



APPENDIX E

Selected statistical tables

List of exhibits

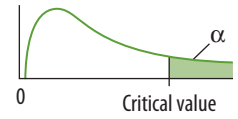
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Exhibit E.2 Critical values of *t* for given probability levels.

<i>d.f.</i>	Level of significance for one-tailed test					
	0.10	0.05	0.025	0.1	.005	.0005
	Level of significance for two-tailed test					
	.20	.10	.05	.02	.01	.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	2.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.767
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

Source: adapted from Table III of R.A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural, and Medical Research*, 6th edn. Edinburgh: Oliver and Boyd Ltd., 1963, with the kind permission of the publisher.

Exhibit E.3 Critical values of the chi-square distribution.



Probability under H_0 that $\chi^2 \geq$ chi-square					
d.f.	.10	.05	.02	.01	.001
1	2.71	3.84	5.41	6.64	10.83
2	4.60	5.99	7.82	9.21	13.82
3	6.25	7.82	9.84	11.34	16.27
4	7.78	9.49	11.67	13.28	18.46
5	9.24	11.07	13.39	15.09	20.52
6	10.64	12.59	15.03	16.81	22.46
7	12.02	14.07	16.62	18.48	24.32
8	13.36	15.51	18.17	20.09	26.12
9	14.68	16.92	19.68	21.67	27.88
10	15.99	18.31	21.16	23.21	29.59
11	17.28	19.68	22.62	24.72	31.62
12	18.55	21.03	24.05	26.22	32.91
13	19.81	22.36	25.47	27.69	34.53
14	21.06	23.68	26.87	29.14	36.12
15	22.31	25.00	28.26	30.58	37.70
16	23.54	26.30	29.63	32.00	39.29
17	24.77	27.59	31.00	33.41	40.75
18	25.99	28.87	32.35	34.80	42.31
19	27.20	30.14	33.69	36.19	43.82
20	28.41	31.41	35.02	37.57	45.32
21	29.62	32.67	36.34	38.93	46.80
22	30.81	33.92	37.66	40.29	48.27
23	32.01	35.17	38.97	41.64	49.73
24	33.20	36.42	40.27	42.98	51.18
25	34.38	37.65	41.57	44.31	52.62
26	35.56	38.88	42.86	45.64	54.05
27	36.74	40.11	44.14	46.96	55.48
28	37.92	41.34	45.42	48.28	56.89
29	39.09	42.56	46.69	49.59	58.30
30	40.26	43.77	47.96	50.89	59.70

Source: adapted from Table IV of R.A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural, and Medical Research*, 6th edn. Edinburgh: Oliver and Boyd Ltd., 1963, with the kind permission of the publisher.

Exhibit E.4 Critical values of t in the Wilcoxon-matched pairs test.

n	Level of significance for one-tailed test		
	.025	.01	.005
	Level of significance for two-tailed test		
	.05	.02	.01
6	0	–	–
7	2	0	–
8	4	2	0
9	6	3	2
10	8	5	3
11	11	7	5
12	14	10	7
13	17	13	10
14	21	16	13
15	25	20	16
16	30	24	20
17	35	28	23
18	40	33	28
19	46	38	32
20	52	43	38
21	59	49	43
22	66	56	49
23	73	62	55
24	81	69	61
25	89	77	68

Source: adapted from Table 1 of F. Wilcoxon, *Some Rapid Approximate Statistical Procedures*. New York: American Cyanamid Company, 1949, p. 13, with the kind permission of the publisher.

Exhibit E.5 Critical values of D in the Kolmogorov–Smirnov one-sample test.

Sample size n	Level of significance for $D = \text{maximum } F_n(X) - S_n(X) $				
	.20	.15	.10	.05	.01
1	.900	.925	.950	.975	.995
2	.684	.726	.776	.842	.929
3	.565	.597	.642	.708	.828
4	.494	.525	.564	.624	.733
5	.446	.474	.510	.565	.669
6	.410	.436	.470	.521	.618
7	.381	.405	.438	.486	.577
8	.358	.381	.411	.457	.543
9	.339	.360	.388	.432	.514
10	.322	.342	.368	.410	.490
11	.307	.326	.352	.391	.468
12	.295	.313	.338	.375	.450
13	.284	.302	.325	.361	.433
14	.274	.292	.314	.349	.418
15	.266	.283	.304	.338	.404
16	.258	.274	.295	.328	.392
17	.250	.266	.286	.318	.381
18	.244	.259	.278	.309	.371
19	.237	.252	.272	.301	.263
20	.231	.246	.264	.294	.356
25	.21	.22	.24	.27	.32
30	.19	.20	.22	.24	.29
35	.18	.19	.21	.23	.27
Over 35	1.07	1.14	1.22	1.36	1.63
	\sqrt{N}	\sqrt{N}	\sqrt{N}	\sqrt{N}	\sqrt{N}

Source: F.J. Massey, Jr., 'The Kolmogorov–Smirnov test for goodness of fit', *Journal of the American Statistical Association* 46, p. 70. Adapted with the kind permission of the publisher.

Exhibit E.6 Critical values of K_D in the Kolmogorov–Smirnov two-samples test (small samples).

n	One-tailed test*		Two-tailed test**	
	$\alpha = .05$	$\alpha = .01$	$\alpha = .05$	$\alpha = .01$
3	3	–	–	–
4	4	–	4	–
5	4	5	5	5
6	5	6	5	6
7	5	6	6	6
8	5	6	6	7
9	6	7	6	7
10	6	7	7	8
11	6	8	7	8
12	6	8	7	8
13	7	8	7	9
14	7	8	8	9
15	7	9	8	9
16	7	9	8	10
17	8	9	8	10
18	8	10	9	10
19	8	10	9	10
20	8	10	9	11
21	8	10	9	11
22	9	11	9	11
23	9	11	10	11
24	9	11	10	12
25	9	11	10	12
26	9	11	10	12
27	9	12	10	12
28	10	12	11	13
29	10	12	11	13
30	10	12	11	13
35	11	13	12	
40	11	14	13	

Source: *abridged from I.A. Goodman, 'Kolmogorov-Smirnov tests for psychological research', *Psychological Bulletin* 51 (1951), p. 167, copyright (1951) by the American Psychological Association. Reprinted with the kind permission of the publisher.

**Derived from Table 1 of F.J. Massey, Jr., 'The distribution of the maximum deviation between two sample cumulative step functions', *Annals of Mathematical Statistics* 23 (1951), pp. 126–27, with the kind permission of the publisher.

Exhibit E.7 Critical values of D in the Kolmogorov–Smirnov two-samples test for large samples (two-tailed).

Level of significance	Value of D so large as to call for rejection of H_0 at the indicated level of significance, where $D = \text{maximum } S_{n_1}(X) - S_2(X) $	
.10	1.22	$\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$
.05	1.36	$\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$
.025	1.48	$\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$
.01	1.63	$\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$
.005	1.75	$\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$
.001	1.95	$\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$

Source: adapted from N. Smirnov, 'Table for estimating the goodness of fit of empirical distribution', *Annals of Mathematical Statistics* 18 (1948), pp. 280–81, with the kind permission of the publisher.

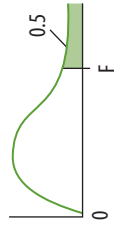
Exhibit E.8 Partial table of critical values of U in the Mann–Whitney test.

Critical values for one-tailed test at $\alpha = .025$ or a two-tailed test at $\alpha = .05$												
$n_1 \setminus n_2$	9	10	11	12	13	14	15	16	17	18	19	20
12	0	0	0	1	1	1	1	1	2	2	2	2
3	2	3	3	4	4	5	5	6	6	7	7	8
4	4	5	6	7	8	9	10	11	11	12	13	13
5	7	8	9	11	12	13	14	15	17	18	19	20
6	10	11	13	14	16	17	19	21	22	24	25	27
7	12	14	16	18	20	22	24	26	28	30	32	34
8	15	17	19	22	24	26	29	31	34	36	38	41
9	17	20	23	26	28	31	34	37	39	42	45	48
10	20	23	26	29	33	36	39	42	45	48	52	55
11	23	26	30	33	37	40	44	47	51	55	58	62
12	26	29	33	37	41	45	49	53	57	61	66	69
13	28	33	37	41	45	50	54	59	63	67	72	76
14	31	36	40	45	50	55	59	64	67	74	78	83
15	34	39	44	49	54	59	64	70	75	80	85	90
16	37	42	47	53	59	64	70	75	81	86	92	98
17	39	45	51	57	63	67	75	81	87	93	99	105
18	42	48	55	61	67	74	80	86	93	99	106	112
19	45	52	58	65	72	78	85	92	99	106	113	119
20	48	55	62	69	76	83	90	98	105	112	119	127

Critical values for one-tailed test at $\alpha = .05$ or a two-tailed test at $\alpha = .10$												
$n_1 \setminus n_2$	9	10	11	12	13	14	15	16	17	18	19	20
1											0	0
2	1	1	1	2	2	2	3	3	3	4	4	4
3	3	4	5	5	6	7	7	8	9	9	10	11
4	6	7	8	9	10	11	12	14	15	16	17	18
5	9	11	12	13	15	16	18	19	20	22	23	25
6	12	14	16	17	19	21	23	25	26	28	30	32
7	15	17	19	21	24	26	28	30	33	35	37	39
8	18	20	23	26	28	31	33	36	39	41	44	47
9	21	24	27	30	33	36	39	42	45	48	51	54
10	24	27	31	34	37	41	44	48	51	55	58	62
11	27	31	34	38	42	46	50	54	57	61	65	69
12	30	34	38	42	47	51	55	60	64	68	72	77
13	33	37	42	47	51	56	61	65	70	75	80	84
14	36	41	46	51	56	61	66	71	77	82	87	92
15	39	44	50	55	61	66	72	77	83	88	94	100
16	42	48	54	60	65	71	77	83	89	95	101	107
17	45	51	57	64	70	77	83	89	96	102	109	115
18	48	55	61	68	75	82	88	95	102	109	116	123
19	51	58	65	72	80	87	94	101	109	116	123	130
20	54	62	69	77	84	92	100	107	115	123	130	138

Source: abridged from D. Auble, 'Extended tables from the Mann-Whitney *Bulletin of the Institute of Educational Research at Indiana University*' 1, no. 2. Reprinted with kind permission of the publisher. For tables for other size samples consult this source.

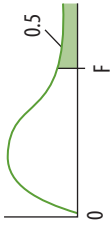
Exhibit E.9 Critical values of the F distribution for $\alpha = .05$.



Degrees of freedom for denominator

n_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	243.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92

Exhibit E.9 Continued



n_2	Degrees of freedom for denominator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

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Exhibit E.10 Random numbers.

97446	30328	05262	77371	13523	62057	44349	85884	94555	23288
15453	75591	60540	77137	09485	27632	05477	99154	78720	10323
69995	77086	55217	53721	85713	27854	41981	88981	90041	20878
69726	58696	27272	38148	52521	73807	29685	49152	20309	58734
23604	31948	16926	26360	76957	99925	86045	11617	32777	38670
13640	17233	58650	47819	24935	28670	33415	77202	92492	40290
90779	09199	51169	94892	34271	22068	13923	53535	56358	50258
71068	19459	32339	10124	13012	79706	07611	52600	83088	26829
55019	79001	34442	16335	06428	52873	65316	01480	72204	39494
20879	50235	17389	25260	34039	99967	48044	05067	69284	53867
00380	11595	49372	95214	98529	46593	77046	27176	39668	20566
68142	40800	20527	79212	14166	84948	11748	69540	84288	37211
42667	89566	20440	57230	35356	01884	79921	94772	29882	24695
07756	78430	45576	86596	56720	65529	44211	18447	53921	92722
45221	31130	44312	63534	47741	02465	50629	94983	05984	88375
20140	77481	61686	82836	41058	41331	04290	61212	60294	95954
54922	25436	33804	51907	73223	66423	68706	36589	45267	35327
48340	30832	72209	07644	52747	40751	06808	85349	18005	52323
23603	84387	20416	88084	33103	41511	59391	71600	35091	52722
12548	01033	22974	59596	92087	02116	63524	00627	41778	24392
15251	87584	12942	03771	91413	75652	19468	83889	98531	91529
65548	59670	57355	18874	63601	55111	07278	32560	40028	36079
48488	76170	46282	76427	41693	04506	80979	26654	62159	83017
02862	15665	62159	15159	69576	20328	68873	28152	66087	39405
67929	06754	45842	66365	80848	15262	55144	37816	08421	30071
73237	07607	31615	04892	50989	87347	14393	21165	68169	70788
13788	20327	07960	95917	75112	01398	26381	41377	33549	19754
43877	66485	40825	45923	74410	69693	76959	70973	26343	63781
14047	08369	56414	78533	76378	44204	71493	68861	31042	81873
88383	46755	51342	13505	55324	52950	22244	28028	73486	98797
29567	16379	41994	65947	58926	50953	09388	00405	29874	44954
20508	60995	41539	26396	99825	25652	28089	57224	35222	58922
64178	76768	75747	32854	32893	61152	58565	33128	33354	16056
26373	51147	90362	93309	13175	66385	57822	31138	12893	68607
10083	47656	59241	73630	99200	94672	59785	95449	99279	25488
11683	14347	04369	98719	75005	43633	24125	30532	54830	95387
56548	76293	50904	88579	24621	94291	56881	35062	48765	22078
35292	47291	82610	27777	43965	31802	98444	88929	54383	93141
51329	87645	51623	08971	50704	82395	33916	95859	99788	97885
51860	19180	39324	68483	78650	74750	64893	58042	82878	20619
23886	01257	07945	71175	31243	87167	42829	44601	08769	26417
80028	82310	43989	09242	15056	48250	04529	96941	48190	69644
83946	46858	09164	18858	12672	55190	02820	45861	29104	75386
00000	41586	25972	25356	54260	95691	99431	89903	22306	43863
90615	12848	23376	29458	48239	37628	59265	50152	30340	40713
42003	10738	55835	48218	23204	19188	13556	06610	77667	88068
86135	26174	07834	17007	97938	96728	15689	77544	89186	41252
54436	10828	41212	19836	89476	53685	28085	22878	71868	35048
14545	72034	32131	38783	58588	47499	50945	97045	42357	53536
43925	49879	13339	78773	95626	67119	93023	96832	09757	98545

Source: The Rand Corporation, *A Million Random Digits with 100,000 Normal Deviates*. Glencoe, IL: Free Press, 1955, p. 225.

