

Simulation

LEARNING OBJECTIVES

After completing this supplement, you should be able to:

- 1 Explain what is meant by the term *simulation*.
- 2 List some of the reasons for simulation's popularity as a tool for decision making.
- 3 Explain how and why random numbers are used in simulation.
- 4 Outline the advantages and limitations of simulation.
- 5 Describe the alternatives that a manager would reject before choosing simulation as a decision-making tool.
- 6 Solve typical problems that require the use of simulation.

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Simulation is a descriptive technique in which a model of a process is developed and then experiments are conducted on the model to evaluate its behavior under various conditions. Unlike many of the other models described in the text, simulation is not an optimizing technique. It does not produce a solution per se. Instead, simulation enables decision makers to test *their* solutions on a model that reasonably duplicates a real process; simulation models enable decision makers to experiment with decision alternatives using a *what if* approach.

The use of simulation as a decision-making tool is fairly widespread, and you are undoubtedly familiar with some of the ways it is used. For instance, space engineers simulate space flight in laboratories to permit future astronauts to become accustomed to working in a weightless environment. Similarly, airline pilots often undergo extensive training with simulated landings and takeoffs before being allowed to try the real thing. Many video games are simulations, and universities use management games as a means of simulating business environments. Tire designers evaluate alternative tread designs using machines that simulate conditions that produce tire wear and handling problems.

Simulation A descriptive technique that enables a decision maker to evaluate the behavior of a model under various conditions.

INTRODUCTION

Simulation has applications across a broad spectrum of operations management problems. In some instances, the simulations are quite modest, while others are rather complex. Their usefulness in all cases depends on the degree to which decision makers are able to successfully answer their *what if* questions.

A list of operations management topics would reveal that most have simulation applications. For instance, simulation is often helpful in product design and testing, facilities layout, line balancing, job design, aggregate planning, testing of alternative inventory policies, scheduling, waiting lines, and project management. Actually, the list is quite long. The books and articles in the bibliography indicate the richness and breadth of those applications and offer the interested reader an opportunity to explore this fascinating and useful subject in more detail.

Generally, analysts use the simulation approach either because optimization techniques are unavailable or because the assumptions required by an optimizing technique are not reasonably satisfied in a given situation. Waiting-line problems are a good example of the latter reason. Although waiting-line problems are pervasive, the rather restrictive assumptions of arrival and service distributions in many cases are simply not met. Very often, analysts will then turn to simulation as a reasonable alternative for obtaining descriptive information about the system in question.

Other reasons for the popularity of simulation include

1. Many situations are too complex to permit development of a mathematical solution; the degree of simplification needed would seriously affect the results. In contrast, simulation models are often able to capture the richness of a situation without sacrificing simplicity, thereby enhancing the decision process.
2. Simulation models are fairly simple to use and understand.
3. Simulation enables the decision maker to conduct experiments on a model that will help in understanding process behavior while avoiding the risks of conducting tests on the model's real-life counterpart.
4. Extensive computer software packages make it easy to use fairly sophisticated models.
5. Simulation can be used for a wide range of situations.
6. There have been numerous successful applications of these techniques.

STEPS IN THE SIMULATION PROCESS

Regardless of the type of simulation involved, certain basic steps are used for all simulation models:

1. Identify the problem and set objectives.
2. Develop the simulation model.
3. Test the model to be sure that it reflects the system being studied.
4. Develop one or more experiments (conditions under which the model's behavior will be examined).
5. Run the simulation and evaluate the results.
6. Repeat steps 4 and 5 until you are satisfied with the results.

The first step in problem solving of any sort is to clearly declare the problem and set objectives that the solution is intended to achieve; simulation is no exception. A clear statement of the objectives can provide not only guidance for model development but also the basis for evaluation of the success or failure of a simulation. In general, the goal of a simulation study is to determine how a system will behave under certain conditions. The more specific a manager is about what he or she is looking for, the better the chances that the simulation model will be designed to accomplish that. Toward that

end, the manager must decide on the *scope* and *level of detail* of the simulation. This indicates the necessary degree of complexity of the model and the information requirements of the study.

The next step is model development. Typically, this involves deciding on the structure of the model and using a computer to carry out the simulations. (For instructional purposes, the examples and problems in this chapter are primarily manual, but in most real-life applications computers are used. This stems from the need for large numbers of runs, the complexity of simulations, and the need for record keeping of results.) Data gathering is a significant part of model development. The amount and type of data needed are a direct function of the scope and level of detail of the simulation. The data are needed for both model development and evaluation. Naturally, the model must be designed to enable evaluation of key decision alternatives.

The validation phase is closely related to model development. Its main purpose is to determine if the model adequately depicts real system performance. An analyst usually accomplishes this by comparing the results of simulation runs with known performance of the system under the same circumstances. If such a comparison cannot be made because, for example, real-life data are difficult or impossible to obtain, an alternative is to employ a test of reasonableness, in which the judgments and opinions of individuals familiar with the system or similar systems are relied on for confirmation that the results are plausible and acceptable. Still another aspect of validation is careful consideration of the assumptions of the model and the values of parameters used in testing the model. Again, the judgments and opinions of those familiar with the real-life system and those who must use the results are essential. Finally, note that model development and model validation go hand in hand: Model deficiencies uncovered during validation prompt model revisions, which lead to the need for further validation efforts and perhaps further revisions.

The fourth step in simulation is designing experiments. Experiments are the essence of a simulation; they help answer the *what if* questions posed in simulation studies. By going through the process, the manager or analyst learns about system behavior.

The fifth step is to run the simulation model. If a simulation model is deterministic and all parameters are known and constant, only a single run will be needed for each *what if* question. But if the model is probabilistic, with parameters subject to random variability, multiple runs will be needed to obtain a clear picture of the results. In this text, probabilistic simulations are the focal point of the discussion, and the comments are limited to them. Probabilistic simulation is essentially a form of random sampling, with each run representing one observation. Consequently, statistical theory can be used to determine appropriate sample sizes. In effect, the larger the degree of variability inherent in simulation results, the greater the number of simulation runs needed to achieve a reasonable level of confidence in the results as true indicators of model behavior.

The last step in the simulation process is to analyze and interpret the results. Interpretation of the results depends to a large extent on the degree to which the simulation model approximates reality; the closer the approximation, the less need to “adjust” the results. Moreover, the closer the approximation of the model to reality, the less the risk inherent in applying the results.

MONTE CARLO SIMULATION

There are many different kinds of simulation techniques. The discussion will focus on probabilistic simulation using the **Monte Carlo method**. The technique gets its name from the famous Mediterranean resort associated with games of chance. The chance element is an important aspect of Monte Carlo simulation, and this approach can be used only when a process has a **random**, or chance, component.

In the Monte Carlo method, an analyst identifies a probability distribution that reflects the random component of the system under study. Random samples taken from this probability distribution are analogous to observations made on the system itself. As the number

Monte Carlo method Probabilistic simulation technique used when a process has a random component.

Random Chance.

TABLE 18S.1
Random numbers

	1	2	3	4	5	6	7	8	9	10	11	12
1	18	20	84	29	91	73	64	33	15	67	54	07
2	25	19	05	64	26	41	20	09	88	40	73	34
3	73	57	80	35	04	52	81	48	57	61	29	35
4	12	48	37	09	17	63	94	08	28	78	51	23
5	54	92	27	61	58	39	25	16	10	46	87	17
6	96	40	65	75	16	49	03	82	38	33	51	20
7	23	55	93	83	02	19	67	89	80	44	99	72
8	31	96	81	65	60	93	75	64	26	90	18	59
9	45	49	70	10	13	79	32	17	98	63	30	05
10	01	78	32	17	24	54	52	44	28	50	27	68
11	41	62	57	31	90	18	24	15	43	85	31	97
12	22	07	38	72	69	66	14	85	36	71	41	58



of observations increases, the results of the simulation will more closely approximate the behavior of the real system, provided an appropriate model has been developed. Sampling is accomplished by the use of random numbers.

The basic steps in the process are as follows:

1. Identify a probability distribution for each random component of the system.
2. Work out an assignment so that intervals of random numbers will correspond to the probability distribution.
3. Obtain the random numbers needed for the study.
4. Interpret the results.

The random numbers used in Monte Carlo simulation can come from any source that exhibits the necessary randomness. Typically, they come from one of two sources: Large studies depend on computer-generated random numbers, and small studies commonly make use of numbers from a table of random digits like the one shown in Table 18S.1. The digits are listed in pairs for convenience, but they can be used singly, in pairs, or in whatever grouping a given problem calls for.

Two important features of the sets of random numbers are essential to simulation. One is that the numbers are uniformly distributed. This means that for any size grouping of digits (e.g., two-digit numbers), every possible outcome (e.g., 34, 89, 00) has the same probability of appearing. The second feature is that there are no discernible patterns in sequences of numbers to enable one to predict numbers further in the sequence (thus the name *random* digits). This feature holds for any sequence of numbers; the numbers can be read across rows and up or down columns.

When using the table, it is important to avoid always starting in the same spot; that would result in the same sequence of numbers each time. Various methods exist for choosing a random starting point. One can use the serial number of a dollar bill to select the row, column, and direction of number selection. Or use rolls of a die. For our purposes, the starting point will be specified in each manual example or problem so that everyone obtains the same results.

The process of simulation will become clearer as we work through some simple problems.

EXAMPLE 18S-1

The manager of a machine shop is concerned about machine breakdowns, and wants to simulate breakdowns for a 10-day period. Historical data on breakdowns over the last 100 days are given in the following table:

Number of Breakdowns	Frequency
0.....	10
1.....	30
2.....	25
3.....	20
4.....	10
5.....	5
	<u>100</u>

Simulate breakdowns for a 10-day period. Read two-digit random numbers from Table 18S.1, starting at the top of column 1 and reading down.

a. Develop cumulative frequencies for breakdowns:

- (1) Convert frequencies into relative frequencies by dividing each frequency by the sum of the frequencies. Thus, 10 becomes $10/100 = .10$, 30 becomes $30/100 = .30$, and so on.
- (2) Develop cumulative frequencies by successive summing. The results are shown in the following table:

Number of Breakdowns	Frequency	Relative Frequency	Cumulative Frequency
0	10	.10	.10
1	30	.30	.40
2	25	.25	.65
3	20	.20	.85
4	10	.10	.95
5	5	.05	1.00
	<u>100</u>	<u>1.00</u>	

b. Assign random-number intervals to correspond with the cumulative frequencies for breakdowns. (Note: Use two-digit numbers because the frequencies are given to two decimal places.) You want a 10 percent probability of obtaining the event “0 breakdowns” in our simulation. Therefore, you must designate 10 percent of the possible random numbers as corresponding to that event. There are 100 two-digit numbers, so you can assign the 10 numbers 01 to 10 to that event.

Similarly, assign the numbers 11 to 40 to “one breakdown,” 41 to 65 to “two breakdowns,” 66 to 85 to “three breakdowns,” 86 to 95 to “4 breakdowns” and 96 to 00 to five breakdowns.

Number of Breakdowns	Frequency	Probability	Cumulative Probability	Corresponding Random Numbers
0	10	.10	.10	01 to 10
1	30	.30	.40	11 to 40
2	25	.25	.65	41 to 65
3	20	.20	.85	66 to 85
4	10	.10	.95	86 to 95
5	5	.05	1.00	96 to 00
	<u>100</u>	<u>1.00</u>		

c. Obtain the random numbers from Table 18S.1, column 1, as specified in the problem:

18 25 73 12 54 96 23 31 45 01

SOLUTION

d. Convert the random numbers into numbers of breakdowns:

18 falls in the interval 11 to 40 and corresponds, therefore, to one breakdown on day 1.

25 falls in the interval 11 to 40, which corresponds to one breakdown on day 2.

73 corresponds to three breakdowns on day 3.

12 corresponds to one breakdown on day 4.

54 corresponds to two breakdowns on day 5.

96 corresponds to five breakdowns on day 6.

23 corresponds to one breakdown on day 7.

31 corresponds to one breakdown on day 8.

45 corresponds to two breakdowns on day 9.

01 corresponds to no breakdowns on day 10.

The following table summarizes these results:

Day	Random Number	Simulated Number of Breakdowns
1	18	1
2	25	1
3	73	3
4	12	1
5	54	2
6	96	5
7	23	1
8	31	1
9	45	2
10	01	0
		17

The mean number of breakdowns for this 10-period simulation is $17/10 = 1.7$ breakdowns per day. Compare this to the *expected* number of breakdowns based on the historical data:

$$0(.10) + 1(.30) + 2(.25) + 3(.20) + 4(.10) + 5(.05) = 2.05 \text{ per day}$$

Several points are worth noting:

1. This simple example is intended to illustrate the basic concept of Monte Carlo simulation. If our only goal were to estimate the average number of breakdowns, we would not have to simulate; we could base the estimate on the historical data alone.
2. The simulation should be viewed as a *sample*; it is quite likely that additional runs of 10 numbers would produce different means.
3. Because of the variability inherent in the results of small samples, it would be unwise to attempt to draw any firm conclusions from them; in an actual study, much larger sample sizes would be used.

In some cases, it is helpful to construct a flowchart that describes a simulation, especially if the simulation will involve periodic updating of system values (e.g., amount of inventory on hand), as illustrated in Example 18S-2.

The Excel spreadsheet formulation for this problem is shown below. Note that the alignment of values in columns B, C, and F must be exactly as shown.

	A	B	C	D	E	F	G
1	EXAMPLE S-1						
2							
3	Probability	Cumulative probability	Number of breakdowns		Day	Random number	Simulated demand
4	0	0	0		1	=RAND()	=VLOOKUP(F4,\$B\$4:\$C\$10,2,1)
5	0.1	0.1	1		2	=RAND()	=VLOOKUP(F4,\$B\$4:\$C\$10,2,1)
6	0.3	0.4	2		3	=RAND()	=VLOOKUP(F4,\$B\$4:\$C\$10,2,1)
7	0.25	0.65	3		4	=RAND()	=VLOOKUP(F4,\$B\$4:\$C\$10,2,1)
8	0.2	0.85	4		5	=RAND()	=VLOOKUP(F4,\$B\$4:\$C\$10,2,1)
9	0.1	0.95	5		6	=RAND()	=VLOOKUP(F4,\$B\$4:\$C\$10,2,1)
10	0.05	1			7	=RAND()	=VLOOKUP(F4,\$B\$4:\$C\$10,2,1)
11					8	=RAND()	=VLOOKUP(F4,\$B\$4:\$C\$10,2,1)
12					9	=RAND()	=VLOOKUP(F4,\$B\$4:\$C\$10,2,1)
13					10	=RAND()	=VLOOKUP(F4,\$B\$4:\$C\$10,2,1)
14						SUM	=SUM(G4:G13)
15						AVG	=AVERAGE(G4:G13)

The simulation results are shown in the following screen. Use key F4 to do a simulation or another simulation.

	A	B	C	D	E	F	G
1	EXAMPLE S-1						
2							
3	Probability	Cumulative probability	Number of breakdowns		Day	Random number	Simulated demand
4	0	0	0		1	0.22433	1
5	0.1	0.1	1		2	0.39772	1
6	0.3	0.4	2		3	0.20817	1
7	0.25	0.65	3		4	0.631	2
8	0.2	0.85	4		5	0.28439	1
9	0.1	0.95	5		6	0.16299	1
10	0.05	1			7	0.9581	5
11					8	0.66967	3
12					9	0.07676	0
13					10	0.06033	0
14						SUM	15
15						AVG	1.5

The manager of a small truck dealership wants to acquire some insight into how a proposed policy for reordering trucks might affect order frequency. Under the new policy, two trucks are to be ordered whenever the number of trucks on hand is five or fewer. Due to the nearness of the dealer to the home office, orders can be filled overnight. According to the dealer's records, the probability distribution for daily demand is

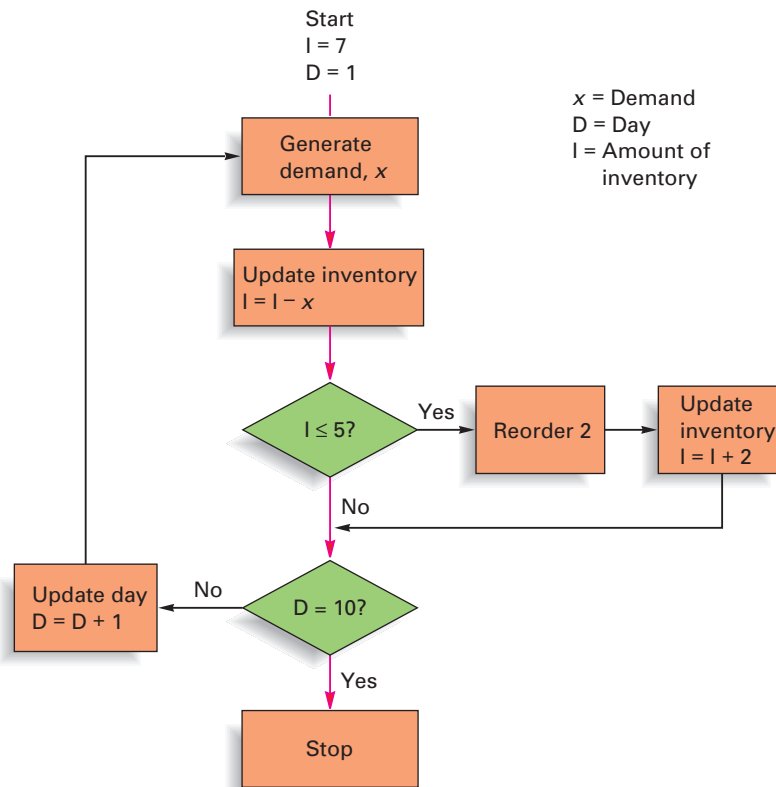
EXAMPLE 18S-2

Demand, x	$P(x)$
050
140
210

- a. Construct a flowchart that describes a 10-day simulation.
- b. Use two-digit random numbers from Table 18S.1, column 11, reading down. Assume a beginning inventory of seven trucks.

SOLUTION

a.



- b. (1) Specify random number ranges for demand:

x	$P(x)$	Cumulative $P(x)$	Ranges
0	.50	.50	01–50
1	.40	.90	51–90
2	.10	1.00	91–00

- (2) Obtain random numbers, convert to demand, update inventory accordingly, and reorder when necessary:

Day	Random Number	Demand, x	Beginning Inventory	Ending Inventory
1	54	1	7	6
2	73	1	6	5 (reorder 2; new beginning inventory = 5 + 2)
3	29	0	7	7
4	51	1	7	6
5	87	1	6	5 (reorder 2; new beginning inventory = 5 + 2)
6	51	1	7	6

7	99	2	6	4 (reorder 2; new beginning inventory = 4 + 2)
8	18	0	6	6
9	30	0	6	6
10	27	0	6	6

Simulating Theoretical Distributions

In many instances, a simulation will involve the use of theoretical distributions. Among the most frequently encountered theoretical distributions are the Poisson, normal, and exponential distributions. Being able to simulate these distributions will greatly enhance your knowledge and appreciation of simulation.

Simulation of a Poisson distribution requires the mean of the distribution. Knowledge of the mean enables one to obtain cumulative probabilities for the distribution from Appendix B, Table C; these, in turn, provide the basis for random-number assignments. You can use Table 18S.1 to obtain random numbers; you must read three-digit random numbers from Table 18S.1 to achieve correspondence. Example 18S-3 illustrates these concepts.

The number of lost-time accidents at a large plant has been determined from historical records to be two per day. Moreover, it has been determined that this accident rate can be well approximated by a Poisson distribution that has a mean of 2.0. Simulate five days of accident experience for the plant. Read random numbers from columns 1 and 2 of Table 18S.1.

EXAMPLE 18S-3

First obtain the cumulative distribution from Appendix B, Table C for a mean of 2.0, and make the range assignments:

SOLUTION

x	Cumulative Probability	Random Number Ranges
0135	001 to 135
1406	136 to 406
2677	407 to 677
3857	678 to 857
4947	858 to 947
5983	948 to 983
6995	984 to 995
7999	996 to 999
8	1.000	000

Next obtain three-digit numbers from Table 18S.1. Reading from columns 1 and 2 as instructed, you find 182, 251, 735, 124, and 549.

Finally, convert the random numbers into number of lost-time accidents using the established set of ranges. Since 182 falls in the second range, it corresponds to one accident on day 1. The second random number, 251, falls in the same range, indicating one accident on day 2. The number 735 falls between 678 and 857, which corresponds to three accidents on day 3; 124 corresponds to 0 accidents on day 4; and 549 corresponds to two accidents on day 5.

The normal distribution can be important in many problems. There are a number of ways to simulate a normal distribution, but perhaps the simplest is to use a table of normally distributed random numbers, such as Table 18S.2. The basis of the table is a normal distribution with a mean of 0 and a standard deviation of 1.00. Like all such tables, the numbers are arranged randomly, so that when they are read in any sequence they exhibit “randomness.” In order to use the table, we must have the parameters of a normal distribution (i.e., its mean and standard deviation) in mind. Numbers obtained from the random number table can then be converted to “actual” values by multiplying the standard deviation by the random number and adding this amount to the mean. That is:

$$\text{Simulated value} = \text{Mean} + \text{Random number} \times \text{Standard deviation} \quad (18S-1)$$

TABLE 18S.2

Normally distributed random numbers

	1	2	3	4	5	6	7	8	9	10
1	1.46	-0.09	-0.59	0.19	-0.52	-1.82	0.53	-1.12	1.36	-0.44
2	-1.05	0.56	-0.67	-0.16	1.39	-1.21	0.45	-0.62	-0.95	0.27
3	0.15	-0.02	0.41	-0.09	-0.61	-0.18	-0.63	-1.20	0.27	-0.50
4	0.81	1.87	0.51	0.33	-0.32	1.19	2.18	-2.17	1.10	0.70
5	0.74	-0.44	1.53	-1.76	0.01	0.47	0.07	0.22	-0.59	-1.03
6	-0.39	0.35	-0.37	-0.52	-1.14	0.27	-1.78	0.43	1.15	-0.31
7	0.45	0.23	0.26	-0.31	-0.19	-0.03	-0.92	0.38	-0.04	0.16
8	2.40	0.38	-0.15	-1.04	-0.76	1.12	-0.37	-0.71	-1.11	0.25
9	0.59	-0.70	-0.04	0.12	1.60	0.34	-0.05	-0.26	0.41	0.80
10	-0.06	0.83	-1.60	-0.28	0.28	-0.15	0.73	-0.13	-0.75	-1.49

In effect, the random number equates to a normal z value, which indicates how far a particular value is above or below the distribution mean.

EXAMPLE 18S-4

It has been determined that the time required to perform a certain task can be described by a normal distribution that has a mean of 30 minutes and a standard deviation of 4 minutes. Simulate times for three jobs using the first three values in column 1 of Table 18S.2.

SOLUTION

The first three values are 1.46, -1.05, and 0.15. The simulated values are

- For 1.46: $30 + 1.46(4) = 35.84$ minutes.
- For -1.05: $30 - 1.05(4) = 25.80$ minutes.
- For 0.15: $30 + 0.15(4) = 30.60$ minutes.

It is important to recognize that Example 18S-4 involves a continuous variable, whereas the previous examples involved discrete variables. (Remember that discrete variables typically take on only integer values, whereas continuous variables can take on integer and noninteger values.) Whenever possible, a model of a continuous variable should be able to simulate noninteger values as well as integer values.

Another continuous type of distribution we can consider is the uniform distribution, in which values may occur anywhere over a continuous range between two extremes, a and b , as illustrated in Figure 18S.1.

$$\text{Simulated value} = a + (b - a) (\text{Random number as a percentage}) \tag{18S-2}$$

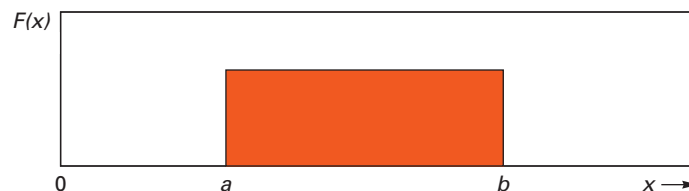
Converting the random number to a percentage simply involves placing a decimal point to the left of the number. For example, 77 becomes .77.

Example 18S-5 illustrates a simulation involving a uniform distribution.

A third continuous distribution is the exponential distribution. We will concern ourselves with simulating values of negative exponential distributions, as portrayed in Figure 18S.2.

FIGURE 18S.1

A uniform distribution



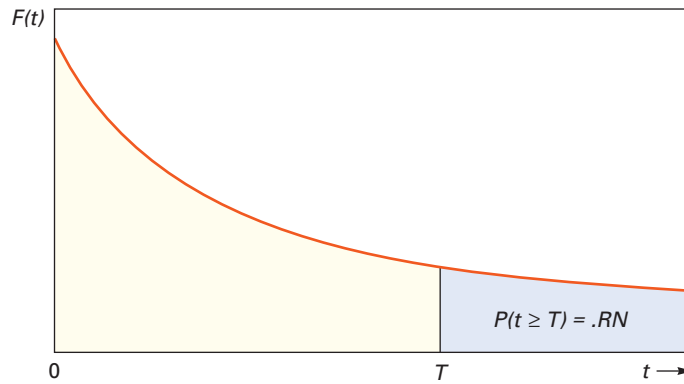


FIGURE 18S.2
A negative exponential distribution

Job times vary uniformly between 10 and 15 minutes. Use Table 18S.1 to simulate job times for four jobs. Read numbers from column 9, going down.

EXAMPLE 18S-5

$$a = 10 \text{ minutes}, \quad b = 15 \text{ minutes}, \quad b - a = 5 \text{ minutes}$$

- a. Obtain the random numbers: 15, 88, 57, and 28.
- b. Convert to simulated values:

SOLUTION

Random Number	Computation	Simulated Value (minutes)
15	$10 + 5(.15) =$	10.75
88	$10 + 5(.88) =$	14.40
57	$10 + 5(.57) =$	12.85
28	$10 + 5(.28) =$	11.40

With a negative exponential distribution, the probability is fairly high that the random variable will assume a value close to zero. Moreover, the probability decreases as the specified value of the random variable increases. The probability that a random variable will take on a value greater than some specified value T , given that the variable can be described by an exponential distribution with a mean equal to $1/\lambda$, is given by the formula

$$P(t \geq T) = e^{-\lambda t} \tag{18S-3}$$

To simulate exponential values, we obtain a random number, place a decimal point to the left of it, set this equal to the probability $P(T)$, and solve Formula 18S-3 for t . The result is a simulated value from an exponential distribution with a mean of $1/\lambda$.

We can obtain an expression for t by taking the natural logarithm of both sides of the equation. Thus, with $P(T) = .RN$ (for *random number*), we have

$$\ln(.RN) = \ln(e^{-\lambda t})$$

The natural logarithm of a power of e is equal to the power itself, so

$$\ln(.RN) = \ln(e^{-\lambda t}) = -\lambda t$$

Then

$$t = -\frac{1}{\lambda} \ln(.RN) \tag{18S-4}$$

This concept is illustrated in Figure 18S.2.

Values of random numbers can be obtained using Table 18S.1, as demonstrated in the following example.

EXAMPLE 18S-6

Times between breakdowns of a certain type of equipment can be described by an exponential distribution with a mean of five hours. Simulate the time between two breakdowns. Read two-digit random numbers from column 3 of Table 18S.1.

SOLUTION

The mean, $1/\lambda$, is 5 hours. The random numbers are 84 and 05. Using Formula 18S-4, the simulated times are

$$\text{For 84: } t = -5[\ln(.84)] = -5[-0.1744] = 0.872 \text{ hour}$$

$$\text{For 05: } t = -5[\ln(.05)] = -5[-2.9957] = 14.979 \text{ hours}$$

Note that the smaller the value of the random number, the larger the simulated value of t .

COMPUTER SIMULATION

Although the emphasis in this supplement has been on manual simulation in order to convey the main concepts, most real-life simulations involve the use of a computer. Computers offer relatively quick and easy means of obtaining results. Many real-life applications involve fairly complex simulation models that have substantial record-keeping requirements for which computers offer a practical solution.

Over the years, programmers have developed a number of simulation languages that make the task of writing simulation programs much simpler. Some of the general-purpose languages are SIMSCRIPT II.5, GPSS/H, GPSS/PC, and RESQ. In addition, a number of other simulation packages are available, some of which have very narrow focuses that relate to queuing or network problems. Most of the simulation packages have certain features in common. For example, they generally provide for random number generation from a variety of statistical distributions, as well as collection and tabulation of simulation results and time keeping.

It should be noted that some managers prefer to write their own simulations or have a member of their staff do so, using spreadsheet software such as MS Excel rather than a simulation language, as illustrated on page 855. In cases where simulation is used infrequently, it can be more practical to use that approach rather than go through the time and effort required to use a specialty language. For simple problems, the slight inefficiency related to the use of a standard language is not usually an important consideration.

ADVANTAGES AND LIMITATIONS OF USING SIMULATIONS

Among the main advantages of simulation are these:

1. It lends itself to problems that are difficult or impossible to solve mathematically.
2. It permits an analyst to experiment with system behavior while avoiding possible risks inherent in experimenting with the actual system.
3. It compresses time so that managers can quickly discern long-term effects.
4. It can serve as a valuable tool for training decision makers by building up their experience and understanding of system behavior under a wide range of conditions.

Certain limitations are also associated with simulation. Chief among these are the following:

1. Simulation does not produce an optimum solution; it merely indicates an *approximate* behavior for a given set of inputs. There are two reasons for this:
 - a. By design, there is inherent randomness (i.e., random numbers) in simulation.
 - b. Simulations are based on models, and models are only approximations of reality.
2. For large-scale simulation, it can require considerable effort to develop a suitable model as well as considerable computer time to obtain simulations.

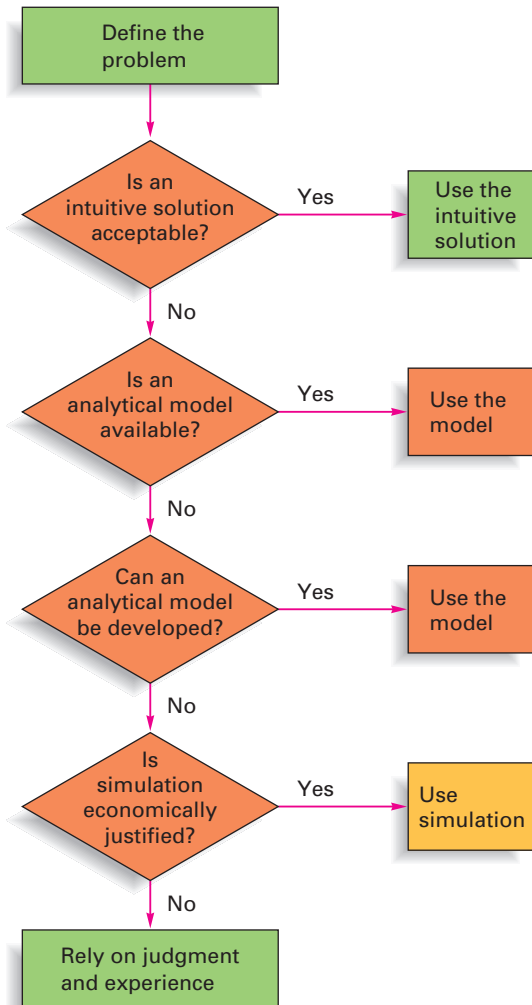


FIGURE 18S.3
The decision-making process

Because simulation produces an approximate answer rather than an exact solution, and because of the cost of running a simulation study, simulation is not usually the first choice of a decision maker. Instead, depending on the complexity of the situation, intuitive or analytical methods should first be investigated. In simple cases, an intuitive solution very often is acceptable. In more complex cases, an analytical solution is preferable, assuming an appropriate technique is available. If not, it may be possible to develop an analytical model that could be used to generate a solution. If these measures do not suffice, simulation becomes the next logical possibility. Of course, if that is not economically justifiable, the decision maker will have to rely on judgment and experience; in effect, after reevaluating all of the alternatives, the decision maker may revert to an intuitive solution, even though initially that approach did not appear acceptable. Figure 18S.3 outlines this process.

SOLVED PROBLEMS

Problem 1

The number of customers who arrive at a transmission repair shop can be described by a Poisson distribution that has a mean of three per hour. Assuming the distribution holds for an entire eight-hour day, simulate customer arrivals for the first four hours of a day. Read random numbers from Table 18S.1, columns 4 and 5, going down.

Solution

- a. Obtain cumulative probabilities of the Poisson distribution from Appendix B, Table C for the mean specified. (Those values are given below for convenience.) Determine the random number ranges.

x	Cumulative Probability
0050
1199
2423
3647
4815
5916
6966
7988
8996
9999
10	1.000

- b. Obtain the random numbers: 299, 642, 350, and 091. (*Note:* Three-digit numbers are needed because the probabilities are given to three decimal places.)
- c. Convert the random numbers to numbers of arrivals. Note where each number falls in the random number range list. For instance, 299 falls between 199 and 423. Interpret this to mean two customers arrive in the first hour. Similarly, 642 is interpreted to mean three customers arrive in the second hour, 350 implies two customers in the third hour, and 091 implies one customer in the fourth hour.

In sum, the number of customers per hour for the four-hour simulation is

Hour	Number of Arrivals
1	2
2	3
3	2
4	1

Problem 2

Jobs arrive at a workstation at fixed intervals of one hour. Processing time is approximately normal and has a mean of 56 minutes per job and a standard deviation of 4 minutes per job. Using the fifth row of the table of normally distributed random numbers (Table 18S.2), simulate the processing times for four jobs, and determine the amount of operator idle time and job waiting time. Assume the first job arrives at Time = 0.

Solution

- a. Obtain the random numbers from the table: 0.74, -0.44, 1.53, and -1.76.
- b. Convert the random numbers to simulated processing times:

Random Number	Computation	Simulated Time
0.74	$56 + 4(0.74) =$	58.96
-0.44	$56 + 4(-0.44) =$	54.24
1.53	$56 + 4(1.53) =$	62.12
-1.76	$56 + 4(-1.76) =$	48.96

Note that three of the times are less than the interarrival times for the jobs, meaning the operator may be idle after those three jobs. One time exceeds the one-hour interval, so the next job must wait, and possibly the job following it if the waiting plus processing time exceeds 60 minutes.

c. Compute waiting and idle times:

Job Number	Arrives at	Processing Time, t (minutes)	$60 - t$ Operator Idle (minutes)	$t - 60$ Next Job Waits (minutes)
1	0	58.96	1.04	—
2	60	54.24	5.76	—
3	120	62.12	—	2.12
4	180	48.96	8.92*	—
			15.72	2.12

* $60 - 2.12 - 48.96 = 8.92$.

The time between mechanics' requests for tools in a large plant is normally distributed with a mean of 10 minutes and a standard deviation of 1 minute. The time to fill requests is also normal with a mean of 9 minutes per request and a standard deviation of 1 minute. Mechanics' waiting time represents a cost of \$2 per minute, and servers represent a cost of \$1 per minute. Simulate arrivals for the first nine mechanic requests and their service times, and determine the mechanics' waiting time, assuming one server. Would it be economical to add another server? Explain. Use Table 18S.2, column 8 for requests and column 9 for service.

Problem 3

a. Obtain random numbers and convert to times [see columns (a) and (b) in the following table for requests and columns (f) and (g) for service].

Solution

CUSTOMER ARRIVALS				SERVICE			
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Random Number	Time between Arrival	Cumulative Arrival Time	($e - c$) Customer Waiting Time	Service Begins	Random Number	Service Time	($e + g$) Service Ends
-1.12	8.88	8.88	0.00	8.88	1.36	10.36	19.24
-0.62	9.38	18.26	0.98	19.24	-0.95	8.05	27.29
-1.20	8.80	27.06	0.23	27.29	0.27	9.27	36.56
-2.17	7.83	34.89	1.67	36.56	1.10	10.10	46.66
0.22	10.22	45.11	1.55	46.66	-0.59	8.41	55.07
0.43	10.43	55.54	0.00	55.54	1.15	10.15	65.69
0.38	10.38	65.92	0.00	65.92	-0.04	8.96	74.88
-0.71	9.29	75.21	0.00	75.21	-1.11	7.89	83.10
-0.26	9.74	84.95	0.00	84.95	0.41	9.41	94.36
			4.43				

- b. Determine arrival times [column (c)] by successive adding to times between arrivals in column (b).
- c. Use arrival times for service start *unless service is still in progress on a previous request*. In that case, determine how long the arrival must wait ($e - c$). Column (e) values are the sum of starting time and service time [column (g)], which is the time service ends [column (h)]. Thus, service on each new request begins [column (e)] at the same time that service on the previous request ends [column (h)].
- d. The simulation and resulting waiting times for the first nine arrivals are shown in the table. Total waiting time is 4.43 minutes.
- e. The total cost for the 94.36 minutes (end of service on the ninth request) of the simulation is

Waiting cost:	4.43 minutes at \$2 per minute	= \$ 8.86
Server cost:	94.36 minutes at \$1 per minutes	= <u>94.36</u>
		\$103.22

- f. Usually, a second simulation with two servers would be needed (but with the same arrival times so that the results are comparable). However, in this case it is apparent that a second server would increase server cost by about \$94 but could not eliminate more than \$8.86 of waiting cost. Hence, the second server would not be justified.

DISCUSSION AND REVIEW QUESTIONS

1. What is a simulation?
2. What are some of the primary reasons for the widespread use of simulation techniques in practice?
3. What are some of the ways managers can use simulation?
4. What role do random numbers play in Monte Carlo simulations?
5. Respond to the following comment: "I ran the simulation several times, and each run gave me a different result. Therefore, the technique does not seem to be useful. I need answers!"
6. List the main advantages of simulation.
7. What are some of the limitations of simulation as a tool for decision making?

PROBLEMS

1. The number of jobs received by a small shop is to be simulated for an eight-day period. The shop manager has collected the following data:

<u>Number of Jobs</u>	<u>Frequency</u>
2 or less	0
3	10
4	50
5	80
6	40
7	16
8	4
9 or more	<u>0</u>
	200

Use the third column of Table 18S.1 and read two-digit numbers, going down. Determine the average number of jobs per day for the eight-day simulation period.

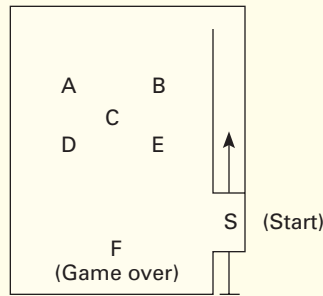
2. Jack M. sells insurance on a part-time basis. His records on the number of policies sold per week over a 50-week period are as follows:

<u>Number Sold</u>	<u>Frequency</u>
0	8
1	15
2	17
3	7
4	<u>3</u>
	50

Simulate three five-day periods. Use Table 18S.1, column 6 for the first simulation, column 7 for the second, and column 8 for the third. In each case, read two-digit numbers, beginning at the *bottom* of the column and going *up*. For each simulation, determine the percentage of days on which two or more policies are sold.

3. After a careful study of requests for a special tool at a large tool crib, an analyst concluded that demand for the tool can be adequately described by a Poisson distribution with a mean of two

- requests per day. Simulate demand for a 12-working-day period for this tool using Table 18S.1. Read three-digit numbers from columns 5 and 6 combined, starting at the *top* and reading *down* (e.g., 917, 264, 045).
4. The number of lost-time accidents at a logging firm can be described using a Poisson distribution that has a mean of four accidents per month. Using the last two columns of Table 18S.1 (e.g. 540, 733, 293), simulate accidents for a 12-month period.
 5. The time a physician spends with patients can be modeled using a normal distribution that has a mean of 20 minutes and a standard deviation of 2 minutes. Using the table of normally distributed random numbers (Table 18S.2), simulate the times the doctor might spend with the next seven patients. Use column 4 of the table; start at the *bottom* of the column and read *up*.
 6. Jobs are delivered to a workstation at random intervals. The time between job arrivals tends to be normally distributed with a mean of 15 minutes and a standard deviation of 1 minute. Job processing time is also normally distributed with a mean of 14 minutes per job and a standard deviation of 2 minutes.
 - a. Using Table 18S.2, simulate the arrival and processing of five jobs. Use column 4 of the table for job arrival times and column 3 for processing times. Start each column at row 4. Find the total times jobs wait for processing.
 - b. The company is considering the use of new equipment that would result in processing time that is normal with a mean of 13 minutes and a standard deviation of 1 minute. Job waiting represents a cost of \$3 per minute, and the new equipment would represent an additional cost of \$.50 per minute. Would the equipment be cost justified? (*Note:* Use the same arrival times and the same random numbers for processing times.)
 7. Daily usage of sugar in a small bakery can be described by a uniform distribution with endpoints of 30 pounds and 50 pounds. Assuming usage is independent of the day of the week, simulate daily usage for a 10-day period. Read four-digit numbers from Table 18S.1, columns 5 and 6, going *up* from the bottom.
 8. Weekly usage of spare parts for a specialized machine can be described by a Poisson distribution with a mean of 2.8 parts per week. Lead time to replenish the supply of spare parts is one week (constant). Simulate the total usage of parts during lead time 10 times, and then determine the frequency of lead time demands (i.e., what percentage of times was the demand equal to 2, 3, 4, etc.?). Read four-digit numbers from Table 18S.1, columns 8 and 9, going down.
 9. (*Computer exercise.*) Repeat Problem 8 for 100 lead time periods.
 10. A repair shop breaks an average of .6 tool per day. The average number of days required to obtain replacements is six. (Parts are delivered by mail once each day.) Both breakages and delivery times can be described by Poisson distributions. Tools are reordered whenever three or more must be replaced.
 - a. Draw a flowchart to describe this process.
 - b. Simulate breakage and ordering for a 12-day period. Read three-digit numbers from Table 18S.1, columns 5 and 6, going down (e.g., 917, 264), for tool breakage, and columns 7 and 8, going down (e.g., 643, 200), for delivery time. Assume zero tools in inventory to start.
 11. (*Computer exercise.*) Repeat Problem 10 for 150 days.
 12. Customers arrive randomly at a catalog department of a large store. The time between arrivals varies uniformly between 10 and 20 minutes. Service time is normal with a mean of 15 minutes and a standard deviation of 2 minutes.
 - a. Simulate processing and waiting times for nine customers. Read three-digit numbers going down columns 9 and 10 of Table 18S.1 for arrivals (e.g., 156, 884, 576). Use column 8, Table 18S.2, for processing time.
 - b. If management can reduce the range of arrival times to between 13 and 17 minutes, what would the impact be on customer waiting times? (Use the same service times and the same random numbers for arrival times from part *a.*) Round arrival times to two decimal places.
 13. Probabilities have been determined for the movements of the ball in a pinball game. These are shown in the accompanying table, along with the points awarded if the ball strikes a given position. Simulate the paths of three balls, and compute the number of points awarded for each ball. Use column 1 of Table 18S.1 for the first ball, column 2 for the second ball, and column 3 for the third ball, reading down the columns.



Position	Points
A	400
B	300
C	200
D	100
E	50
F	20

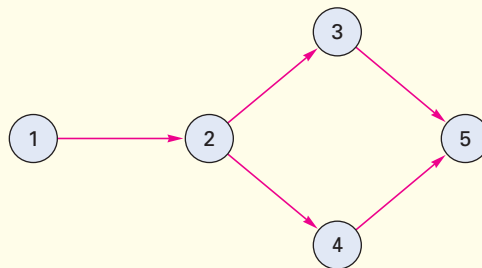
Path	Probability	Path	Probability	Path	Probability
S to A	.30	B to A	.30	D to A	.10
B	.30	B	.20	B	.05
C	.25	C	.20	C	.25
D	.10	D	.15	D	.20
E	.04	E	.10	E	.25
F	.01	F	.05	F	.15
A to A	.25	C to A	.10	E to A	.05
B	.25	B	.15	B	.10
C	.25	C	.20	C	.15
D	.10	D	.15	D	.15
E	.10	E	.25	E	.20
F	.05	F	.15	F	.35

(Hint: Construct a cumulative probability distribution for each of the six cases. Each new random number indicates the next position to move to. Each ball must start at S.)

14. Repeat problem 13, using columns 1, 2, and 3 for the next three balls, reading up from the bottom.
15. An analyst found that the length of telephone conversations in an office could be described by an exponential distribution with a mean of four minutes. Reading two-digit random numbers from Table 18S.1, column 6, simulate the length of five calls and compute the simulated average time. Why is the simulated average different from the mean of four minutes?
16. The length of time between calls for service of a certain piece of equipment can be described by an exponential distribution with a mean of 40 minutes. Service time can be described by a normal distribution with a mean of eight minutes and a standard deviation of two minutes. Simulate the time until the first breakdown and the times between three breakdowns, as well as the four service times. For breakdowns, read two-digit numbers from Table 18S.1, column 7; for service times, read numbers from Table 18S.2, column 8.
17. The number of jobs per day that a repair shop gets can be described by a Poisson distribution with a mean of 3.0. Repair time per job can be described by an exponential distribution with a mean of six hours. Simulate the number of jobs received for a four-day period and the repair time for each job. What is the simulated total repair time per day? For number of jobs received, read three-digit numbers from Table 18S.1, row 1; for repair times, read two-digit numbers from row 2.
18. A service operation consists of three steps. The first step can be described by a uniform distribution that ranges between five and nine minutes. The second step can be described by a normal distribution with a mean of seven minutes and a standard deviation of one minute, and the third step can be described by an exponential distribution with a mean of five minutes. Simulate three cycles using two-digit numbers from Table 18S.1, row 4 for step 1; Table 18S.2, row 6 for step 2; and two-digit numbers from column 4 of Table 18S.1 for step 3. Determine the simulated time for each of the three cycles.
19. A project consists of five major activities, as illustrated in the accompanying diagram. Activity times are normally distributed with means and standard deviations as shown in the following table. Note that there are two paths through the project: 1-2-3-5 and 1-2-4-5. Project duration is defined as the largest sum of times along a path. Simulate 12 times for each activity. Use columns

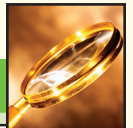
1 and 2 of Table 18S.2 for activity 1-2, columns 3 and 4 for activity 2-3, columns 5 and 6 for activity 2-4, columns 7 and 8 for activity 3-5, and columns 9 and 10 for activity 4-5. Determine the project duration for each of the 12 sets, and prepare a frequency distribution of project duration. Use categories of 25 to less than 30, 30 to less than 35, 35 to less than 40, 40 to less than 45, and 45 or more. Determine the proportion of time that a simulated duration of less than 40 days occurred. How might this information be used?

Activity	Mean (days)	Standard Deviation (days)
1-2	10	2
2-3	12	2
2-4	15	3
3-5	14	2
4-5	8	1



Coquille Refinery Corporation

CASE



The Coquille Refinery Corporation is contemplating building a crude-oil storage and docking facility on the southern coast of France. They plan to import crude oil by ship from the Middle East and distribute this crude oil by pipeline to refineries in the area.

The construction of this facility represents a substantial capital investment. Furthermore, the cost of such a facility is principally determined by its crude-oil storage capacity. You have been asked to study the problem and recommend an appropriate storage capacity; bear in mind that too large a capacity represents an unnecessary expense, but too small a capacity will result in costly later additions to the facility.

A long-term contract has been made with the Middle East supplier to furnish an average daily supply of 300,000 barrels of crude oil. Because its fleet of ships consists of 200,000-barrel tankers, the supplier expects that the arrival of its tankers will follow the distribution below:

Tanker Arrivals per Day	Probability
0	0.1
1	0.5
2	0.2
3	0.2

A review of past production records of refineries in the area suggests the following distribution of crude-oil demand per day:

Barrels per Day	Probability
100,000	0.1
200,000	0.2
300,000	0.3
400,000	0.4

Questions

- Consider the following issues before you simulate:
 - What is the expected daily demand for crude oil? Why must this be so?
 - What assumptions concerning the timing of crude-oil receipts and deliveries would require the greatest oil storage capacity?
 - What assumption concerning receipts and deliveries would require the least oil storage capacity?
 - Give a reason based on systems-analysis considerations why back orders should be filled from the next day's receipts rather than considered as lost sales.
- Develop a Monte Carlo simulation model for Coquille that will generate information useful for resolving the storage capacity problem and simulate 10 days of capacity.

(continued)

(concluded)

3. Assume a computer simulated 10,000 days of activity under the assumptions listed in question 1*b*. From the results of the simulation, the following distribution of oil in storage, after a day's receipts, is determined. (Note: Negative figures represent back orders.)

Oil in Storage, in Thousands	Probability
-300	0.01
-200	0.04
-100	0.06
0	0.07
⋮	⋮
1,000	0.09
1,100	0.08
1,200	0.07
1,300	0.05
1,400	0.03
1,500	0.02
	1.00

- a. What level of crude-oil safety stock should be carried to ensure 95 percent protection against stock-outs (1 stock-out in 20 days)?
 - b. If Coquille decides to use the above safety stock, what should be the oil storage capacity to ensure 90 percent protection against overruns (i.e., sufficient capacity to completely accommodate receipts 9 days out of 10)?
4. Note two ways one might determine if the simulation run length of 10,000 days is adequate.
 5. Make a list of cost factors that would influence the final selection of oil storage capacity.

Source: James A. Fitzsimmons and Robert S. Sullivan, *Service Operations Management* (New York: McGraw-Hill, 1982), pp. 78–80. Reprinted by permission.

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