Spearman's rho	Product moment correlation for ranked data
Nominal	Phi	Chi-square (CS) based for 2×2 tables
	Cramer's *V*	CS based; adjustment when one table dimension > 2
	Contingency coefficient *C*	CS based; flexible data and distribution assumptions
	Lambda	PRE-based interpretation
	Goodman–Kruskal's tau	PRE-based with table marginals emphasis
	Uncertainty coefficient	Useful for multidimensional tables
	Kappa	Agreement measure

All these questions may be evaluated by means of measures of association. And all call for different techniques based on the level at which the variables were measured or the intent of the question. The first three use nominal, ordinal and interval data, respectively. The last one is answered through simple bivariate regression.

With correlation, one calculates an index to measure the nature of the relationship between variables. With regression, an equation is developed to predict the values of a dependent variable. Both are affected by the assumptions of measurement level and the distributions that underlie the data.

Exhibit 19.1 lists some common measures and their uses. The chapter follows the progression of the exhibit, first covering bivariate linear correlation, then simple regression and concluding with non-parametric measures of association. Exploration of data through visual inspection and diagnostic evaluation of assumptions continues to be emphasized.

19.2 Bivariate correlation analysis

Bivariate correlation analysis differs from non-parametric measures of association and regression analysis in two important ways. First, parametric correlation requires two continuous variables measured on an interval or ratio scale. Second, the coefficient does not distinguish between independent and dependent variables. It treats the variables symmetrically since the coefficient r_{xy} has the same interpretation as r_{yx}.

Pearson's product moment coefficient *r*

The **Pearson (product moment) correlation coefficient** varies over a range of +1 through 0 to −1. The designation *r* symbolizes the coefficient's estimate of linear association based on sampling data. The coefficient *r* represents the population correlation.

Correlation coefficients reveal the magnitude and direction of relationships. The magnitude is the degree to which variables move in unison or opposition. The size of a correlation of +.40 is the same as one of −.40. The sign says nothing about size. The degree of correlation is modest. The coefficient's sign signifies the direction of the relationship. Direction tells us whether large values on one variable are associated with large values on the other (and small values with small values). When the values correspond in this way, the two variables have a positive relationship: as one increases, the other also increases. Family income, for example, is positively related to household food expenditures. As income increases, food expenditures increase. Other variables are inversely related. Large values on the first variable are associated with small values on the second (and vice versa). The prices of products and services are inversely related to their scarcity. In general, as products decrease in available quantity, their prices rise. The absence of a relationship is expressed by a coefficient of approximately zero.

Scatterplots for exploring relationships

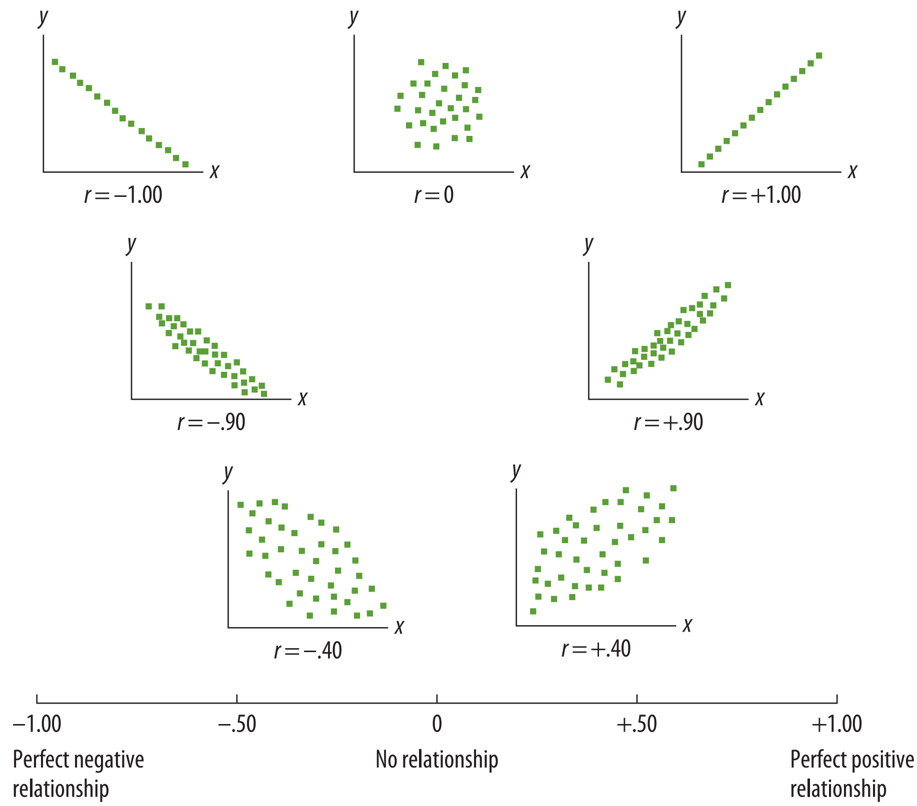
Scatterplots are essential for understanding the relationships between variables. They provide a means for visual inspection of data that a list of values for two variables cannot. Both the direction and the shape of a relationship are conveyed in a plot. With a little practice, the magnitude of the relationship can be seen.

Exhibit 19.2 contains a series of scatterplots that depict some relationships across the range *r*. The three plots on the left side of the figure have their points sloping from the upper left to the lower right of each $x - y$ plot.[1] They represent different magnitudes of negative relationships. On the right side of the figure, three plots have opposite patterns and show positive relationships.

When stronger relationships are apparent (e.g. the ±.90 correlations), the points cluster close to an imaginary straight line passing through the data. The weaker relationships (±.40) depict a more diffuse data cloud with points spread farther from the line.

The shape of linear relationships is characterized by a straight line, whereas non-linear relationships have curvilinear, parabolic and compound curves representing their shapes. Pearson's *r* measures relationships in variables that are linearly related. It cannot distinguish nonlinear data. Summary statistics alone do not reveal the appropriateness of the data for the model, as the following example illustrates.

Exhibit 19.2 ***Scatterplots of correlation between two variables.***



One author constructed four small datasets possessing identical summary statistics but displaying strikingly different patterns.[2] Exhibit 19.3 contains these data and Exhibit 19.4 plots them. In plot 1 of the figure, the variables are positively related. Their points follow a superimposed straight line through the data. This example is well suited to correlation analysis. In plot 2, the data are curvilinear in relation to the line, and r is an inappropriate measure of their relationship. Plot 3 shows the presence of an influential point that changed a coefficient that would have otherwise been a perfect +1.0. The last plot displays constant values of x (similar to what you might find in an animal or quality-control experiment). One leverage point establishes the fitted line for these data.

We will return to these concepts and the process of drawing the line when we discuss regression. For now, comparing plots 2 through 4 with plot 1 suggests the importance of visually inspecting correlation data for underlying patterns. Careful analysts make scatterplots an integral part of the inspection and exploration of their data. Although small samples may be plotted by hand, statistical software packages save time and offer a variety of plotting procedures.

The assumptions of r

Like other parametric techniques, correlation analysis makes certain assumptions about the data. Many of these assumptions are necessary to test hypotheses about the coefficient.

The first requirement for r is **linearity**. All the examples in Exhibit 19.2 with the exception of $r = 0$ illustrate a relationship between variables that can be described by a straight line passing through the data cloud. When $r = 0$, no pattern is evident that could be described with a single line. Parenthetically, it is also possible to find coefficients of 0 where the variables are highly related but in a non-linear form. As we have seen, plots make such findings evident.

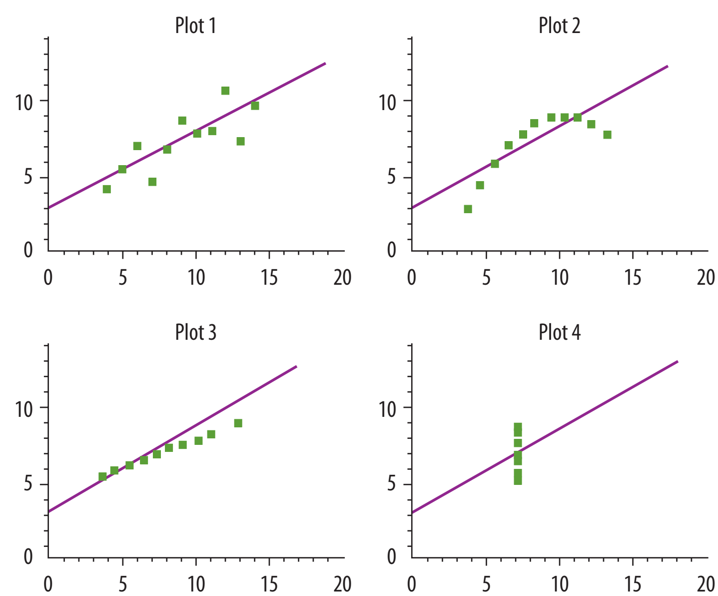
The second assumption for correlation is a bivariate normal distribution – that is, the data are from a random sample of a population where the two variables are normally distributed in a joint manner.

Exhibit 19.3 ***Four datasets with the same summary statistics.***

S_S	X_1	Y_1	X_2	Y_2	X_3	Y_3	X_4	Y_4
1	10	8.04	10	9.14	10	7.46	8	6.58
2	8	6.95	8	8.14	8	6.77	8	5.76
3	13	7.58	13	8.74	13	12.74	8	7.71
4	9	8.81	9	8.77	9	7.11	8	8.84
5	11	8.33	11	9.26	11	7.81	8	8.47
6	14	9.96	14	8.10	14	8.84	8	7.04
7	6	7.24	6	6.13	6	6.08	8	5.25
8	4	4.26	4	3.10	4	5.39	19	12.50
9	12	10.84	12	9.13	12	8.15	8	5.56
10	7	4.82	7	7.26	7	6.42	8	7.91
11	5	5.68	5	4.74	5	5.73	8	6.89
Pearson's r	.816 42		.816 24		.816 24		.816 52	
r^2	.666 54		.666 24		.666 24		.666 71	
Adjusted r^2	.629 49		.629 16		.629 16		.629 67	
Standard error	1.236 6		1.237 2		1.237 2		1.235 7	
	0		1		1		0	

Often these assumptions or the required measurement level cannot be met. Then the analyst should select a non-linear or non-parametric measure of association, many of which are described later in this chapter.

Exhibit 19.4 ***Different scatterplots for the same summary statistics.***



Computation and testing of r

The formula for calculating Pearson's r is

$$r = \frac{\Sigma(X - \bar{X})(Y - \bar{Y})}{(N - 1)s_x s_y}$$

where

N = The number of pairs of cases.
s_x, s_y = The standard deviations for X and Y.

Alternatively:

$$r = \frac{\Sigma xy}{\sqrt{(\Sigma x^2)(\Sigma y^2)}}$$

since

$$s_x = \sqrt{\frac{\Sigma x^2}{N}} \qquad s_y = \sqrt{\frac{\Sigma y^2}{N}}$$

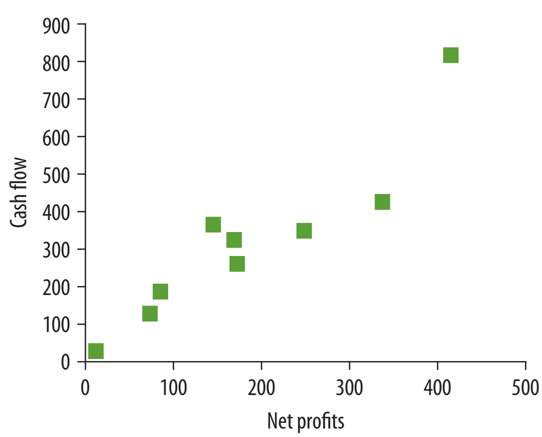
If the numerator of the equation $\Sigma xy/\sqrt{(\Sigma x^2)(\Sigma y^2)}$ is divided by N, we have the covariance, the amount of deviation that the X and Y distributions have in common. With a positive covariance, the variables move in unison; with a negative one, they move in opposition. When the covariance is 0, there is no relationship. The denominator of the equation above represents the maximum potential variation that the two distributions share. Thus, correlation may be thought of as a ratio.

Exhibit 19.5 contains a random subsample of 10 firms of the *Forbes* 500 sample. The variables chosen to illustrate the computation of *r* are cash flow and net profits. Beneath each variable is its mean and standard deviation. In columns 4 and 5 we obtain the deviations of the *X* and *Y* values from their means, and in column 6 we find the product. Columns 7 and 8 are the squared deviation scores.

Exhibit 19.5 ***Computation of Pearson's product moment correlation.****

Corporation	Net profits ($ mil.) X	Cash flow ($ mil.) Y	Deviations from means				
			$(X-\bar{X})x$	$(Y-\bar{Y})y$	xy	x^2	Y^2
1	82.6	126.5	−93.84	−178.64	16 763.58	8 805.95	31 912.25
2	89.0	191.2	−87.44	−113.94	9 962.91	7 645.75	12 982.32
3	176.0	267.0	−0.44	−38.14	16.78	0.19	1 454.66
4	82.3	137.1	−94.14	−168.04	15 819.29	8 862.34	28 237.44
5	413.5	806.8	237.06	501.66	11 9923.52	56 197.44	251 602.56
6	19.1	35.2	158.34	−269.94	42 742.30	25 071.56	72 867.60
7	337.3	425.5	160.86	120.36	19 361.11	25 875.94	14 486.60
8	145.8	380.0	−30.64	74.86	−2 293.71	938.81	5 604.02
9	172.6	326.6	−3.84	21.36	−82.02	14.75	456.25
10	247.2	355.5	70.76	50.36	3 563.47	5 006.98	2 536.13
	$\bar{X}=176.44$	$\bar{Y}=305.14$			$\Sigma XY=224\,777.23$		
	$s_x=216.59$	$s_y=124.01$				$\Sigma X^2=138\,419.71$	$\Sigma Y^2=422\,139.76$

Exhibit 19.6 ***Plot of net profit against cash flow.***



Substituting into the formula, we get:

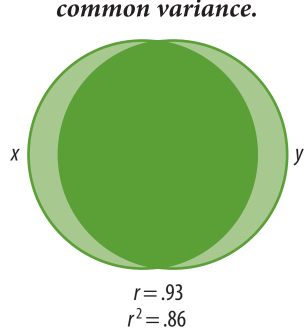
$$r=\frac{224\,777.23}{\sqrt{138\,419.71}\times\sqrt{442\,139.76}}=.9298$$

In this subsample, net profits and cash flow are positively related and have a very high coefficient. As net profits increase, cash flow increases; the opposite is also true. Linearity of the variables may be examined with a scatterplot such as the one shown in Exhibit 19.6. The data points fall along a straight line.

Common variance as an explanation

The amount of common variance in *X* (net profits) and *Y* (cash flow) may be summarized by r^2, the **coefficient of determination**. As Exhibit 19.7 shows, the overlap between the two variables is the proportion of their common or shared variance.

Exhibit 19.7 ***Diagram of common variance.***



The area of overlap represents the percentage of the total relationship accounted for by one variable or the other. So 86 per cent of the variance in *X* is explained by *Y*, and vice versa.

Testing the significance of r

Is the coefficient representing the relationship between net profits and cash flow real, or does it occur by chance? This question tries to discover whether our *r* is a chance deviation from a population *p* of zero. In other situations, the researcher may wish to know if significant differences exist between two or more rs. In either case, r's significance should be checked before r is used in other calculations or comparisons. For this test, we must have independent random samples from a **bivariate normal distribution**. Then the Z or t-test may be used for the null hypothesis, $p = 0$.

The formula for small samples is:

$$t = \frac{r}{\sqrt{\dfrac{1 - r^2}{n - 2}}}$$

where

$r = .93$
$n = 10$

Substituting into the equation, we calculate t:

$$t = \frac{.93}{\sqrt{\dfrac{1 - .86}{8}}} = 7.03$$

With $n - 2$ degrees of freedom, the statistical program calculates the value of t (7.03) at a probability less than .005 for the one-tailed alternative, H^A: $\tilde{n} > 0$. We reject the hypothesis that there is no linear relationship between net profits and cash flow in the population. The above statistic is appropriate when the null hypothesis states a correlation of 0. It should be used only for a one-tailed test.[3] However, it is often difficult to know in advance whether the variables are positively or negatively related, particularly when a computer removes our contact with the raw data. Software programs produce two-tailed tests for this eventuality. The observed significance level for a one-tailed test is half of the printed two-tailed version in most programs.

Correlation matrix

A **correlation matrix** is a table used to display coefficients for more than two variables. Exhibit 19.8 shows the intercorrelations among six variables for the full *Forbes* 500 dataset.[4]

Exhibit 19.8 ***Correlation matrix for Forbes 500 sample.***

	Assets ($ m)	Cash flow ($ m)	Number employed (thousands)	Market value ($ m)	Net profits ($ m)	Sales ($ m)
Assets	1.0000					
Cash flow	.3426	1.0000				
Employed	.3898	.8161	1.0000			
Market value	.3642	.9353	.8106	1.0000		
Net profits	.2747	.9537	.7467	.9101	1.0000	
Sales	.5921	.7990	.8831	.7485	.7261	1.0000

Notes: All coefficients are statistically significant, $p < 0.1$; $n = 100$.

It is conventional for a symmetrical matrix to report findings in the triangle below the diagonal. The diagonal contains coefficients of 1.00 that signify the relationship of each variable with itself. Journal articles and management reports often show matrices with coefficients at different probability levels. A symbol beside the coefficient keys the description of differences to a legend. The practice of reporting tests of the null hypothesis, $r = 0$, was followed in Exhibit 19.8.

Correlation matrices have utility beyond bivariate correlation studies. Interdependence among variables is a common characteristic of most multivariate techniques. Matrices form the basis for computation and understanding of the nature of relationships in multiple regression, discriminant analysis, factor analysis and many others. Such applications call for variations on the standard matrix. Pooled within-groups covariance matrices average the separate covariances for several groups and array the results as coefficients. Total or overall correlation matrices treat coefficients as if they came from a single sample.

Interpretation of correlations

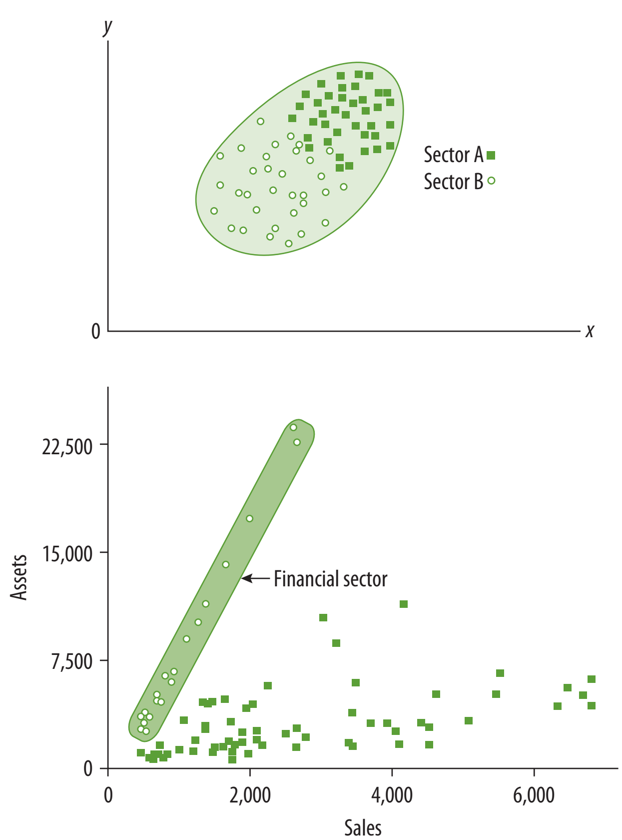
A correlation coefficient of any magnitude or sign, whatever its statistical significance, does not imply causation. Increased net profits may cause an increase in market value, or improved satisfaction may cause improved performance in certain situations, but correlation provides no evidence of cause and effect. Several alternate explanations may be provided for correlation results:

- *X* causes *Y*
- *Y* causes *X*
- *X* and *Y* are activated by one or more other variables
- *X* and *Y* influence each other reciprocally.

Ex-post facto studies seldom possess sufficiently powerful designs to demonstrate which of these conditions could be true. By controlling variables under an experimental design, we may obtain more rigorous evidence of causality.

Take care to avoid so-called **artefact correlations**, where distinct groups combine to give the impression of one. The upper panel of Exhibit 19.9 shows data from two business sectors. If all the data points for the *X* and *Y* variables are aggregated and a correlation is computed for a single group, a positive correlation results. Separate calculations for each sector (note that dark-coloured dots for Sector A form a circle, as do the light-coloured dots for Sector B) reveal no relationship between the *X* and *Y* variables. A second example shown in the lower panel contains a plot of data on assets and sales. We have enclosed and highlighted the data for the financial sector. This is shown as a narrow band enclosed by an ellipse. These companies score high on assets and low in sales – all are banks. When the data for banks are removed and treated separately, the correlation is nearly perfect (.99). When banks are returned to the sample and the correlation is recalculated, the overall relationship drops to the mid-.80s. In short, data hidden or nested within an aggregated set may present a radically different picture.

Exhibit 19.9 ***Artefact correlations.***



Another issue affecting interpretation of coefficients concerns practical significance. Even when a coefficient is statistically significant, it must be practically meaningful. In many relationships, other factors combine to make the coefficient's meaning misleading. For example, in nature we expect rainfall and the height of reservoirs to be positively correlated. But in states where water management and flood control mechanisms are complex, an apparently simple relationship may not hold. Techniques like partial and multiple correlation or multiple regression are helpful in sorting out confounding effects.

With large samples, even exceedingly low coefficients can be statistically significant. This 'significance' only reflects the likelihood of a linear relationship in the population. Should magnitudes less than .30 be reported when they are significant? It all depends. We might consider the correlations between variables such as cash flow, sales, market value or net profits to be interesting revelations of a particular phenomenon whether they were high, moderate or low. The nature of the study, the characteristics of the sample or other reasons will be determining factors. But a coefficient is not remarkable simply because it is statistically significant.

By probing the evidence of direction, magnitude, statistical significance and common variance together with the study's objectives and limitations, we reduce the chances of reporting trivial findings. Simultaneously, the communication of practical implications to the reader will be improved.

SPSS reference

Pallant (2013) discusses how to calculate correlation coefficients in SPSS in Chapter 11.

Bivariate linear regression[5]

In the previous section, we focused on relationships between variables. The product moment correlation was found to represent an index of the magnitude of the relationship, the sign governed the direction and r^2 explained the common variance. Relationships also serve as a basis for estimation and prediction.

When we take the observed values of X to estimate or predict corresponding Y values, the process is called simple prediction.[6] When more than one X variable is used, the outcome is a function of multiple predictors. Simple and multiple predictions are made with a technique called **regression analysis**.

The similarities and differences of regression and correlation are summarized in Exhibit 19.10. Their relatedness would suggest that beneath many correlation problems is a regression analysis that could provide further insight about the relationship of Y with X.

The basic model

A straight line is fundamentally the best way to model the relationship between two continuous variables. The bivariate linear regression may be expressed as:

$$Y = \beta_0 + \beta_1 X_i$$

where the value of the dependent variable Y is a linear function of the corresponding value of the independent variable X_i in the ith observation. The slope, β_1, and the Y intercept, β_0, are known as **regression coefficients**. The **slope**, β_1, is the change in Y for a one-unit change in X. It is sometimes called the 'rise over run'. This is defined by the formula:

$$\beta_1 = \frac{\Delta Y}{\Delta X}$$

This is the ratio of change (.) in the rise of the line relative to the run or travel along the X axis. Exhibit 19.11 shows a few of the many possible slopes you may encounter.

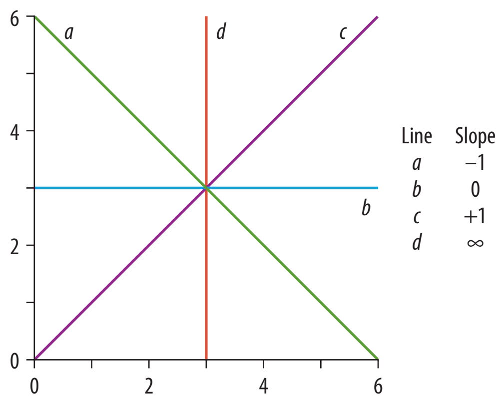
The **intercept**, β_0, is the value for the linear function when it crosses the Y axis; it is the estimate of Y when $X = 0$. A formula for the intercept based on the mean scores of the X and Y variables is:

$$\beta = \bar{Y} - \beta_1 \bar{X}$$

***Exhibit 19.10* Comparison of bivariate linear correlation and regression.**

	Correlation	Regression
Measurement level	Interval or ratio scale	Interval or ratio scale
Nature of variables	Both continuous, linearly related	Both continuous, linearly related
$X-Y$ relationship	X and Y are symmetric; $r_{xy} = r_{yx}$	Y is dependent, X is independent; regression of X on Y differs from Y on X
Correlation	The correlation of X and Y produces an estimate of linear association based on sampling data	Correlation of $Y-X$ is the same as the correlation between the predicted values of Y and observed values of Y
Coefficient of determination	Explains common variance of X and Y	Proportion of variability of Y explained by its least-squares regression on X

Exhibit 19.11 ***Examples of different slopes.***



Concept application

The price of investment-grade red wine is influenced in several ways, not the least of which is tasting. Tasting from the barrel is a major determinant of market 'en primeur' or futures contracts, which represent about 60 per cent of the harvest. After the wine rests for 19–24 months in oak casks, further tasting occurs and the remaining stock is released.

Weather is widely regarded as being responsible for pronouncements about wine quality. A Princeton economist has elaborated on that notion. He suggested that just a few facts about local weather conditions may be better predictors of vintage French red wines than the most refined palates and noses.[7] The regression model developed predicts an auction price index for about 80 wines from winter and harvest rainfall amounts and average growing-season temperatures. Interestingly, the calculations suggested that the 1989 Bordeaux would be one of the best since 1893. The 'guardians of tradition' reacted hysterically to these methods yet agreed with the conclusion.

Our first example uses one predictor with highly simplified data. Let X represent the average growing-season temperature in degrees Celsius and Y the price of a 12-bottle case in French Francs (FF). The data appear in Exhibit 19.12.

Exhibit 19.12 ***Plot of wine price by average growing temperature.***

X Average temperature Celsius	Y Price per case (FF)
12	2,000
16	3,000
20	4,000
24	5,000
$\bar{X} = 19$	$\bar{Y} = 3{,}500$

The plotted data in Exhibit 19.12 show a linear relationship between the pairs of points and a perfect positive correlation, $r_{yx} = 1.0$. The slope of the line is calculated:

$$\beta = \frac{Y_i - Y_j}{X_i - X_j} = \frac{4000 - 3000}{20 - 16} = \frac{1000}{4} = 250$$

where the X_iY_i values are the data points (20, 4000) and X_iY_i are points (16, 3000). The intercept β_0 is −1000, the point at which $X = 0$ in this plot. This area is off the graph and appears in an insert on the figure.

$$\beta = \bar{Y} - \beta_1\bar{X} = 3500 - 250(19) = -1000$$

Substituting into the formula, we have the simple regression equation:

$$Y = -1000 + 250X_i$$

We could now predict that a warm growing season with 25.5°C temperature would bring a case price of 5,375 French Francs. $\hat{Y}$ (called Y-hat) is the predicted value of Y.

$$\hat{Y} = -1000 + 250(25.5) = 5375$$

Unfortunately, one rarely comes across a dataset composed of four paired values, a perfect correlation and an easily drawn line. A model based on such data is deterministic in that for any value of X, there is only one possible corresponding value of Y. It is more likely that we will collect data where the values of Y vary for each X value. Considering Exhibit 19.13, we should expect a distribution of price values for the temperature $X = 16$, another for $X = 20$ and another for each value of X. The means of these Y distributions will also vary in some systematic way with X. These variabilities lead us to construct a probabilistic model that also uses a linear function.[8] This function is written:

$$Y_i = \beta_0\beta_1X_1 + \varepsilon_1$$

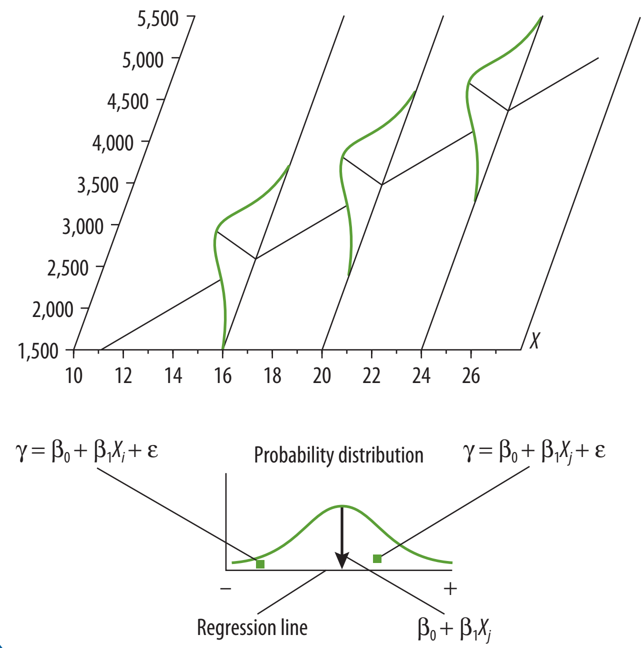
where ε symbolizes the deviation of the ith observation from the mean, $\beta_0 + \beta_1X_1$.

As shown in Exhibit 19.13, the actual values of Y may be found above or below the regression line represented by the mean value of Y ($\beta_0 + \beta_1 X_i$) for a particular value of X. These deviations are the error in fitting the line and are often called the **error term**.

SPSS reference

The exercise presented here could be replicated in SPSS, and if you want to do so, read Chapter 11 and the very short Chapter 12 on partial correlation in Pallant (2013).

***Exhibit 19.13* Distribution of Y for observations of X.**



Method of least squares

Exhibit 19.14 contains a new dataset for the wine price example. Our prediction of Y from X must now account for the fact that the X and Y pairs do not fall neatly along the line. Actually, the relationship could be summarized by several lines. Exhibit 19.15 suggests a few alternatives based on visual inspection – all of which produce errors, or vertical distances from the observed values to the line. The **method of least squares** allows us to find a regression line, or line of best fit, which will keep these errors to a minimum. It uses the criterion of minimizing the total squared errors of estimate. When we predict values of Y for each X_i, the difference between the actual Y_i and the predicted N is the error. This error is squared and then summed. The line of best fit is the one that minimizes the total squared errors of prediction.[9]

$$\sum_{i=1}^{n} e_i^2 \text{ minimized}$$

***Exhibit 19.14* Data for wine price study.**

	Y price (FF)	X temperature (°C)	XY	Y^2	X^2
1	1 913.00	11.80	21 393.40	3 286 969.00	139.24
2	2 558.00	15.70	40 160.60	6 543 364.00	246.49
3	2 628.00	14.00	36 792.00	6 906 384.00	196.00
4	3 217.00	22.90	73 669.30	10 349 089.00	524.41
5	3 228.00	20.00	64 560.00	10 419 984.00	400.00
6	3 629.00	20.10	72 942.90	13 169 641.00	404.01
7	3 886.00	17.90	69 559.40	15 100 996.00	320.41
8	4 897.00	23.40	114 589.80	23 980 609.00	547.56
9	4 933.00	24.60	121 351.80	24 334 489.00	605.16
10	5 199.00	25.70	133 614.30	27 029 601.00	660.49
Σ	35 988.00	196.10	748 633.50	141 121 126.00	4 043.77
Mean	3 598.80	19.61			
s	1 135.66	4.69			
Sum of squares (SS)	11 607 511.59	198.25	42 908.82		

Regression coefficients β_0 and β_1 are used to find the least-squares solution. They are computed as follows:

$$\beta = \frac{\Sigma XY - \frac{(\Sigma X)(\Sigma Y)}{n}}{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}$$

Substituting data from Exhibit 19.14 into both formulas, we get:

$$\beta_1 = \frac{748633.5 - \frac{(196.1)(35988)}{10}}{4043.77 - \frac{(19.61)^2}{10}} = 216.439$$

$$\beta_0 = 3598.8 - (1216.439)(19.61) = -645.569$$

Exhibit 19.15 ***Scatterplot and possible regression line based on visual inspection: wine price study.***

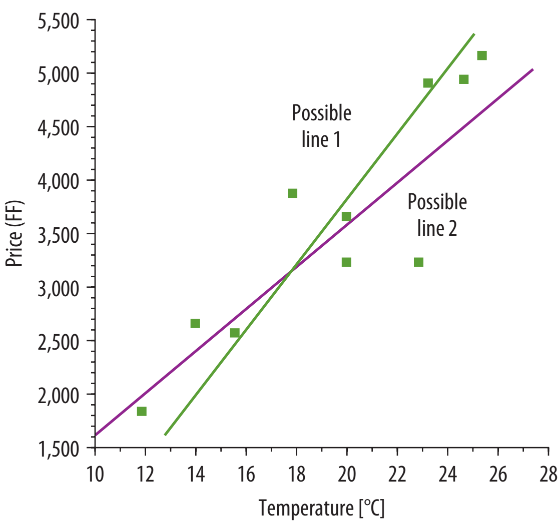
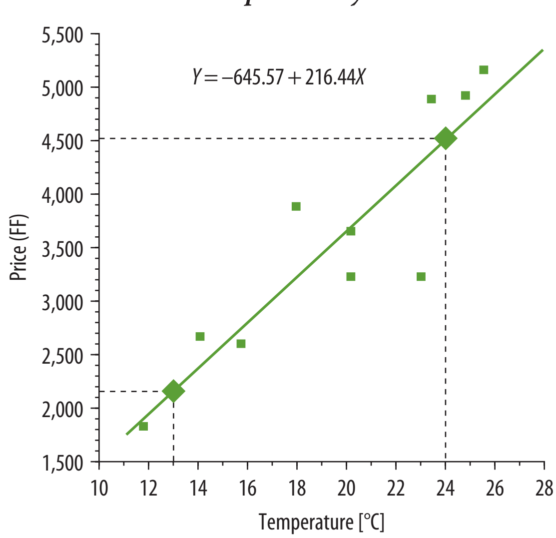


Exhibit 19.16 ***Drawing the least-squares line: wine price study.***



The predictive equation is now $\bar{Y} = -645.57 + 216.44X_i$.

Drawing the regression line

Before drawing the regression line, we select two values of X to compute. Using values 13 and 24 for X_i, the points are:

$$\bar{Y} = -645.57 + 216.44(13) = 2168.15$$

$$\bar{Y} = -645.57 + 216.44(24) = 4548.99$$

Comparing the line drawn in Exhibit 19.16 to the trial lines in Exhibit 19.15, one can readily see the success of the least-squares method in minimizing the error of prediction.

Residuals

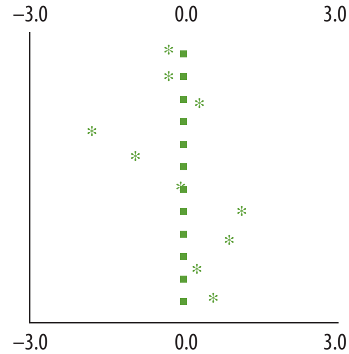
We now turn our attention to the plot of standardized residuals in Exhibit 19.17. A **residual** is what remains after the line is fit or ($Y_i = \bar{Y}_i$). When standardized, residuals are comparable to Z scores with a mean of 0 and a standard deviation of 1. In this plot, the standardized residuals should fall between 2 and −2, be randomly distributed about zero and show no discernible pattern. All these conditions say the model is applied correctly.

In our example, we have one residual at −2.2, a random distribution about zero and few indications of a sequential pattern. It is important to apply other diagnostics to verify that the regression assumptions are met. Various software programs provide plots and other checks of normality, linearity, equality of variance and independence of error.[10]

Predictions

If we wanted to predict the price of a case of investment-grade red wine for a growing season that averages 21°C, our prediction would be:

Exhibit 19.17 ***Plot of standardized residuals: wine price study.***



Case	Y Price	Predicted price	Residual
1	1813	1908.4112	–95.4112
2	2558	2752.5234	–194.5234
3	2628	2384.5771	243.4229
4	3217	4310.8844	–1093.8844
5	3228	3683.2112	–455.2112
6	3629	3704.8551	–75.8551
7	3886	3228.6893	657.3107
8	4897	4419.1039	477.8961
9	4933	4678.8307	254.1693
10	5199	4916.9137	282.0863

$$\hat{Y} = -645.57 + 216.44(21)\bar{Y} = 3899.67$$

This is a point prediction of Y and should be corrected for greater precision.

As with other confidence estimates, we establish the degree of confidence desired and substitute into the formula:

$$\hat{Y} \pm t_{\alpha/2^s}\sqrt{1+\frac{1}{10}+\frac{(X-\bar{X})^2}{SS_x}}$$

where

$t_{\alpha/2}$ = The two-tailed critical value for t at the desired level (95 per cent in this example).

s = The standard error of estimate (also the square root of the mean square error from the analysis of variance of the regression model) (see Exhibit 19.14).

SS_x = The sum of squares for X (Exhibit 19.14).

$$3899.67 \pm (2.306)(538.559)\sqrt{\frac{1}{10}+\frac{(21-19.61)^2}{198.25}}$$

$$3899.67 \pm 1308.29$$

We are 95 per cent confident of our prediction that a case of investment-quality French red wine grown in a particular year at 21°C average temperatures will be initially priced at 3899.67 ± 1308.29, or from approximately FF2,591 to FF5,208. The comparatively large bandwidth results from the amount of error in the model (reflected by r^2), some peculiarities in the Y values and the use of a single predictor.

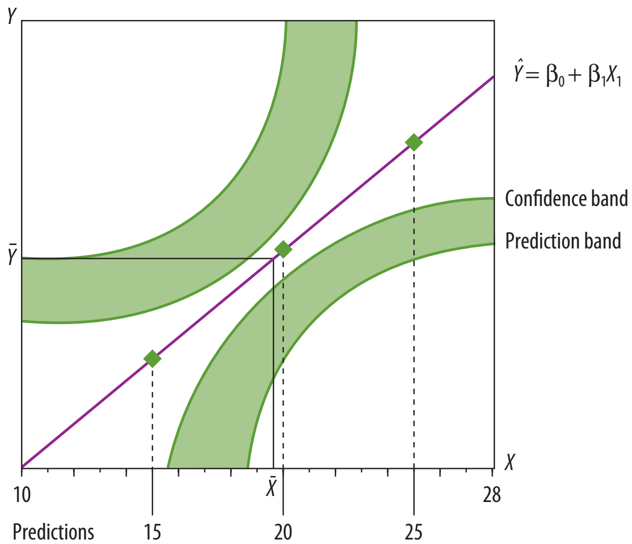
It is more likely that we would want to predict the average price of all cases grown at 21°C. This prediction would use the same basic formula omitting the first digit (the 1) under the radical. A narrower confidence band is the result since the average of all Y values is being predicted from a given X. In our example, the confidence interval for 95 per cent is 3899.67 ± 411.42, or from FF3,488 to FF4,311.

The predictor we selected, 21°C, was close to the mean of X (19.61). Because the **prediction and confidence bands** are shaped like a bow tie, predictors farther from the mean have larger bandwidths. For example, X values of 15, 20 and 25 produce confidence bands of ±565, ±397 and ±617, respectively. This is illustrated in Exhibit 19.18. The farther one's selected predictor is from X, the wider is the prediction interval.

Testing the goodness of fit

With the regression line plotted and a few illustrative predictions, we should now gather some evidence of **goodness of fit** – how well the model fits the data. The most important test in bivariate linear regression is whether the slope, β_1, is equal to zero.[11] We have already observed a slope of zero in Exhibit 19.11, line b. Zero slopes result from various conditions:

Exhibit 19.18 ***Prediction and confidence bands based on proximity to x.***



- Y is completely unrelated to X and no systematic pattern is evident
- there are constant values of Y for every value of X
- the data are related but represented by a non-linear function.

The t-test

To test whether $\beta_1 = 0$, we use a two-tailed test (since the actual relationship is positive, negative or zero). The test follows the t distribution for $n - 2$ degrees of freedom.

$$t = \frac{b_1}{s(b_1)} = \frac{216.439}{34.249} = 5.659$$

where

b_1 was previously defined as the slope β_1.
$s(b_1)$ is the standard error of β_1.[12]

We reject the null, $\beta_1 = 0$, because the calculated t is greater than any t value for 8° of freedom and $\alpha = .01$.

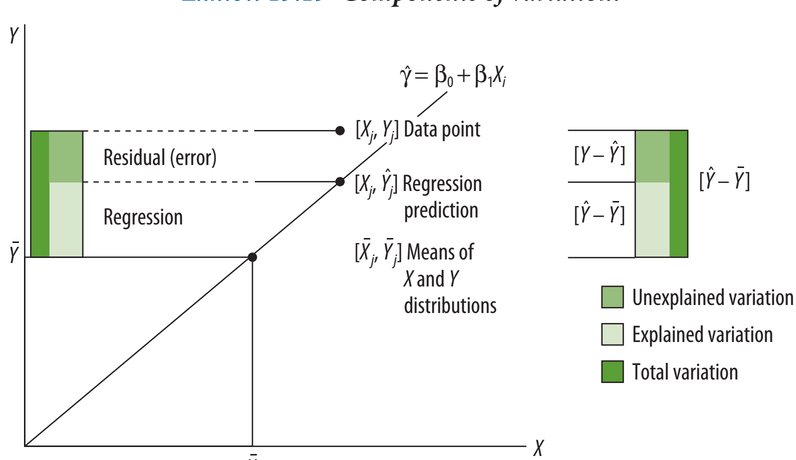
SPSS reference

T-tests and parametric tests are discussed in Chapter 17 of Pallant (2013).

The F-test

Computer printouts generally contain an analysis of variance (ANOVA) table with an *F*-test of the regression model. In bivariate regression, *t*- and *F*-tests produce the same results since t^2 is equal to F. In multiple regression, the *F*-test has an overall role for the model and each of the independent variables is evaluated with a separate *t*-test. From the last chapter, recall that ANOVA partitions variance into component parts. For regression, it comprises explained deviations, $\hat{Y} - \bar{Y}$, and unexplained deviations, $Y - \hat{Y}$. Together they constitute the total deviation, $Y - \bar{Y}$. This is shown graphically in Exhibit 19.19. These sources of deviation are squared for all observations and summed across the data points.

Exhibit 19.19 ***Components of variation.***



In Exhibit 19.20, we develop this concept sequentially concluding with the *F*-test of the regression model for the wine data. Based on the results presented in that table, we find statistical evidence of a linear relationship between variables. The alternative hypothesis, $r^2 \neq 0$, is accepted with $F = 32.02$, *d.f.*, (1,8), $p < .005$. The null hypothesis for the *F*-test had the same effect as $\beta_1 = 0$ since we could select either test.

Exhibit 19.20 Progressive applications of partitioned variance concepts.

General concept				
$(\hat{Y} - \bar{Y})$ Explained variation (the regression relationship between X and Y)	+	$(Y - \hat{Y})$ Unexplained variation (cannot be explained by the regression relationship)	=	$(Y - \bar{Y})$ Total variation
ANOVA application				
$\sum_{i=1}^{n}(\hat{Y} - \bar{Y})^2$ SS_r Sum of squares regression	+	$\sum_{i=1}^{n}(Y - \bar{Y})^2$ SS_r Sum of squares error	=	$\sum_{i=1}^{n}(Y - \bar{Y})^2$ SS_r Sum of squares total
Content of summary table				
Source	Degree of freedom	Sum of squares	Mean square	*F* ratio
Regression	1	SS_r	$MS_r = \frac{SS_r}{1}$	MS_r
Error	$n - 2$	SS_e	$MS_r = \frac{SS_e}{n-2}$	$\overline{MS}_e$
Total		SS_r		
ANOVA summary table: test of regression model				
Source	Degree of freedom	Sum of squares	Mean square	*F* ratio
Regression	1	9 287 143.11	9 287 143.11	32.02
Residual (error)	8	2 320 368.49	290 046.06	
Total		11 607 511.60	Significance of $F = .0005$	

Coefficient of determination

In predicting the values of *Y* without any knowledge of *X*, our best estimate would be $\bar{Y}$, its mean. Each predicted value that does not fall on *Y* contributes to an error of estimate, $(Y - \bar{Y})$. The total squared error for several predictions would be $\Sigma(\bar{Y}_i - \bar{Y})^2$. By introducing known values of *X* into a regression equation, we attempt to reduce this error even further. Naturally, this is an improvement over using $\bar{Y}$, and the result is $(\bar{Y} - \bar{Y})$. The total improvement based on several estimates is $\Sigma(\bar{Y}_i - \bar{Y})^2$, the amount of variation explained by the relationship between *X* and *Y* in the regression. Based on the formula, the coefficient of determination is the ratio of the line of best fit's error over that incurred by using *Y*. One purpose of testing, then, is to discover whether the regression equation is a more effective predictive device than the mean of the dependent variable.

As in correlation, the coefficient of determination is symbolized by r^2.[13] It has several purposes. As an index of fit, it is interpreted as the total proportion of variance in *Y* explained by *X*. As a measure of linear relationship, it tells us how well the regression line fits the data. It is also an important indicator of the predictive accuracy of the equation. Typically, we would like to have an r^2 that explains 80 per cent or more of the variation. Lower than that, predictive accuracy begins to fall off. The coefficient of determination, r^2, is calculated like this:

$$r^2 = \frac{\sum_{j=1}^{n}(\hat{Y} - \bar{Y})^2}{\sum_{j=1}^{n}(Y - \bar{Y})^2} = \frac{SS_r}{SS_e} = 1 - \frac{SS_e}{SS_t}$$

For the wine price study, r^2 was found by using the data from the bottom of Exhibit 19.20.

$$r^2 = 1 - \frac{2320368.49}{11607511.60} = .80$$

A total of 80 per cent of the variance in price may be explained by growing-season temperatures. With actual data and multiple predictors, our results would improve.

19.3 Non-parametric measures of association[14]

Measures for nominal data

Nominal measures are used to assess the strength of relationships in cross-classification tables. They are often used with chi-square or may be used separately. In this section, we provide examples of three statistics based on chi-square and two that follow the proportional reduction in error approach.

There is no fully satisfactory all-purpose measure for categorical data. Some are adversely affected by table shape and number of cells; others are sensitive to sample size or marginals. It is perturbing to find similar statistics reporting different coefficients for the same data. This occurs because of a statistic's particular sensitivity or the way it was devised.

Technically, we would like to find two characteristics with nominal measures:

1 When there is no relationship at all, the coefficient should be 0.
2 When there is a complete dependency, the coefficient should display unity or 1.

This does not always happen. In addition to the sensitivity problem, analysts should be alerted to the need for careful selection of tests.

Chi-square-based measures

Exhibit 19.21 reports a 2 × 2 variation of the Containers Ltd shipping study on smoking and job-related accidents introduced in Chapter 17. In this example, the observed significance level is less than the testing level ($\alpha = .05$) and the null hypothesis is rejected. A correction to chi-square is provided. We now turn to measures of association to

Exhibit 19.21 ***Chi-square-based measures of association.***

		On-the-job accident		
	Count	Yes	No	Row total
Smoker	Yes	21	10	31
	No	13	22	35
	Column total	34	32	66

Chi-square	*Value*	*d.f.*	*Significance*
Pearson	6.16257	1	.01305
Community correction	4.99836	1	.02537
Minimal expected frequency	15.030		

Statistic	*Value*	*Approximate significance*
Phi	.30557	.01305*
Cramer's *V*	.30557	.01305*
Contingency coefficient *C*	.29223	.01305*

Note: *Pearson chi-square probability.

detect the strength of the relationship. Notice that the exhibit also provides an approximate significance of the coefficient based on the chi-square distribution. This is a test of the null hypothesis that no relationship exists between the variables of accidents and smoking.

The first **chi-square-based measure** is applied to smoking and on-the-job accidents. It is called **phi (ϕ)**. Phi ranges from 0 to +1.0 and attempts to correct χ^2 proportionately to N. Phi is best employed with 2×2 tables like this one since its coefficient can exceed +1.0 when applied to larger tables. Phi is calculated:

$$\phi = \sqrt{\frac{\chi^2}{N}} = \sqrt{\frac{6.616257}{66}} = .3056$$

Phi's coefficient shows a moderate relationship between smoking and job-related accidents. There is no suggestion in this interpretation that one variable causes the other, nor is there an indication of the direction of the relationship.

Cramer's V is a modification of phi for larger tables and has a range up to 1.0 for tables of any shape. It is calculated like this:

$$V = \sqrt{\frac{\chi^2}{N(k-1)}} = \sqrt{\frac{6.616257}{66(1)}} = .3056$$

where

k = the lesser number of rows or columns.

In Exhibit 19.21, the coefficient is the same as phi.

The **contingency coefficient C** is reported last. It is not comparable to other measures and has a different upper limit for various table sizes. The upper limits are determined as:

$$\sqrt{\frac{k-1}{k}}$$

where

k = the number of columns.

For a 2×2 table, the upper limit is .71; for a 3×3, .82; and for a 4×4, .87. Although this statistic operates well with tables having the same number of rows as columns, its upper-limit restriction is not consistent with a criterion of good association measurement. C is calculated as:

$$C = \sqrt{\frac{\chi^2}{\chi^2 + N}} = \sqrt{\frac{6.616257}{6.616257 + 66}} = .2922$$

The chief advantage of C is its ability to accommodate data in almost every form: skewed or normal, discrete or continuous, and nominal or ordinal.

Proportional reduction in error

Proportional reduction in error (PRE) statistics are the second type used with contingency tables. Lambda and tau are the examples discussed here. The coefficient **lambda (λ)** is based on how well the frequencies of one nominal variable offer predictive evidence about the frequencies of another. Lambda is asymmetrical – allowing calculation for the direction of prediction – and symmetrical, predicting row and column variables equally.

The computation of lambda is straightforward. In Exhibit 19.22, we have results from an opinion survey with a sample of 400 shareholders. Only 180 out of 400 (45 per cent) favour capping executives' salaries; 220 (55 per cent) do not favour it. With this information alone, if asked to predict the opinions of an individual in the sample, we would achieve the best prediction record by always choosing the modal category. Here it is 'do not favour'. By doing so, however, we would be wrong 180 out of 400 times. The probability estimate for an incorrect classification is .45, $P(1) = (1 - .55)$.

Exhibit 19.22 ***Proportional reduction in error measures.***

What is your opinion about capping executives' salaries?

	Count Row Pct.	Favour	Do not favour	Row total
Occupational class	Managerial	90 82.0%	20 18.0%	110
	White collar	60 43.0%	80 57.0%	140
	Blue collar	30 20.0%	120 80.0%	150
	Column total	180 45.0%	220 55.0%	400 100.0%

Chi-square	*Value*	*d.f.*	*Significance*
Pearson	98.38646	2	.00000
Likelihood ratio	104.96542	2	.00000
Minimal expected frequency	49.500		

Statistic	*Value*	*ASEI*	*T value*	*Approximate significance*
Lambda:				
symmetric	.30233	.03955	6.77902	
with occupation dependent	.24000	.03820	5.69495	
with opinion dependent	.38889	.04555	7.08010	
Goodman–Kruskal tau:				
with occupation dependent	.11669	.02076		.00000*
with opinion dependent	.24597	.03979		.00000*

Note: *Based on chi-square approximation.

Now suppose we have prior information about the respondents' occupational status and are asked to predict opinion. Would it improve predictive ability? Yes, we would make the predictions by summing the probabilities of all cells that are not the modal value for their rows (for example, cell [2, 1] is 20/400 or .05):

$$P(2) = \text{cell } (1, 2)\ .05 + \text{cell } (2, 1)\ .15 + \text{cell } (3, 1)\ .075 = .275$$

Lambda is then calculated:

$$\lambda = \frac{P(1) - P(2)}{P(1)} = \frac{.45 - .275}{.45} = .3889$$

Note that the asymmetric lambda in Exhibit 19.22, where opinion is the dependent variable, reflects this computation. As a result of knowing the respondents' occupational classification, we improve our prediction by 39 per cent. If we wish to predict occupational classification from opinion instead of the opposite, a λ of .24 would be secured. This means that 24 per cent of the error in predicting occupational class is eliminated by knowledge of opinion on the executives' salary question. Lambda varies between 0 and 1, corresponding with no ability to eliminate errors to elimination of all errors of prediction.

Goodman and Kruskal's **tau (τ)** uses table marginals to reduce prediction errors. In predicting opinion on executives' salaries without any knowledge of occupational class, we would expect a 50.5 per cent correct classification and a 49.5 per cent probability of error. These are based on the column marginal percentages in Exhibit 19.22.

Column marginal		Column per cent		Correct cases
180	*	45	=	81
220	*	55	=	121
Total correct classifications				210
Correct classifications of the opinion variable $= .505 = \frac{202}{400}$				
Probability of error, $P(1) = (1 - .505) = .495$				

When additional knowledge of occupational class is used, information for correct classification of the opinion variable is improved to 62.7 per cent with a 37.3 per cent probability of error. This is obtained by using the cell counts and marginals for occupational class (see Exhibit 19.22), as shown below:

Row 1	=	73.6364 + 3.6364	=	77.2727
Row 2	=	25.7143 + 45.7142	=	71.4286
Row 3	=	6.0 + 96.0	=	102.0000

Total correct classifications (with additional information on occupational classes) 250.7013

Correct classifications of the opinion variable $= .627 = \frac{250.7}{400}$

Probability of error, $P(2) = (1 - .505) = .373$

Tau is then computed like this:

$$\tau \frac{P(1) - P(2)}{P(1)} = \frac{.495 - .373}{.495} = .246$$

Exhibit 19.22 shows that the information about occupational class has reduced error in predicting opinion to approximately 25 per cent. The table also contains information on the test of the null hypothesis that tau = 0 with an approximate observed significance level and asymptotic error (for developing confidence intervals). Based on the small observed significance level, we would conclude that tau is significantly different from a coefficient of 0 and that there is an association between opinion on executives' salaries and occupational class in the population from which the sample was selected. We can also establish the confidence level for the coefficient at the 95 per cent level as approximately .25 ±.04.

Measures for ordinal data

When data require **ordinal measures**, there are several statistical alternatives. In this section we will illustrate:

- gamma
- Kendall's tau *b* and tau *c*
- Somers's *d*
- Spearman's rho.

All but Spearman's rank-order correlation are based on the concept of **concordant** and **discordant** pairs. None of these statistics requires the assumption of a bivariate normal distribution, yet by incorporating order, most produce a range from +1.0 (a perfect negative relationship) to −1.0 (a perfect positive one). Within this range, a coefficient with a larger magnitude (absolute value of the measure) is interpreted as having a stronger relationship. These characteristics allow the analyst to interpret both the direction and the strength of the relationship.

Exhibit 19.23 presents data for 70 managerial employees of KeyDesign, a large industrial design firm. All 70 employees have been evaluated for coronary risk by the firm's health insurer. The management levels are ranked, as are the fitness assessments by the physicians. If we were to use a nominal measure of association with this data (such as Cramer's V), the computed value of the statistic would be positive since order is not present in nominal data. But using ordinal measures of association reveals the actual nature of the relationship. In this example, all coefficients have negative signs.

***Exhibit 19.23* Tabled ranks for management and fitness levels at KeyDesign.**

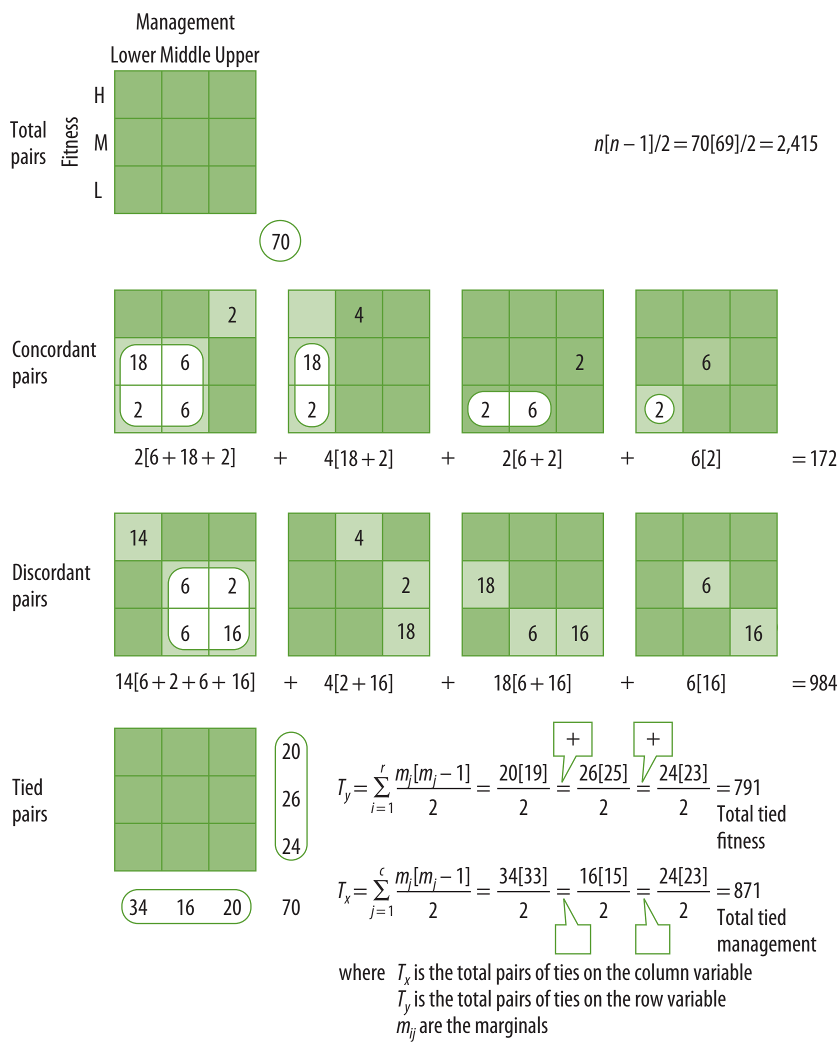
		Management level			
	Count	Lower	Middle	Upper	
Fitness	High	14	4	2	20
	Moderate	18	6	2	26
	Low	2	6	16	24
		34	16	20	70

Statistic	*Value**
Gamma	−.70242
Kendall's tau *b*	−.51279
Kendall's tau *c*	−.49714
Somers's *d*	
Symmetric	−.51263
With fitness dependent	−.52591
With management-level dependent	−.50000

Note: *The t value for each coefficient is −5.86451.

The information in the exhibit has been arranged so that the number of concordant and discordant pairs of individual observations may be calculated. When a subject that ranks higher on one variable also ranks higher on the other variable, the pairs of observations are said to be concordant. If a higher ranking on one variable is accompanied by a lower ranking on the other variable, the pairs of observations are discordant. Let *P* stand for concordant pairs and *Q* stand for discordant. When concordant pairs exceed discordant pairs in a $P - Q$ relationship, the statistic reports a positive association between the variables under study. As discordant pairs increase over concordant pairs, the association becomes negative. A balance indicates no relationship between the variables. Exhibit 19.24 summarizes the procedure for calculating the summary terms needed in all the statistics we are about to discuss.[15]

Exhibit 19.24 ***Calculation of concordant (p), discordant (q), tied (t_x, t_y) and total paired observations: KeyDesign example.***



Goodman and Kruskal's **gamma (γ)** is a statistic that compares concordant and discordant pairs and then standardizes the outcome by maximizing the value of the denominator. It has a proportional PRE interpretation that connects nicely with what we already know about PRE nominal measures. Gamma is defined as:

$$\gamma = \frac{P - Q}{P + Q} = \frac{172 - 984}{172 + 984} = \frac{-812}{1152} = -.7024$$

For the fitness data, we conclude that as management level increases, fitness decreases. This is immediately apparent from the larger number of discordant pairs.

A more precise explanation for gamma takes its absolute value (ignoring the sign) and relates it to PRE. Hypothetically, if one was trying to predict whether the pairs were concordant or discordant, one might flip a coin and classify the outcome. A better way is to make the prediction based on the preponderance of concordance or discordance; the absolute value of gamma is the proportional reduction in error when prediction is done the second way. For example, you would get a 50 per cent hit ratio using the coin.

A PRE of .70 improves your hit ratio to 85 per cent $(.50 \times .70) + (.50) = .85$.

With a γ of $-.70$, 85 per cent of the pairs are discordant and .15 per cent are concordant.[16] There are almost six times as many discordant pairs as concordant pairs. In situations where the data call for a 2×2 table, the appropriate modification of gamma is Yule's Q.[17]

Kendall's **tau b (τb)** is a refinement of gamma that considers tied pairs. A tied pair occurs when subjects have the same value on the X variable, on the Y variable or on both. For a given sample size, there are $n(n-1)/2$ pairs of observations.[18] After concordant pairs and discordant pairs are removed, the remainder are tied. Tau b does not have a PRE interpretation but does provide a range of -1.0 to $+1.0$ for square tables. Its compensation for ties uses the information found in Exhibit 19.24. It may be calculated as:

$$\tau_b = \frac{P-Q}{\sqrt{\left(\frac{n(n-1)}{2} - T_x\right)\left(\frac{n(n-1)}{2} - T_y\right)}} = \frac{172-984}{\sqrt{(2415-871)(2415-791)}} = -.5218$$

Kendall's **tau c (τ_c)** is another adjustment to the basic $P - Q$ relationship of gamma. This approach to ordinal association is suitable for tables of any size. Although we illustrate tau c, we would select tau b since the cross-classification table for the fitness data is square. The adjustment for table shape is seen in the formula:

$$\tau_c = \frac{2m(P-Q)}{N^2(M-1)} = \frac{2(3)(172-984)}{(70)^2(3-3)} = -.4971$$

where m is the smaller number of rows or columns.

Somers's d rounds out our coverage of statistics employing the concept of concordant pairs. This statistics utility comes from its ability to compensate for tied ranks and adjust for the direction of the dependent variable. Again, we refer to the preliminary calculations provided in Exhibit 19.24 to compute the symmetric and asymmetric ds. As before, the symmetric coefficient (equation 1) takes the row and column variables into account equally. The second and third calculations show fitness as the dependent and management level as the dependent, respectively.

1 $$d_{sym} = \frac{(P-Q)}{n(n-1) - T_xT_y/2} = \frac{-812}{1584} = -.5126$$

2 $$d_{y-x} = \frac{(P-Q)}{\frac{n(n-1)}{2} - T_x} = \frac{-812}{2415-871} = -.5259$$

3 $$d_{x-y} = \frac{(P-Q)}{\frac{n(n-1)}{2} - T_y} = \frac{-812}{2415-791} = -.5000$$

The **Spearman's rho (ρ)** correlation is a popular ordinal measure. Along with Kendall's tau, it is among the most widely used of ordinal techniques. Rho correlates ranks between two ordered variables. Occasionally, researchers find continuous variables with too many abnormalities to correct. Then scores may be reduced to ranks and calculated with Spearman's rho.

As a special form of Pearson's product moment correlation, rho's strengths outweigh its weaknesses. When data are transformed by logs or squaring, rho remains unaffected. Second, outliers or extreme scores that were troublesome before ranking no longer pose a threat since the largest number in the distribution is equal to the sample size. Third, it is an easy statistic to compute. The major deficiency is its sensitivity to tied ranks. Too many ties distort the coefficient's size. However, there are rarely too many ties to justify the correction formulas available.

To illustrate the use of rho, consider a situation where Dean Merrill, a brokerage firm, is recruiting account executive trainees. Assume that the field has been narrowed to 10 applicants for final evaluation. They arrive at the company headquarters, go through a battery of tests and are interviewed by a panel of three executives. The test results are evaluated by an industrial psychologist who then ranks the 10 candidates. The executives produce a composite ranking based on the interviews. Your task is to decide how well these two sets of ranking agree. Exhibit 19.25 contains the data and preliminary calculations. Substituting into the equation, we get:

$$\rho_s = 1 - \frac{6\Sigma d^2}{n^3 - n} = \frac{6(57)}{(10)^3 - 10} = .654$$

where n is the number of subjects being ranked.

Exhibit 19.25 ***Dean Merrill data for Spearman's rho.***

Applicant	Rank by			
	Panel *x*	Psychologist *y*	*d*	d^2
1	3.5	6	−2.5	6.25
2	10	5	5	25.00
3	6.5	8	−1.5	2.25
4	2	1.5	0.5	0.25
5	1	3	−2	4.00
6	9	7	2	4.00
7	3.5	1.5	2	4.00
8	6.5	9	−2.5	6.25
9	8	10	−2	4.00
10	5	4	1	1.00
				57.00

Note: Tied ranks were assigned the average (of ranks) as if no ties had occurred.

The relationship between the panel's and the psychologist's ranking is moderately high, suggesting agreement between the two measures. The test of the null hypothesis that there is no relationship between the measures ($r_s = 0$) is rejected at the .05 level with $n - 2$ degrees of freedom.

$$t = r_s\sqrt{\frac{n-2}{1-r_s^2}} = \sqrt{\frac{8}{1-.4277}} = 2.45$$

SPSS reference

Pallant (2013) is an accessible source to help you get acquainted with SPSS. It covers most of the statistical procedures and tests available in SPSS, and provides you with a grounding to use the system for more advanced procedures.

Summary

1 Management questions frequently involve relationships between two or more variables. Correlation analysis may be applied to study such relationships. A correct correlational hypothesis states that the variables occur together in some specified manner without implying that one causes the other.

2 Parametric correlation requires two continuous variables measured on an interval or ratio scale. The product moment correlation coefficient represents an index of the magnitude of the relationship: its sign governs the direction and its square explains the common variance. Bivariate correlation treats X and Y variables symmetrically and is intended for use with variables that are linearly related.

Scatterplots allow the researcher to visually inspect relationship data for appropriateness of the selected statistic. The direction, magnitude and shape of a relationship are conveyed in a plot. The shape of linear relationships is characterized by a straight line, whereas non-linear relationships are curvilinear or parabolic, or have other curvature. The assumptions of linearity and bivariate normal distribution may be checked through plots and diagnostic tests.

A correlation matrix is a table used to display coefficients for more than two variables. Matrices form the basis for computation and understanding of the nature of relationships in multiple regression, discriminant analysis, factor analysis and many multivariate techniques.

A correlation coefficient of any magnitude or sign, regardless of statistical significance, does not imply causation. Similarly, a coefficient is not remarkable simply because it is statistically significant. Practical significance should be considered in interpreting and reporting findings.

3 Regression analysis is used to further our insight into the relationship of Y with X. When we take the observed values of X to estimate or predict corresponding Y values, the process is called simple prediction. When more than one X variable is used, the outcome is a function of multiple predictors. Simple and multiple predictions are made with regression analysis.

A straight line is fundamentally the best way to model the relationship between two continuous variables. The method of least squares allows us to find a regression line, or line of best fit, that minimizes errors in drawing the line. It uses the criterion of minimizing the total squared errors of estimate. Point predictions made from well-fitted data are subject to error. Prediction and confidence bands may be used to find a range of probable values for Y based on the chosen predictor. The bands are shaped in such a way that predictors farther from the mean have larger bandwidths.

4 We test regression models for linearity and to discover whether the equation is effective in fitting the data. An important test in bivariate linear regression is whether the slope is equal to zero. In bivariate regression, t-tests and F-tests of the regression produce the same result since t^2 is equal to F.

5 Often the assumptions or the required measurement level for parametric techniques cannot be met. Non-parametric measures of association offer alternatives. Nominal measures of association are used to assess the strength of relationships in cross-classification tables. They are often used in conjunction with chi-square or may be based on the PRE approach.

Phi ranges from 0 to +1.0 and attempts to correct chi-square proportionately to N. Phi is best employed with 2×2 tables. Cramer's V is a modification of phi for larger tables and has a range up to 1.0 for tables of any configuration. Lambda, a PRE statistic, is based on how well the frequencies of one nominal variable offer predictive evidence about the frequencies of another. Goodman and Kruskal's tau uses table marginals to reduce prediction errors.

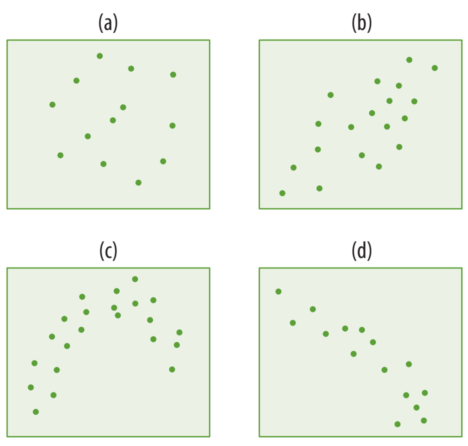
Measures for ordinal data include gamma, Kendall's tau b and tau c, Somers's d and Spearman's rho. All but Spearman's rank-order correlation are based on the concept of concordant and discordant pairs. None of these statistics require the assumption of a bivariate normal distribution, yet by incorporating order, most produce a range from −1 to +1.

Discussion questions

Terms in review

1 Distinguish between the following:
 a regression coefficient and correlation coefficient
 b $r = 0$ and $\rho = 0$
 c the test of the true slope, the test of the intercept and $r^2 = 0$
 d r^2 and r
 e a slope of 0 and $\beta_0 = 0$
 f F and t^2.

2 Describe the relationship between the two variables in the four plots below.



Making research decisions

3 A tax on the market value of stock and bond transactions has been proposed as one remedy for the budget deficit. The following data were collected on a sample of 60 registered voters by a polling organization.
 a Compute gamma for the table.
 b Compute tau b or tau c for the same data.
 c What accounts for the differences?
 d Decide which is more suitable for this data.

Opinion about market tax	Education		
	High school	Bachelor's degree	MBA
Favourable	15	5	0
Undecided	10	8	2
Unfavourable	0	2	19

4 Using the table data in question 3, compute Somers's d symmetric and then use opinion as the dependent variable. Decide which approach is best for reporting the decision.

5 A research team conducted a study of voting preferences on a referendum on the European Constitution among 260 members of political parties, 130 members of the Labour Party and 130 members of the Conservative Party. They secured the following results.

	Favour	Against
Labour Party	80	50
Conservative Party	40	90

Calculate an appropriate measure of association and decide how to present your results.

From concept to practice

6 Using the data below:
- **a** create a scatterplot
- **b** find the least-squares line
- **c** plot the line on the diagram
- **d** predict Y if X is 10, Y if X is 17.

X	3	6	9	12	15	21
Y	6	10	15	24	21	20

7 A home pregnancy test claims to be 97 per cent accurate when consumers obtain a positive result. To what extent are the variables of 'actual clinical condition' and 'test readings' related?
- **a** Compute phi, Cramer's V and the contingency coefficient for the table below. What can you say about the strength of the relationship between the two variables?
- **b** Compute lambda for this data. What does this statistic tell you?

		Test readings of in-vitro pregnancy diagnostics		
		Positive (pregnant)	*Negative (not pregnant)*	*Total*
Actual clinical condition	*Pregnant*	451	36	487
	Not pregnant	15	193	198
	Total	466	219	685

8 Fill in the question marks for the ANOVA summary table below on net profits and market value used with regression analysis.
- **a** What does the F tell you? ($\alpha = .05$)
- **b** What is the t value? Explain its meaning.

	d.f.	Sum of squares	Mean square	*F*
Regression	1	11 116 995.47	?	?
Error	?	?	116 104.63	
Total	9	12 045 832.50		

9 Using a computer program, produce a correlation matrix for the following data (the data are also included as an Excel file Entitled 'dutch_banks' on the website).

Revenue	Expenses	Result B.T.	Net profit	Total assets	Net equity	No. of foreign countries	Total employees
19 469	13 744	4 725	2 498	543 169	15 523	73	115 098
70 109	42 642	27 467	1 739	3 922 875	156 529	5	298
21 087	14 216	6 871	4 442	274 889	72 454	0	85
3 515	947	2 568	16 839	2 094 225	49 119	1	97
56 043	46 262	9 781	4 124	694 037	21 613	86	80 000
328 005	167 504	160 501	92 154	16 921 968	763 115	2	1 238
77 003	38 689	38 314	25 514	3 068 801	205 567	0	389
228 486	17 369	117 658	76 929	3 600 246	2 423 025	0	6 092
50 009	23 316	26 693	1 735	3 673 928	339 689	33	39 044
111 963	88 905	23 058	1 682	2 473 537	2 038	1	612
27 839	21 037	6 802	4 949	940 033	131 000	42	98 000
64 206	25 931	38 275	2 488	1 303 115	146 642	3	206
77 908	22 178	5 573	36 286	4 229 011	345 096	0	199
43 617	23 968	19 649	12 761	157 934	80 828	8	217
474 275	432 926	41 349	27 676	4 380 827	151 968	64	62 881
158 274	10 794	50 334	57 205	562 453	35 892	0	1034
58 584	44 615	13 969	11 984	650 172	25 274	64	92 650
12 378	79 686	44 094	29 365	5 417 193	176 419	4	792
15 089 057	12 801 053	2 288 004	812 284	14 772 538	5 222 542	21	22 000
161 425	64 813	96 612	66 221	1 079 605	219 636	5	370

10 Secure Spearman's rank-order correlations for the largest Pearson coefficient in the matrix from question 9. Explain the differences between the two findings.

11 Using the matrix data in question 9, select a pair of variables and run a simple regression. Then investigate the appropriateness of the model for the data using diagnostic tools for evaluating assumptions.

12 For the data in the table below:

a calculate the correlation between X and Y

b interpret the sign of the correlation

c interpret the square of the correlation

d plot the least-squares line

e test for a linear relationship

i $\beta_1 = 0$

ii $r = 0$

iii an F-test.

X	25	19	17	14	12	9	8	7	3
Y	5	7	12	23	20	25	26	28	20

Recommended further reading

Aczel, Amir D. and Jayauel Sounderpandian, *Complete Business Statistics* (7th edn). Chicago: McGraw-Hill, 2008. The chapter on simple regression/correlation has impeccable exposition and examples and is highly recommended.

Agresti, Alan and Barbara Finlay, *Statistical Methods for the Social Sciences* (4th edn). Upper Saddle River, NJ: Pearson, 2008. Very clear coverage of non-parametric measures of association.

Chatterjee, Samprit and Ali S. Hadi, *Regression Analysis by Example* (5th edn). New York: Wiley, 2012. Updated version of widely used examples textbook.

Cohen, Jacob, Patricia Cohen, Stephen G. West and Leona S. Aiken, *Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences* (3rd edn). Mahwah, NJ: Lawrence Erlbaum Associates, 2003. A classic reference work.

Neter, John, Michael H. Kutner, Christopher J. Nachtsheim and William Wasserman, *Applied Linear Statistical Models* (5th edn). New York: McGraw-Hill, 2004, Chapters 1–10 and 15 provide an excellent introduction to regression and correlation analysis.

Siegel, S. and N.J. Castellan Jr., *Nonparametric Statistics for the Behavioral Sciences* (2nd edn). New York: McGraw-Hill, 1988.

Online LearningCentre

Get started with understanding statistical techniques!

When you have read this chapter, log on to the Online Learning Centre website at *www.mcgraw-hill.co.uk/textbooks/blumberg* to explore chapter-by-chapter test questions, additional case studies, a glossary and more online study tools for *Business Research Methods*.

Notes

1 Typically, we plot the x (independent) variable on the horizontal axis and the y (dependent) variable on the vertical axis. Although correlation does not distinguish between independent and dependent variables, the convention is useful for consistency in plotting and will be used later with regression.

2 F.J. Anscombe, 'Graphs in statistical analysis', *American Statistician* 27 (1973), pp. 17–21. Cited in Samprit Chatterjee and Bertram Price, *Regression Analysis by Example*. New York: Wiley, 1977, pp. 7–9.

3 Amir D. Aczel, *Complete Business Statistics* (2nd edn). Homewood, IL: Irwin, 1993, p. 433.

4 The coefficient for net profits and cash flow in the example calculation used a subsample (n = 10) and was found to be .93. The matrix shows the coefficient as .95. The matrix calculation was based on the larger sample (n = 100).

5 This section is partially based on the concepts developed by Emanuel J. Mason and William J. Bramble, *Understanding and Conducting Research*. New York: McGraw-Hill, 1989, pp. 172–82; and elaborated in greater detail by Aczel, *Complete Business Statistics*, pp. 414–29.

6 Technically, estimation uses a concurrent criterion variable where prediction uses a future criterion. The statistical procedure is the same in either case.

7 Peter Passell, 'Can math predict a wine? An economist takes a swipe at some noses', *International Herald Tribune*, 5 March 1990, p. 1; Jacques Neher, 'Top quality Bordeaux cellar is an excellent buy', *International Herald Tribune*, 9 July 1990, p. 8.

8 See Alan Agresti and Barbara Finlay, *Statistical Methods for the Social Sciences*. San Francisco: Dellen Publishing, 1986, pp. 248–9. See also the discussion of basic regression models in John Neter, William Wasserman and Michael H. Kutner, *Applied Linear Statistical Models*. Homewood, IL: Irwin, 1990, pp. 23–49.

9 We distinguish between the error terms $e_1 = Y_i - \mathrm{E}[Y_i]$ and the residual $e_i = (Y_i - \bar{Y}_i)$. The first is based on the vertical deviation of Y_i from the true regression line. It is unknown and estimated. The second is the vertical deviation of Y_i from the fitted N on the estimated line. See Neter et al., *Applied Linear Statistical Models*, p. 47.

10 For further information on software-generated regression diagnostics, see the most current release of software manuals for SPSS, MINITAB, BMDP and SAS.

11 Aczel, *Complete Business Statistics*, p. 434.

12 This calculation is normally listed as the standard error of the slope (SE B) on computer printouts. For these data it is further defined as:

$$s(b_1) = \frac{8}{\sqrt{SS_x}} + \frac{538.559}{198.249} = 38.249$$

where

s = The standard error of estimate (and the square root of the mean square error of the regression)
SS_x = The sum of squares for the X variable

13 Computer printouts use uppercase (R2) because most procedures are written to accept multiple and bivariate regression.

14 The table output for this section has been modified from SPSS and is described in Norusis/SPSS, Inc., *SPSS Base System User's Guid*e. For further discussion and examples of non-parametric measures of association, see S. Siegel and N.J. Castellan Jr., *Nonparametric Statistics for the Behavioral Sciences* (2nd edn). New York: McGraw-Hill, 1988.

15 Calculation of concordant and discordant pairs is adapted from Agresti and Finlay, *Statistical Methods for the Social Sciences*, pp. 221–3.

16 We know that the percentage of concordant plus the percentage of discordant pairs sums to 1.0. We also know their difference is −.70. The only numbers satisfying these two conditions are .85 and .15 (.85 + .15 = 1.0, .15 − .85 = −.70).

17 G.U. Yule and M.G. Kendall, *An Introduction to the Theory of Statistics*. New York: Hafner, 1950.

18 M.G. Kendall, *Rank Correlation Methods* (4th edn). London: Charles W. Griffin, 1970.