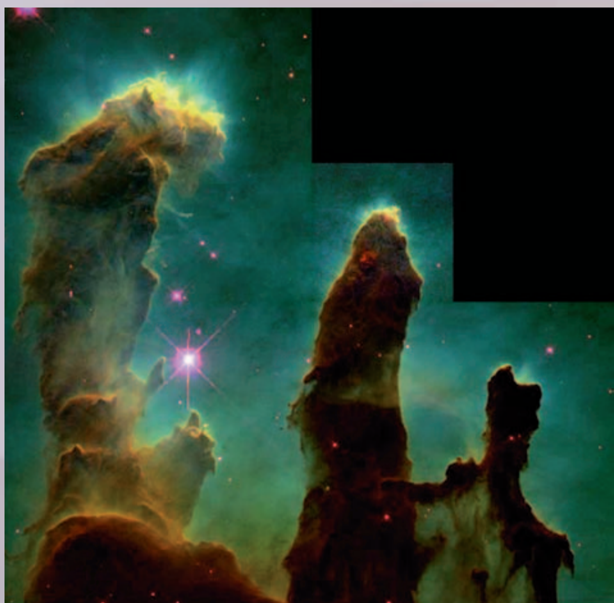
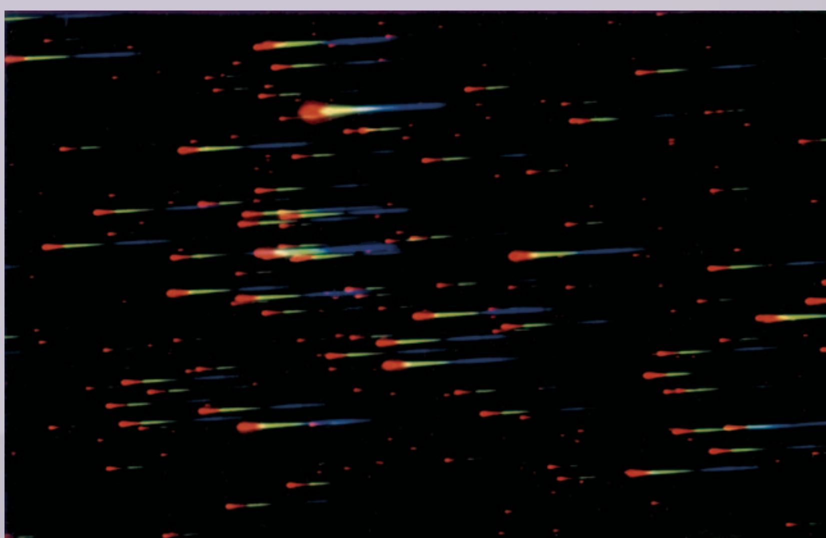
12 Measuring the Properties of Stars



A region where stars are forming from interstellar dust and gas clouds.



?What Is This?
(See chapter summary for the answer.)

Chapter Objectives

- Astronomers can learn much about stars, such as their temperature and composition, from the spectra of their light.
- Many stars are members of binary star systems in which one star orbits another.
- Study of binary star motions gives information about the stars' mass.
- Classification of stars by their temperature and intrinsic brightness shows that stars can be divided into three main groups:
    - Main-sequence stars.
    - Red giants.
    - White dwarfs.
- Main-sequence stars obey the mass–luminosity law:
    - More-massive stars are more luminous.
- Some stars pulsate, so that their size and brightness change.

Concepts and Skills to Remember

- Relation between temperature of a hot object and the color of the light it emits (Wien's law) (3.2)
- Doppler shift (3.6)
- Modified form of Kepler's third law (2.7)

It is hard to believe that the stars we see in the night sky as tiny glints of light are in reality huge, dazzling balls of gas and that many are vastly larger and brighter than our Sun. They look dim to us only because they are so far away—several light-years (trillions of miles) to even the nearest. Such remoteness creates tremendous difficulties for astronomers trying to understand the nature of stars. We cannot physically travel to the stars, but in this chapter we will see how astronomers overcome the distance barrier that separates us from stars and how they learn many of the physical properties of these distant objects. How far away are they? How big? What are they made of?

The answers to these questions show us that most stars are remarkably like the Sun. For example, like the Sun, they are composed mostly of hydrogen and helium and have about the same mass. A small percentage, however, are as massive as 30 times the Sun's mass (30 $M_\odot$) and are much hotter than the Sun and blue in color. Others are much less massive than the Sun, only one-tenth its mass, and are cool, red, and dim. Moreover, even stars similar to the Sun in composition and mass may differ enormously from it in their radii and density. For example, some giant stars have a radius hundreds of times larger than the Sun's—so big that were the Sun their size, it would extend beyond the Earth's orbit. On the other hand, some stars are white dwarfs, with as much material as the Sun packed into a volume the size of the Earth.

Astronomers can learn all these properties of stars by using physical laws and theories to interpret measurements made from the Earth. For example, theories of light yield the surface temperature, distance, and motion of a star; theories of atoms yield the composition of a star; and a modified form of Kepler's third law yields the mass of a star. In using such laws, astronomers may sometimes employ more than one method to determine a desired property of a star. For example, a star's temperature may be measured from either its color or its spectrum. Such alternative methods serve as checks on the correctness of the procedures astronomers use to determine the properties of stars.

12.1 MEASURING A STAR'S DISTANCE

One of the most difficult problems astronomers face is measuring the distance to stars and galaxies. Yet knowing the distances to such bodies is vital if we are to understand their size and structure. For example, without knowledge of a star's distance, astronomers find it difficult to learn many of the star's other properties, such as its mass, radius, or energy output.

Measuring Distance by Triangulation and Parallax

Astronomers have several methods for measuring a star's distance, but for nearby stars the fundamental technique is **triangulation,** the same method we described for finding the distance to the Moon. In triangulation, we construct a triangle in which one side is the distance we seek but cannot measure directly and another side is a distance we can measure—a baseline, as shown in figure 12.1A. For example, to measure the distance across a deep gorge, we construct an imaginary triangle with one side spanning the gorge and another side at right angles to it and running along the edge we are on, as shown in figure 12.1B. By measuring the length of the side along the gorge edge and the angle *A*, we can determine the distance across the gorge either by a trigonometric calculation or from a scale drawing of the triangle.

Using parallax to find distance may seem unfamiliar, but parallax creates our stereovision, the ability to see things three dimensionally. When we look at something, each eye sends a slightly different image to the brain, which then processes the pictures to determine the object's distance. You can demonstrate the importance of the two images by covering one eye and trying to pick up a pencil quickly. You will probably not succeed in grasping it on the first attempt.

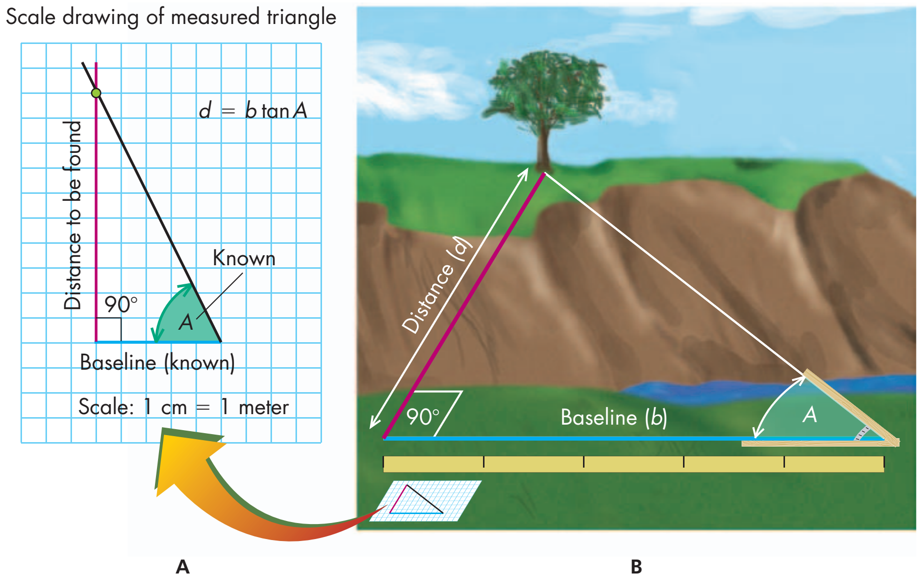


FIGURE 12.1
Sketch illustrating the principle of triangulation.

ANIMATION
Parallax

Astronomers use a method of triangulation called **parallax** to measure the distance to stars. Parallax is a change in an object's apparent position caused by a change in the observer's position. An example—easy to demonstrate in your room—is to hold your hand motionless at arm's length and shift your head from side to side. Your hand seems to move against the background even though in reality it is your head that has changed position, not your hand.

This simple demonstration illustrates how parallax gives a clue to an object's distance. If you hold your hand at different distances from your face, you will notice that the apparent shift in your hand's position—its parallax—is larger if it is close to your face than if it is at arm's length. That is, *nearby objects exhibit more parallax than remote ones,* for a given motion of the observer, a result true for your hand and for stars.

To observe stellar parallax, astronomers take advantage of the Earth's motion around the Sun, as shown in figure 12.2A. They observe a star and carefully measure its position against background stars. They then wait 6 months until the Earth has moved to the other side of its orbit, a known distance of 2 AU (about 300 million kilometers), and make a second measurement. As figure 12.2B shows, the star will have a slightly different position compared to the background of stars as seen from the two points. The amount by which the star's apparent position changes depends on its distance from us. The change is larger for nearby stars than for remote stars, but for all stars it is extremely small—so small that it is measured not in degrees but in fractions of a degree called "arc seconds."*

For convenience, astronomers define a star's parallax, p, not by the angle its position appears to shift, but by half that angle (see fig. 12.2C). With that definition for parallax, the star's distance, d, is simply $1/p$ if we measure p in arc seconds and d, not in kilometers or light-years, but in a new unit called "parsecs" (abbreviated pc). That is,

$$d_{\text{pc}} = \frac{1}{p_{\text{arc seconds}}}$$

With this choice of units, one **parsec** equals 3.26 light-years (3.09×10^{13} kilometers). The word *parsec* comes from a combination of *par*allax and "arc *sec*ond."

Keep in mind that the change in angle we are talking about here is *very* small. A shift of one second of arc is equivalent to looking at one edge of a U.S. penny 4 km (2.5 miles) away and then looking at its opposite edge.

*One arc second is $\frac{1}{3600}$ of a degree because an arc second is $\frac{1}{60}$ of an arc minute, which is $\frac{1}{60}$ of a degree.

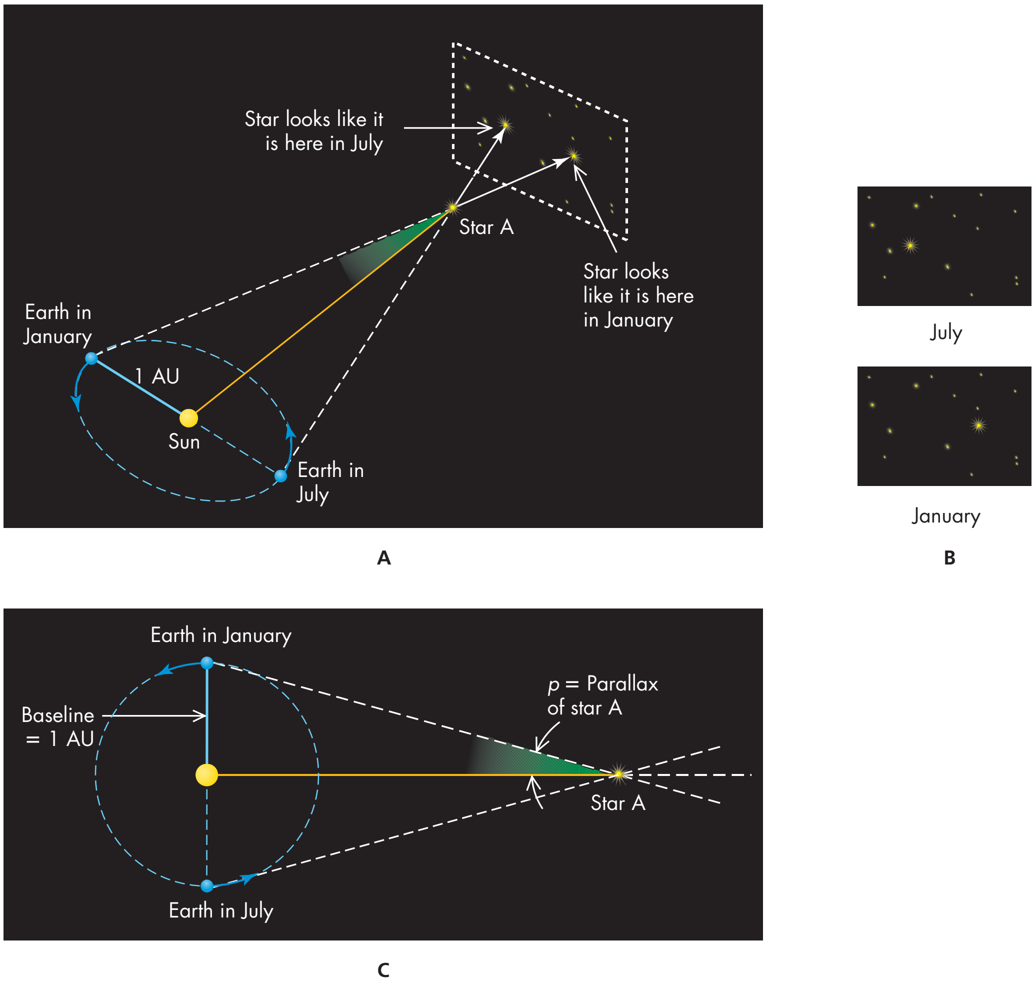


FIGURE 12.2
(A) Triangulation to measure a star's distance. The radius of the Earth's orbit is the baseline. (B) As the Earth moves around the Sun, the star's position changes as seen against background stars. (C) Parallax is defined as one-half the angle by which the star's position shifts. Sizes of bodies and their separation are exaggerated for clarity.

To determine a star's distance, we measure its parallax, p, and use $d_{\text{pc}} = 1/p_{\text{arc seconds}}$. For example, suppose from the shift in position of a nearby star we find its parallax is 0.25 arc seconds. Its distance is then $d = \frac{1}{0.25} = 4$ parsecs. Similarly, a star whose parallax is 0.1 arc second is $\frac{1}{0.1} = 10$ parsecs from the Sun. From this technique, astronomers have discovered that at present the nearest star is Proxima Centauri (See "Looking Up at Centaurus and Crux, The Southern Cross."), which lies 1.3 parsecs (4.3 light-years) from the Sun.* This spacing (about 1 parsec) is typical for stars near the Sun.

Although the parallax–distance relation is mathematically a very simple formula, measuring a star's parallax to use in the formula is very difficult because the angle by which the star shifts is extremely small. It was not until the 1830s that the first parallax was measured by the German astronomer Friedrich Bessel at Königsberg Observatory (now in Kaliningrad). Even now, the method fails for most stars farther away than about 100 parsecs because the Earth's atmosphere blurs the tiny angle of their shift, making it unmeasurable.† Astronomers can avoid such blurring effects by

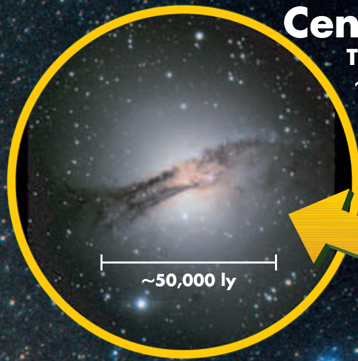
*Proxima Centauri has two companions only slightly more distant. The bigger companion, Alpha Centauri A, is a bright star visible in the southern sky.

†With electronic techniques, astronomers can measure, from the ground, distances to some bright stars as far away as 250 parsecs.

LOOKING UP AT...
CENTAURUS AND CRUX, THE SOUTHERN CROSS

These constellations are best observed from the southern hemisphere. Northern hemisphere viewers can see Centaurus low in the southern sky in May – July. Crux may be seen just above the southern horizon in May and June from the extreme southern US (Key West and South Texas)

Centaurus A



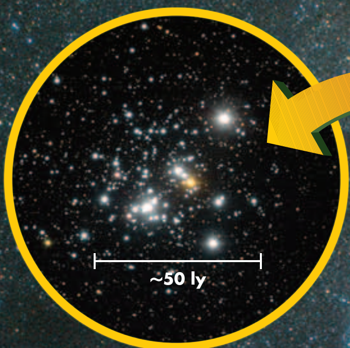
This unusual galaxy, ~11 million ly distant, is one of the brightest radio sources in the sky

Alpha Centauri

Proxima Centauri

This dim star is the nearest star to the Sun, 4.22 ly distant

Crux
The Southern Cross



The Jewel Box

NGC 4755, an open star cluster (Unit 69) ~500 ly from us.

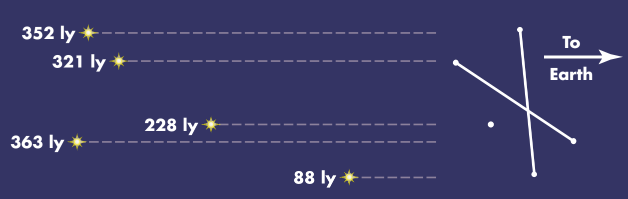
The Coal Sack

An interstellar dust cloud

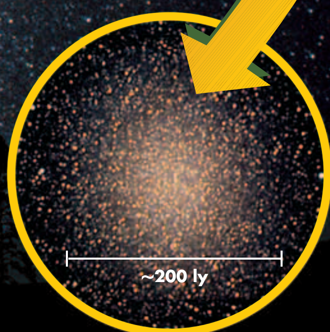
Eta Carinae

A very high-mass star doomed to die young ~8000 ly distant

Southern Cross in 3-D



1 light year (ly) ≈ 10 trillion km ≈ 6 trillion miles



Omega Centauri

The largest globular cluster in the Milky Way, ~17,000 ly distant and containing millions of stars

EXTENDING OUR REACH

MEASURING THE DISTANCE TO SIRIUS

We can measure the distance to Sirius (or any nearby star) using a geometric construction similar to the one we used in chapter 1 to relate an object's true diameter to its angular diameter and distance. We begin by measuring the star's apparent shift in the sky resulting from our planet's motion around the Sun, as illustrated in figure 12.2A. Let p be half that angle, as shown in figure 12.2C and box figure 12.1, below. To determine the star's distance from the parallax angle, p, we construct the triangle *ACS*. The line *AC* is the radius of the Earth's orbit, with point *A* chosen so that an imaginary line from Sirius (*S*) to the Sun, *C*, will be perpendicular to *AC*. This makes *ACS* a right triangle, with angle *C* equal to 90° and the side *AC*—the distance from the Earth to the Sun—1 astronomical unit. To calculate Sirius's distance from the Sun, *d*, we draw a circle centered on Sirius and passing through the Earth and the Sun. We call the radius of the circle *d*, to stand for Sirius's distance from the Sun. We next form the following proportion: *AC*, the radius of the Earth's orbit, is to the circumference of the circle as p is to the total number of degrees around the circle, which we know is 360. That is,

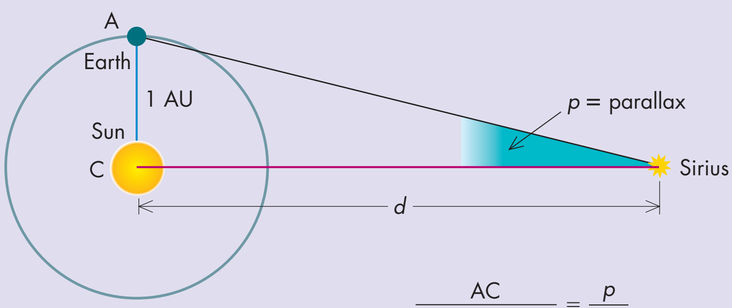
$$\frac{AC}{2\pi d} = \frac{p}{360}.$$

If we measure d and *AC* in astronomical units, we can set $AC = 1$. Then, solving for d, we obtain

$$d = \frac{360}{2\pi p}\ \text{AU}$$

This equation takes on a much simpler form if we express p in arc seconds rather than degrees and d in parsecs (as defined above) rather than in AU. With these new units, the factors 360 and 2π disappear, leaving $d = 1/p$ as we claimed earlier. (In the above, we assume that Sirius itself has not changed its location significantly compared to its distance in so brief a time as 6 months—a very good assumption.)

Measurements show that for Sirius, the angle p is 0.377 arc seconds. Thus, its distance is $1/0.377 = 2.65$ parsecs. To express this in light-years, we multiply by 3.26 (the number of light-years in a parsec) to get Sirius's distance as about 8.6 light-years.



$$\frac{AC}{\text{Circumference}} = \frac{p}{360}$$

$$\text{Circumference} = 2\pi \times \text{Radius} = 2\pi d$$

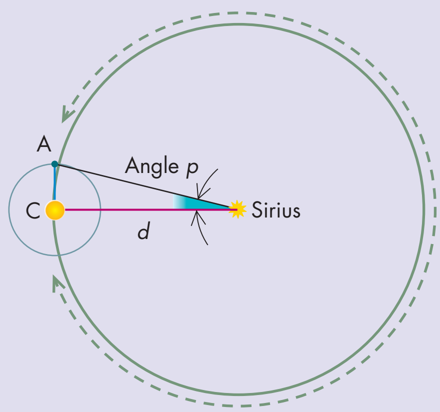
$$\frac{AC}{2\pi d} = \frac{p}{360}$$

Cross multiply to get

$$\frac{360\ AC}{2\pi p} = d$$

or

$$d = \frac{360\ AC}{2\pi p}$$



BOX FIGURE 12.1
How to determine the relation between a star's distance and its parallax.

observing from above the atmosphere, and an orbiting satellite, *Hipparcos,* has done just that: making parallax measurements from space. With its data, astronomers are able to accurately measure distances to stars as far away as 250 parsecs. For more remote stars, they must use a different method, one based on how bright objects look to us.

Measuring Distance by the Standard-Candles Method

If you look at an object of known brightness, you can estimate its distance from how bright it appears. For example, if you look at two 100-watt light bulbs, one close and one far away, you can tell fairly accurately how much farther away the dim one is. In fact, if you drive at night, your life depends on making such distance estimates when you see traffic lights or oncoming cars. Astronomers call such distance measurements the **method of standard candles** and use a similar but more refined version of it to find the distance to stars and galaxies. But to use this method to measure an object's distance, astronomers must first determine its true brightness. So until we learn ways to make such brightness determinations, we must set this method aside, but we will return to it later in the chapter.

12.2 MEASURING THE PROPERTIES OF STARS FROM THEIR LIGHT

If we were studying flowers or butterflies, we would want to know something about their appearance, size, shape, colors, and structure. So, too, astronomers want to know the sizes, colors, and structure of stars. Such knowledge not only helps us better understand the nature of stars but also is vital in unraveling their life story. Determining a star's physical properties is not easy, however, because we cannot directly probe it. But by analyzing a star's light, astronomers can deduce many of its properties, such as its temperature, composition, radius, mass, and motions.

Temperature

Stars are extremely hot by earthly standards. The surface temperature of even cool stars is far above the temperatures at which most substances vaporize, and so using a physical probe to take a star's temperature would not succeed, even if we had the technology to send the probe to the star. So, if astronomers want to know how hot a star is, they must once more rely on indirect methods. Yet the method used is familiar. You use it yourself in judging the temperature of an electric stove burner.

An object's temperature can often be deduced from the color of its emitted light. As we saw in chapter 3, hotter objects emit more blue light than red. Thus, hot objects tend to glow blue and cooler ones red. You can see such color differences if you look carefully at stars in the night sky. Some, such as Rigel in Orion, have a blue tint. Others, such as Antares in Scorpius, are obviously reddish. Thus, even our naked eye can tell us that stars differ in temperature.

We can use color in a more precise way to measure a star's temperature with Wien's law, which we discussed in chapter 3. It states that an object's temperature, T, in Kelvin is given by the following:

T = Temperature of star
λ_m = Strongest emitted wavelength of starlight

$$T = \frac{3 \times 10^6}{\lambda_m}$$

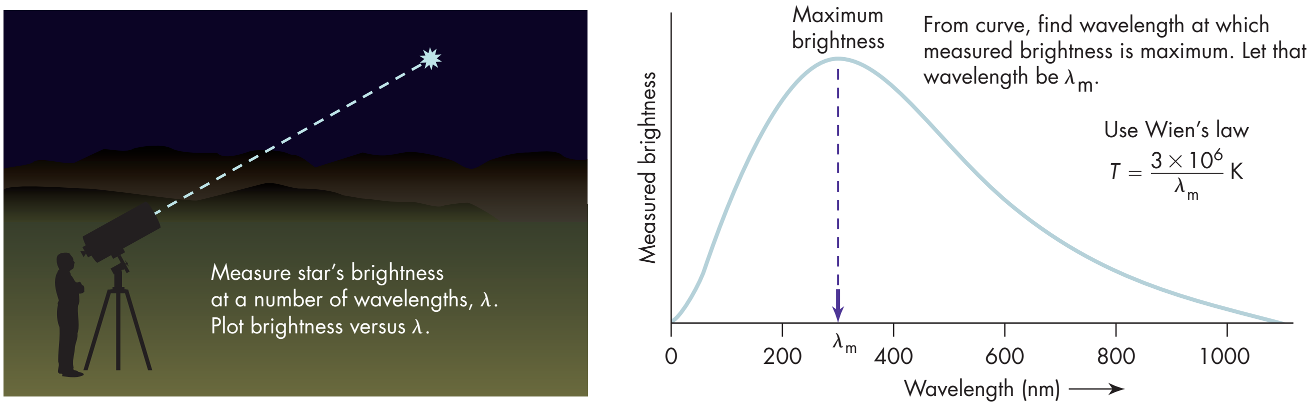


FIGURE 12.3
Measuring a star's surface temperature using Wien's law.

where λ_m is the wavelength in nanometers (nm) at which it radiates most strongly.* Thus, the longer the wavelength of the maximum emitted energy, the lower is the temperature of the radiating object.

For an illustration of how we can use Wien's law to measure a star's temperature, suppose we pick Sirius, a bright nearby star. To determine the wavelength at which it radiates most strongly, we need to measure its brightness at many different wavelengths, as illustrated in figure 12.3. We find that the strongest emission is at 300 nanometers, in the ultraviolet part of the spectrum. That is, $\lambda_m = 300$ nanometers. Inserting this value in Wien's law, we find

$$T = \frac{3 \times 10^6}{300} = 10{,}000 \text{ K}$$

For another example, we pick the red star Betelgeuse, which radiates most strongly at about 1000 nanometers. Its temperature is therefore about

$$\frac{3 \times 10^6}{1000} = \frac{3 \times 10^6}{10^3} = 3 \times 10^{6-3} = 3 \times 10^3 = 3000 \text{ K}$$

A star's temperature is only one of several properties we can deduce from its light. For example, in some cases we can also measure the star's radius, but before we discuss how, we need to discuss briefly another general property of stars and other hot objects—the amount of energy they radiate.

Luminosity

Astronomers call the amount of energy an object radiates each second its **luminosity** (abbreviated as L). An everyday example of luminosity is the wattage of a light bulb: a typical table lamp has a luminosity of 100 watts, whereas a bulb for an outdoor parking lot light may have a luminosity of 1500 watts. Stars, of course, are enormously more luminous. For example, the Sun has a luminosity of about 4×10^{26} watts, which it obtains by "burning" its hydrogen into helium. Thus, a star's luminosity measures how fast it consumes its fuel, a vital quantity for determining its lifetime. Knowing a star's luminosity is also important because from it astronomers can measure a star's radius and distance. But to understand how such measurements are made, we must first discuss a relation between how bright an object appears, how bright it really is (its luminosity), and its distance—a relation known as the *inverse-square law*.

*The subscript m on λ is to remind us that it is the wavelength at which the star's emitted energy is at a *maximum*.

The Inverse-Square Law and Measuring a Star's Luminosity

ANIMATION

The inverse-square law

The **inverse-square law** relates an object's luminosity to its distance and its apparent brightness, that is, how bright it looks to us. We all know that a light looks brighter when we are close to it than when we are farther from it. The inverse-square law puts that everyday experience into a mathematical form, describing how light energy spreads out from a source such as a star. As the light travels outward, it moves in straight lines, spreading its energy uniformly in all directions, as shown in figure 12.4A. Near the source, the light will have spread only a little, and so more enters an observer's eye, making it look brighter than if the observer were far away. That is, the farther away from us a source of light is, the less of its light enters our eyes. As a result, more-distant objects look dimmer.

This decrease in brightness with increasing distance can also be understood if you think of light as photons. Photons leaving a light source such as a star spread evenly in all directions. Now imagine a series of progressively larger spheres drawn around the source. If nothing absorbs the light, the same number of photons pass through each sphere. But because more-distant spheres are larger, the number of photons passing through any given *area* on any one sphere grows smaller as the spheres become more distant and larger, as shown in figure 12.4B. Similarly, as you move away from a light source, the photons reaching you are spread more widely—therefore fewer enter your eyes, and the source appears dimmer. Thus, a more distant source is dimmer because more of its light has spread along lines that never reach you.

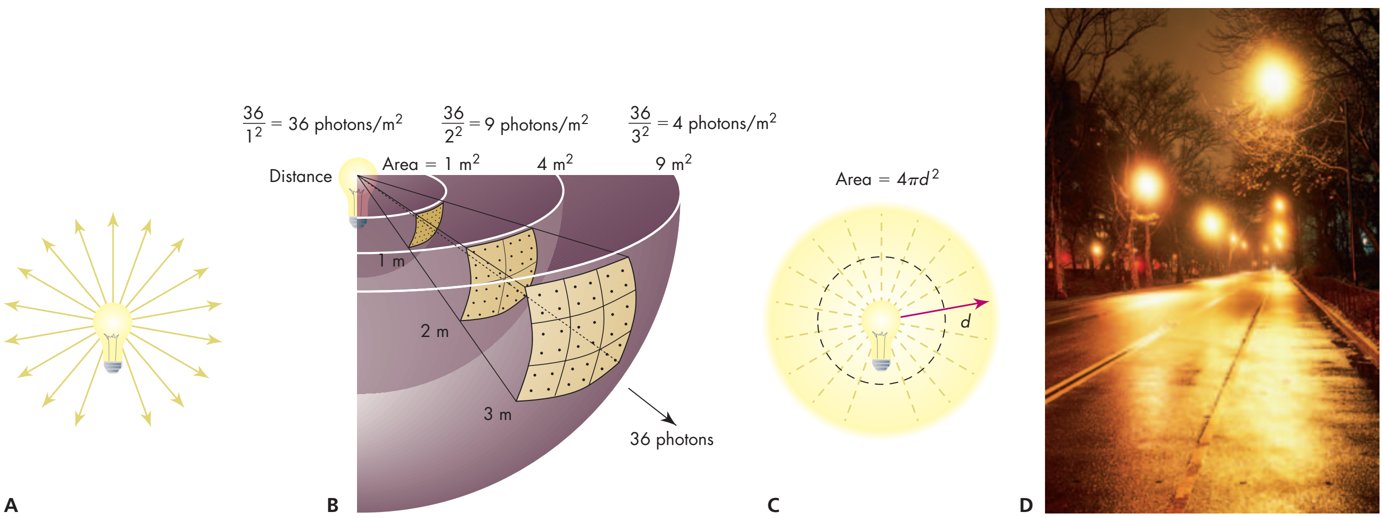


FIGURE 12.4
The inverse-square law. (A) Light spreads out from a point source in all directions. (B) As photons move out from a source, they are spread over a progressively larger area as the distance from the source increases. Thus, a *given* area intercepts fewer photons the farther it is from the source. (C) The area over which light a distance d from the source is spread is $4\pi d^2$. (D) You can see the inverse-square law at work in this picture of street lights.

Q★ Count the "photons" (dots) in a small square at each distance to verify the inverse-square law.

A★ In the inner sphere, there are 36 dots in a small square (although this is hard to see). In the next sphere there are 9 dots, and in the outermost sphere there are 4 dots. Nine is one-fourth of 36, and 4 is one-ninth of 36. Thus, the number of spots drops as the square of the distance.

We can use this argument to show the following: at a distance d from a light source, the source's luminosity, L, has spread over a sphere whose radius is d, as shown in figure 12.4C. (We use d to stand for radius here to emphasize that we are finding a distance.) Any sphere has a surface area given by $4\pi d^2$, and so the brightness we observe, B, is just

$$B = \frac{L}{4\pi d^2}$$

This relationship is called the inverse-square law because distance appears in the denominator as a square.

The inverse-square law is one of the most useful mathematical tools available to astronomers not only for measuring a star's luminosity but also its distance. To find a star's luminosity, astronomers measure its distance, d, by parallax, as described in section 12.1. Next, with a photometer, a device similar to the electric exposure meter in a camera, they measure how bright the star appears from Earth, B. Finally, with B and d known, they calculate the star's luminosity, L, using the inverse-square law. Such measurements reveal that the average star has a luminosity similar to that of our Sun. However, astronomers also find some stars that are millions of times more luminous than the Sun and other stars that are thousands of times less luminous than the Sun. Much of this range in luminosity comes from the great range in stellar radii. That is, some stars are vastly larger than others, as we will now show.

Radius

Common sense tells us that if we have two objects of the same temperature but of different sizes, the larger one will emit more energy than the smaller one. For example, three glowing charcoal briquettes in a barbecue emit more energy than just one briquette at the same temperature. Similarly,

if two stars have the same temperature but one is more luminous than the other, the brighter star must have a larger surface area, therefore a larger radius than the dimmer star.

Thus, if we know a star's temperature, we can infer its size from the amount of energy it radiates. To calculate the star's radius, however, we need a mathematical relation between luminosity, temperature, and radius—a relation known as the "Stefan-Boltzmann law."

The Stefan-Boltzmann Law

Imagine watching an electric stove burner heat up. When the burner is on low and is relatively cool, it glows dimly red and gives off only a slight amount of heat. When the burner is on high and is very hot, it glows bright yellow-orange and gives off far more heat. Thus, you can both see and feel that raising a body's temperature increases the amount of radiation it emits per second—that is, its luminosity. This familiar situation is an example of a law deduced in the late 1800s by two German scientists, Josef Stefan and Ludwig Boltzmann, who showed that the luminosity of a hot object depends on its temperature.

The Stefan-Boltzmann law, as their discovery is now called, states that an object of temperature T radiates an amount of energy each second equal to σT^4 per square meter, as shown in figure 12.5A. The quantity σ is called the "Stefan-Boltzmann constant," and its value is 5.67×10^{-8} watts $\text{m}^{-2}\,\text{K}^{-4}$. The Stefan-Boltzmann law affords a mathematical explanation of what we noticed for the electric stove burner. That is, a hotter burner has a larger T, and therefore, according to the law, the burner radiates

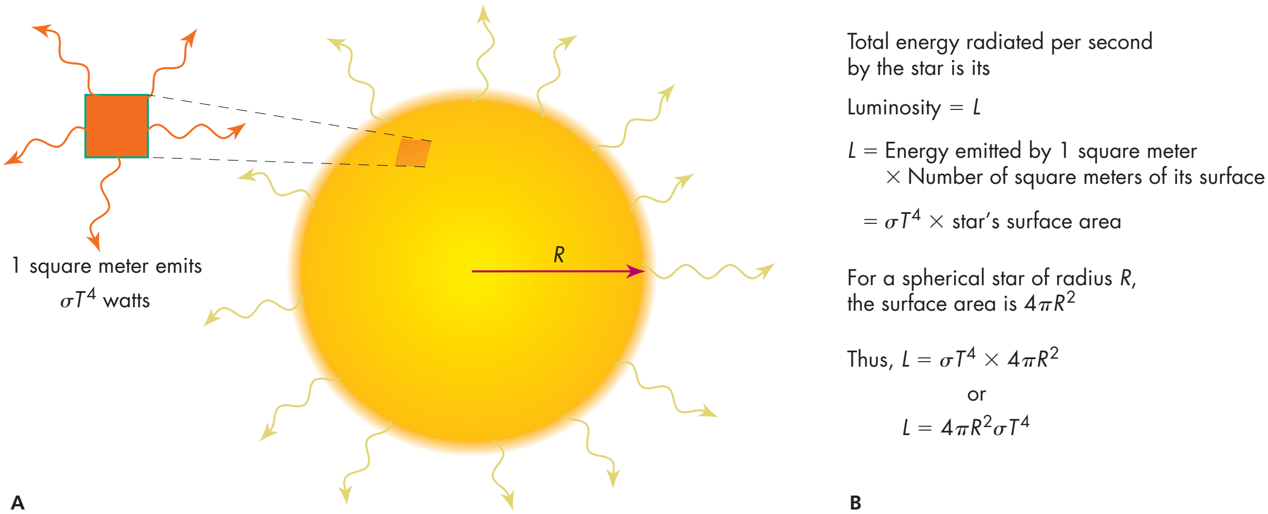


FIGURE 12.5
The Stefan-Boltzmann law can be used to find a star's radius. (A) Each part of the star's surface radiates σT^4. (B) Multiplying σT^4 by the star's surface area ($4\pi R^2$) gives its total power output—its luminosity, $L = 4\pi R^2 \sigma T^4$. To find the star's radius, we solve this equation to get $R = \sqrt{L/(4\pi\sigma T^4)}$. Finally, we can use this equation and measured values of the star's L and T to determine R.

more strongly. Similarly, the Stefan-Boltzmann law is the basis for dimmer switches on lights: changing the amount of electricity going to the lamp changes its temperature and thereby its brightness. However, the Stefan-Boltzmann law does not apply to all hot objects. For example, it does not accurately describe the radiation from hot, low-density gas, such as that in fluorescent light bulbs or interstellar clouds.

We can apply the Stefan-Boltzmann law to determine a star's luminosity as follows. According to the law, if a star has a temperature T, each square meter of its surface radiates an amount of energy per second given by σT^4. We can find the total energy the star radiates per second—its luminosity, L—by multiplying the energy radiated from 1 square meter (σT^4) by the number of square meters of its surface area (fig. 12.5B). If we assume that the star is a sphere, its area is $4\pi R^2$, where R is its radius. Its luminosity, L, is therefore

$$L = 4\pi R^2 \sigma T^4$$

That is, a star's luminosity equals its surface area times σT^4. This relation between L, R, and T, may at first appear complex, but its content is very simple: increasing either the temperature or the radius of a star makes it more luminous. Making T larger makes each square meter of the star brighter. Making R larger increases the number of square meters.

In the above expression, L is measured in watts, but the "wattage" of stars is so enormous that it is more convenient to use the Sun's luminosity as a standard unit. Likewise, it is easier to use the Sun's radius as a standard size unit rather than meters or even kilometers. If we need to convert to watts or meters, we simply remember that one solar luminosity, $L_\odot$, is 4×10^{26} watts and one solar radius, $R_\odot$, is 7×10^5 kilometers.

Because a star's luminosity depends on its radius and temperature, if we know its luminosity and temperature, we can find its radius, R. The method works because if we have a mathematical relation between three quantities (in this case L, T, and R) we can find any one of these given the other two. Thus, if we know a star's luminosity and its temperature, we can use the Stefan-Boltzmann law to solve for its radius. Extending Our Reach: "Measuring the Radius of the Star Sirius" explains how.

EXTENDING OUR REACH

MEASURING THE RADIUS OF THE STAR SIRIUS

From its color (the wavelength at which it radiates most strongly), we found that the temperature (T_s) of Sirius is about 10,000 K. From the amount of energy we receive from Sirius and its distance, which can be found by parallax, we can find that its luminosity (L_s) is about 25 $L_\odot$. We solve for the radius of Sirius (R_s) as follows. First write down the Stefan-Boltzmann law for Sirius to get $L_s = 4\pi R_s^2 \sigma T_s^4$. Then write down the same relation for the Sun: $L_\odot = 4\pi R_\odot^2 \sigma T_\odot^4$.

Next, we divide the expression for Sirius by the expression for the Sun to get

$$\frac{L_s}{L_\odot} = \frac{4\pi R_s^2 \sigma T_s^4}{4\pi R_\odot^2 \sigma T_\odot^4}$$

We can simplify this expression by canceling the identical 4π and σ factors to get

$$\frac{L_s}{L_\odot} = \frac{R_s^2 T_s^4}{R_\odot^2 T_\odot^4}$$

We next collect the R's and T's as separate factors, giving us

$$L_s/L_\odot = (R_s/R_\odot)^2 (T_s/T_\odot)^4$$

We now solve this expression for $(R_s/R_\odot)^2$ to get

$$(R_s/R_\odot)^2 = (L_s/L_\odot)(T_\odot/T_s)^4$$

Finally, we take the square root of both sides to get

$$R_s/R_\odot = (L_s/L_\odot)^{1/2} (T_\odot/T_s)^2$$

We can now evaluate $R_s/R_\odot$ by inserting the values for the luminosity and temperature of Sirius and the Sun. Notice that $T_\odot = 6000$ K, and by definition of our units for luminosity, $L_\odot = 1$. This gives us

$$\frac{R_s}{R_\odot} = \left(\frac{25}{1}\right)^{1/2} \left(\frac{6000\text{ K}}{10{,}000\text{ K}}\right)^2 = 5(0.6)^2 = 1.8$$

That is, the radius of Sirius* is 1.8 $R_\odot$, a little less than twice the Sun's radius.

*In classical times, the Persians, Greeks, and Romans sometimes called Sirius the Dog Star. It is the brightest star in the constellation Canis Major, the Big Dog, and was associated by many early people with misfortune and fevers. The ancient Greeks and Romans also blamed Sirius for the extreme heat of July and August, because at that time of year, it rose at about the same time as the Sun. They therefore believed it added its brilliance to the Sun's, making the season extra warm. The heat we receive from Sirius is, of course, negligible, but, like the ancient Greeks and Romans, we still refer to the hot days of late summer when the Dog Star shines with the Sun as the "dog days."

Astronomers can also measure a star's radius from its angular size, the technique we used in chapter 11 to measure the radius of the Sun. Unfortunately, the angular size of all stars except the Sun is extremely tiny because they are so far from the Earth, and so even in powerful telescopes stars generally look like a smeary spot of light. That smearing is caused by the blurring effects of our atmosphere and by a physical limitation of telescopes called "diffraction," which we discussed briefly in chapter 4, "Telescopes."

Diffraction limits a telescope's ability to measure tiny angular size, hopelessly blurring the light by an amount that depends on the diameter of the telescope's lens or mirror. Although such diffraction effects are less severe on bigger diameter telescopes, to measure the angular size of most stars, a truly immense diameter telescope is needed. For example, to measure the angular size of a star like the Sun if it were 50 light-years away would require a telescope 300 meters in diameter, about three times the size of a football field. To avoid the need for such enormous (and expensive) telescopes, astronomers have therefore devised an alternative way to measure the angular size of stars by using not one huge telescope but two (or more) smaller ones separated by a large distance (for example, several hundred meters). Such a device is called an "interferometer" (discussed more fully in chapter 4), and its ability to measure angular sizes is equivalent to that of a single telescope whose diameter is equal to the distance that separates the two smaller ones. (Of course, a single enormous telescope would gather more light and enable astronomers to measure the sizes of very faint stars, but it may never be possible to build such a huge instrument.) A computer then combines the information from the two telescopes to give a crude picture of the star. Such interferometric observations are still hampered by blurring caused by our atmosphere. Those effects can be partially offset by a technique called "speckle

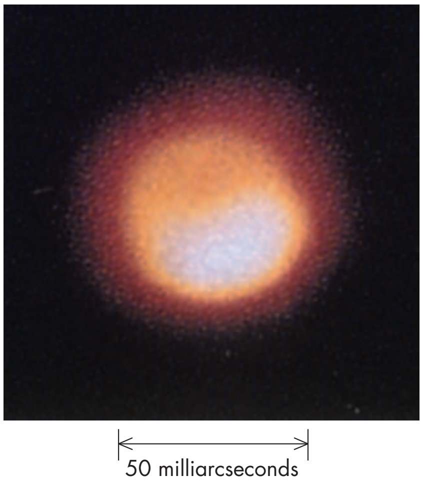


FIGURE 12.6
Image of the star Betelgeuse made by interferometry. The lack of detail in this image reflects the current difficulty in observing the disk of any star but our Sun. One milliarcsecond (mas) is approximately 2.8×10^{-7} degrees.

Use this image of Betelgeuse to measure its diameter. You will need the formula developed in the box in chapter 1, section 1.2, describing how to determine the diameter of a body from its angular size and distance. Betelgeuse is about 6×10^{15} kilometers away (200 parsecs).

An angle of 1 arc second spans 1 AU at 1 parsec (by definition of the parsec). If Betelegeuse is 200 parsecs away, 1 arc second will span 200 AU. (An object's linear size is proportional to its angular size times its distance). An angle of 50 mas (milliarcseconds) is 50/1000 = 1/20th of an arc second. Thus, Betelgeuse's diameter is 1/20 of 200 AU = 10 AU.

interferometry" in which a high-speed camera takes many very short exposures of the star's light. A computer then combines the separate short exposures into a "deblurred" image, such as that of Betelgeuse, illustrated in figure 12.6. With interferometers, astronomers can measure the radius of several dozen nearby stars and a hundred or so more-distant giant stars.

The Stefan-Boltzmann law and interferometer observations show that stars differ enormously in radius. Although most stars have approximately the same radius as the Sun has, some are hundreds of times larger, and so astronomers call them **giants.** Smaller stars (our Sun included) are called **dwarfs.**

In using the Stefan-Boltzmann law to measure a star's radius, we have seen that L can be measured either in watts or in solar units for L. Astronomers sometimes use a different set of units, however, to measure stellar brightness.

The Magnitude System

About 150 B.C., the ancient Greek astronomer Hipparchus measured the apparent brightness of stars in the night sky using units he called **magnitudes.** He designated the stars that looked brightest as magnitude 1 and the dimmest ones (the ones he could just barely see) as magnitude 6. For example, Betelgeuse, a bright red star in the constellation Orion, is magnitude 1, while the somewhat dimmer stars in the Big Dipper's handle are approximately magnitude 2. Astronomers still use this scheme to measure the brightness of astronomical objects, but they now use the term *apparent magnitude* to emphasize that they are measuring how bright a star *looks* to an observer. A star's apparent magnitude depends on its luminosity and its distance; it may look dim either because it has a small luminosity—it does not emit much energy—or because it is very far away.

Astronomers use the magnitude system for many purposes (for example, to indicate the brightness of stars on star charts), but it has several confusing properties. First, the scale is "backward" in the sense that bright stars have small magnitudes, while dim stars have large magnitudes. Moreover, modern measurements show that Hipparchus underestimated the magnitudes of the brightest stars, and so the magnitudes now assigned them are negative numbers.

Second, magnitude *differences* correspond to brightness *ratios*. That is, if we measure the brightness of a first-magnitude star and a sixth-magnitude star, the former is 100 times brighter than the latter. Thus, a *difference* of 5 magnitudes corresponds to a brightness *ratio* of 100; so, when we say a star is 5 magnitudes brighter than another, we mean it is a *factor* of 100 brighter. Each magnitude difference corresponds to a factor of 2.512 . . .—the fifth root of 100—in brightness. Thus, a first-magnitude star is 2.512 times brighter than a second-magnitude star and is 2.512×2.512, or 6.310, times brighter than a third-magnitude star. Table 12.1 lists the ratios that correspond to various differences in magnitude. For example, let us compare the apparent

TABLE 12.1

Relating Magnitudes to Brightness Ratios

Magnitude Difference	Ratio of Brightness
1	2.512:1
2	$2.512^2 = 6.31$:1
3	$2.512^3 = 15.85$:1
4	$2.512^4 = 39.8$:1
5	$2.512^5 = 100$:1
⋮	⋮
10	$2.512^{10} = 10^4$:1

TABLE 12.2

Relating Absolute Magnitude to Luminosity

Absolute Magnitude	Approximate Luminosity in Solar Units
−5	10,000
0	100
5	1
10	0.01

brightness of the planet Venus with the star Aldebaran. At its brightest, Venus has an apparent magnitude of −4.2; Aldebaran's apparent magnitude is 0.8, and so the difference in their magnitudes is $0.8 - (-4.2) = 5.0$. Therefore we see from table 12.1 that Venus is 100 times brighter to our eye than Aldebaran.

Third, astronomers often use a quantity called "absolute magnitude" to measure a star's luminosity. Recall that a star's apparent magnitude is how bright it *looks* to an observer. But the apparent magnitude of a star depends on the distance from which we observe it. The same star will have one apparent magnitude if it is near to us but a different apparent magnitude if it is far from us. Astronomers, therefore find it useful to have a way to describe a star's brightness that does not depend on its distance. One way to do that is to imagine how bright a star would look if we were to observe it from some *standard* distance. Astronomers have chosen 10 parsecs as that standard distance, and they call the apparent magnitude of a star seen from 10 pc its *absolute magnitude*. Because the absolute magnitude does not depend on distance, it is a measure of a star's true brightness. In other words, absolute magnitude is a measure of a star's luminosity. Table 12.2 illustrates how absolute magnitude is related to a star's luminosity.

12.3 SPECTRA OF STARS

A star's spectrum depicts the energy it emits at each wavelength and is perhaps the single most important thing we can know about the star. From the spectrum we can find the star's composition, temperature, luminosity, velocity in space, rotation speed, and some other properties as well. For example, under some circumstances, we may also deduce the star's mass and radius.

INTERACTIVE

Stellar spectroscopy

Figure 12.7 shows the spectra of three stars. You can easily see differences between them. Spectrum B is from a star similar to the Sun; spectrum A is from a star hotter than the Sun; and spectrum C is from a star cooler than the Sun. Spectrum A has only a few lines, but they are very strong and their spacing follows a regular pattern. Spectrum C, however, shows a welter of lines with no apparent regularity. Understanding such differences and what they tell us about a star is one of the goals of studying stellar spectra.

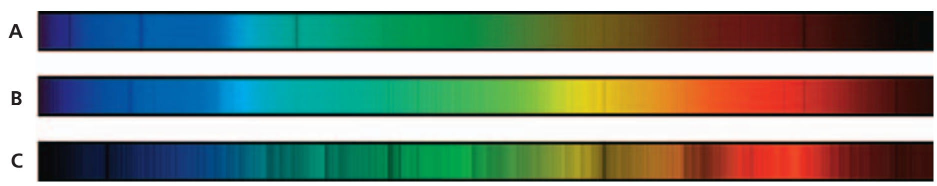


FIGURE 12.7
Spectra of three stars. A is hotter than the Sun. B is similar to the Sun. C is cooler than the Sun.

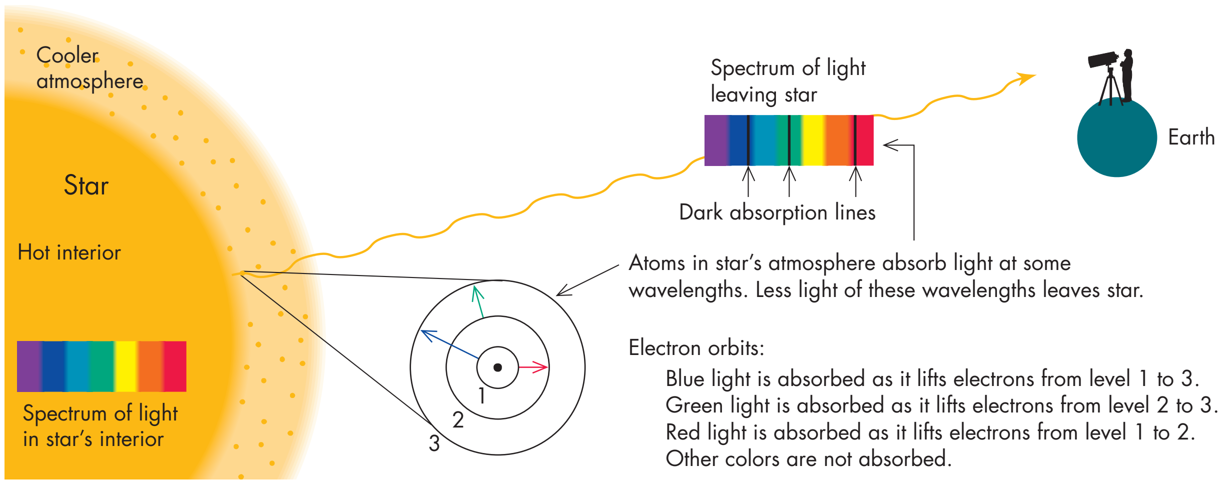


FIGURE 12.8
Formation of stellar absorption lines. Atoms in the cooler atmospheric gas absorb radiation at wavelengths corresponding to jumps between electron orbits.

Measuring a Star's Composition

As light moves from a star's core through the gas in its surface layers, atoms there absorb the radiation at some wavelengths, creating dark absorption lines in the star's spectrum, as shown in figure 12.8. Each type of atom—hydrogen, helium, calcium, and so on—absorbs at a unique set of wavelengths. For example, hydrogen absorbs at 656, 486, and 434 nanometers, in the red, blue, and violet part of the spectrum, respectively. Gaseous calcium, on the other hand, absorbs strongly at 393.3 and 396.8 nanometers, producing a strong double line in the violet. Because each atom absorbs a unique combination of wavelengths of light, each has a unique set of absorption lines. From such absorption lines, we can determine what a star is made of.

To measure a star's composition, we make a spectrum of its light and then compare the absorption lines we see with tables that list the lines made by each atom. When we find a match between an absorption line in the star and a line in the table, we can deduce that the element listed exists in the star. To find the quantity of each atom in the star—each element's abundance—we use the darkness of the absorption line, as discussed in chapter 3. Such determinations of the identity and quantity of elements present are difficult, however, because even though an element may be present in the star, the element's atoms may, because of temperature effects, be unable to absorb light and make a spectral line.

How Temperature Affects a Star's Spectrum

To see why temperature affects a star's spectrum, recall that light is absorbed when its energy matches the energy difference between two electron orbits, as we discussed in chapter 3. For an atom to absorb light, its electrons must therefore be in the proper orbit, or, more technically, energy level. An atom may be abundant in a star's atmosphere and create only very weak lines at a particular wavelength simply because the gas is either too hot or too cold, so that its electrons are in the "wrong" level to absorb light at that wavelength. Hydrogen illustrates this situation dramatically.

The absorption lines of hydrogen that we see at visible wavelengths are made by electrons orbiting in its second level. These lines are sometimes called the "Balmer lines"* to distinguish them from other hydrogen lines in the ultraviolet and infrared wavelengths. Balmer lines occur at wavelengths where light has exactly the amount

*Balmer lines are named for Johann Balmer, the scientist who first studied their pattern.

of energy needed to lift an electron from hydrogen's second energy level to the third level, or a higher level. These lines are especially important because their wavelengths are in the visible spectrum and are therefore easily observable. If the hydrogen atoms in a star have no electrons orbiting in level 2, no Balmer absorption lines will appear even though hydrogen may be the most abundant element in the star.

Hydrogen Balmer lines appear weakly or not at all if a star is either very cold or very hot. In a cool star, most hydrogen atoms have their electrons in level 1 and so they cannot absorb Balmer line radiation, which can be absorbed only if the electron is in level 2. In a hot star the atoms move faster, and when they collide, electrons may be excited ("knocked") into higher orbits: the hotter the star, the higher the orbit. Moreover, in very hot stars, the radiation is so energetic that the photons can knock electrons out of the atom, in which case the atom is said to be ionized. As a result of this excitation, proportionally more of the electrons in a very hot star will be in level 3 or higher. Only if the hydrogen has a temperature between about 8000 K and 15,000 K will enough atoms have their electron in level 2 to make strong hydrogen Balmer lines. If we are therefore to deduce correctly the abundance of elements in a star, we must correct for such temperature effects. With these corrections, we discover that virtually all stars are composed mainly of hydrogen: their composition is similar to that of our Sun—about 71% H, 27% He, and a 2% mix of the remaining elements. But despite their uniform composition, stars exhibit a wide range in the appearance of their spectra, as astronomers noted when they began to study stellar spectra.

Classification of Stellar Spectra

Stellar spectroscopy, the study and classification of spectra, was born early in the 19th century when the German scientist Joseph Fraunhofer discovered dark lines in the spectrum of the Sun. He later observed similar lines in the spectra of stars and noted that different stars had different patterns of lines. In 1866, Pietro Angelo Secchi, an Italian priest and scientist, noticed that the line patterns depended on the star's color. He assigned stars to four color classes—white, yellow, red, and deep red—and considered these classes as evidence for stellar evolution—hot stars cooling from white heat to deep red as they aged.

Technological improvements have allowed astronomers to refine Secchi's system further. The first such improvement came in 1872, when Henry Draper,* a physician and amateur astronomer, recorded spectra on photographs. On Draper's death, his widow endowed a project at Harvard to create a compilation of stellar spectra, known as the Henry Draper Catalog.

The use of photography in compiling the Draper Catalog made it possible to obtain many more spectra and with better details than could be observed visually. In fact, details were so good that E. C. Pickering, the astronomer in charge of the project, began to use letters to subdivide the spectral classes, assigning the letters *A* to *D* to Secchi's white class, *E* to *L* to the yellow class, and *M* and *N* to the red classes.

FIGURE 12.9
Annie Jump Cannon.

About 1901, Annie Jump Cannon (fig. 12.9), the astronomer actually doing the classification for the Draper Catalog, discovered that the classes fell in more orderly sequence of appearance if rearranged by temperature. She therefore reordered Secchi's classes to obtain the sequence *O*, *B*, *A*, *F*, *G*, *K*, and *M*, with *O* stars being the hottest and *M* stars being the coolest. Her work is the basis for the stellar **spectral classes** we use today.

Stellar spectral classes are based on the appearance of the spectrum. For example, *A*-type stars show extremely strong hydrogen lines, while *B*-type stars show helium and weak hydrogen lines. This scheme, however, despite its wide adoption, lacked a physical

*Draper's father was a professor of chemistry and physiology, and the first person we know of to photograph an astronomical object—the Moon.

Confirming Pages

SCIENCE AT WORK

NEW SPECTRUM CLASSES

Since the establishment of the spectrum classes *O* through *M*, astronomers, aided by new technology, have made further refinements in classifying stellar spectra. For example, much classification is now done automatically by computers that scan a star's spectrum and match it against standard spectra stored in memory. Other improvements come from more sensitive infrared detectors that allow astronomers to detect dim stars far cooler than any previously known. Some of these stars are so cool that not only do molecules form in their atmospheres but solid dust particles condense there as well. With atmospheres so different from those of ordinary stars, these extremely cool objects have spectra that are also radically different. Therefore, astronomers have devised new spectrum classes—*L* and *T*—to describe these cool, dim objects. For example, *L* stars show strong molecular lines of iron hydride and chromium hydride, while *T* stars show strong absorption lines of methane.*

*Can you think up a mnemonic for the spectral classes that includes these new classes?

FIGURE 12.10
Cecilia Payne.

basis in that it employed the appearance of the spectra rather than the physical properties of the stars that produced them. For example, astronomers at that time did not know what made the spectra of *A* and *F* stars differ. That understanding came in the 1920s from the work of the American astronomer Cecilia Payne (later Payne-Gaposhkin) (fig. 12.10), who explained why the strength of the hydrogen lines depended on the star's temperature, as we described above. Her discovery was based on work by the Indian astronomer M. Saha, who showed how to calculate the level in which an atom's electrons were most likely to be found. Saha showed that the levels occupied by electrons—a quantity crucial for interpreting the strength of spectral lines—depend on the star's temperature and density. With Saha's equation, Payne then showed mathematically the correctness and reasonableness of Cannon's order for the spectral classes. She thereby demonstrated that a star's spectral class is determined mainly by its temperature.*

Payne's theory unfortunately left astronomers with Cannon's odd nonalphabetical progression for the spectral classes, which—from hot to cold—ran *O*, *B*, *A*, *F*, *G*, *K*, and *M*. So much effort, however, had been invested in classifying stars using this system that it was easier to keep the classes as assigned with their odd order than to reclassify them (Cannon, in her life, classified some quarter million stars). As a help to remember the peculiar order, astronomers have devised mnemonics. One of the first is "Oh, be a fine girl/guy. Kiss me." A more violent version is "Oh, big and furry gorilla, kill my roommate."† A third version is "Only brilliant, artistic females generate killer mnemonics." Choose whichever appeals most, or make up your own version, to help you learn this important sequence.

Definition of the Spectral Classes

A star's spectral class is determined by the lines in its spectrum. Figure 12.11 illustrates the seven main types—*O* through *M*—and the differences in the line patterns are easy to see even without knowing the identity of the lines. However, knowing which element makes

*Payne also made clear that stars are composed mainly of hydrogen, as mentioned earlier.

†Devised by an astronomy class at Harvard, the *R* for "roommate" is one of four rare spectral classes: *R*, *N*, *S*, and *W*.

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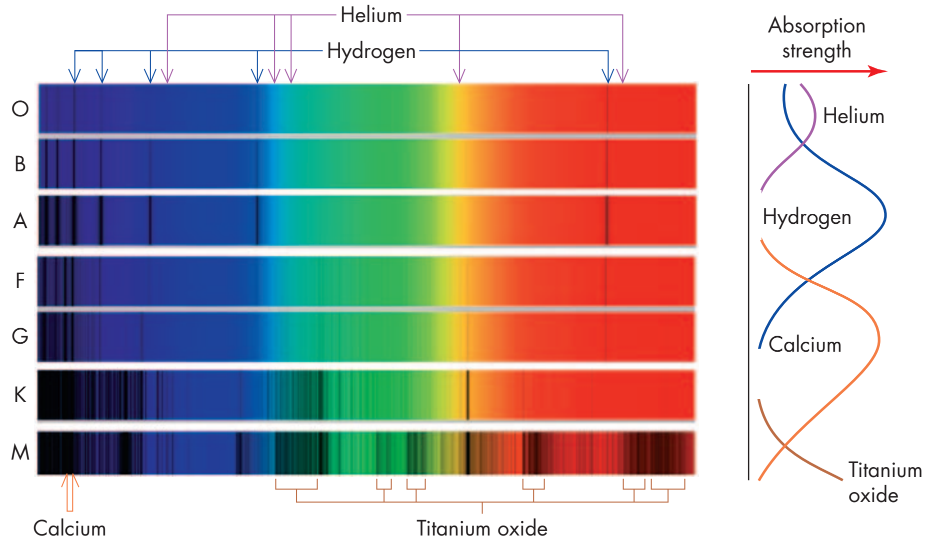


FIGURE 12.11
The stellar spectral classes. Notice how the hydrogen line at right is weak in the top spectrum and grows stronger down to *A* stars, but then essentially disappears in *K* and *M* stars. The separate curves to the right of the spectra show how the strength of the hydrogen absorption lines and those of other chemical elements change with the spectrum class.

which line can increase our understanding of the different line patterns. As mentioned earlier, almost all stars are made of the same mix of elements, but differing conditions, especially their surface temperature, result in differing spectra. For example, *O* stars have weak absorption lines of hydrogen but strong absorption lines of helium, the second most abundant element, because they are so hot. At their high temperature, an *O* star's hydrogen atoms collide so violently and are excited so much by the star's intense radiation that the electrons are stripped from most of the hydrogen, ionizing it. With its electron missing, a hydrogen atom cannot absorb light. Because most of the *O* star's hydrogen is ionized, such stars have extremely weak hydrogen absorption lines. Helium atoms are more tightly bound, however, and most retain at least one of their electrons, allowing them to absorb light. Thus, *O* stars have absorption lines of helium but only weak lines of hydrogen.

Stars of spectral class *A* have very strong hydrogen lines. Their temperature is just right to put lots of electrons into orbit 2 of hydrogen, which makes for strong Balmer lines. Balmer lines also appear in *F* stars but are weaker. *F* stars are distinguished by the multitude of lines from metals such as calcium and iron, elements that also appear strongly in *G* and *K* stars. Such elements are present in hotter stars but are ionized and generally create only very weak spectral lines under those conditions. In cooler stars, however, metal lines, particularly ionized calcium, are moderately strong, and hydrogen lines become weak.

In the very cool *K* and *M* stars, hydrogen is almost invisible because its electrons are mostly in level 1 and therefore cannot make Balmer lines. These stars have such cool atmospheres that molecules form, and as we saw in chapter 3, they produce very complex spectra. As a result, *K* and *M* stars have numerous lines from such substances as the carbon compound cyanogen (CN) and the carbon radical methylidyne (CH), as well as gaseous titanium oxide (TiO). The small graph at the right-hand side of figure 12.11 shows how the strength of the absorption lines of hydrogen, calcium, and so on, change with spectrum class. The changing strengths of these lines and others are summarized in table 12.3.

We have seen that *O* stars are hot and *M* stars are cool, but what, in fact, are their temperatures? Application of Wien's law and theoretical calculations based on Saha's law of how electrons are distributed in atomic orbits show that temperatures range from more than 25,000 K for *O* stars to less than 3500 K for *M* stars, with *A* stars being about 10,000 K and *G* stars, such as our Sun, being about 6000 K.

Because a star's spectral class is set by its temperature, its class also indicates its color. We know that hot objects are blue and cool objects are red (recall Wien's law), and so we find that *O* and *B* stars (hot classes) are blue, while *K* and *M* stars (cool classes) are red.

To distinguish finer gradations in temperature, astronomers subdivide each class by adding a numerical suffix—for example, *B*1, *B*2, . . . *B*9—with the smaller numbers indicating higher temperatures. With this system, the temperature of a *B*1 star is about 20,000 K, whereas that of a *B*5 star is about 13,500 K. Similarly, our Sun, rather than being just a *G* star, is a *G*2 star.

TABLE 12.3

Summary of Spectral Classes

Spectral Class	Temperature Range (K)	Features	Representative Star
O	Hotter than 25,000	Ionized helium, weak hydrogen	
B	11,000–25,000	Neutral helium, hydrogen stronger	Rigel
A	7500–11,000	Hydrogen very strong	Sirius
F	6000–7500	Hydrogen weaker, metals—especially ionized Ca—moderate	Canopus
G	5000–6000	Ionized Ca strong, hydrogen even weaker	The Sun
K	3500–5000	Metals strong, CH and CN molecules appearing	Aldebaran
M	2200–3500	Molecules strong, especially TiO and water	Betelgeuse
L	1300–2200	TiO disappears. Strong lines of metal hydrides, water, and reactive metals such as potassium and cesium	
T	900?–1300?	Strong lines of water and methane	

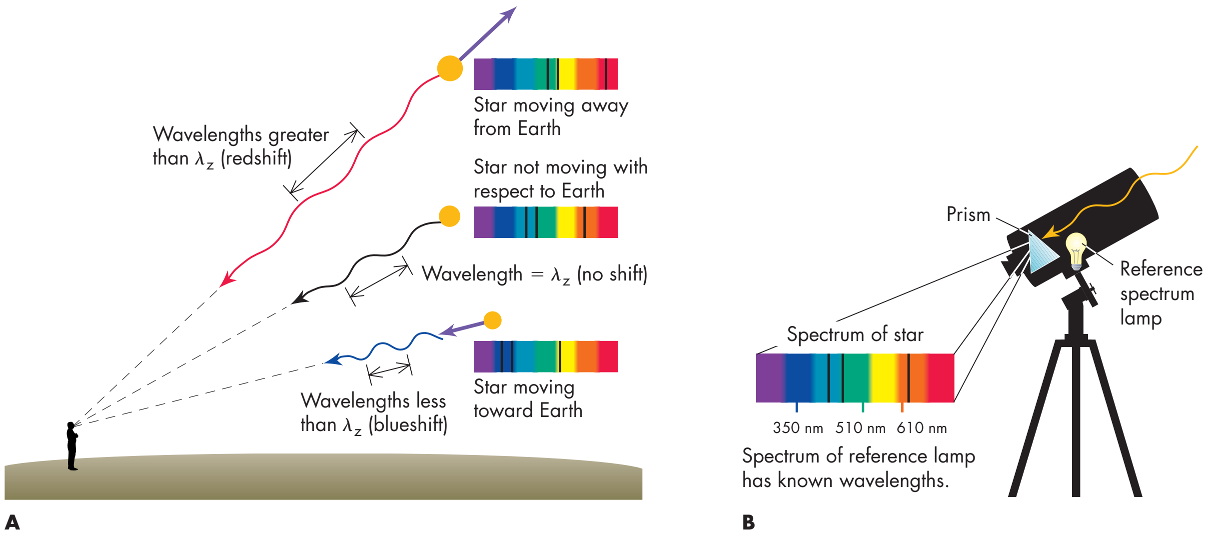


FIGURE 12.12
Measuring a star's radial velocity from its Doppler shift. (A) Spectrum lines from a star moving away from Earth are shifted to longer wavelengths (a redshift). Spectrum lines from a star approaching Earth are shifted to shorter wavelengths (a blueshift). (B) A standard lamp attached inside the telescope serves as a comparison spectrum to allow the wavelength shift of the star's light to be found.

Measuring a Star's Motion

All stars move through space and probably, like our Sun, also spin on their rotation axes. Such motions turn out to be important clues to a star's history and are vital in determining star masses and the structure of galaxies.

We find a star's motion from its spectrum using the Doppler shift, which, as we saw in chapter 3, is the alteration of the wavelength of light from a moving source. If a source moves toward us, its wavelengths are shortened, while if it moves away, they are lengthened, as illustrated in figure 12.12A. The amount of the wavelength shift depends on the source's speed along the line of sight, what astronomers call **radial velocity.** We find the shift by subtracting the wavelength the star would absorb (or emit) if it were not moving relative to the Earth, λ_z, from the new wavelength, λ. We will call that difference $\Delta\lambda$. With these definitions, the Doppler shift law states that

$$\frac{\Delta\lambda}{\lambda_z} = \frac{V}{c}$$

where V is the source's radial velocity, and c is the speed of light. That is, the change in wavelength created by motion of the star (or observer) divided by the wavelength in the absence of motion is just the radial velocity of the star divided by the speed of light. Thus, if we can measure $\Delta\lambda$, we can solve for V. The quantity $\Delta\lambda$ may turn out to be either positive or negative depending on whether the wavelengths of the source are increased or decreased. If the wavelengths are increased, the distance between the source and observer is increasing, meaning that from the observer's point of view the source is moving away. If the wavelengths are decreased, the distance between the source and observer is decreasing, and so from the observer's point of view, the source is approaching. Thus, from the observer's point of view, a positive $\Delta\lambda$ implies the source is receding, while a negative $\Delta\lambda$ implies it is approaching.

Astronomers measure the Doppler shift of a star by recording its spectrum directly beside the spectrum of a standard, nonmoving source of light attached to the telescope, as shown in figure 12.12B. They then measure the wavelength shift by comparing the spectrum of the star to the standard. From that shift, they calculate the star's radial velocity with the Doppler shift formula. Notice that a star will have a radial velocity only if its motion has some component toward or away from us. Stars moving across our line of sight, maintaining a constant distance from Earth, have no Doppler shift.

We illustrate how to measure a star's motion with the following example. Suppose we observe a star and see a strong absorption line in the red part of the spectrum with a wavelength of 656.380 nanometers. From laboratory measurements on nonmoving sources, we know that hydrogen has a strong red spectral line at a wavelength of 656.280 nanometers. Assuming these are the same lines, we infer that the wavelength is shifted by a $\Delta\lambda$ of 0.100 nanometers. The Doppler shift law tells us that

$\Delta\lambda$ = Change in wavelength due to motion of light source
λ_z = Wavelength of unmoving light source
V = Radial velocity of light source*
c = Speed of light (300,000 km/s)

$$\frac{\Delta\lambda}{\lambda_z} = \frac{V}{c}$$

Therefore,

$$\frac{V}{c} = \frac{0.100}{656.28}$$

To find the radial velocity, V, we multiply both sides by c and obtain

$$V = \frac{0.100c}{656.28} = 45.7 \text{ kilometers per second}$$

Because $\Delta\lambda$ is positive, the star is moving *away* from us.

Such measurements reveal that all stars are moving and that those near the Sun share approximately its direction and speed of revolution (about 200 kilometers per second) around the center of our galaxy, the Milky Way. Superimposed on this orbital motion, however, are small random motions of about 20 kilometers per second. Stars therefore move rather like cars on a freeway—some move a little faster and overtake slower-moving ones, which gradually fall behind. Stars are so far away, however, that their motion is imperceptible to the eye, much as a distant airplane seems to hang motionless in the sky. In fact, stellar motions across the sky are difficult to detect even in pictures of the sky taken years apart.

*We assume here that $V \ll C$.

12.4 BINARY STARS

ANIMATION

Stages in the evolution of a low-mass binary

Many stars have a special motion: they orbit around each other, held together as stellar companions by their mutual gravitational attraction, as illustrated in figure 12.13. Astronomers call such stellar pairs **binary stars** and value them greatly because they offer one of the few ways to measure stellar masses. To understand how, recall that the gravitational force between two bodies depends on their masses. The gravitational force, in turn, determines the stars' orbital motion. If that motion can be measured, we can work backwards to find the mass.

A star's mass, as we will discover in chapter 13, critically controls its existence, determining both how long it lasts and how its structure changes as it ages. Thus, if we are to understand the nature of stars, we must know their mass. Fortunately, this is not difficult to determine for stars belonging to a binary system, and such stars are relatively common. Among the hottest stars (*O* and *B* type), roughly 80% have orbiting companions. For Sun-like stars, a little over half have companions. But for the coolest stars (*M* type, in particular), the majority lack companions. Because such cool, dim stars vastly outnumber more luminous stars, single stars are the rule, although binary stars predominate among the brighter stars. Although we have described binaries as star pairs, in many cases more than two stars are involved. Some stars are triples, others quadruples, and in at least one case a six-member system is known.

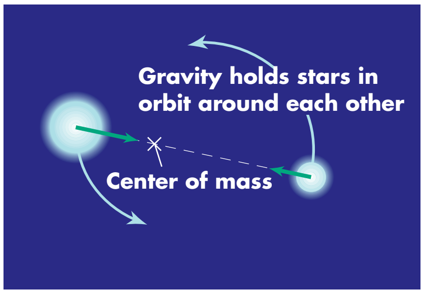


FIGURE 12.13
Two stars orbiting in a binary system, held together as a pair by their mutual gravitational attraction.

Most binary stars are only a few astronomical units apart. A few, however, are so close that they "touch," orbiting in contact with each other in a common envelope of gas. These contact binaries and other close pairs probably formed at the same time from a common parent gas cloud; that is, rather than becoming a single star surrounded by planets as our Sun did, the cloud formed a close star pair. Widely spaced binaries, on the other hand, perhaps formed when one star captured another that happened to pass nearby, but astronomers still have many unanswered questions about how binary stars form.

INTERACTIVE

Binary stars

A few binary stars are easy to see with even a small telescope. Mizar, the middle star in the handle of the Big Dipper, is a good example. When you look at this star with the naked eye, you will first see a dim star, Alcor, which is not a true binary companion but simply a star lying in the same direction as Mizar. With a telescope, however, Mizar can be seen to be two very close stars: a true binary. You can see where Alcor and Mizar are in the sky in "Looking Up at Ursa Major" on page 368.

Visual and Spectroscopic Binaries

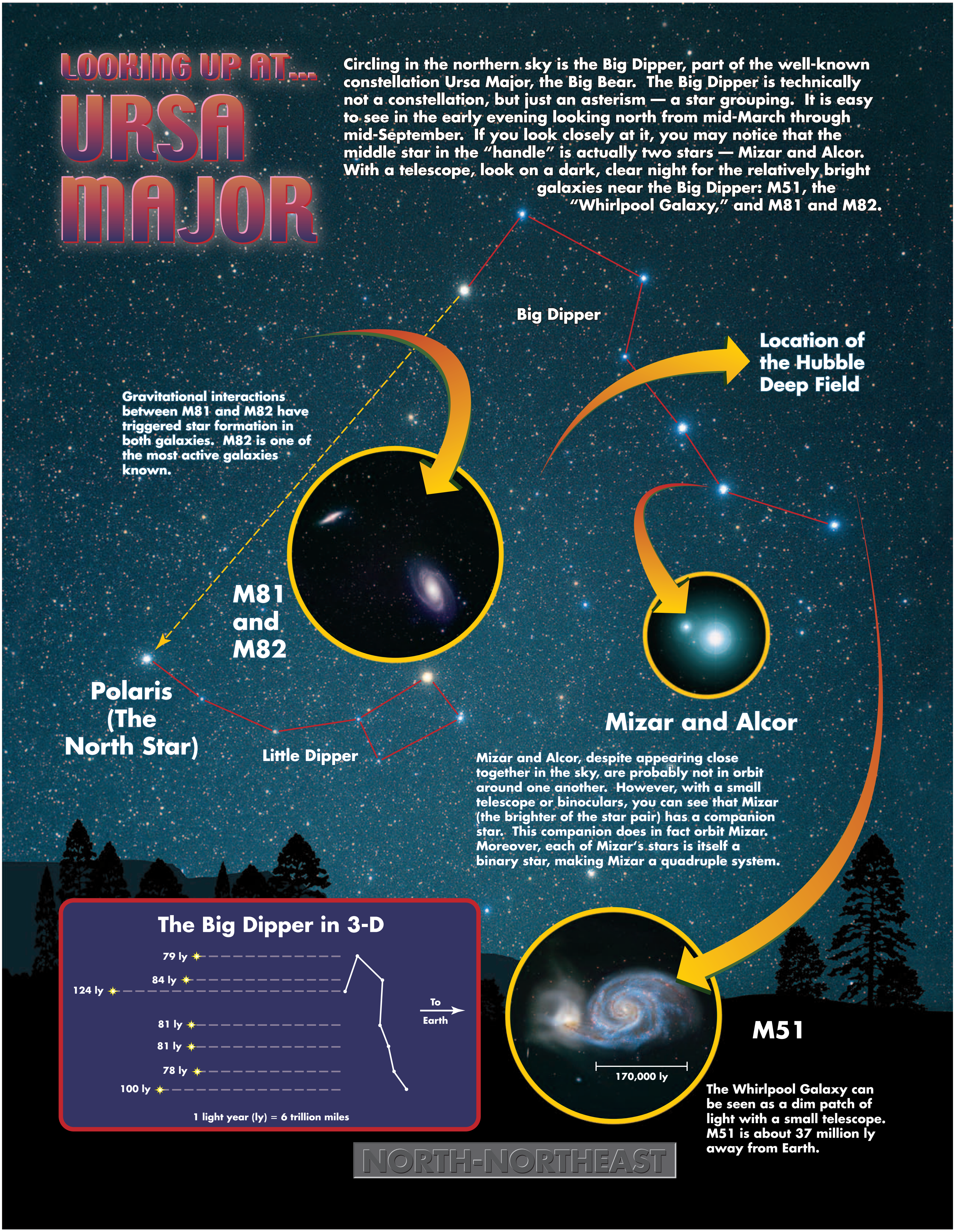
For some stars (such as Mizar) we can directly see orbital motion of one star about the other* by comparing images made several years apart. Such star pairs are called **visual binaries** because we can see two separate stars and their individual motion.

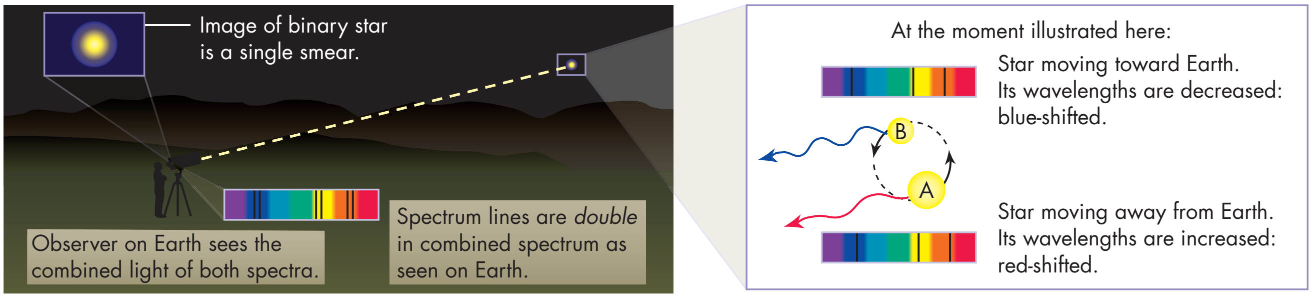
ANIMATION

Orbital motion is reflected in the shifting spectral lines of a spectroscopic binary.

Some binary stars may be so close together, however, that their light blends into a single blob that defies separation with even the most powerful telescopes. In such cases, the orbital motion cannot be seen directly, but it may nevertheless be inferred from their combined spectra. Figure 12.14A shows a typical **spectroscopic binary,** as such stars are called. As the stars orbit each other about their shared center of mass, each star alternately moves toward and then away from Earth. This motion creates a Doppler shift, and so the spectrum of the star pair shows lines that shift—figure 12.14B—first to the red as the star swings away from us and then toward the blue as it approaches. From a series of spectra, astronomers can measure the orbital speed from the Doppler shift of the star, and by observing a full cycle of the motion, they can find the orbital period. From the orbital period and speed, they can find the size of the orbit, and with the size of the orbit and the period, they can find the stars' masses using a modified form of Kepler's third law, as we will now show.

*More technically, the two stars orbit a common center of mass located on the imaginary line joining them. The exact location of the center of mass depends on the star masses—if the masses are the same, the center of mass lies exactly halfway between them.


LOOKING UP AT...
URSA MAJOR
Circling in the northern sky is the Big Dipper, part of the well-known constellation Ursa Major, the Big Bear. The Big Dipper is technically not a constellation, but just an asterism — a star grouping. It is easy to see in the early evening looking north from mid-March through mid-September. If you look closely at it, you may notice that the middle star in the "handle" is actually two stars — Mizar and Alcor. With a telescope, look on a dark, clear night for the relatively bright galaxies near the Big Dipper: M51, the "Whirlpool Galaxy," and M81 and M82.
Big Dipper
Location of the Hubble Deep Field
Gravitational interactions between M81 and M82 have triggered star formation in both galaxies. M82 is one of the most active galaxies known.
M81 and M82
Polaris (The North Star)
Little Dipper
Mizar and Alcor
Mizar and Alcor, despite appearing close together in the sky, are probably not in orbit around one another. However, with a small telescope or binoculars, you can see that Mizar (the brighter of the star pair) has a companion star. This companion does in fact orbit Mizar. Moreover, each of Mizar's stars is itself a binary star, making Mizar a quadruple system.
The Big Dipper in 3-D
79 ly
84 ly
124 ly
81 ly
81 ly
78 ly
100 ly
To Earth
1 light year (ly) = 6 trillion miles
170,000 ly
M51
The Whirlpool Galaxy can be seen as a dim patch of light with a small telescope. M51 is about 37 million ly away from Earth.
NORTH-NORTHEAST



A **B**

FIGURE 12.14
Spectroscopic binary star. (A) The two stars are generally too close to be separated by even powerful telescopes. (B) Their orbital motion creates a different Doppler shift for the light from each star. Thus, the spectrum of the stellar pair contains two sets of lines, one from each star.

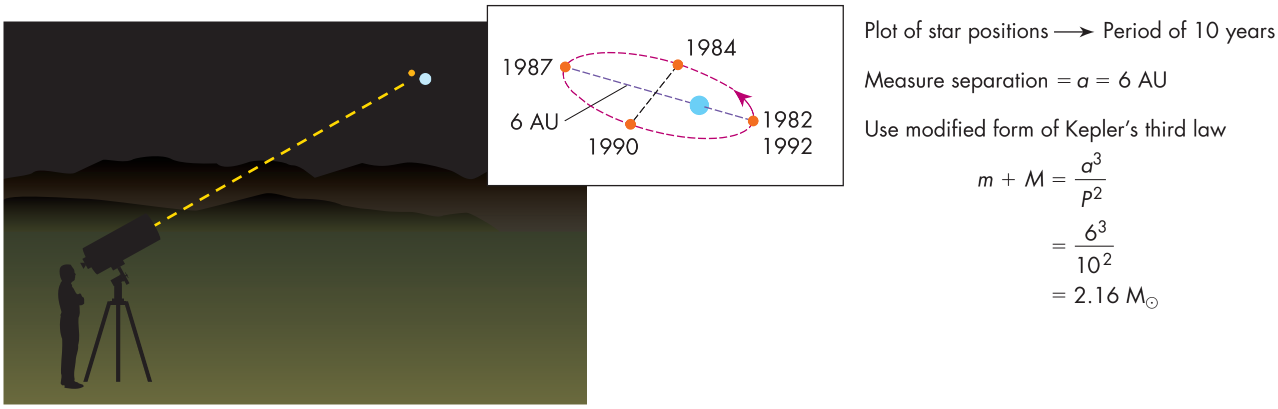


FIGURE 12.15
Measuring the combined mass of two stars in a binary system using the modified form of Kepler's third law.

Measuring Stellar Masses with Binary Stars

In the early seventeenth century, Kepler showed that the time required for a planet to orbit the Sun is related to its distance from the Sun. If P is the orbital period in years and a is the semimajor axis (half the long dimension) of the planet's orbit in AU, then $P^2 = a^3$, a relation called Kepler's third law, as we discussed in chapter 1.

Newton discovered that Kepler's third law in a generalized form applies to any two bodies in orbit around each other. If their masses are m and M and they follow an elliptical path of semimajor axis, a, relative to each other with an orbital period, P, then

$$(m + M)P^2 = a^3$$

where P is expressed in years, a in astronomical units, and m and M in solar masses. This relation is our basic tool for measuring stellar masses, as we will now describe.

To find the mass of the stars in a visual binary, astronomers first plot their orbital motion, as depicted in figure 12.15. It may take many years to observe the entire orbit, but eventually the time required for the stars to complete an orbit, P, can be determined. From the plot of the orbit and with knowledge of the star's distance from the Sun, astronomers next measure the semimajor axis, a, of the orbit of one star about the other. Suppose P = 10.0 years and a turns out to be 6.00 AU. We can then find their combined mass, $m + M$, by solving the modified form of Kepler's law, and we get (after dividing both sides by P^2)

$$m + M = \frac{a^3}{P^2}$$

Inserting the measured values for P and a, we see that

$$\begin{aligned} m + M &= \frac{6.00^3}{10.0^2} \\ &= \frac{216}{100} \\ &= 2.16 \ M_\odot \end{aligned}$$

That is, the combined mass of the stars is 2.16 times the Sun's mass.

Additional observations of the stars' orbits allow us to find their individual masses, but for simplicity we will omit the details here. From analyzing many such star pairs, astronomers have discovered that star masses fall within a fairly narrow range from about 30 to 0.1 $M_\odot$. We will discover in the next chapter why the range is so narrow, but once again, we find our Sun is about midway between the extremes.

In the above discussion, we assumed that we can actually see the two stars as separate objects, that is, that they are a visual binary. If the stars are spectroscopic binaries and cannot be distinguished as separate stars, their period and separation can be determined from a series of spectra.

Eclipsing Binary Stars

On rare occasions, the orbit of a binary star will be almost exactly edge on as seen from the Earth. Then, as the stars orbit, one will eclipse the other as it passes between its companion and the Earth. Such systems are called **eclipsing binary** stars, and if we watch such a system, its light will periodically dim. During most of the orbit, we see the combined light of both stars, but at the times of eclipse, the brightness of the system decreases as one star covers the other, producing a cycle of variation in light intensity called a "light curve." Figure 12.16A is a graph of such change in brightness over time.

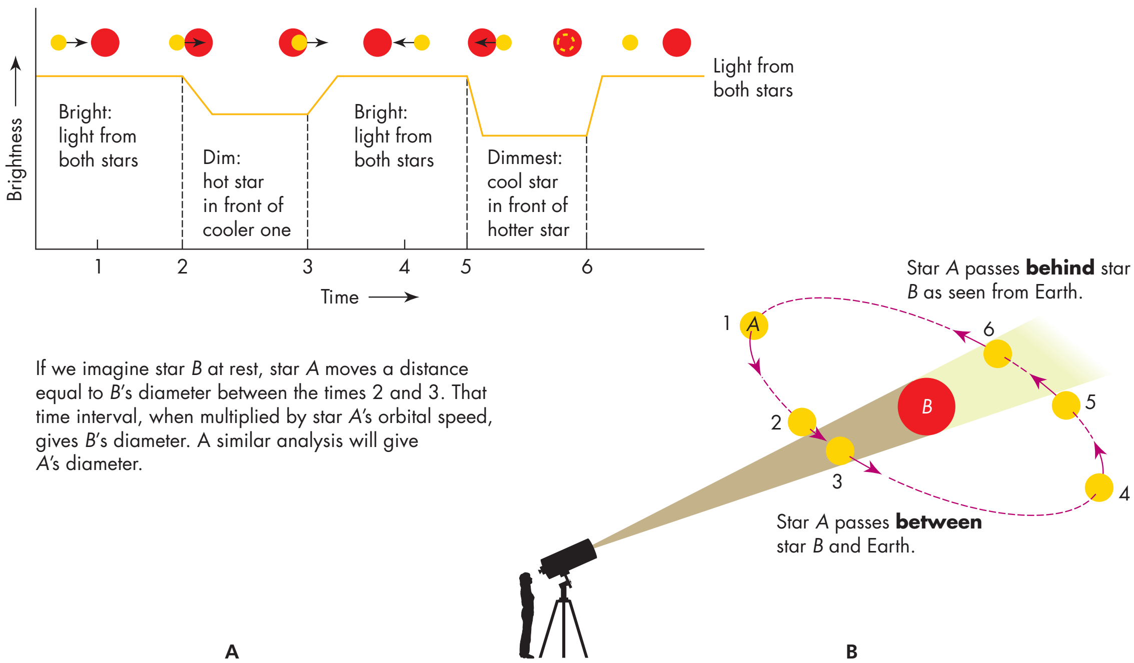


FIGURE 12.16
An eclipsing binary and its light curve. From the duration of the eclipse, the diameter of each star may be found. Note: In this illustration, the yellow star is hotter than the red star.

Eclipsing binary stars are useful to astronomers because the duration of the eclipses depends on the stars' diameter. Figure 12.16 shows why. The eclipse begins as the edge of one star first lines up with the edge of the other, as shown in the figure. The eclipse ends when the opposite edges of each star line up. During the eclipse, the covering star has moved relative to the other a distance equal to its diameter plus that of the star it has covered. From such observations and knowledge of the orbital velocities of the eclipsing stars, astronomers can calculate the diameter of each star.

Light curves of eclipsing stars can also give information about the stars' shape and the distribution of brightness across their disks. For example, "star spots"—analogous to sunspots—can be detected on some stars by this method. If, in addition, astronomers can obtain spectra of the eclipsing system, the stars' masses may be found by use of the method we described above for spectroscopic binaries. Thus, astronomers can obtain especially detailed information about eclipsing binary stars.

ANIMATION

An eclipsing binary and its light curve

12.5 SUMMARY OF STELLAR PROPERTIES

Measuring the properties of stars is not easy and is possible for only a tiny fraction of the stars known. Nevertheless, using the methods we have just described, astronomers can determine a star's distance, temperature, composition, radius, mass, and radial velocity. Some of the methods used are summarized in table 12.4. We find from such measurements that all stars have nearly the same composition of about 71% hydrogen and 27% helium, with a trace of the heavier elements. Most have surface temperatures between about 30,000 and 3000 K and masses between about 30 and 0.1 $M_{\odot}$. We will see in the next chapter that the need to balance their internal gravity is what gives stars their properties, but before we do, we will discuss how astronomers illustrate diagrammatically many of the features we have described above.

TABLE 12.4

Methods for Determining Stellar Quantities

Quantity	Method
Distance	1. Parallax (triangulation)—for nearby stars (distance less than 250 pc) 2. Standard-candle method for more distant stars
Temperature	1. Wien's law (color-temperature relation) 2. Spectral class (*O* hot; *M* cool)
Luminosity	1. Measure star's apparent brightness and distance and then calculate with inverse square law. 2. Luminosity class of spectrum (to be discussed later)
Composition	Spectral lines observed in star
Radius	1. Stefan-Boltzmann law (measure *L* and *T*, solve for *R*) 2. Interferometer (gives angular size of star; from distance and angular size, calculate radius) 3. Eclipsing binary light curve (duration of eclipse phases)
Mass	Modified form of Kepler's third law applied to binary stars
Radial velocity	Doppler shift of spectrum lines

12.6 THE H-R DIAGRAM

By the early 1900s, astronomers had learned to *measure* stellar temperatures, masses, radii, and motions but had *understood* little of how stars worked. What made an *A* star different from a *K* star? Why were some *M* stars more luminous and others much less luminous? What supplied the power to make stars shine? We have now reached a similar point in our study of stars. We know that there are many varieties of stars. But what creates that variety and what does it mean? A fundamental step in the discovery of the nature of stars remains: the Hertzsprung-Russell diagram.

In 1912, the Danish astronomer Ejnar Hertzsprung and the American astronomer Henry Norris Russell working independently found that if stars are plotted on a diagram according to their luminosity and their temperature (or, equivalently, spectral class), then most of them lie along a smooth curve. The nearly simultaneous discovery of this relation by astronomers working on opposite sides of the Atlantic Ocean is an interesting example of how conditions can be "ripe" for scientific advances.

A typical Hertzsprung-Russell diagram, now generally called an **H-R diagram** for short, is shown in figure 12.17. Stars fall in the diagram along a diagonal line, with hot luminous stars at the upper left and cool dim stars at the lower right. The Sun lies almost in the middle. The H-R diagram does *not* depict the position of stars at some location in space. It shows merely a correlation between stellar properties much as a height-weight table does for people.

INTERACTIVE

Stellar evolution and the H-R diagram

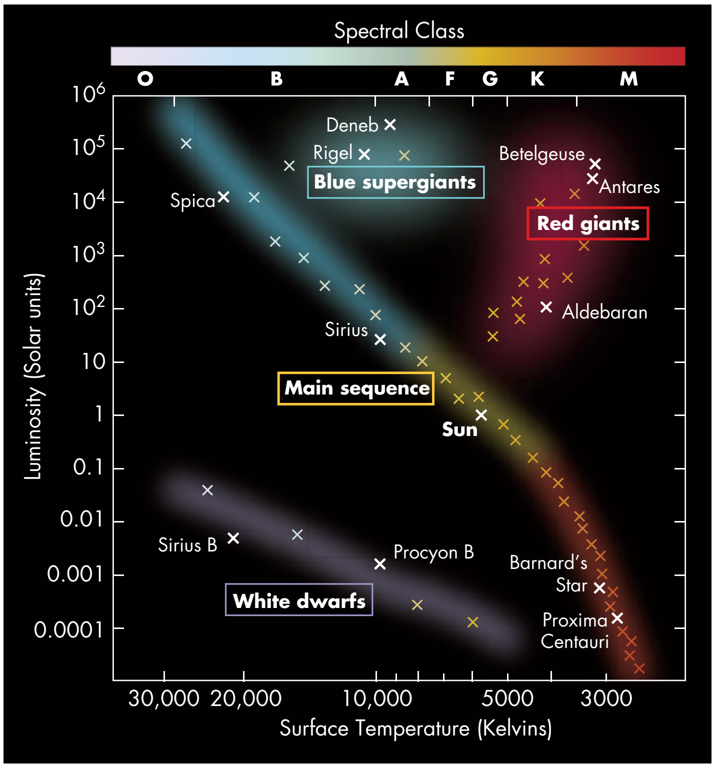


FIGURE 12.17
The H-R diagram. The long diagonal line from upper left to lower right is the main sequence. Giant stars lie above the main sequence. White dwarf stars lie below it. Notice that bright stars are at the top of the diagram and dim stars at the bottom. Also notice that hot (blue) stars are on the left and cool (red) stars are on the right.

http://www.mhhe.com/arny

Constructing the H-R Diagram

To construct an H-R diagram of a group of stars, astronomers plot each according to its temperature (or spectral class) and luminosity, as shown in figure 12.18. By tradition, they put bright stars at the top and dim stars at the bottom—hot stars on the left and cool stars on the right. Equivalently, blue stars lie to the left and red stars to the right. Notice that temperature therefore increases to the left, rather than to the right, as is conventional in graphs. The approximately straight line along which the majority of stars lies is called the **main sequence.**

When a group of stars is plotted on an H-R diagram, generally about 90% will lie along the main sequence, but some will lie off it. Of these, a few will be in the upper right, where stars are cool but very luminous, and some will be below the main sequence to the lower center, where stars are hot but dim. What makes these stars different?

Analyzing the H-R Diagram

The Stefan-Boltzmann law, described in section 12.2, gives us the answer. A star's luminosity depends on its surface area and its temperature. If two stars have the same temperature, any difference in luminosity must reflect a difference in area. But a star's area depends on its radius—a large area means a large radius. Stars that lie in the upper right of the H-R diagram have the same temperature as those on the lower main sequence, but because they are more luminous, they must be larger. In fact, they must be immensely larger because some of these bright cool stars emit thousands of times more energy than main-sequence stars of

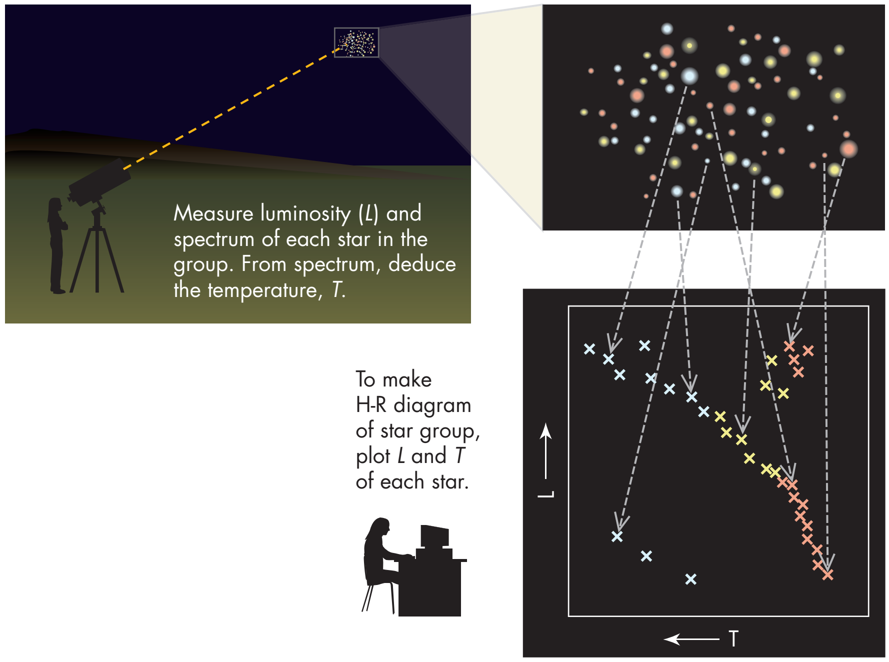


FIGURE 12.18
Constructing an H-R diagram from the spectra and luminosities of stars in a cluster.

the same temperature, implying that their surface areas are thousands of times larger. Astronomers therefore call these bright, cool stars "giants." The region in the H-R diagram populated by giants is sometimes called the "giant branch." Because many of the stars are cool and therefore red (recall that color is related to temperature), these huge cool stars are called **red giants.** For example, the bright star Aldebaran in the constellation Taurus is a red giant. Its temperature is about 4000 Kelvin and its radius is about 30 times larger than the Sun's.

A similar analysis shows that the stars lying below the main sequence must have very tiny radii if they are both hot and dim. In fact, the radius of a typical hot, dim star is roughly 100 times smaller than the Sun's, making it about the same size as the Earth. Because these stars are so hot, they glow with a white heat and are therefore called **white dwarfs.** Sirius B, a dim companion of the bright star Sirius, is a white dwarf. Its radius is about 0.008 the Sun's—a little smaller than the Earth's radius—and its surface temperature is about 27,000 Kelvin.

We can illustrate such extremes in size by drawing lines in the H-R diagram that represent a star's radius. That is, we use the Stefan-Boltzmann law, which relates the star's luminosity L, radius R, and temperature T. From this law, $L = 4\pi R^2 \sigma T^4$, we plot the values of L that result from a fixed choice of R as we vary T. Any stars lying along the plotted line must have equal radii. Figure 12.19 shows that such lines run diagonally from upper left to lower right. Notice that as we move to the upper right, we cross lines of progressively larger radius, implying that stars there are very large. That is, not only are they giants in luminosity, as we mentioned earlier, but they are also giants in radius. Comparing now the radii of stars across the H-R diagram, we see three main types: main-sequence stars, red giants, and white dwarfs.

Giants and Dwarfs

Giants, white dwarfs, and main-sequence stars differ in more than just diameter. They also differ dramatically in density.* Recall that density is a body's mass divided by its volume. For a given mass, a larger body will therefore have a lower density, and so a giant star is much less dense than a main-sequence star if their masses are similar. For example, the

*Technically, we should use the term *average density* here and in the following.

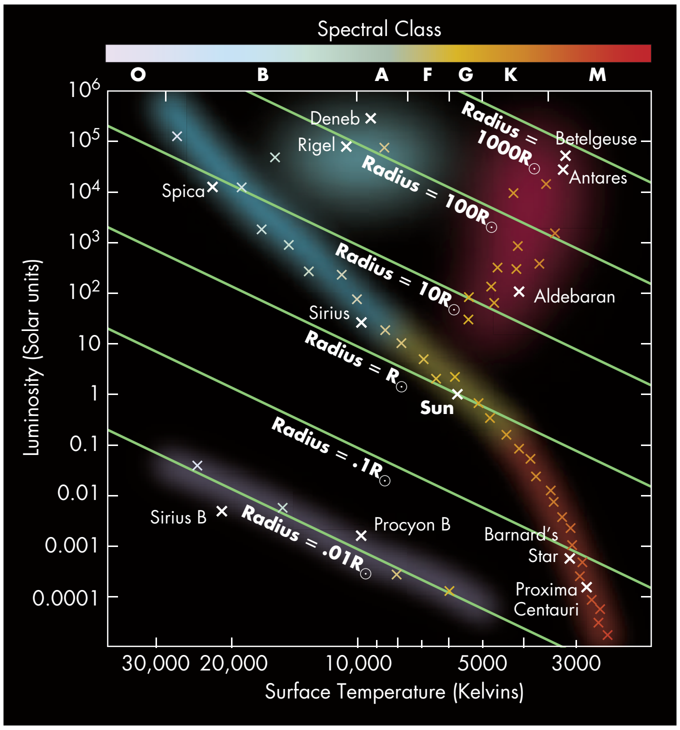


FIGURE 12.19
Lines showing where in the H-R diagram stars of a given radius will lie.

Q On the basis of these lines, what is the approximate radius of Betelgeuse? Does that agree with the number you found in figure 12.6?

A Betelgeuse lies just a little above the line for a radius of 1000 $R_\odot$, or about 7×10^8 km. We found in figure 12.6 that Betelegeuse's *diameter* is about 10 AU $= 1.5 \times 10^9$ km. Its radius will be half that number, or about 7.5×10^8 km, pretty close to what we find from reading it off the graph.

density of the Sun, a main-sequence star, is about 1 gram per cubic centimeter, while the density of a typical giant star is about 10^{-6} gram per cubic centimeter—1 million times less.

Why do stars have such disparate densities? Russell suggested about 1918 that as a star evolves, its density changes. He believed that stars began their lives as diffuse, low-density red giants. Gravity then drew the stars' atoms toward one another, compressing their matter and making stars smaller, denser, and hotter so that they turned into hot, blue, main-sequence stars. Russell, in fact, believed that gravity compressed these gaseous bodies so much that by the time they were on the main sequence, they had become liquid. Because liquids are difficult to compress, he believed that once a star reached the main sequence, it could no longer shrink. Unable to be compressed further, the star would have no source of heat and would therefore simply cool off and gradually evolve down the main sequence, growing dimmer and dimmer.

As we will discuss in the next chapter, today we know that stars evolve in the opposite direction. They begin life as main-sequence stars and turn into giants as they age. But although Russell was wrong about how stars evolve, his ideas led to many other important discoveries about stars, such as the mass-luminosity relation.

The Mass–Luminosity Relation

In 1924, the English astrophysicist A. S. Eddington discovered that the luminosity of a main-sequence star is determined by its mass. That is, main-sequence stars obey a **mass–luminosity relation** such that the larger a star's mass, the larger its luminosity is, as illustrated in figure 12.20. A consequence of this relation is that stars near the top of the main sequence, brighter stars, are more massive than stars lower down.

Eddington made his discovery by measuring the masses of stars in binary systems and determining their luminosity from their distance and apparent brightness. Moreover, he discovered that the mass–luminosity relation could be expressed by a simple formula. If $\mathscr{M}$ and $\mathscr{L}$ are given in solar units, then for many stars

$\mathscr{L}$ = Luminosity of stars in solar units
$\mathscr{M}$ = Mass of stars in solar units

$$\mathscr{L} = \mathscr{M}^3$$

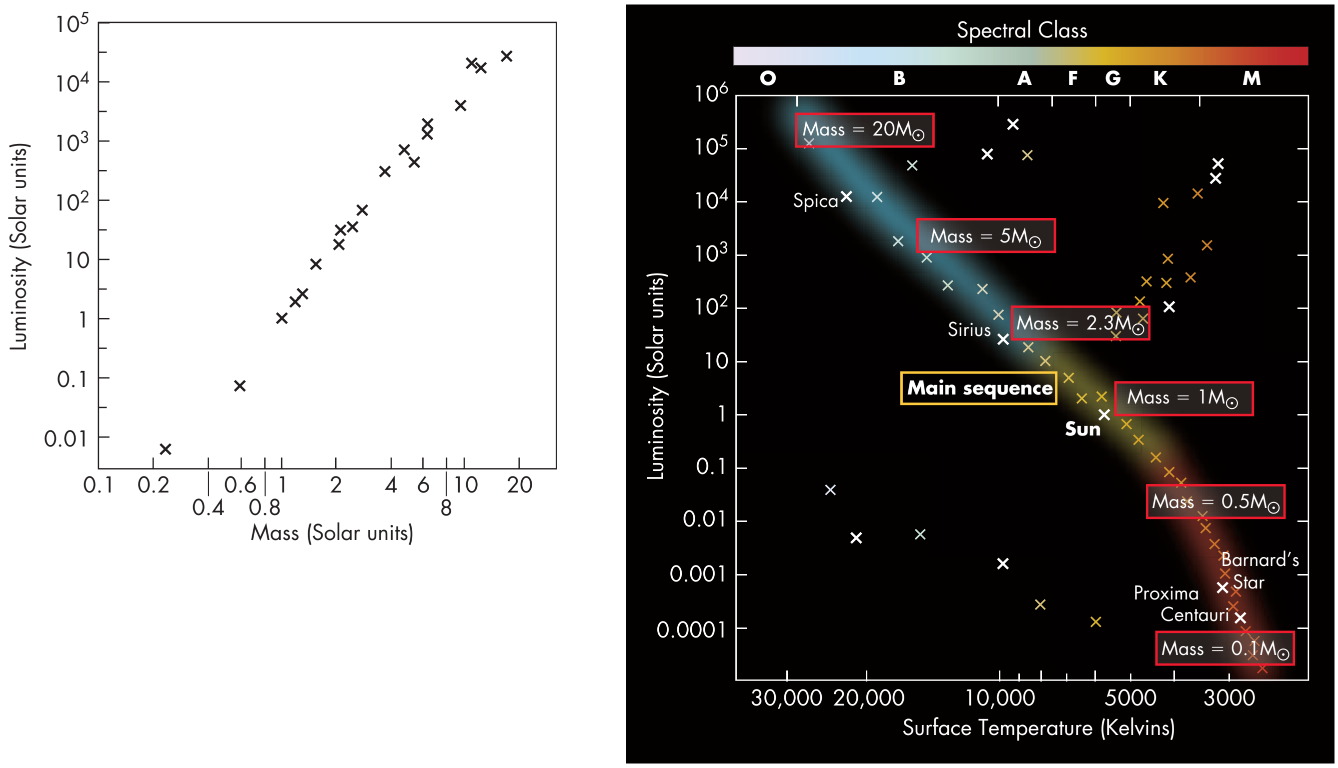
approximately. If we apply the relation to the Sun, its mass is 1 and its luminosity is 1 in these units, and the relation is clearly obeyed because $1 = 1^3$. If we consider instead a more massive star, for example, a 2 $M_\odot$ star $\mathscr{L}$ (according to the relation) should be

$$2^3\ (= 8)\ L_\odot$$

Eddington went on to show from the physics of stellar structure that just such a mass–luminosity relationship is required if a star is to avoid collapsing under its own gravitational forces and if its energy flows through its interior by radiation. In chapter 13, we will see how this relation arises and its importance in determining how long a star can live.

Luminosity Classes

Because astronomers use a star's luminosity to find its distance, radius, and life span, they have sought other ways to measure it. In the late 1800s, Antonia Maury, an early



A B

FIGURE 12.20
(A) The mass-luminosity relation shows that along the main sequence more massive stars are more luminous. The law does not work for red giants or white dwarfs. (B) On the H-R diagram, high-mass stars lie higher on the main sequence than low-mass stars.

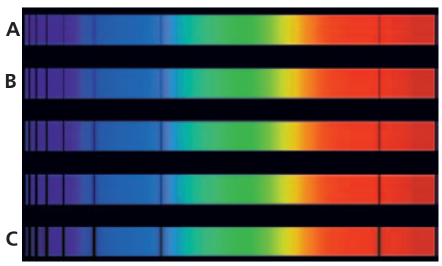


FIGURE 12.21
Spectral lines are narrow in giant (luminous) stars (A and B) and wide in main-sequence (dim) stars (C).

Suppose that the spectrum of the giant star illustrated here is of Deneb and the spectrum of the main-sequence star is of Sirius. Is that consistent with their position in the H-R diagram (see fig. 12.17)?

Deneb is actually a supergiant, but the point is that the narrowness of its spectrum lines shows that it must be much brighter than Sirius.

stellar spectroscopist at Harvard, noticed that absorption lines were extremely narrow in some stars compared with other stars of the same temperature. In the early 1900s, Hertzsprung, too, noticed that narrowness and, even before developing the H-R diagram, recognized that luminous stars had narrower lines than less luminous stars, as illustrated in figure 12.21.

The width of the absorption lines in a star's spectrum turns out to depend on the star's density: the lines are wide in high-density stars and narrow in low-density ones. The density of a star's gas is related in turn to its luminosity because a large-diameter star has—other things being equal—a large surface area, causing it to emit more light, and a large volume, giving it a lower density. A small-diameter star, on the other hand, generally has a high density because its gas is compressed into a small volume. Such a star also tends to be less luminous because its surface area is small and thus emits less light.

Using this relationship between spectral line width and luminosity, astronomers divide stars into five luminosity classes (fig. 12.22), denoted by the Roman numerals I to V. Class V stars are the dimmest, and class I stars are the brightest. In fact, class I stars are split into two classes, Ia and Ib. Table 12.5 shows the correspondence between class and luminosity, and although the scheme is not very precise, it allows astronomers to get an indication of a star's luminosity from its spectrum.

A star's luminosity class is often added to its spectral class to give a more complete description of its light. For example, our Sun is a *G*2V star, while the blue giant Rigel is a *B*8Ia star. The luminosity class is especially useful for indicating the difference in luminosity between main-sequence and giant stars of the same spectral class, such as the low-luminosity, nearby, main-sequence star 40 Eridani (*K*1V) and the high-luminosity giant star Arcturus (*K*1III).

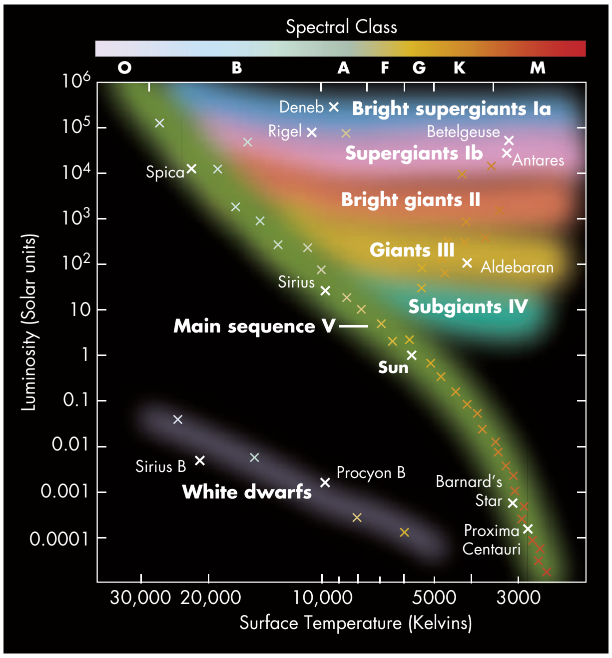


FIGURE 12.22
Stellar luminosity classes.

TABLE 12.5

Stellar Luminosity Classes

Class	Description	Example
Ia	Supergiants	Betelgeuse, Rigel
Ib	Dimmer supergiants	Polaris (the North Star)
II	Bright giants	Mintaka (a star in Orion's belt)
III	Ordinary giants	Arcturus
IV	Subgiants	Achernar (a bright southern star)
V	Main sequence	The Sun, Sirius

Summary of the H-R Diagram

The H-R diagram offers a simple, pictorial way to summarize stellar properties. Most stars lie along the main sequence, with hotter stars being more luminous. Of these, the hottest are blue, and the coolest are red. Yellow stars, such as the Sun, have an intermediate temperature.

Along the main sequence, a star's mass determines its position, as described by the mass-luminosity relation. Massive stars lie near the top of the main sequence; they are hotter and more luminous than low-mass stars. Thus, from top to bottom along the main sequence, the star masses decrease but nevertheless lie in a relatively narrow range, from about 30 to 0.1 $M_{\odot}$.

Our study of the H-R diagram shows us that most stars fall nicely into one of three regions in the diagram: main-sequence stars, giants, or white dwarfs. But the H-R diagram also clearly shows that some stars fall into yet a fourth region—that of variable stars.

12.7 VARIABLE STARS

Not all stars have a constant luminosity. Stars that change in brightness are called **variable stars.** Probably all stars vary slightly in brightness—even the Sun does in the course of its magnetic activity cycle—but relatively few change enough to be

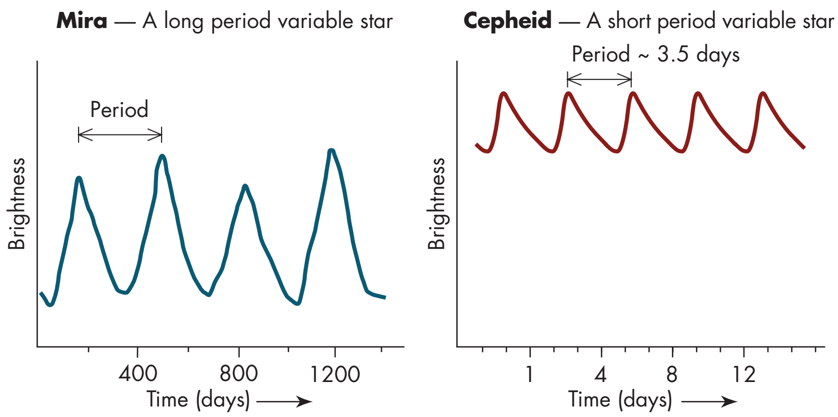


FIGURE 12.23
Two variable star light curves. Left: Sketch of the light variation of a long period Mira type variable. Right: Light variation of a Cepheid variable.

Approximately how long is the Mira-type pulsation period?

Measure the distance between the peaks and compare with the scale on the diagram. The time between peaks is about 11 months (340 days, or so).

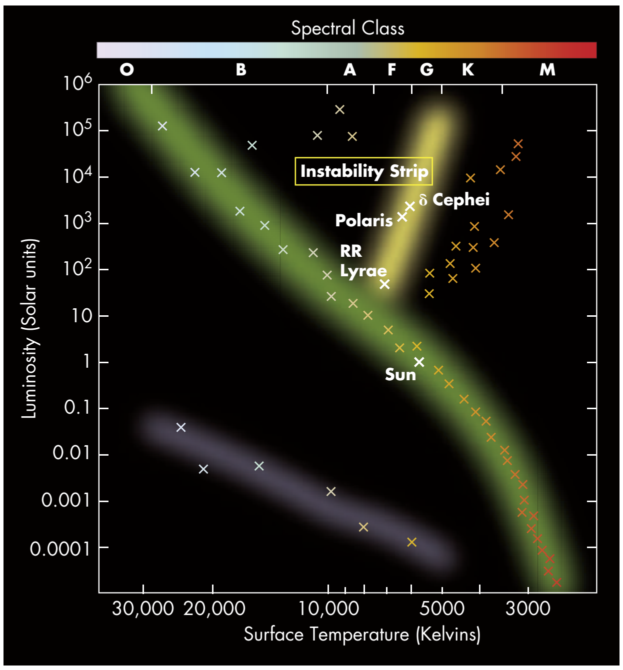


FIGURE 12.24
The instability strip in the H-R diagram. Stars that fall in this band generally pulsate.

noticeable to the naked eye. Among those that do vary, however, are several varieties of stars that are very valuable to astronomers as distance indicators, as we will discover in chapter 13. Especially valuable are the pulsating variables—stars whose radius changes, rhythmically swelling and shrinking. As the radius changes, the star's luminosity must also change (recall that a star's luminosity depends on its radius and temperature). The luminosity change, however, is complicated because, as the star pulsates, its temperature also changes, sometimes by as much as a factor of about 2. The result of both changes is that, as the star pulsates, its luminosity changes by a factor of 2 or 3. Although such large variations in luminosity are typical for many pulsating variables, ordinary stars also pulsate but much less dramatically. For them, the pulsations are more like the vibrations of a ringing bell. For example, our Sun's surface vibrates up and down in a complex pattern, as shown in figure 11.13.

The variation in luminosity and temperature is periodic for the majority of pulsating stars; that is, they repeat over some definite time interval called the **period,** as shown in figure 12.23. Astronomers call such a depiction of a star's brightness change its "light curve," and they use the shape of the curve and the time for the star to complete a cycle of brightness—the period—to classify variable stars. On the basis of such light curves, astronomers identify more than a dozen types of regularly pulsating variable stars. For example, some stars pulsate with a period of about half a day, while others take more than a year to complete a pulsation. If variable stars are plotted on the H-R diagram according to their average brightness, most lie in a narrow region called the "instability strip," illustrated in figure 12.24. In chapter 13, we will learn why pulsating stars are confined to this region.

Although many variable stars pulsate regularly, some variable stars change their brightness erratically. Such stars are called "irregular variables," and many very young and very old stars are of this category.

12.8 FINDING A STAR'S DISTANCE BY THE METHOD OF STANDARD CANDLES

In our discussion at the start of this chapter, we claimed that astronomers can determine a star's distance from knowledge of its luminosity. As an example of the method, we mentioned using the brightness of an oncoming car's headlights to estimate its distance. We saw that this method works, however, only if we know the luminosity of the light source with some assurance. That is, the luminosity of the light source is some "standard" value. For that reason, astronomers sometimes call this distance-finding scheme the *method of standard candles,* and it is perhaps their single most powerful tool for finding distances.

To measure a star's distance, an astronomer first measures its apparent brightness, B, with a photometer, as mentioned in section 12.1. Next, the star's luminosity, L, must be determined by any of the number of ways astronomers have to deduce it. For example, they may use the star's spectrum, its position in the H-R diagram, or, for a pulsating variable star, its period (as we will discuss in chapter 13) to reveal the star's luminosity. With B and L known, the astronomers can then calculate the distance to the star, d, using the inverse-square law. Unfortunately, all of the above methods to determine the luminosity have uncertainties, and so the distance calculated is uncertain, too.

In using the inverse-square law, however, it is generally much easier to find a star's distance by comparing it to the distance of a known star. For example, suppose we have two stars whose luminosity we know to be the same because they have identical spectra. Suppose one star, the nearer one, is close enough that we can find its distance by parallax to be 10 parsecs. Furthermore, suppose the star whose distance we seek looks 25 times dimmer. We find its distance by writing out the inverse-square law for both stars:

L = Luminosity of star
B = Brightness of star
d = Distance to star

$$B_{\text{near}} = \frac{L_{\text{near}}}{4\pi d_{\text{near}}^2}$$

$$B_{\text{far}} = \frac{L_{\text{far}}}{4\pi d_{\text{far}}^2}$$

Next, we divide the bottom expression into the top one to obtain after cancellation

$$\frac{B_{\text{near}}}{B_{\text{far}}} = \frac{4\pi L_{\text{near}} d_{\text{far}}^2}{4\pi L_{\text{far}} d_{\text{near}}^2} = \frac{L_{\text{near}} d_{\text{far}}^2}{L_{\text{far}} d_{\text{near}}^2}$$

If the two stars have the same luminosity, we can also cancel the Ls, leaving us with

$$\frac{B_{\text{near}}}{B_{\text{far}}} = \left(\frac{d_{\text{far}}}{d_{\text{near}}}\right)^2$$

That is, for two stars of equal luminosity, the brightness ratio is the inverse of the distance ratio squared. We can therefore find the distance ratio by taking the square root of both sides to get $d_{\text{far}}/d_{\text{near}} = \sqrt{B_{\text{near}}/B_{\text{far}}}$. For the problem we are solving, the brightness ratio is 25. Its square root is 5. Therefore, the farther star is 5 times more distant than the nearer one. Because we know that the nearer star is 10 parsecs away, the further star is 50 parsecs from us.

This method may seem to be rather roundabout, but astronomers have few other options for finding star distances. Moreover, with a little practice, such calculations can be done quickly. **To find a star's distance compared to another star of the same luminosity, find the apparent brightness ratio and take its square root.** This is then the distance ratio. If the stars have different luminosities, the calculation is slightly more difficult. In that case, we must use the full equation above.

SUMMARY

Astronomers use many techniques to measure the properties of a star (fig. 12.25). Parallax—triangulation of the star from opposite sides of the Earth's orbit—gives a distance for nearby stars, with the nearest being about 1.3 parsecs (about 4 light-years) away from the Sun. The parallax method fails beyond about 250 parsecs because the angle becomes too small to measure. Astronomers must therefore use the inverse-square law, comparing apparent brightness and luminosity to find the distance of more remote stars.

Stars have a wide range of surface temperature as measured by their color. According to Wien's law, hot stars will be blue and cool stars red. Once the temperature and luminosity of a star are known, we can calculate its radius with the Stefan-Boltzmann law.

We can find a star's composition by looking at the lines in its spectrum. The spectrum can also be used to assign stars to the spectral classes: *O*, *B*, *A*, *F*, *G*, *K*, *M*, where *O* stars are the hottest and *M* stars are the coolest.

We can find a star's mass if it is in a binary system—two stars bound together gravitationally. For such stars, the orbital period and size when combined in the modified form of Kepler's third law give the masses of the orbiting stars.

Once a star's luminosity and temperature are known, it may be plotted on an H-R diagram. In this plot, most stars lie along a diagonal line called the main sequence, which runs from hot, blue, luminous stars to cool, red, dim ones. Main-sequence stars obey a mass-luminosity relation such that high-mass stars are more luminous than low-mass ones.

A small percentage of stars fall above the main sequence, being very luminous but cool. Their high luminosity implies they must have a large diameter, and so they are called red giants. Similarly, a few stars fall below the main sequence and are hot but dim. Their low luminosity implies a very small diameter, comparable to that of the Earth. These hot, small stars are called white dwarfs.

Some stars vary in brightness and are called variable stars. Many of these variables lie in a narrow region of the H-R diagram called the "instability strip."

?What It Is.

This is a photograph of the Hyades star cluster made through a thin prism. The prism spreads light from each star into a spectrum. The brightest spectrum toward the top is that of Aldebaran, a red giant.

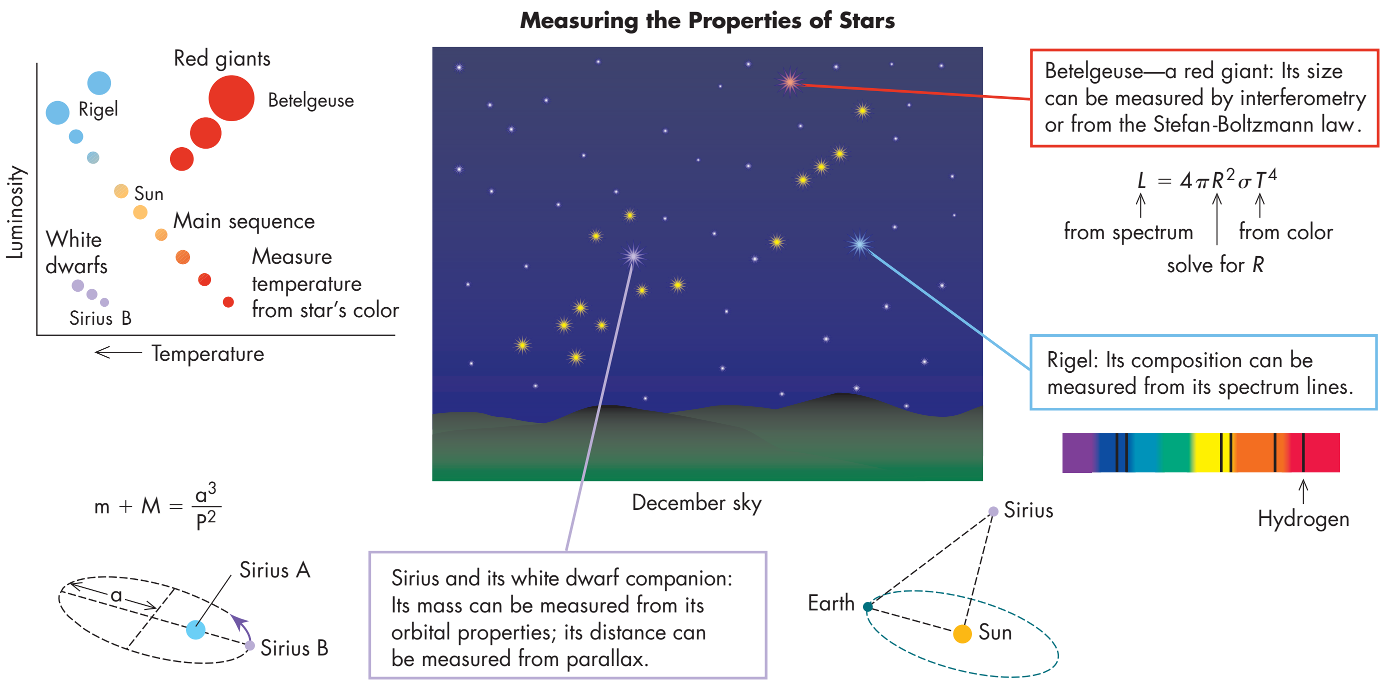


FIGURE 12.25
Summary of how astronomers measure the distance, temperature, mass, composition, and radius of stars.

QUESTIONS FOR REVIEW

1. Describe the method of distance measurement by triangulation.
2. How do astronomers triangulate a star's distance?
3. How is the parsec defined? How big is a parsec compared with a light-year?
4. How do astronomers measure a star's temperature?
5. Why do stars have dark lines in their spectra?
6. What are the stellar spectral classes? Which are hot and which cool?
7. What distinguishes the spectral classes of stars?
8. What is a binary star?
9. How do visual and spectroscopic binaries differ?
10. Why are binary stars useful to astronomers?
11. What is an eclipsing binary? What can be learned from eclipsing binaries?
12. What is the H-R diagram? What are its axes?
13. What is the main sequence?
14. How do we know that giant stars are big and dwarfs small?
15. How does mass vary along the main sequence?
16. What is the mass–luminosity relation?
17. What is a variable star?
18. What is meant by the period of a variable star?
19. Where in the H-R diagram are variable stars found?

THOUGHT QUESTIONS

1. Measure approximately the distance from where you live to the neighboring building by triangulation. Make a scale drawing, perhaps letting 1/4 inch be 1 foot, and choose a base line. Construct a scale triangle on your drawing and find how far apart the buildings are.
2. Would it be easier to measure a star's parallax from Pluto? Why?
3. Suppose a binary star's orbit is in a plane perpendicular to our line of sight. Can we measure its mass using the methods described in this chapter? Why?
4. Use the data in table 12.6 to plot an H-R diagram. Which star is a red giant? Which is a white dwarf? Note: Plotting will be much easier if you plot the logarithm of the luminosity; that is, express it in powers of 10 and use the power. For example, if the luminosity is 100, plot it as 2 for 10^2. Alternatively, use a pocket calculator as follows. Enter the luminosity in solar units and hit the "log" key. If the luminosity is 300,000, the answer you get should be 5.477 . . . .
5. Why are the hydrogen Balmer lines weaker in *O* and *B* stars than they are in *A* stars?

TABLE 12.6

Data for Plotting an H-R Diagram*

Star Name	Temperature	Spectrum	Luminosity (Solar Units)
Sun	6000	*G2*	1
Sirius	10,000	*A1*	25
Rigel	12,000	*B8*	60,000†
Betelgeuse	3600	*M2*	60,000†
40 Eridani B	15,000	*DA*‡	0.01
Barnard's Star B	3000	*M5*	0.001
Spica	20,000	*B1*	2000

*These data have been rounded off to make plotting easier.
†The luminosities of Rigel and Betelgeuse are not accurately known in part because the distance to them is uncertain.
‡Given the low luminosity of this star, what do you think the *D* in its spectral class stands for?

PROBLEMS

1. The star Rigel radiates most strongly at about 200 nanometers. How hot is it?
2. The bright southern star Alpha Centauri radiates most strongly at about 500 nanometers. What is its temperature? How does this compare to the Sun's?
3. Arcturus is about half as hot as the Sun but is about 100 times more luminous. What is its radius compared to the Sun's?
4. A stellar companion of Sirius has a temperature of about 27,000 K and a luminosity of about 10^{-2} $L_\odot$. What is its radius compared to the Sun's? What is its radius compared to the Earth's?
5. Sirius has a parallax of 0.377 arc seconds. How far away is it?
6. The parallax of Proxima Centauri is about 0.763 arc seconds. How far away is it?
7. The parallax of the red giant Betelgeuse is just barely measurable and has a value of about 0.005 arc seconds. What is its distance? Suppose the measurement is in error by + or −0.003 arc seconds. What limits can you set on its distance?
8. Two stars in a binary system have an orbital period, P, of 5 years and an orbital separation, a, of 10 AU. What is their combined mass?
9. Two stars in a binary system have an orbital period, P, of 2 years and an orbital separation, a, of 4 AU. What is their combined mass?
10. Two stars in a binary system are determined from their position on the H-R diagram and the mass-luminosity relation to have a combined mass of 8 $M_\odot$. Their orbital period, P, is 1 year. What is their orbital separation, a?
11. A line in a star's spectrum lies at 400.0 nanometers. In the laboratory, that same line lies at 400.2 nanometers. How fast is the star moving along the line of sight; that is, what is its radial velocity? Is it moving toward or away from us?

TEST YOURSELF

1. A star has a parallax of 0.04 arc seconds. What is its distance?
   (a) 4 light years
   (b) 4 parsecs
   (c) 40 parsecs
   (d) 25 parsecs
   (e) 250 parsecs
2. A star radiates most strongly at 400 nanometers. What is its surface temperature?
   (a) 400 K
   (b) 4000 K
   (c) 40,000 K
   (d) 75,000 K
   (e) 7500 K
3. Which of the following stars is hottest?
   (a) An *M* star
   (b) An *F* star
   (c) A *G* star
   (d) A *B* star
   (e) An *O* star
4. A star that is cool and very luminous must have
   (a) a very large radius.
   (b) a very small radius.
   (c) a very small mass.
   (d) a very great distance.
   (e) a very low velocity.
5. In what part of the H-R diagram do white dwarfs lie?
   (a) Upper left
   (b) Lower center
   (c) Upper right
   (d) Lower right
   (e) Just above the Sun on the main sequence

FURTHER EXPLORATIONS

Berger, David H., Jason P. Aufdenberg, and Nils H. Turner. "Resolving the Faces of Stars." *Sky and Telescope* 113 (February 2007): 40.

Cannizzo, John K., and Ronald H. Kaitchuck. "Accretion Disks in Interacting Binary Stars." *Scientific American* 266 (January 1992): 92.

Davis, John. "Measuring the Stars." *Sky and Telescope* 82 (October 1991): 361.

Hearnshaw, John B. "Origin of the Stellar Magnitude Scale." *Sky and Telescope* 84 (November 1992): 494.

Kaler, James B. *Stars and Their Spectra: An Introduction to the Stellar Spectral Sequence*. New York: Cambridge University Press, 1989. (Much of this material is also covered in a series of articles that appeared in *Sky and Telescope* 71 [February 1986]: 129 and continued at 3-month intervals, culminating with "Journeys on the H-R Diagram," *Sky and Telescope* 75 [May 1988]: 482.)

———. “Stars in the Cellar: Classes Lost and Found.” *Sky and Telescope* 100 (September 2000): 38.

Kidwell, P. A. “Three Women of American Astronomy.” *American Scientist* 78 (May/June 1990): 244.

MacRobert, Alan M. “The Spectral Types of Stars.” *Sky and Telescope* 92 (October 1996): 48.

———. “The Stellar Magnitude System.” *Sky and Telescope* 91 (January 1996): 42.

Steffey, Philip C. “The Truth about Star Colors.” *Sky and Telescope* 84 (September 1992): 266.

Terrell, Dirk. “Demon Variables (the Algol Family).” *Astronomy* 20 (October 1992): 34.

Trimble, Virginia. “White Dwarfs: The Once and Future Suns.” *Sky and Telescope* 72 (October 1986): 348.

Website

Please visit the *Explorations* website at **http://www.mhhe.com/arny** for additional online resources on these topics.

PLANETARIUM EXERCISE

1. Go outside and, with the help of a sketch pad and the star charts located at the back of your text, locate a constellation. Sketch that constellation and estimate the brightness of each star. Rank the stars in order of decreasing brightness, and use the magnitude scale to label your estimates for their brightness. Load the Starry Sky planetarium software and center on the constellation you chose. Place the cursor on each of the stars and compare your estimate of magnitude and order of brightness to the real numbers in the information boxes. Are there any discrepancies in your estimates? Any surprises?

KEY TERMS

PROJECT

With the help of the star charts, find five of the following stars: Aldebaran, Altair, Antares, Betelgeuse, Capella, Castor, Deneb, Pollux, Procyon, Regulus, Rigel, Spica, Vega. What color does each star look to you? (Try not to let its twinkling affect the color you decide on.) Do your colors match those that you infer from the star’s spectrum class as listed in appendix table 8? Which of these stars are giants? How many times bigger a radius does each have compared to the Sun’s? Using the information in the appendix, about how long ago did the light from each of the stars start on its journey to Earth? Where were you when the light started its trip? Were you even born yet?