CHAPTER 3

Logic

Outline

MATH IN The Art of Persuasion

Everywhere you turn in modern society, somebody is trying to convince you of something. "Vote for me!" "Buy my product!" "Lease a car from our dealership!" "Bailing out the auto industry is a bad idea!" "You should go out with me this weekend!" "Join our fraternity!" Logic is sometimes defined as correct thinking or correct reasoning. Some people refer to logic by a more casual name: *common sense*. Regardless of what you call it, the ability to think logically is crucial for all of us because our lives are inundated daily with advertisements, contracts, product and service warranties, political debates, and news commentaries, to name just a few. People often have problems processing these things because of misinterpretation, misunderstanding, and faulty logic.

You can look up the truth or falseness of a fact on the Internet, but that won't help you in analyzing whether a certain claim is logically valid. The term *common sense* is misleading, because evaluating logical arguments can be a challenging and involved process. This chapter introduces the basic concepts of formal symbolic logic and shows how to determine whether arguments are valid or invalid by using truth tables. One of our biggest goals will be to examine the form of an argument and determine if its conclusion follows logically from the statements in the argument.

To that end, we've written some claims below. Your job is to determine which of the arguments is valid, meaning that a conclusion can be logically drawn from a set of statements. The skills you learn in this chapter will help you to do so.

- Where there's smoke, there's fire.
- Having a lot of money makes people happy. My neighbor is a really happy guy, so he must have a lot of money.
- Every team in the SEC is good enough to play in a bowl game. Florida State is not in the SEC, so they're not good enough to play in a bowl game.
- Scripture is the word of God. I know this because it says so in the Bible.
- If Iraq has weapons of mass destruction, we should go to war. It turns out that they don't have them, so we should not go to war.
- It will be a snowy day in Hawaii before Tampa Bay makes it to the World Series. Tampa Bay played in the 2008 World Series, so it must have snowed in Hawaii.

For answers, see Math in the Art of Persuasion Revisited on Page 141

Section 3-1 Statements and Quantifiers

LEARNING OBJECTIVES

- ❑ 1. Define and identify statements.
- ❑ 2. Define the logical connectives.
- ❑ 3. Write the negation of a statement.
- ❑ 4. Write statements symbolically.

As the world gets more complex and we are bombarded with more and more information, it becomes more important than ever to be able to make sensible, objective evaluations of that information. One of the most effective tricks that advertisers, politicians, and con artists use is to encourage emotions to enter into these evaluations. They use carefully selected words and images that are designed to keep you from making decisions objectively. The field of **symbolic logic** was designed exactly for this reason. Symbolic logic uses letters to represent statements and special symbols to represent words like *and, or,* and *not*. Use of this symbolic notation in place of the statements themselves allows us to analytically evaluate the validity of the logic behind an argument without letting bias and emotion cloud our judgment. And these unbiased evaluations are the main goal of this chapter.

Statements

In the English language there are many types of sentences, including factual statements, commands, opinions, questions, and exclamations. In the objective study of logic, we will use only factual statements.

A **statement** is a declarative sentence that can be objectively determined to be true or false, but not both.

For example, sentences like

It is raining.

The United States has sent a space probe to Mars.

$2 + 2 = 4$

$10 - 5 = 4$

are statements because they are either true or false and you don't use an opinion to determine this.

Whether a sentence is true or false doesn't matter in determining if it is a statement. Notice in the above example the last statement "$10 - 5 = 4$" is false, but it's still a statement.

The following sentences, however, are not statements:

Give me onion rings with my order.

What operating system are you running?

Sweet!

The guy sitting next to me is kind of goofy.

The first is not a statement because it is a command. The second is not a statement because it is a question. The third is not a statement because it is an exclamation, and the fourth is not a statement because the word *goofy* is subjective; that is, it requires an opinion.

EXAMPLE 1 Recognizing Statements

Decide which of the following are statements and which are not.

(a) Most scientists agree that global warming is a threat to the environment.
(b) Is that your laptop?
(c) Man, that hurts!
(d) $8 - 2 = 6$
(e) This book is about database management.
(f) Everybody should watch reality shows.

SOLUTION

Parts (a), (d), and (e) are statements because they can be judged as true or false in a nonsubjective manner.
Part (b) is not a statement because it is a question.
Part (c) is not a statement because it is an exclamation.
Part (f) is not a statement because it requires an opinion.

Try This One 1

1. Define and identify statements.

Decide which of the following are statements and which are not.

(a) Cool!
(b) $12 - 8 = 5$
(c) Ryan Seacrest is the host of *American Idol*.
(d) Cat can send text messages with her cell phone.
(e) When does the party start?
(f) History is interesting.

Simple and Compound Statements

Statements can be classified as *simple* or *compound*. A **simple statement** contains only one idea. Each of these statements is an example of a simple statement.

These cargo pants are khaki.

My dorm room has three beds in it.

Daytona Beach is in Florida.

A statement such as "I will take chemistry this semester, and I will get an A" is called a **compound statement** since it consists of two simple statements.

Compound statements are formed by joining two simple statements with what is called a *connective*.

The basic **connectives** are *and, or, if . . . then,* and *if and only if*.

Each of the connectives has a formal name: *and* is called a **conjunction,** *or* is called a **disjunction,** *if . . . then* is called a **conditional,** and *if and only if* is called a **biconditional.** Here are some examples of compound statements using connectives.

John studied for 5 hours, and he got an A. (conjunction)

Luisa will run in a mini triathlon or she will play in the campus tennis tournament. (disjunction)

If I get 80% of the questions on the LSAT right, then I will get into law school. (conditional)

We will win the game if and only if we score more points than the other team. (biconditional)

Math Note

In standard usage, the word *then* is often omitted from a conditional statement; instead of "If it snows, then I will go skiing," you'd probably just say, "If it snows, I'll go skiing."

EXAMPLE 2 Classifying Statements as Simple or Compound

Math Note

Technically, we've given the names *conjunction, disjunction, conditional,* and *biconditional* to the connectives, but from now on, we'll refer to whole statements using these connectives by those names. For example, we would call the compound statement in Example 2d a disjunction.

Classify each statement as simple or compound. If it is compound, state the name of the connective used.

(a) Our school colors are red and white.
(b) If you register for WiFi service, you will get 3 days of free access.
(c) Tomorrow is the last day to register for classes.
(d) I will buy a hybrid or I will buy a motorcycle.

SOLUTION

(a) Don't let use of the word *and* fool you! This is a simple statement.
(b) This if . . . then statement is compound and uses a conditional connective.
(c) This is a simple statement.
(d) This is a compound statement, using a disjunction.

☑ 2. Define the logical connectives.

Try This One 2

Classify each statement as simple or compound. If it is compound, state the name of the connective used.

(a) The jacket is trendy and it is practical.
(b) This is an informative website on STDs.
(c) If it does not rain, then I will go windsurfing.
(d) I will buy a flash drive or I will buy a zip drive.
(e) Yesterday was the deadline to withdraw from a class.

Sidelight A BRIEF HISTORY OF LOGIC

The basic concepts of logic can be attributed to Aristotle, who lived in the fourth century BCE. He used words, sentences, and deduction to prove arguments using techniques we will study in this chapter. In addition, in 300 BCE. Euclid formalized geometry using deductive proofs. Both subjects were considered to be the "inevitable truths" of the universe revealed to rational people.

In the 19th century, people began to reject the idea of inevitable truths and realized that a deductive system like Euclidean geometry is only true based on the original assumptions. When the original assumptions are changed, a new deductive system can be created. This is why there are different types of geometry. (See the Sidelight entitled "Non-Euclidean Geometry" in Chapter 10.)

Eventually, several people developed the use of symbols rather than words and sentences in logic. One such person was George Boole (1815–1864). Boole created the symbols used in this chapter and developed the theory of symbolic logic. He also used symbolic logic in mathematics. His manuscript, entitled "An Investigation into the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities," was published when he was 39 in 1854.

Boole was a friend of Augustus De Morgan, who formulated De Morgan's laws, which we studied in Chapter 2. Much earlier, Leonhard Euler (1707–1783) used circles to represent logical statements and proofs. The idea was refined into Venn diagrams by John Venn (1834–1923).

Quantified Statements

Quantified statements involve terms such as *all, each, every, no, none, some, there exists,* and *at least one*. The first five (*all, each, every, no, none*) are called *universal quantifiers* because they either include or exclude every element of the universal set. The latter three (*some, there exists, at least one*) are called *existential quantifiers* because they show the existence of something, but do not include the entire universal set. Here are some examples of quantified statements:

Every student taking Math for Liberal Arts this semester will pass.

Some people who are Miami Hurricane fans are also Miami Dolphin fans.

There is at least one professor in this school who does not have brown eyes.

No Marlin fan is also a Yankee fan.

The first and the fourth statements use universal quantifiers, and the second and third use existential quantifiers. Note that the statements using existential quantifiers are not "all inclusive" (or all exclusive) as the other two are.

Math Note

We'll worry later about determining whether statements involving quantifiers and connectives are true or false. For now, focus on learning and understanding the terms.

Negation

The *negation* of a statement is a corresponding statement with the opposite truth value. For example, for the statement "My dorm room is blue," the negation is "My dorm room is not blue." It's important to note that the truth values of these two are completely opposite: one is true, and the other is false—period. You can't negate "My dorm room is blue" by saying "My dorm room is yellow," because it's completely possible that *both* statements are false. To make sure that you have a correct negation, check that if one of the statements is true, the other must be false, and vice versa. The typical way to negate a simple statement is by adding the word *not,* as in these examples:

Statement	**Negation**
Auburn will win Saturday.	Auburn will not win Saturday.
I took a shower today.	I did not take a shower today.
My car is clean.	My car is not clean.

You have to be especially careful when negating quantified statements. Consider the example statement "All dogs are fuzzy." It's not quite right to say that the negation is "All dogs are not fuzzy," because if some dogs are fuzzy and others aren't, then both statements are false. All we need for the statement "All dogs are fuzzy" to be false is to find at least one dog that is not fuzzy, so the negation of the statement "All dogs are fuzzy" is "Some dogs are not fuzzy." (In this setting, we define the word *some* to mean *at least one*.)

We can summarize the negation of quantified statements as follows:

Statement Contains . . .	**Negation**
All do	Some do not, or not all do
Some do	None do, or all do not
Some do not	All do
None do	Some do

Math Note

The words *each, every,* and *all* mean the same thing, so what we say about *all* in this section applies to the others as well. Likewise, *some, there exists,* and *at least one* are considered to be the same and are treated that way as well.

EXAMPLE 3 Writing Negations

Write the negation of each of the following quantified statements.

(a) Every student taking Math for Liberal Arts this semester will pass.
(b) Some people who are Miami Hurricane fans are also Miami Dolphin fans.
(c) There is at least one professor in this school who does not have brown eyes.
(d) No Marlin fan is also a Yankee fan.

SOLUTION

(a) Some student taking Math for Liberal Arts this semester will not pass (or, not every student taking Math for Liberal Arts this semester will pass).
(b) No people who are Miami Hurricane fans are also Miami Dolphin fans.
(c) All professors in this school have brown eyes.
(d) Some Marlin fan is also a Yankee fan.

CAUTION

Be especially careful when negating statements. Remember that the negation of "Every student will pass" is *not* "Every student will fail".

3. Write the negation of a statement.

Try This One 3

Write the negation of each of the following quantified statements.

(a) All cell phones have cameras.
(b) No woman can win the lottery.
(c) Some professors have PhDs.
(d) Some students in this class will not pass.

Math Note

For three of the four connectives in Table 3-1, the order of the simple statements doesn't matter: for example, $p \wedge q$ and $q \wedge p$ represent the same compound statement. The same is true for the connectives $\vee$ (disjunction) and $\leftrightarrow$ (biconditional). The one exception is the conditional ($\rightarrow$), where order is crucial.

Symbolic Notation

Recall that one of our goals in this section is to write statements in symbolic form to help us evaluate logical arguments objectively. Now we'll introduce the symbols and methods that will be used. The symbols for the connectives *and, or, if . . . then,* and *if and only if* are shown in Table 3-1.

Simple statements in logic are usually denoted with lowercase letters like p, q, and r. For example, we could use p to represent the statement "I get paid Friday" and q to represent the statement "I will go out this weekend." Then the conditional statement "If I get paid Friday, then I will go out this weekend" can be written in symbols as $p \rightarrow q$.

The symbol ~ (tilde) represents a negation. If p still represents "I get paid Friday," then $\sim p$ represents "I do not get paid Friday."

We often use parentheses in logical statements when more than one connective is involved in order to specify an order. (We'll deal with this in greater detail in the next section.) For example, there is a difference between the compound statements $\sim p \wedge q$ and $\sim(p \wedge q)$. The statement $\sim p \wedge q$ means to negate the statement p first, then use the negation of p in conjunction with the statement q. For example, if p is the statement "Fido is a dog" and q is the statement "Pumpkin is a cat," then $\sim p \wedge q$ reads, "Fido is not a dog and Pumpkin is a cat." The statement $\sim p \wedge q$ could also be written as $(\sim p) \wedge q$.

TABLE 3-1 Symbols for the Connectives

Connective	Symbol	Name
and	$\wedge$	Conjunction
or	$\vee$	Disjunction
if . . . then	$\rightarrow$	Conditional
if and only if	$\leftrightarrow$	Biconditional

The statement $\sim(p \wedge q)$ means to negate the conjunction of the statement p and the statement q. Using the same statements for p and q as before, the statement $\sim(p \wedge q)$ is written, "It is not the case that Fido is a dog and Pumpkin is a cat."

The same reasoning applies when the negation is used with other connectives. For example, $\sim p \rightarrow q$ means $(\sim p) \rightarrow q$.

Example 4 illustrates in greater detail how to write statements symbolically.

EXAMPLE 4 Writing Statements Symbolically

Let p represent the statement "It is cloudy" and q represent the statement "I will go to the beach." Write each statement in symbols.

(a) I will not go to the beach.
(b) It is cloudy, and I will go to the beach.
(c) If it is cloudy, then I will not go to the beach.
(d) I will go to the beach if and only if it is not cloudy.

SOLUTION

(a) This is the negation of statement q, which we write as $\sim q$.
(b) This is the conjunction of p and q, written as $p \wedge q$.
(c) This is the conditional of p and the negation of q: $p \rightarrow \sim q$.
(d) This is the biconditional of p and not q: $p \leftrightarrow \sim q$.

▼ Try This One 4

Let p represent the statement "I will buy a Coke" and q represent the statement "I will buy some popcorn." Write each statement in symbols.

(a) I will buy a Coke, and I will buy some popcorn.
(b) I will not buy a Coke.
(c) If I buy some popcorn, then I will buy a Coke.
(d) I will not buy a Coke, and I will buy some popcorn.

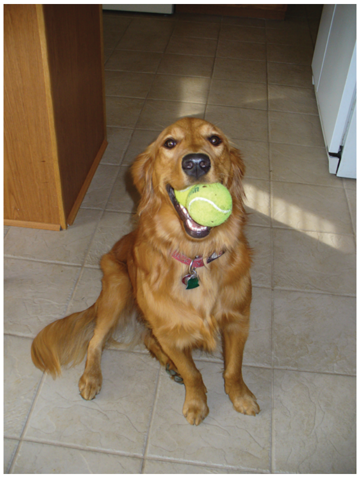
You probably noticed that some of the compound statements we've written sound a little awkward. It isn't always necessary to repeat the subject and verb in a compound statement using *and* or *or*. For example, the statement "It is cold, and it is snowing" can be written "It is cold and snowing." The statement "I will go to a movie, or I will go to a play" can be written "I will go to a movie or a play." Also the words *but* and *although* can be used in place of *and*. For example, the statement "I will not buy a television set, and I will buy a CD player" can also be written as "I will not buy a television set, but I will buy a CD player."

Statements written in symbols can also be written in words, as shown in Example 5.

EXAMPLE 5 Translating Statements from Symbols to Words

Write each statement in words. Let p = "My dog is a golden retriever" and q = "My dog is fuzzy."

(a) $\sim p$ (b) $p \vee q$ (c) $\sim p \rightarrow q$ (d) $q \leftrightarrow p$ (e) $q \wedge p$



If this is your dog (which it's not, because it's mine), statement (e) describes it pretty well.

SOLUTION

(a) My dog is not a golden retriever.
(b) My dog is a golden retriever or my dog is fuzzy.
(c) If my dog is not a golden retriever, then my dog is fuzzy.
(d) My dog is fuzzy if and only if my dog is a golden retriever.
(e) My dog is fuzzy, and my dog is a golden retriever.

▼ Try This One 5

Write each statement in words. Let p = "My friend is a football player" and q = "My friend is smart."

(a) $\sim p$ (b) $p \vee q$ (c) $\sim p \rightarrow q$ (d) $p \leftrightarrow q$ (e) $p \wedge q$

4. Write statements symbolically.

In this section, we defined the basic terms of symbolic logic and practiced writing statements using symbols. These skills will be crucial in our objective study of logical arguments, so we're off to a good start.

Answers to Try This One

1 (b), (c), and (d) are statements.

2 (a) (conjunction), (c) (conditional), and (d) (disjunction) are compound; (b) and (e) are simple.

3
(a) Some cell phones don't have cameras.
(b) Some women can win the lottery.
(c) No professors have Ph.Ds.
(d) All students in this class will pass.

4 (a) $p \wedge q$ (b) $\sim p$ (c) $q \rightarrow p$ (d) $\sim p \wedge q$

5
(a) My friend is not a football player.
(b) My friend is a football player or my friend is smart.
(c) If my friend is not a football player, then my friend is smart.
(d) My friend is smart if and only if my friend is a football player.
(e) My friend is smart and my friend is a football player.

EXERCISE SET 3-1

Writing Questions

1. Define the term *statement* in your own words.
2. Explain the difference between a simple and a compound statement.
3. Describe the terms and symbols used for the four connectives.
4. Explain why the negation of "All spring breaks are fun" is not "All spring breaks are not fun."

Real-World Applications

For Exercises 5–14, state whether the sentence is a statement or not.

5. Please do not use your cell phone in class.
6. $5 + 9 = 14$
7. $9 - 3 = 2$
8. Nicki is a student in vet school.
9. Who will win the student government presidency?
10. Neither Sam nor Mary arrives to the exam on time.
11. You can carry a cell phone with you.
12. Bill Gates is the creator of Microsoft.
13. Go with the flow.
14. Math is not hard.

For Exercises 15–24, decide if each statement is simple or compound.

15. He goes to parties and hangs out at the coffee shop.
16. Sara got her hair highlighted.
17. Raj will buy an iMac or a Dell computer.
18. Euchre is fun if and only if you win.
19. February is when Valentine's Day occurs.
20. Diane is a chemistry major.
21. If you win the Megabucks multistate lottery, then you will be rich.
22. He listened to his iPod and he typed a paper.
23. $8 + 9 = 12$
24. Malcolm and Alisha will both miss the spring break trip.

For Exercises 25–32, identify each statement as a conjunction, disjunction, conditional, or biconditional.

25. Bob and Tom like stand-up comedians.
26. Either he passes the test, or he fails the course.
27. A number is even if and only if it is divisible by 2.
28. Her nails are long, and they have rhinestones on them.
29. I will go to the big game, or I will go to the library.
30. If a number is divisible by 3, then it is an odd number.
31. A triangle is equiangular if and only if three angles are congruent.
32. If your battery is dead, then you need to charge your phone overnight.

For Exercises 33–38, write the negation of the statement.

33. The sky is blue.
34. It is not true that your computer has a virus.
35. The dorm room is not large.
36. The class is not full.
37. It is not true that you will fail this class.
38. He has large biceps.

For Exercises 39–50, identify the quantifier in the statement as either universal or existential.

39. All fish swim in water.
40. Everyone who passes algebra has studied.
41. Some people who live in glass houses throw stones.
42. There is at least one person in this class who won't pass.
43. Every happy dog wags its tail.
44. No men can join a sorority.
45. There exists a four-leaf clover.
46. Each person who participated in the study will get \$100.
47. No one with green eyes wears glasses.
48. Everyone in the class was bored by the professor's lecture.
49. At least one of my friends has an iPhone.
50. No one here gets out alive.

For Exercises 51–62, write the negation of the statements in Exercises 39–50.

For Exercises 63–72, write each statement in symbols. Let p = "Sara is a political science major" and let q = "Jane is a quantum physics major."

63. Sara is a political science major, and Jane is a quantum physics major.
64. Sara is not a political science major.
65. If Jane is not a quantum physics major, then Sara is a political science major.
66. It is not true that Jane is a quantum physics major or Sara is a political science major.
67. It is false that Jane is a quantum physics major.
68. It is not true that Sara is a political science major.
69. Jane is a quantum physics major, or Sara is not a political science major.
70. Jane is not a quantum physics major, or Sara is a political science major.
71. Jane is a quantum physics major if and only if Sara is a political science major.
72. If Sara is a political science major, then Jane is a quantum physics major.

For Exercises 73–82, write each statement in symbols. Let p = "My dad is cool" and q = "My mom is cool." Let nerdy *mean* not cool.

73. My mom is not cool.
74. Both my dad and my mom are nerdy.
75. If my mom is cool, then my dad is cool.
76. It is not true that my dad is cool.
77. Either my mom is nerdy, or my dad is cool.
78. It is not true that my mom is nerdy and my dad is cool.
79. My mom is cool if and only if my dad is cool.
80. Neither my mom nor my dad is cool.
81. If my mom is nerdy, then my dad is cool.
82. My dad is nerdy if and only if my mom is not cool.

For Exercises 83–92, write each statement in words. Let p = "The plane is on time." Let q = "The sky is clear."

83. $p \wedge q$
84. $\sim p \vee q$
85. $q \rightarrow p$
86. $q \rightarrow \sim p$
87. $\sim p \wedge \sim q$
88. $q \leftrightarrow p$
89. $p \vee \sim q$
90. $\sim p \leftrightarrow \sim q$
91. $q \rightarrow (p \vee \sim p)$
92. $(p \rightarrow q) \vee \sim p$

For Exercises 93–102, write each statement in words. Let p = "Mark lives on campus." Let q = "Trudy lives off campus."

93. $\sim q$
94. $p \rightarrow q$
95. $p \vee \sim q$
96. $q \leftrightarrow p$
97. $\sim p \rightarrow \sim q$
98. $\sim p$
99. $p \vee q$
100. $(\sim p \vee q) \vee \sim q$
101. $q \vee p$
102. $(p \vee q) \rightarrow \sim(\sim q)$

Critical Thinking

103. Explain why the sentence "All rules have exceptions" is not a statement.

104. Explain why the sentence "This statement is false" is not a statement.

Section 3-2 Truth Tables

LEARNING OBJECTIVES

- ❑ 1. Construct truth tables for negation, disjunction, and conjunction.
- ❑ 2. Construct truth tables for the conditional and biconditional.
- ❑ 3. Construct truth tables for compound statements.
- ❑ 4. Identify the hierarchy of logical connectives.
- ❑ 5. Construct truth tables by using an alternative method.

"You can't believe everything you hear." Chances are you were taught this when you were younger, and it's pretty good advice. In an ideal world, everyone would tell the truth all the time, but in the real world, it is extremely important to be able to separate fact from fiction. When someone is trying to convince you of some point of view, the ability to logically evaluate the validity of an argument can be the difference between being informed and being deceived—and maybe between you keeping and you being separated from your hard-earned money!

This section is all about deciding when a compound statement is or is not true, based not on the situation itself, but simply on the structure of the statement and the truth of the underlying components. We learned about logical connectives in Section 3-1. In this section, we'll analyze these connectives using *truth tables*. A **truth table** is a diagram in table form that is used to show when a compound statement is true or false based on the truth values of the simple statements that make up the compound statement. This will allow us to analyze arguments objectively.

Negation

According to our definition of *statement,* a statement is either true or false, but never both. Consider the simple statement p = "Today is Tuesday." If it is in fact Tuesday, then p is true, and its negation ($\sim p$) "Today is not Tuesday" is false. If it's not Tuesday, then p is false and $\sim p$ is true. The truth table for the negation of p looks like this.

p	$\sim p$
T	F
F	T

There are two possible conditions for the statement p—true or false—and the table tells us that in each case, the negation $\sim p$ has the opposite truth value.

Conjunction

If we have a compound statement with two component statements p and q, there are four possible combinations of truth values for these two statements:

Possibilities	**Symbolic value of each** p	q
1. p and q are both true.	T	T
2. p is true and q is false.	T	F
3. p is false and q is true.	F	T
4. p and q are both false.	F	F

So when setting up a truth table for a compound statement with two component statements, we'll need a row for each of the four possibilities.

Now we're ready to analyze conjunctions. Recall that a conjunction is a compound statement involving the word *and*. Suppose a friend who's prone to exaggeration tells you, "I bought a new computer and a new iPod." This compound statement can be symbolically represented by $p \wedge q$, where p = "I bought a new computer" and q = "I bought a new iPod." When would this conjunctive statement be true? If your friend actually had made both purchases, then of course the statement "I bought a new computer and a new iPod" would be true. In terms of a truth table, that tells us that if p and q are both true, then the conjunction $p \wedge q$ is true as well, as shown below.

p	q	$p \wedge q$
T	T	T

On the other hand, suppose your friend bought only a new computer or only a new iPod, or maybe neither of those things. Then the statement "I bought a new computer and a new iPod" would be false. In other words, if either or both of p and q are false, then the compound statement $p \wedge q$ is false as well. With this information, we complete the truth table for a basic conjunction:

	p	q	$p \wedge q$
Bought computer and iPod	T	T	T
Bought computer, not iPod	T	F	F
Bought iPod, not computer	F	T	F
Bought neither	F	F	F

Truth Values for a Conjunction

The conjunction $p \wedge q$ is true only when both p and q are true.

Disjunction

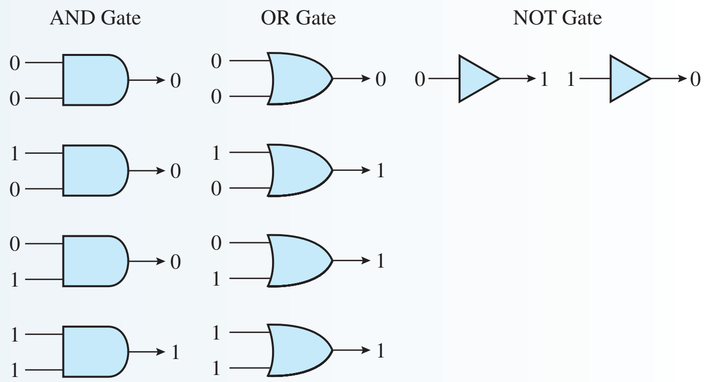
Next, we'll look at truth tables for *or* statements. Suppose your friend from the previous example made the statement, "I bought a new computer *or* a new iPod" (as opposed to *and*). If your friend actually did buy one or the other, then this statement would be true. And if he or she bought neither, then the statement would be false. So a partial truth table looks like this:

	p	q	$p \vee q$
Bought computer and iPod	T	T	
Bought computer, not iPod	T	F	T
Bought iPod, not computer	F	T	T
Bought neither	F	F	F

Sidelight Logical Gates and Computer Design

Logic is used in electrical engineering in designing circuits, which are the heart of computers. The truth tables for *and*, *or,* and *not* are used for computer gates. These gates determine whether electricity flows through a circuit. When a switch is closed, the current has an uninterrupted path and will flow through the circuit. This is designated by a 1. When a switch is open, the path at the current is broken, and it will not flow. This is designated by a 0. The logical gates are illustrated here—notice that they correspond exactly with our truth tables.

This simple little structure is responsible for the operation of almost every computer in the world—at least until quantum computers become a reality. If you're interested, do a Google search for *quantum computer* to read about the future of computing.



But what if the person actually bought both items? You might lean toward the statement "I bought a new computer or a new iPod" being false. Believe it or not, it depends on what we mean by the word *or*. There are two interpretations of that word, known as the *inclusive or* and the *exclusive or*. The inclusive or has the possibility of both statements being true; but the exclusive or does not allow for this, that is, exactly one of the two simple statements must be true. In English when we use the word *or,* we typically think of the exclusive or. If I were to say, "I will go to work or I will go to the beach," you would assume I am doing one or the other, but not both. In logic we generally use the inclusive or. When the inclusive or is used, the statement "I will go to work or I will go to the beach" would be true if I went to both work and the beach. For the remainder of this chapter we will assume the symbol $\vee$ represents the inclusive or and will drop *inclusive* and just say *or.*

The completed truth table for the disjunction is

	p	q	$p \vee q$
Bought computer and iPod	T	T	T
Bought computer, not iPod	T	F	T
Bought iPod, not computer	F	T	T
Bought neither	F	F	F

1. Construct truth tables for negation, disjunction, and conjunction.

Truth Values for a Disjunction

The disjunction $p \vee q$ is true when either p or q or both are true. It is false only when both p and q are false.

Conditional Statement

A conditional statement, which is sometimes called an *implication,* consists of two simple statements using the connective if . . . then. For example, the statement "If I bought a ticket, then I can go to the concert" is a conditional statement. The first component, in this case "I bought a ticket," is called the *antecedent*. The second component, in this case "I can go to the concert," is called the *consequent*.

Conditional statements are used commonly in every area of math, including logic. You might remember statements from high school algebra like "If two lines are parallel, then they have the same slope." Remember that we represent the conditional statement "If p, then q" by using the symbol $p \rightarrow q$.

To illustrate the truth table for the conditional statement, think about the following simple example: "If it is raining, then I will take an umbrella." We'll use p to represent "It is raining" and q to represent "I will take an umbrella," then the conditional statement is $p \rightarrow q$. We'll break the truth table down into four cases.

Case 1: It is raining and I do take an umbrella (p and q are both true). Since I am doing what I said I would do in case of rain, the conditional statement is true. So the first line in the truth table is

	p	q	$p \rightarrow q$
Raining, take umbrella	T	T	T

Case 2: It is raining and I do not take an umbrella (p is true and q is false). Since I am not doing what I said I would do in case of rain, I'm a liar and the conditional statement is false. So the second line in the truth table is

	p	q	$p \rightarrow q$
	T	T	T
Raining, do not take umbrella	T	F	F

Case 3: It is not raining and I do take an umbrella (p is false and q is true). This requires some thought. I never said in the original statement what I would do if it were not raining, so there's no reason to regard my original statement as false. Based on the information given, we consider the original statement to be true, and the third line of the truth table is

	p	q	$p \rightarrow q$
	T	T	T
	T	F	F
Not raining, take umbrella	F	T	T

Case 4: It is not raining, and I do not take my umbrella (p and q are both false). This is essentially the same as case 3—I never said what I would do if it did not rain, so there's no reason to regard my statement as false based on what we know. So we consider the original statement to be true, and the last line of the truth table is

	p	q	$p \rightarrow q$
	T	T	T
	T	F	F
	F	T	T
Not raining, do not take umbrella	F	F	T

For cases 3 and 4, it might help to think of it this way: unless we have definite proof that a statement is false, we will consider it to be true.

Truth Values for a Conditional Statement

The conditional statement $p \rightarrow q$ is false only when the antecedent p is true and the consequent q is false.

Biconditional Statement

A biconditional statement is really two statements in a way; it's the conjunction of two conditional statements. For example, the statement "I will stay in and study Friday if and only if I don't have any money" is the same as "If I don't have any money, then I will stay in and study Friday *and* if I stay in and study Friday, then I don't have any money." In symbols, we can write either $p \leftrightarrow q$ or $(p \rightarrow q) \wedge (q \rightarrow p)$. Since the biconditional is a conjunction, for it to be true, both of the statements $p \rightarrow q$ and $q \rightarrow p$ must be true. We will once again look at cases to build the truth table.

Case 1: Both p and q are true. Then both $p \rightarrow q$ and $q \rightarrow p$ are true, and the conjunction $(p \rightarrow q) \wedge (q \rightarrow p)$, which is also $p \leftrightarrow q$, is true as well.

p	q	$p \leftrightarrow q$
T	T	T

Case 2: p is true and q is false. In this case, the implication $p \rightarrow q$ is false, so it doesn't even matter whether $q \rightarrow p$ is true or false—the conjunction has to be false.

p	q	$p \leftrightarrow q$
T	T	T
T	T	F

A technician who designs an automated irrigation system needs to decide whether the system should turn on *if* the water in the soil falls below a certain level or *if and only if* the water in the soil falls below a certain level. In the first instance, other inputs could also turn on the system.

Case 3: p is false and q is true. This is case 2 in reverse. The implication $q \rightarrow p$ is false, so the conjunction must be as well.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F

Case 4: p is false and q is false. According to the truth table for a conditional statement, both $p \rightarrow q$ and $q \rightarrow p$ are true in this case, so the conjunction is as well. This completes the truth table.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

2. Construct truth tables for the conditional and biconditional.

Truth Values for a Biconditional Statement

The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth value and is false when they have opposite truth values.

Table 3-2 on the next page provides a summary of the truth tables for the basic compound statements and the negation. The last thing you should do is to try and memorize these tables! If you understand how we built them, you can rebuild them on your own when you need them.

Truth Tables for Compound Statements

Once we know truth values for the basic connectives, we can use truth tables to find the truth values for any logical statement. The key to the procedure is to take it step

TABLE 3-2 Truth Tables for the Connectives and Negation

Conjunction "and"

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction "or"

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional "if . . . then"

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional "if and only if"

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Negation "not"

p	$\sim p$
T	F
F	T

by step, so that in every case, you're deciding on truth values based on one of the truth tables in Table 3-2.

EXAMPLE 1 Constructing a Truth Table

"My leg isn't better, or I'm taking a break" is an example of a statement that can be written as $\sim p \vee q$.

Construct a truth table for the statement $\sim p \vee q$.

SOLUTION

Step 1 Set up a table as shown.

p	q
T	T
T	F
F	T
F	F

The order in which you list the Ts and Fs doesn't matter as long as you cover all the possible combinations. For consistency in this book, we'll always use the order TTFF for p and TFTF for q when these are the only two letters in the logical statement.

Step 2 Find the truth values for $\sim p$ by negating the values for p, and put them into a new column, column 3, marked $\sim p$.

p	q	$\sim p$
T	T	F
T	F	F
F	T	T
F	F	T
	2	3

Truth values for $\sim p$ are opposite those for p.

Step 3 Find the truth values for the disjunction $\sim p \vee q$. Use the T and F values for $\sim p$ and q in columns 2 and 3, and use the disjunction truth table from earlier in the section.

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T
			4

The disjunction is true unless $\sim p$ and q are both false.

The truth values for the statement $\sim p \vee q$ are found in column 4. The statement is true unless p is true and q is false.

▼ Try This One 1

Construct a truth table for the statement $p \vee \sim q$.

When a statement contains parentheses, we find the truth values for the statements in parentheses first, as shown in Example 2. This is similar to the order of operations used in arithmetic and algebra.

EXAMPLE 2 Constructing a Truth Table

"It is not true that if it rains, then we can't go out" is an example of a statement that can be written as $\sim(p \rightarrow \sim q)$.

Construct a truth table for the statement $\sim(p \rightarrow \sim q)$.

SOLUTION

Step 1 Set up the table as shown.

p	q
T	T
T	F
F	T
F	F

Step 2 Find the truth values for $\sim q$ by negating the values for q, and put them into a new column.

p	q	$\sim q$
T	T	F
T	F	T
F	T	F
F	F	T
1		3

Truth values for $\sim q$ are opposite those for q.

Step 3 Find the truth values for the implication $p \to \sim q$, using the values in columns 1 and 3 and the implication truth table from earlier in the section.

p	q	$\sim q$	$p \to \sim q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T
			4

The conditional is true unless p is true and $\sim q$ is false.

Step 4 Find the truth values for the negation $\sim(p \to \sim q)$ by negating the values for $p \to \sim q$ in column 4.

p	q	$\sim q$	$p \to \sim q$	$\sim(p \to \sim q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	T	F
F	F	T	T	F
			4	5

The negation has opposite values from column 4.

The truth values for $\sim(p \to \sim q)$ are in column 5. The statement is true only when p and q are both true.

Try This One 2

Construct a truth table for the statement $p \leftrightarrow (\sim p \wedge q)$.

We can also construct truth tables for compound statements that involve three or more components. For a compound statement with three simple statements p, q, and r, there are eight possible combinations of Ts and Fs to consider. The truth table is set up as shown in step 1 of Example 3.

EXAMPLE 3 Constructing Truth Table with Three Components

"I'll do my math assignment, or if I think of a good topic, then I'll start my English essay" is an example of a statement that can be written as $p \vee (q \to r)$.

Construct a truth table for the statement $p \vee (q \to r)$.

SOLUTION

Step 1 Set up the table as shown.

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F
	2	3

Again, the order of the Ts and Fs doesn't matter as long as all the possible combinations are listed. Whenever there are three letters in the statement, we'll use the order shown above for consistency.

Step 2 Find the truth value for the statement in parentheses, $q \rightarrow r$. Use the values in columns 2 and 3 and the conditional truth table from earlier in the section. Put those values in a new column.

p	q	r	$q \rightarrow r$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	T
1			4

The conditional is true unless q is true and r is false.

Step 3 Find the truth values for the disjunction $p \vee (q \rightarrow r)$, using the values for p from column 1 and those for $q \rightarrow r$ from column 4. Use the truth table for disjunction from earlier in the section, and put the results in a new column.

p	q	r	$q \rightarrow r$	$p \vee (q \rightarrow r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T
				5

The disjunction is true unless both p and $q \rightarrow r$ are false.

The truth values for the statement $p \vee (q \rightarrow r)$ are found in column 5. The statement is true unless p and r are false while q is true.

3. Construct truth tables for compound statements.

Try This One 3

Construct a truth table for the statement $(p \wedge q) \vee \sim r$.

In the method we've demonstrated for constructing truth tables, we begin by setting up a table with all possible combinations of Ts and Fs for the component letters from the statement. Then we build new columns, one at a time, by writing truth values for parts of the compound statement, using the basic truth tables we built earlier in the section.

We have seen that when we construct truth tables, we find truth values for statements inside parentheses first. To avoid having to always use parentheses, a hierarchy of connectives has been agreed upon by those who study logic.

1. Biconditional $\leftrightarrow$
2. Conditional $\rightarrow$
3. Conjunction $\wedge$, disjunction $\vee$
4. Negation $\sim$

When we find the truth value for a compound statement without parentheses, we find *the truth value of a lower-order connective first*. For example, $p \vee q \rightarrow r$ is a conditional statement since the conditional ($\rightarrow$) is of a higher order than the disjunction ($\vee$). If you were constructing a truth table for the statement, you would find the truth value for $\vee$ first. The statement $p \leftrightarrow q \wedge r$ is a biconditional statement since the biconditional ($\leftrightarrow$) is of a higher order than the order of the conjunction ($\wedge$). When constructing a truth table for the statement, the truth value for the conjunction ($\wedge$) would be found first. The conjunction and disjunction are of the same order; the statement $p \wedge q \vee r$ cannot be identified unless parentheses are used. In this case, $(p \wedge q) \vee r$ is a disjunction and $p \wedge (q \vee r)$ is a conjunction.

Math Note

When parentheses are used to emphasize order, the statement $p \vee q \rightarrow r$ is written as $(p \vee q) \rightarrow r$. The statement $p \leftrightarrow q \wedge r$ is written as $p \leftrightarrow (q \vee r)$.

EXAMPLE 4 Using the Hierarchy of Connectives

For each, identify the type of statement using the hierarchy of connectives, and rewrite by using parentheses to indicate order.

(a) $\sim p \vee \sim q$ (b) $p \rightarrow \sim q \wedge r$ (c) $p \vee q \leftrightarrow q \vee r$ (d) $p \rightarrow q \leftrightarrow r$

SOLUTION

(a) The $\vee$ is higher than the $\sim$; the statement is a disjunction and looks like $(\sim p) \vee (\sim q)$ with parentheses.
(b) The $\rightarrow$ is higher than the $\wedge$ or $\sim$; the statement is a conditional and looks like $p \rightarrow (\sim q \wedge r)$ with parentheses.
(c) The $\leftrightarrow$ is higher than $\vee$; the statement is a biconditional and looks like $(p \vee q) \leftrightarrow (q \vee r)$ with parentheses.
(d) The $\leftrightarrow$ is higher than the $\rightarrow$; the statement is a biconditional and looks like $(p \rightarrow q) \leftrightarrow r$ with parentheses.

Try This One 4

For each, identify the type of statement using the hierarchy of connectives, and rewrite by using parentheses to indicate order.

(a) $\sim p \vee q$ (c) $p \vee q \leftrightarrow \sim p \vee \sim q$ (e) $p \leftrightarrow q \rightarrow r$
(b) $p \vee \sim q \rightarrow r$ (d) $p \wedge \sim q$

☑ 4. Identify the hierarchy of logical connectives.

EXAMPLE 5 An Application of Truth Tables

Use the truth value of each simple statement to determine the truth value of the compound statement.

p: O. J. Simpson was convicted in California in 1995.
q: O. J. Simpson was convicted in Nevada in 2008.
r: O. J. Simpson gets sent to prison.

Statement: $(p \vee q) \rightarrow r$

SOLUTION

In probably the most publicized trial of recent times, Simpson was acquitted of murder in California in 1995, so statement p is false. In 2008, however, Simpson was convicted of robbery and kidnapping in Nevada, so statement q is true. Statement r is also true, as Simpson was sentenced in December 2008.

Now we'll analyze the compound statement. First, the disjunction $p \vee q$ is true when either p or q is true, so in this case, $p \vee q$ is true. Finally, the implication $(p \vee q) \rightarrow r$ is true when both r and $p \vee q$ are true, which is the case here. So the compound statement $(p \vee q) \rightarrow r$ is true.

▼ Try This One 5

Using the simple statements in Example 5, find the truth value of the compound statement $(\sim p \wedge \sim q) \rightarrow r$.

An Alternative Method for Constructing Truth Tables

In the next two examples, we will illustrate a second method for constructing truth tables so that you can make a comparison. The problems are the same as Examples 2 and 3.

EXAMPLE 6 Constructing a Truth Table by Using an Alternative Method

Construct a truth table for $\sim(p \rightarrow \sim q)$.

SOLUTION

Step 1 Set up the table as shown.

p	q	$\sim(p \rightarrow \sim q)$
T	T	
T	F	
F	T	
F	F	

Step 2 Write the truth values for p and q underneath the respective letters in the statement as shown, and label the columns as 1 and 2.

p	q	$\sim(p$	$\rightarrow$	$\sim$	$q)$
T	T	T			T
T	F	T			F
F	T	F			T
F	F	F			F
		1			2

Step 3 Find the negation of q since it is inside the parentheses, and place the truth values in column 3. Draw a line through the truth values in column 2 since they will not be used again.

p	q	$\sim(p$	$\rightarrow$	$\sim$	$q)$
T	T	T		F	~~T~~
T	F	T		T	~~F~~
F	T	F		F	~~T~~
F	F	F		T	~~F~~
		1		3	2

"It is not true that if it rains, then we can't go out," is an example of a statement that can be written as $\sim(p \rightarrow \sim q)$.

Step 4 Find the truth values for the conditional ($\rightarrow$) by using the T and F values in columns 1 and 3 and the conditional truth table from earlier in the section.

Place these values in column 4 and draw a line through the T and F values in columns 1 and 3, as shown.

p	q	$\sim(p$	$\rightarrow$	$\sim$	$q)$
T	T	T	F	F	T
T	F	T	T	T	F
F	T	F	T	F	T
F	F	F	T	T	F
		1	4	3	2

The conditional is true unless p is true and ~q is false.

Step 5 Find the negations of the truth values in column 4 (since the negation sign is outside the parentheses).

p	q	$\sim$	$(p$	$\rightarrow$	$\sim$	$q)$
T	T	T	T	F	F	T
T	F	F	T	T	T	F
F	T	F	F	T	F	T
F	F	F	F	T	T	F
		5	1	4	3	2

The negation has values opposite those in column 4.

The truth value of $\sim(p \rightarrow \sim q)$ is found in column 5. Fortunately, these are the same values we found in Example 2.

Math Note

It isn't necessary to label the columns with numbers or to draw a line through the truth values in the columns when they are no longer needed; however, these two strategies can help reduce errors.

▼ Try This One 6

Construct a truth table for the statement $p \leftrightarrow (\sim p \wedge q)$, using the alternative method.

EXAMPLE 7 Constructing a Truth Table by Using an Alternative Method

Construct a truth table for the statement $p \vee (q \rightarrow r)$.

SOLUTION

Step 1 Set up the table as shown.

p	q	r	$p \vee (q \rightarrow r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Step 2 Recopy the values of p, q, and r under their respective letters in the statement as shown.

p	q	r	p	$\vee$	$(p$	$\to$	$r)$
T	T	T	T		T		T
T	T	F	T		T		F
T	F	T	T		F		T
T	F	F	T		F		F
F	T	T	F		T		T
F	T	F	F		T		F
F	F	T	F		F		T
F	F	F	F		F		F
			1		2		3

Step 3 Using the truth values in columns 2 and 3 and the truth table for the conditional ($\to$), find the values inside the parentheses for the conditional and place them in column 4.

p	q	r	p	$\vee$	$(q$	$\to$	$r)$
T	T	T	T		T	T	T
T	T	F	T		T	F	F
T	F	T	T		F	T	T
T	F	F	T		F	T	F
F	T	T	F		T	T	T
F	T	F	F		T	F	F
F	F	T	F		F	T	T
F	F	F	F		F	T	F
			1		2	4	3

The conditional is true unless q is true and r is false.

Step 4 Complete the truth table, using the truth values in columns 1 and 4 and the table for the disjunction ($\vee$), as shown.

p	q	r	p	$\vee$	$(q$	$\to$	$r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	F
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	F
F	T	T	F	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	T	F	T	T
F	F	F	F	T	F	T	F
			1	5	2	4	3

The disjunction is true unless p and $q \to r$ are both false.

The truth value for $p \vee (q \to r)$ is found in column 5. These are the same values we found in Example 3.

▼ Try This One 7

Construct a truth table for the statement $(p \wedge q) \vee {\sim}r$ using the alternative method.

☑ 5. Construct truth tables by using an alternative method.

The best approach to learning truth tables is to try each of the two methods and see which one is more comfortable for you. In any case, we have seen that truth tables are an effective way to organize truth values for statements, allowing us to determine the truth values of some very complicated statements in a systematic way.

Answers to Try This One

1

p	q	${\sim}q$	$p \vee {\sim}q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

2

p	q	${\sim}p$	${\sim}p \wedge q$	$p \leftrightarrow ({\sim}p \wedge q)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	T

3

p	q	r	$p \wedge q$	${\sim}r$	$(p \wedge q) \vee {\sim}r$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

4
(a) Disjunction; $({\sim}p) \vee q$
(b) Conditional; $(p \vee {\sim}q) \rightarrow r$
(c) Biconditional; $(p \vee q) \leftrightarrow ({\sim}p \vee {\sim}q)$
(d) Conjunction; $p \wedge ({\sim}q)$
(e) Biconditional; $p \leftrightarrow (q \rightarrow r)$

5 True

6

p	q	p	$\leftrightarrow$	$({\sim}$	p	$\wedge$	$q)$
T	T	T	F	F	T	F	T
T	F	T	F	F	T	F	F
F	T	F	F	T	F	T	T
F	F	F	T	T	F	F	F
		1	5	3	1	4	2

7

p	q	r	$(p$	$\wedge$	$q)$	$\vee$	${\sim}$	r
T	T	T	T	T	T	T	F	T
T	T	F	T	T	T	T	T	F
T	F	T	T	F	F	F	F	T
T	F	F	T	F	F	T	T	F
F	T	T	F	F	T	F	F	T
F	T	F	F	F	T	T	T	F
F	F	T	F	F	F	F	F	T
F	F	F	F	F	F	T	T	F
			1	4	2	6	5	3

EXERCISE SET 3-2

Writing Exercises

1. Explain the purpose of a truth table.
2. Explain the difference between the inclusive and exclusive disjunctions.
3. Explain the difference between the conditional and biconditional statements.
4. Describe the hierarchy for the basic connectives.

Computational Exercises

For Exercises 5–34, construct a truth table for each.

5. $\sim(p \vee q)$
6. $q \rightarrow p$
7. $\sim p \wedge q$
8. $\sim q \rightarrow \sim p$
9. $\sim p \leftrightarrow q$
10. $(p \vee q) \rightarrow \sim p$
11. $\sim(p \wedge q) \rightarrow p$
12. $(p \vee q) \wedge (q \wedge p)$
13. $(\sim q \wedge p) \rightarrow \sim p$
14. $q \wedge \sim p$
15. $(p \wedge q) \leftrightarrow (q \vee \sim p)$
16. $p \rightarrow (q \vee \sim p)$
17. $(p \wedge q) \vee p$
18. $(q \rightarrow p) \vee \sim r$
19. $(r \wedge q) \vee (p \wedge q)$
20. $(r \rightarrow q) \vee (p \rightarrow r)$
21. $\sim(p \vee q) \rightarrow \sim(p \wedge r)$
22. $(\sim p \vee \sim q) \rightarrow \sim r$
23. $(\sim p \vee q) \wedge r$
24. $p \wedge (q \vee \sim r)$
25. $(p \wedge q) \leftrightarrow (\sim r \vee q)$
26. $\sim(p \wedge r) \rightarrow (q \wedge r)$
27. $r \rightarrow \sim(p \vee q)$
28. $(p \vee q) \vee (\sim p \vee \sim r)$
29. $p \rightarrow (\sim q \wedge \sim r)$
30. $(q \vee \sim r) \leftrightarrow (p \wedge \sim q)$
31. $\sim(q \rightarrow p) \wedge r$
32. $q \rightarrow (p \wedge r)$
33. $(r \vee q) \wedge (r \wedge p)$
34. $(p \wedge q) \leftrightarrow \sim r$

Real-World Applications

For Exercises 35–40, use the truth value of each simple statement to determine the truth value of the compound statement. Use the Internet if you need help determining the truth value of a simple statement.

35. p: Japan bombs Pearl Harbor.
    q: The United States stays out of World War II.
    Statement: $p \rightarrow q$
36. p: Barack Obama wins the Democratic nomination in 2008.
    q: Mitt Romney wins the Republican nomination in 2008.
    Statement: $p \wedge q$
37. p: NASA sends a manned spacecraft to the Moon.
    q: NASA sends a manned spacecraft to Mars.
    Statement: $p \vee q$
38. p: Michael Phelps wins eight gold medals.
    q: Michael Phelps gets a large endorsement deal.
    Statement: $p \rightarrow q$
39. p: Apple builds a portable MP3 player.
    q: Apple stops making computers.
    r: Microsoft releases the Vista operating system.
    Statement: $(p \vee q) \wedge r$
40. p: Hurricane Katrina hits New Orleans.
    q: New Orleans Superdome is damaged.
    r: New Orleans Saints play home games in 2006 in Baton Rouge.
    Statement: $(p \wedge q) \rightarrow r$

Exercises 41–46 are based on the compound statement below.

A new weight loss supplement claims that if you take the product daily and cut your calorie intake by 10%, you will lose at least 10 pounds in the next 4 months.

41. This compound statement is made up of three simple statements. Identify them and assign a letter to each.
42. Write the compound statement in symbolic form, using conjunctions and the conditional.
43. Construct a truth table for the compound statement you wrote in Exercise 42.
44. If you take this product daily and don't cut your calorie intake by 10%, and then don't lose 10 pounds, is the claim made by the advertiser true or false?
45. If you take the product daily, don't cut your calorie intake by 10%, and do lose 10 pounds, is the claim true or false?
46. If you don't take the product daily, cut your calorie intake by 10%, and do lose 10 pounds, is the claim true or false?

Exercises 47–52 are based on the compound statement below.

The owner of a professional baseball team publishes an open letter to fans after another losing season. He claims that if attendance for the following season is over 2 million, then he will add $20 million to the payroll and the team will make the playoffs the following year.

47. This compound statement is made up of three simple statements. Identify them and assign a letter to each.
48. Write the compound statement in symbolic form, using conjunction and the conditional.
49. Construct a truth table for the compound statement you wrote in Exercise 48.
50. If attendance goes over 2 million the next year and the owner raises payroll by $20 million, but the team fails to make the playoffs, is the owner's claim true or false?
51. If attendance is less than 2 million but the owner still raises the payroll by $20 million and the team makes the playoffs, is the owner's claim true or false?
52. If attendance is over 2 million, the owner doesn't raise the payroll, but the team still makes the playoffs, is the owner's claim true or false?

Critical Thinking

53. Using the hierarchy for connectives, write the statement $p \rightarrow q \vee r$ by using parentheses to indicate the proper order. Then construct truth tables for $(p \rightarrow q) \vee r$ and $p \rightarrow (q \vee r)$. Are the resulting truth values the same? Are you surprised? Why or why not?

54. The hierarchy of connectives doesn't distinguish between conjunctions and disjunctions. Does that matter? Construct truth tables for $(p \vee q) \wedge r$ and $p \vee (q \wedge r)$ to help you decide.
55. In 2003, New York City Council was considering banning indoor smoking in bars and restaurants. Opponents of the ban claimed that it would have a negligible effect on indoor pollution, but a huge negative effect on the economic success of these businesses. Eventually, the ban was enacted, and a 2004 study by the city department of health found that there was a sixfold decrease in indoor air pollution in bars and restaurants, but jobs, liquor licenses, and tax revenues all increased. Assign truth values to all the premises of the opponents' claim; then write the claim as a compound statement and determine its validity.

Section 3-3 Types of Statements

LEARNING OBJECTIVES

- ❑ 1. Classify a statement as a tautology, a self-contradiction, or neither.
- ❑ 2. Identify logically equivalent statements.
- ❑ 3. Write negations of compound statements.
- ❑ 4. Write the converse, inverse, and contrapositive of a statement.

It's no secret that weight loss has become big business in the United States. It seems like almost every week, a new company pops into existence with the latest miracle pill to turn you into a supermodel.

A typical advertisement will say something like "Use of our product may result in significant weight loss." That sounds great, but think about what that statement really means. If use of the product "may" result in significant weight loss, then it also may not result in weight loss at all! The statement could be translated as "You will lose weight or you will not lose weight." Of course, this statement is always true. In this section, we will study statements of this type.

Tautologies and Self-Contradictions

In our study of truth tables in Section 3-2, we saw that most compound statements are true in some cases and false in others. What we have not done is think about whether that's true for *every* compound statement. Some simple examples should be enough to convince you that this is most definitely not the case.

Consider the simple statement "I'm going to Cancun for spring break this year." Its negation is "I'm not going to Cancun for spring break this year." Now think about these two compound statements:

> "I'm going to Cancun for spring break this year, or I'm not going to Cancun for spring break this year."
>
> "I'm going to Cancun for spring break this year, and I'm not going to Cancun for spring break this year."

Hopefully, it's pretty clear to you that the first statement is always true, while the second statement is always false (whether you go to Cancun or not). The first is an example of a *tautology,* while the second is an example of a *self-contradiction*.

When a compound statement is always true, it is called a **tautology.**
When a compound statement is always false, it is called a **self-contradiction.**

CAUTION

Don't make the mistake of thinking that every statement is either a tautology or a self-contradiction. We've seen many examples of statements that are sometimes true and other times false.

The sample statements above are simple enough that it's easy to tell that they are always true or always false based on common sense. But for more complicated statements, we'll need to construct a truth table to decide if a statement is a tautology, a self-contradiction, or neither.

EXAMPLE 1 Using a Truth Table to Classify a Statement

Let p = "I am going to a concert" and q = "I will wear black." Translate each statement in Example 1 into a word statement using this choice of p and q. Can you predict which statements are tautologies, self-contradictions, or neither?

Determine if each statement is a tautology, a self-contradiction, or neither.

(a) $(p \wedge q) \rightarrow p$ (b) $(p \wedge q) \wedge (\sim p \wedge \sim q)$ (c) $(p \vee q) \rightarrow q$

SOLUTION

(a) The truth table for statement (a) is

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Since the truth table value consists of all Ts, the statement is always true, that is, a tautology.

(b) The truth table for statement (b) is

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge \sim q$	$(p \wedge q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	T	F

Since the truth value consists of all Fs, the statement is always false, that is, a self-contradiction.

(c) The truth table for statement (c) is

p	q	$p \vee q$	$(p \vee q) \rightarrow q$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	T

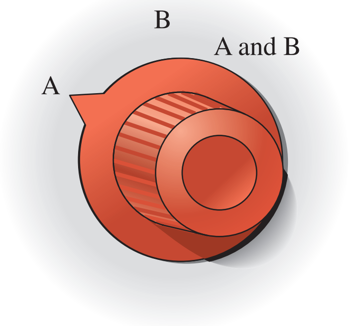
Since the statement can be true in some cases and false in others, it is neither a tautology nor a self-contradiction.

Try This One 1

Determine if each statement is a tautology, a self-contradiction, or neither.

(a) $(p \vee q) \wedge (\sim p \rightarrow q)$ (b) $(p \wedge \sim q) \wedge \sim p$ (c) $(p \rightarrow q) \vee \sim q$

☑ 1. Classify a statement as a tautology, a self-contradiction, or neither.



The statements “If the red dial is set to A, then use only speaker A” and “The red dial is not set to speaker A, or only speaker A is used” can be modeled with the logic statements $p \rightarrow q$ and $\sim p \vee q$.

Logically Equivalent Statements

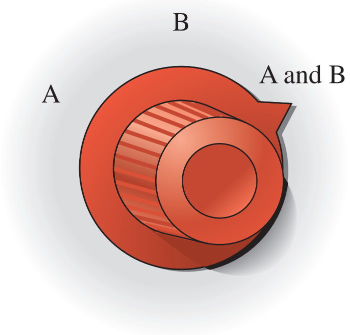
Next, consider the two logical statements $p \rightarrow q$ and $\sim p \vee q$. The truth table for the two statements is

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Notice that the truth values for both statements are *identical,* that is, TFTT. When this occurs, the statements are said to be *logically equivalent;* that is, both compositions of the same simple statements have the same meaning. For example, the statement “If it snows, I will go skiing” is logically equivalent to saying “It is not snowing or I will go skiing.” Formally defined,

> Two compound statements are **logically equivalent** if and only if they have the same truth table values. The symbol for logically equivalent statements is $\equiv$.

EXAMPLE 2 Identifying Logically Equivalent Statements



In the red dial example, $\sim q \rightarrow \sim p$ would be “If speaker A is not the only one used, then the red dial is not set to A.”

☑ 2. Identify logically equivalent statements.

Determine if the two statements $p \rightarrow q$ and $\sim q \rightarrow \sim p$ are logically equivalent.

SOLUTION

The truth table for the statements is

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim q \rightarrow \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Since both statements have the same truth values, they are logically equivalent.

▼ Try This One 2

Determine which two statements are logically equivalent.

(a) $\sim(p \wedge \sim q)$ (b) $\sim p \wedge q$ (c) $\sim p \vee q$

De Morgan’s laws for logic give us an example of equivalent statements.

De Morgan’s Laws for Logic

For any statements p and q,

$\sim(p \vee q) \equiv \sim p \wedge \sim q$ and $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Notice the similarities between De Morgan's laws for sets and De Morgan's laws for logic. De Morgan's laws can be proved by constructing truth tables; the proofs will be left to you in the exercises.

De Morgan's laws are most often used to write the negation of conjunctions and disjunctions. For example, the negation of the statement "I will go to work or I will go to the beach" is "I will not go to work and I will not go to the beach." Notice that when you negate a conjunction, it becomes a disjunction; and when you negate a disjunction, it becomes a conjunction—that is, the *and* becomes an *or,* and the *or* becomes an *and*.

EXAMPLE 3 Using De Morgan's Laws to Write Negations

Write the negations of the following statements, using De Morgan's laws.

(a) Studying is necessary and I am a hard worker.
(b) It is not easy or I am lazy.
(c) I will pass this test or I will drop this class.
(d) She is angry or she's my friend, and she is cool.

SOLUTION

(a) Studying is not necessary or I am not a hard worker.
(b) It is easy and I am not lazy.
(c) I will not pass this test and I will not drop this class.
(d) She is not angry and she is not my friend, or she is not cool.

Try This One 3

Write the negations of the following statements, using De Morgan's laws.

(a) I will study for this class or I will fail.
(b) I will go to the dance club and the restaurant.
(c) It is not silly or I have no sense of humor.
(d) The movie is a comedy or a thriller, and it is awesome.

Earlier in this section, we saw that the two statements $p \to q$ and $\sim p \vee q$ are logically equivalent. Now that we know De Morgan's laws, we can use this fact to find the negation of the conditional statement $p \to q$.

$$\begin{aligned} \sim(p \to q) &\equiv \sim(\sim p \vee q) \\ &\equiv \sim(\sim p) \wedge \sim q \qquad \text{Note: } \sim(\sim p) \equiv p \\ &\equiv p \wedge \sim q \end{aligned}$$

This can be checked by using a truth table as shown.

p	q	$\sim q$	$p \to q$	$\sim(p \to q)$	$p \wedge \sim q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

So the negation of $p \to q$ is $p \wedge \sim q$.

For example, if you say, "It is false that if it is sunny, then I will go swimming," this is equivalent to the statement "It's sunny and I will not go swimming."

EXAMPLE 4 Writing the Negation of a Conditional Statement

Write the negation of the statement "If I have a computer, then I will use the Internet."

SOLUTION

Let p = "I have a computer" and q = "I will use the Internet." The statement $p \rightarrow q$ can be negated as $p \wedge \sim q$. This translates to "I have a computer and I will not use the Internet."

☑ 3. Write negations of compound statements.

Try This One 4

Write the negation of the statement "If the video is popular, then it can be found on YouTube."

Table 3-3 summarizes the negations of the basic compound statements.

TABLE 3-3 Negation of Compound Statements

Statement	Negation	Equivalent Negation
$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
$p \rightarrow q$	$\sim(p \rightarrow q)$	$p \wedge \sim q$

Variations of the Conditional Statement

From the conditional statement $p \rightarrow q$, three other related statements can be formed: the **converse,** the **inverse,** and the **contrapositive.** They are shown here symbolically.

Statement	$p \rightarrow q$
Converse	$q \rightarrow p$
Inverse	$\sim p \rightarrow \sim q$
Contrapositive	$\sim q \rightarrow \sim p$

Using the statement "If Tessa is a chocolate Lab, then Tessa is brown" as our original conditional statement, we find the related statements are as follows:

Converse: If Tessa is brown, then Tessa is a chocolate Lab.

Inverse: If Tessa is not a chocolate Lab, then Tessa is not brown.

Contrapositive: If Tessa is not brown, then Tessa is not a chocolate Lab.

Notice that the original statement is true, but of the three related statements, only the contrapositive is also true. This is an important observation—one that we'll elaborate on shortly.

EXAMPLE 5 Writing the Converse, Inverse, and Contrapositive

Write the converse, the inverse, and the contrapositive for the statement "If you earned a bachelor's degree, then you got a high-paying job."

SOLUTION

It's helpful to write the original implication in symbols: $p \rightarrow q$, where p = "You earned a bachelor's degree" and q = "You got a high-paying job."

Converse: $q \rightarrow p$. "If you got a high-paying job, then you earned a bachelor's degree."

Inverse: $\sim p \rightarrow \sim q$. "If you did not earn a bachelor's degree, then you did not get a high-paying job."

Contrapositive: $\sim q \rightarrow \sim p$. "If you did not get a high-paying job, then you did not earn a bachelor's degree."

▼ Try This One 5

Write the converse, the inverse, and the contrapositive for the statement "If you do well in math classes, then you are intelligent."

Math Note

Many people using logic in real life assume that if a statement is true, the converse is automatically true. Consider the following statement: "If a person earns more than $200,000 per year, that person can buy a Corvette." The converse is stated as "If a person can buy a Corvette, then that person earns more than $200,000 per year." This may be far from the truth since a person may have to make very large payments, live in a tent, or work three jobs in order to afford the expensive car.

The relationships between the variations of the conditional statements can be determined by looking at the truth tables for each of the statements.

p	q	$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Since the original statement ($p \rightarrow q$) and the contrapositive statement ($\sim q \rightarrow \sim p$) have the same truth values, they are equivalent. Also note that the converse ($q \rightarrow p$) and the inverse ($\sim p \rightarrow \sim q$) have the same truth values, so they are equivalent as well. Finally, notice that the original statement is not equivalent to the converse or the inverse since the truth values of the converse and inverse differ from those of the original statement.

Since the conditional statement $p \rightarrow q$ is used so often in logic as well as mathematics, a more detailed analysis is helpful. Recall that the conditional statement $p \rightarrow q$ is also called an *implication* and consists of two simple statements; the first is the *antecedent* p and the second is the *consequent* q. For example, the statement "If I jump into the pool, then I will get wet" consists of the antecedent p, "I jump into the pool," and the consequent q, "I will get wet," connected by the if . . . then connective.

The conditional can also be stated in these other ways:

p implies q

q if p

p only if q

p is sufficient for q

q is necessary for p

All p are q

In four of these six forms, the antecedent comes first, but for "q if p" and "q is necessary for p," the consequent comes first. So identifying the antecedent and consequent is important.

For example, think about the statement "If you drink and drive, you get arrested." Writing it in the different possible forms, we get:

Drinking and driving implies you get arrested.
You get arrested if you drink and drive.
You drink and drive only if you get arrested.
Drinking and driving is sufficient for getting arrested.
Getting arrested is necessary for drinking and driving.
All those who drink and drive get arrested.

Of course, these all say the same thing. To illustrate the importance of getting the antecedent and consequent in the correct order, consider the "q if p" form, in this case "You get arrested if you drink and drive." If we don't start with the consequent, we get "You drink and drive if you get arrested." This is completely false—there are any number of things you could get arrested for other than drinking and driving.

EXAMPLE 6 Writing Variations of a Conditional Statement

Write each statement in symbols. Let p = "A person is over 6′ 6″" and q = "A person is tall."

(a) If a person is over 6′6″, then the person is tall.
(b) Being tall is necessary for being over 6′6″.
(c) A person is over 6′6″ only if the person is tall.
(d) Being 6′6″ is sufficient for being tall.
(e) A person is tall if the person is over 6′6″.

SOLUTION

(a) If p, then q; $p \rightarrow q$
(b) q is necessary for p; $p \rightarrow q$
(c) p only if q; $p \rightarrow q$
(d) p is sufficient for q; $p \rightarrow q$
(e) q if p; $p \rightarrow q$

Actually, these statements all say the same thing!

4. Write the converse, inverse, and contrapositive of a statement.

Try This One 6

Write each statement in symbols. Let p = "A student comes to class every day" and q = "A student gets a good grade."

(a) A student gets a good grade if a student comes to class every day.
(b) Coming to class every day is necessary for getting a good grade.
(c) A student gets a good grade only if a student comes to class every day.
(d) Coming to class every day is sufficient for getting a good grade.

In this section, we saw that some statements are always true (tautologies) and others are always false (self-contradictions). We also defined what it means for two statements to be logically equivalent—they have the same truth values. Now we're ready to analyze logical arguments to determine if they're legitimate or not.

Answers to Try This One

1 (a) Neither (b) Self-contradiction (c) Tautology

2 (a) and (c)

3 (a) I will not study for this class and I will not fail.
(b) I will not go to the dance club or the restaurant.
(c) It is silly and I have a sense of humor.
(d) The movie is not a comedy and it is not a thriller, or it is not awesome.

4 The video is popular and it cannot be found on YouTube.

5 Converse: If you are intelligent, then you do well in math classes.
Inverse: If you do not do well in math classes, then you are not intelligent.
Contrapositive: If you are not intelligent, then you do not do well in math classes.

6 (a) $p \to q$ (b) $q \to p$ (c) $q \to p$ (d) $p \to q$

EXERCISE SET 3-3

Writing Questions

1. Explain the difference between a tautology and a self-contradiction.
2. Is every statement either a tautology or a self-contradiction? Why or why not?
3. Describe how to find the converse, inverse, and contrapositive of a conditional statement.
4. How can you decide if two statements are logically equivalent?
5. How can you decide if one statement is the negation of another?
6. Is a statement always logically equivalent to its converse? Explain.

Computational Exercises

For Exercises 7–16, determine which statements are tautologies, self-contradictions, or neither.

7. $(p \vee q) \vee (\sim p \wedge \sim q)$
8. $(p \to q) \wedge (p \vee q)$
9. $(p \wedge q) \wedge (\sim p \vee \sim q)$
10. $\sim p \vee (p \to q)$
11. $(p \leftrightarrow q) \vee \sim(q \leftrightarrow p)$
12. $(p \wedge q) \leftrightarrow (p \to \sim q)$
13. $(p \vee q) \wedge (\sim p \vee \sim q)$
14. $(p \wedge q) \vee (p \vee q)$
15. $(p \leftrightarrow q) \wedge (\sim p \leftrightarrow \sim q)$
16. $(p \to q) \wedge (\sim p \vee q)$

For Exercises 17–26, determine if the two statements are logically equivalent statements, negations, or neither.

17. $\sim q \to p$; $\sim p \to q$
18. $p \wedge q$; $\sim q \vee \sim p$
19. $\sim(p \vee q)$; $p \to \sim q$
20. $\sim(p \to q)$; $\sim p \wedge q$
21. $q \to p$; $\sim(p \to q)$
22. $p \vee (\sim q \wedge r)$; $(p \wedge \sim q) \vee (p \wedge r)$
23. $\sim(p \vee q)$; $\sim(\sim p \wedge \sim q)$
24. $(p \vee q) \to r$; $\sim r \to \sim(p \vee q)$
25. $(p \wedge q) \vee r$; $p \wedge (q \vee r)$
26. $p \leftrightarrow \sim q$; $(p \wedge \sim q) \vee (\sim p \wedge q)$

For Exercises 27–32, write the converse, inverse, and contrapositive of each.

27. $p \to q$
28. $\sim p \to \sim q$
29. $\sim q \to p$
30. $\sim p \to q$
31. $p \to \sim q$
32. $q \to p$

Real-World Applications

In Exercises 33–42, use De Morgan's laws to write the negation of the statement.

33. The concert is long or it is fun.
34. The soda is sweet or it is not carbonated.
35. It is not cold and I am soaked.
36. I will walk in the Race for the Cure walkathon and I will be tired.
37. I will go to the beach and I will not get sunburned.
38. The coffee is a latte or an espresso.
39. The student is a girl or the professor is not a man.
40. I will go to college and I will get a degree.
41. It is right or it is wrong.
42. Our school colors are not blue or they are not green.

For Exercises 43–49, let p = "I need to talk to my friend" and q = "I will send her a text message." Write each of the following in symbols (see Example 6).

43. If I need to talk to my friend, I will send her a text message.
44. If I will not send her a text message, I do not need to talk to my friend.
45. Sending a text message is necessary for needing to talk to my friend.
46. I will send her a text message if I need to talk to my friend.
47. Needing to talk to my friend is sufficient for sending her a text message.

48. I need to talk to my friend only if I will send her a text message.
49. I do not need to talk to my friend only if I will not send her a text message.
50. Are any of the statements in Exercises 43–49 logically equivalent?

For Exercises 51–56, write the converse, inverse, and contrapositive of the conditional statement.

51. If he graduated with a bachelor's degree in management information systems, then he will get a good job.
52. If she does not earn $5,000 this summer as a barista at the coffeehouse, then she cannot buy the green Ford Focus.
53. If the *American Idol* finale is today, then I will host a party in my dorm room.
54. If my cell phone will not charge, then I will replace the battery.
55. I will go to Nassau for spring break if I lose 10 pounds by March 1.
56. The politician will go to jail if he gets caught taking kickbacks.

Critical Thinking

57. In this section, we wrote the negation of $p \rightarrow q$ by using a disjunction. See if you can write the negation of $p \rightarrow q$ by using a conjunction.
58. Try to write the negation of the biconditional $p \leftrightarrow q$ by using only conjunctions, disjunctions, and negations.
59. Can you think of a true conditional statement about someone you know so that the converse is true as well? How about so that the converse is false?
60. Can you think of a true conditional statement about someone you know so that the inverse is true as well? How about so that the inverse is false?
61. Prove the first of De Morgan's laws by using truth tables:

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

62. Prove the second of De Morgan's laws by using truth tables:

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

Section 3-4 Logical Arguments

LEARNING OBJECTIVES

- ❑ 1. Define *valid argument* and *fallacy*.
- ❑ 2. Use truth tables to determine the validity of an argument.
- ❑ 3. Identify common argument forms.
- ❑ 4. Determine the validity of arguments by using common argument forms.

Common sense is a funny thing in our society: we all think we have it, and we also think that most other people don't. This thing that we call common sense is really the ability to think logically, to evaluate an argument or situation and decide what is and is not reasonable. It doesn't take a lot of imagination to picture how valuable it is to be able to think logically. We're pretty well protected by parents for our first few years of life, but after that the main tool we have to guide us through the perils of life is our brain. The more effectively that brain can analyze and evaluate information, the more successful we're likely to be. The work we've done in building the basics of symbolic logic in the first three sections of this chapter has prepared us for the real point: analyzing logical arguments objectively. That's the topic of this important section.

Valid Arguments and Fallacies

A logical argument is made up of two parts: a premise or premises and a conclusion based on those premises. We will call an argument **valid** if assuming the premises are true guarantees that the conclusion is true as well. An argument that is not valid is called **invalid** or a **fallacy.**

Let's look at an example.

Premise 1:	All students in this class will pass.
Premise 2:	Rachel is a student in this class.
Conclusion:	Rachel will pass this class.

We can easily tell that if the two premises are true, then the conclusion is true, so this is an example of a valid argument.

Sidelight LOGIC AND THE ART OF FORENSICS

Many students find it troubling that an argument can be considered valid even if the conclusion is clearly false. But arguing in favor of something that you don't necessarily believe to be true isn't a new idea by any means—lawyers do it all the time, and it's commonly practiced in the area of formal debate, a style of intellectual competition that has its roots in ancient times.

In formal debate (also known as forensics), speakers are given a topic and asked to argue one side of a related issue. Judges determine which speakers make the most effective arguments and declare the winners accordingly. One of the most interesting aspects is that in many cases, the contestants don't know which side of the issue they will be arguing until right before the competition begins. While that aspect is intended to test the debater's flexibility and preparation, a major consequence is that opinion, and sometimes truth, is taken out of the mix, and contestants and judges must focus on the validity of arguments.

A variety of organizations sponsor national competitions in formal debate for colleges. The largest is an annual championship organized by the National Forensics Association. Students from well over 100 schools participate in a wide variety of categories. The 2008 team champions were Tennessee State University, Kansas State University, California State Long Beach, and Western Kentucky University.

It's very important at this point to understand the difference between a true statement and a conclusion to a valid argument. A statement that is known to be false can still be a valid conclusion if it follows logically from the given premises. For example, consider this argument:

Los Angeles is in California or Mexico.

Los Angeles is not in California.

Therefore, Los Angeles is in Mexico.

1. Define *valid argument* and *fallacy*.

This is a valid argument: if the two premises are true, then the conclusion, "Los Angeles is in Mexico," must be true as well. We know, however, that Los Angeles isn't actually in Mexico. That's the tricky part. In determining whether an argument is valid, *we will always assume that the premises are true*. In this case, we're assuming that the premise "Los Angeles is not in California" is true, even though in fact it is not. We will discuss this aspect of logical arguments in greater depth later in this section.

Truth Table Method

One method for determining the validity of an argument is by using truth tables. We will use the following procedure.

Procedure for Determining the Validity of Arguments

Step 1 Write the argument in symbols.

Step 2 Write the argument as a conditional statement; use a conjunction between the premises and the implication ($\Rightarrow$) for the conclusion. (*Note:* The $\Rightarrow$ is the same as $\rightarrow$ but will be used to designate an argument.)

Step 3 Set up and construct a truth table as follows:

Symbols	Premise $\wedge$ Premise $\Rightarrow$ Conclusion

Step 4 If all truth values under $\Rightarrow$ are Ts (i.e., a tautology), then the argument is valid; otherwise, it is invalid.

EXAMPLE 1 Determining the Validity of an Argument

Determine if the following argument is valid or invalid.

If a figure has three sides, then it is a triangle.
This figure is not a triangle.
Therefore, this figure does not have three sides.

SOLUTION

Step 1 *Write the argument in symbols*. Let p = "The figure has three sides," and let q = "The figure is a triangle."

Translated into symbols:

$p \to q$ (Premise)
$\sim q$ (Premise)
$\therefore \sim p$ (Conclusion)

A line is used to separate the premises from the conclusion and the three triangular dots $\therefore$ mean "therefore."

Step 2 *Write the argument as an implication* by connecting the premises with a conjunction and implying the conclusion as shown.

Premise 1		**Premise 2**		**Conclusion**
$(p \to q)$	$\wedge$	$\sim q$	$\Rightarrow$	$\sim p$

Step 3 *Construct a truth table* as shown.

p	q	$\sim p$	$\sim q$	$p \to q$	$(p \to q) \wedge \sim q$	$[(p \to q) \wedge \sim q] \Rightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Step 4 *Determine the validity of the argument*. Since all the values under the $\Rightarrow$ are true, the argument is valid.

▼ Try This One 1

Determine if the argument is valid or invalid.

I will run for student government or I will join the athletic boosters.
I did not join the athletic boosters.
Therefore, I will run for student government.

EXAMPLE 2 Determining the Validity of an Argument

Determine the validity of the following argument. "If a professor is rich, then he will buy an expensive automobile. The professor bought an expensive automobile. Therefore, the professor is rich."

SOLUTION

Step 1 *Write the argument in symbols*. Let p = "The professor is rich," and let q = "The professor buys an expensive automobile."

$$\begin{array}{l} p \to q \\ \underline{q \qquad} \\ \therefore p \end{array}$$

Step 2 *Write the argument as an implication*.

$$(p \to q) \wedge q \Rightarrow p$$

Step 3 *Construct a truth table* for the argument.

p	q	$p \to q$	$(p \to q) \wedge q$	$[(p \to q) \wedge q] \Rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Step 4 *Determine the validity of the argument*. This argument is invalid since it is not a tautology. (Remember, when the values are not all Ts, the argument is invalid.) In this case, it cannot be concluded that the professor is rich.

Try This One 2

Determine the validity of the following argument. "If John blows off work to go to the playoff game, he will lose his job. John lost his job. Therefore, John blew off work and went to the playoff game."

CAUTION

Remember that in symbolic logic, whether or not the conclusion is true is not important. The main concern is whether the conclusion follows from the premises.

Consider the following two arguments.

1. Either $2 + 2 \neq 4$ or $2 + 2 = 5$

$$\begin{array}{l} \underline{\quad 2 + 2 = 4} \\ \therefore 2 + 2 = 5 \end{array}$$

2. If $2 + 2 \neq 5$, then I passed the math quiz.

$$\begin{array}{l} \underline{\text{I did not pass the quiz.}} \\ \therefore 2 + 2 \neq 5 \end{array}$$

The truth tables on the next page show that the first argument is valid even though the conclusion is false, and the second argument is invalid even though the conclusion is true!

Let p be the statement "2 + 2 = 4" and q be the statement "2 + 2 = 5." Then the first argument is written as

$$\begin{array}{l} (\sim p \vee q) \\ \underline{p \qquad\quad} \\ \therefore\ q \end{array}$$

Truth table for argument 1

p	q	$\sim p$	$\sim p \vee q$	$(\sim p \vee q) \wedge p$	$[(\sim p \vee q) \wedge p] \Rightarrow q$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	F	T
F	F	T	T	F	T

Let p be the statement "2 + 2 ≠ 5" and q be the statement "I passed the math quiz." The second argument is written as

$$\begin{array}{l} p \rightarrow q \\ \underline{\sim q \qquad} \\ \therefore\ p \end{array}$$

Truth table for argument 2

p	q	$\sim q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \sim q$	$[(p \rightarrow q) \wedge \sim q] \Rightarrow p$
T	T	F	T	F	T
T	F	T	F	F	T
F	T	F	T	F	T
F	F	T	T	T	F

The validity of arguments that have three variables can also be determined by truth tables, as shown in Example 3.

EXAMPLE 3 Determining the Validity of an Argument

Determine the validity of the following argument.

$$\begin{array}{l} p \rightarrow r \\ p \wedge r \\ \underline{p \qquad\qquad} \\ \therefore\ \sim q \rightarrow p \end{array}$$

SOLUTION

Step 1 *Write the argument in symbols*. This has been done already.

Step 2 *Write the argument as an implication*. Make a conjunction of all three premises and imply the conclusion:

$$(p \rightarrow r) \wedge (q \wedge r) \wedge p \Rightarrow (\sim q \rightarrow p)$$

Step 3 *Construct a truth table*. When there are three premises, we will begin by finding the truth values for each premise and then work the conjunction from left to right as shown.

p	q	r	$\sim q$	$p \rightarrow r$	$q \wedge r$	$\sim q \rightarrow p$	$(p \rightarrow r) \wedge (q \wedge r) \wedge p$	$[(p \rightarrow r) \wedge (q \wedge r) \wedge p] \Rightarrow (\sim q \rightarrow p)$
T	T	T	F	T	T	T	T	T
T	T	F	F	F	F	T	F	T
T	F	T	T	T	F	T	F	T
T	F	F	T	F	F	T	F	T
F	T	T	F	T	T	T	F	T
F	T	F	F	T	F	T	F	T
F	F	T	T	T	F	F	F	T
F	F	F	T	T	F	F	F	T

Since the truth value for ⇒ is all T 's, the argument is valid.

2. Use truth tables to determine the validity of an argument.

Try This One 3

Determine whether the following argument is valid or invalid.

$$\begin{array}{l} p \lor q \\ \underline{p \lor \sim r} \\ \therefore q \end{array}$$

Common Valid Argument Forms

We have seen that truth tables can be used to test an argument for validity. But some argument forms are common enough that they are recognized by special names. When an argument fits one of these forms, we can decide if it is valid or not just by knowing the general form, rather than constructing a truth table.

We'll start with a description of some commonly used valid arguments.

1. *Law of detachment* (also known by Latin name *modus ponens*):

$$\begin{array}{l} p \to q \\ \underline{p \qquad} \\ \therefore q \end{array}$$

Example:

If our team wins Saturday, then they go to a bowl game.
Our team won Saturday.
Therefore, our team goes to a bowl game.

2. *Law of contraposition* (Latin name *modus tollens*):

$$\begin{array}{l} p \to q \\ \underline{\sim q \qquad} \\ \therefore \sim p \end{array}$$

Example:

If I try hard, I'll get an A.
I didn't get an A.
Therefore, I didn't try hard.

3. *Law of syllogism,* also known as *law of transitivity:*

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$

Example:

If I make an illegal U-turn, I'll get a ticket.
If I get a ticket, I'll get points on my driving record.
Therefore, if I make an illegal U-turn, I'll get points on my driving record.

4. *Law of disjunctive syllogism:*

$$\begin{array}{l} p \lor q \\ \underline{\sim p \quad} \\ \therefore q \end{array}$$

Example:

You're either brilliant or insane.
You're not brilliant.
Therefore, you're insane.

Common Fallacies

Math Note

It's more common to mistakenly think that an invalid argument is valid rather than the other way around, so you should pay special attention to the common fallacies listed.

Next, we will list some commonly used arguments that are invalid.

1. *Fallacy of the converse:*

$$\begin{array}{l} p \to q \\ \underline{q \qquad} \\ \therefore p \end{array}$$

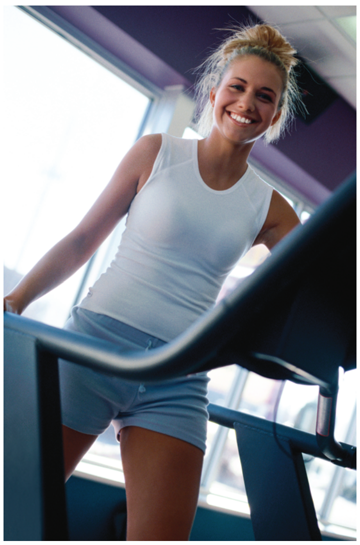
Example:

If it's Friday, then I will go to happy hour.
I am at happy hour.
Therefore, it is Friday.

This is not valid! You can go to happy hour other days, too.

2. *Fallacy of the inverse:*

$$\begin{array}{l} p \to q \\ \underline{\sim p \qquad} \\ \therefore \sim q \end{array}$$



Example:

If I exercise every day, then I will lose weight.
I don't exercise every day.
Therefore, I won't lose weight.

This is also not valid. You could still lose weight without exercising *every* day.

3. *Fallacy of the inclusive or:*

$$\begin{array}{l} p \vee q \\ \underline{p \qquad} \\ \therefore \sim q \end{array}$$

Example:

I'm going to take chemistry or physics.
I'm taking chemistry.
Therefore, I'm not taking physics.

Remember, we've agreed that by *or* we mean *one or the other, or both*. So you could be taking both classes.

You will be asked to prove that some of these are invalid by using truth tables in the exercises.

Sidelight CIRCULAR REASONING

Circular reasoning (sometimes called "begging the question") is a sneaky type of fallacy in which the premises of an argument contain a claim that the conclusion is true, so naturally if the premises are true, so is the conclusion. But this doesn't constitute evidence that a conclusion is true. Consider the following example: A suspect in a criminal investigation tells the police detective that his statements can be trusted because his friend Sue can vouch for him. The detective asks the suspect how he knows that Sue can be trusted, and he says, "I can assure you of her honesty." Ultimately, the suspect becomes even more suspect because of circular reasoning: his argument boils down to "I am honest because I am honest."

While this example might seem blatantly silly, you'd be surprised how often people try to get away with this fallacy. A Google search for the string *circular reasoning* brings up hundreds of arguments that are thought to be circular.

EXAMPLE 4 Recognizing Common Argument Forms

Determine whether the following arguments are valid, using the given forms of valid arguments and fallacies.

(a) $\begin{array}{l} p \rightarrow q \\ \underline{p \qquad} \\ \therefore q \end{array}$ (b) $\begin{array}{l} \sim p \rightarrow q \\ \underline{\sim q \qquad} \\ \therefore p \end{array}$ (c) $\begin{array}{l} \sim p \rightarrow \sim q \\ \underline{\sim q \qquad\quad} \\ \therefore \sim p \end{array}$ (d) $\begin{array}{l} \sim r \rightarrow s \\ \underline{\quad s \rightarrow t \quad} \\ \therefore \sim r \rightarrow t \end{array}$

SOLUTION

(a) This is the law of detachment, therefore a *valid* argument.
(b) This fits the law of contraposition with the statement $\sim p$ substituted in place of p, so it is valid.
(c) This fits the fallacy of the converse, using statement $\sim p$ and $\sim q$ rather than p and q, so it is an invalid argument.
(d) This is the law of syllogism, with statements $\sim r$, s, and t, so the argument is valid.

Try This One 4

3. Identify common argument forms.

Determine whether the arguments are valid, using the commonly used valid arguments and fallacies.

(a) $\begin{array}{l} \sim p \vee q \\ \underline{p \qquad\quad} \\ \therefore q \end{array}$ (b) $\begin{array}{l} r \vee s \\ \underline{s \qquad} \\ \therefore \sim r \end{array}$ (c) $\begin{array}{l} \sim p \rightarrow q \\ \underline{q \qquad\quad} \\ \therefore \sim p \end{array}$ (d) $\begin{array}{l} (p \wedge q) \rightarrow \sim r \\ \underline{r \qquad\qquad\qquad} \\ \therefore \sim(p \wedge q) \end{array}$

EXAMPLE 5 Determining the Validity of an Argument by Using Common Argument Forms

Determine whether the following arguments are valid, using the given forms of valid arguments and fallacies.

(a) If you like dogs, you will live to be 120.
You like dogs.
____
Therefore, you will live to be 120.

(b) If the modem is connected, then you can access the Web.
The modem is not connected.
____
Therefore, you cannot access the Web.

(c) If you watch *Big Brother,* you watch reality shows.
If you watch reality shows, you have time to kill.
___
Therefore, if you have time to kill, you watch *Big Brother*.

(d) The movie *Scream* is a thriller or a comedy.
The movie *Scream* is a thriller.
___
Therefore, the movie *Scream* is not a comedy.

(e) My iPod is in my backpack or it is at my friend's house.
My iPod is not in my backpack.
___
Therefore, my iPod is at my friend's house.

SOLUTION

(a) In symbolic form this argument is $(p \to q) \wedge p \Rightarrow q$. We can see that this is the law of detachment, so the argument is *valid*.
(b) In symbolic form this argument is $(p \to q) \wedge {\sim}p \Rightarrow {\sim}q$. This is the fallacy of the inverse, so the argument is *invalid*.
(c) In symbolic form this argument is $(p \to q) \wedge (q \to r) \Rightarrow (r \to p)$. We know by the law of transitivity that if $(p \to q) \wedge (q \to r)$, then $p \to r$. The given conclusion, $r \to p$, is the converse of $p \to r$, so is not equivalent to $p \to r$ (the valid conclusion). So the argument is *invalid*.
(d) In symbolic form the argument is $(p \vee q) \wedge p \Rightarrow {\sim}q$. This is the fallacy of the inclusive or, so the argument is *invalid*.
(e) In symbolic form the argument is $(p \vee q) \wedge {\sim}p \Rightarrow q$. This is the law of disjunctive syllogism, so the argument is *valid*.

4. Determine the validity of arguments by using common argument forms.

Try This One 5

Determine whether the following arguments are valid using the given forms of valid arguments and fallacies.

(a) If Elliot is a freshman, he takes English.
Elliot is a freshman.
___
Therefore, Elliot takes English.

(b) If you work hard, you will be a success.
You are not a success.
___
Therefore, you do not work hard.

(c) Jon is cheap or financially broke.
He is not financially broke.
___
Therefore, he is cheap.

(d) If Jose asks me out, I will not study Friday.
I didn't study Friday.
___
Therefore, Jose asked me out.

There are two big lessons to be learned in this section. First, sometimes an argument can appear to be legitimate superficially, but if you study it carefully, you may find out that it's not. Second, the validity of an argument is not about whether the conclusion is true or false—it's about whether the conclusion follows logically from the premises.

Answers to Try This One

1 Valid

2 Invalid

3 Invalid

4 (a) Valid (b) Invalid (c) Invalid (d) Valid

5 (a) Valid (b) Valid (c) Valid (d) Invalid

EXERCISE SET 3-4

Writing Exercises

1. Describe the structure of an argument.
2. Is it possible for an argument to be valid, yet have a false conclusion? Explain your answer.
3. Is it possible for an argument to be invalid, yet have a true conclusion? Explain your answer.
4. When you are setting up a truth table to determine the validity of an argument, what connective is used between the premises of an argument? What connective is used between the premises and the conclusion?
5. Describe what the law of syllogism says, in your own words.
6. Describe why the fallacy of the inclusive or is a fallacy.

Computational Exercises

For Exercises 7–16, using truth tables, determine whether each argument is valid.

7. $p \to q$; $\underline{p \wedge q}$; $\therefore p$
8. $p \vee q$; $\underline{\sim q}$; $\therefore p$
9. $\sim p \vee q$; $\underline{p}$; $\therefore p \wedge \sim q$
10. $p \leftrightarrow \sim q$; $\underline{p \wedge \sim q}$; $\therefore p \vee q$
11. $\sim q \vee p$; $\underline{q}$; $\therefore \sim p$
12. $p \vee q$; $\underline{\sim p \wedge \sim q}$; $\therefore p$
13. $p \leftrightarrow q$; $\underline{\sim q}$; $\therefore \sim p$
14. $p \vee \sim q$; $\underline{\sim q \to p}$; $\therefore p$
15. $p \wedge \sim q$; $\underline{\sim r \to q}$; $\therefore q$
16. $p \leftrightarrow q$; $\underline{q \leftrightarrow r}$; $\therefore p \wedge q$
17. Write the law of detachment in symbols; then prove that it is a valid argument by using a truth table.
18. Write the law of contraposition in symbols; then prove that it is a valid argument by using a truth table.
19. Write the law of syllogism in symbols; then prove that it is a valid argument by using a truth table.
20. Write the law of disjunctive syllogism in symbols; then prove that it is a valid argument by using a truth table.
21. Write the fallacy of the converse in symbols; then prove that it is not a valid argument by using a truth table.
22. Write the fallacy of the inclusive or in symbols; then prove that it is not a valid argument by using a truth table.

For Exercises 23–32, determine whether the following arguments are valid, using the given forms of valid arguments and fallacies.

23. $p \to q$; $\underline{\sim q}$; $\therefore \sim p$
24. $p \vee q$; $\underline{q}$; $\therefore \sim p$
25. $\sim p \to q$; $\underline{\sim q}$; $\therefore \sim p$
26. $p \vee \sim q$; $\underline{q}$; $\therefore p$
27. $p \to q$; $\underline{r \to \sim q}$; $\therefore p \to \sim r$
28. $p \to \sim q$; $\underline{\sim q}$; $\therefore p$
29. $\sim p \vee q$; $\underline{\sim q}$; $\therefore \sim p$
30. $\sim p \to q$; $\underline{\sim q}$; $\therefore p$
31. $p \vee \sim q$; $\underline{q}$; $\therefore \sim p$
32. $p \to \sim q$; $\underline{\sim r \to q}$; $\therefore p \to r$

Real-World Applications

For Exercises 33–42, identify p, q, and r if necessary. Then translate each argument to symbols and use a truth table to determine whether the argument is valid or invalid.

33. If it rains, then I will watch the Real World marathon.
    It did not rain.
    ∴ I did not watch the Real World marathon.
34. Ted will get a Big Mac or a Whopper with cheese.
    Ted did not get a Whopper with cheese.
    ∴ Ted got a Big Mac.
35. If Julia uses monster.com to send out her resume, she will get an interview.
    Julia got an interview.
    ∴ Julia used monster.com to send out her resume.
36. If it snows, I can go snowboarding.
    It did not snow.
    ∴ I cannot go snowboarding.
37. I will go to the party if and only if my ex-boyfriend is not going.
    My ex-boyfriend is not going to the party.
    ∴ I will go to the party.
38. If you are superstitious, then do not walk under a ladder.
    If you do not walk under a ladder, then you are superstitious.
    ∴ You are superstitious and you do not walk under a ladder.
39. Either I did not study or I passed the exam.
    I did not study.
    ∴ I failed the exam.
40. I will run the marathon if and only if I can run 30 miles by Christmas.
    I can run 30 miles by Christmas or I will not run the marathon.
    ∴ If I ran the marathon, then I was able to run 30 miles by Christmas.
41. If you eat seven pieces of pizza, then you will get sick.
    If you get sick, then you will not be able to go to the game.
    ∴ If you eat seven pieces of pizza, then you will not be able to go to the game.
42. If you back up your hard drive, then you are protected.
    Either you are protected or you are daring.
    ∴ If you are daring, you won't back up your hard drive.

For Exercises 43–50, write the argument in symbols; then decide whether the argument is valid by using the common forms of valid arguments and fallacies.

43. If the cheese doesn't melt, the nachos are ruined.
    The nachos are ruined.
    ∴ The cheese didn't melt.
44. I studied or I failed the class.
    I did not fail the class.
    ∴ I studied.
45. If I go to the student symposium on environmental issues, I will fall asleep.
    If the speaker is interesting, I will not fall asleep.
    ∴ If I go to the student symposium on environmental issues, the speaker will not be interesting.
46. If it is sunny, I will wear SPF 50 sun block.
    It is not sunny.
    ∴ I will not wear SPF 50 sun block.
47. If I get an A in this class, I will do a dance.
    I did a dance.
    ∴ I got an A in this class.
48. I will not wear a Speedo at the beach or I will be embarrassed.
    I am not embarrassed.
    ∴ I did not wear a Speedo at the beach.
49. If I see the movie at an IMAX theater, I will get dizzy.
    I see the movie at an IMAX theater.
    ∴ I get dizzy.
50. I will backpack through Europe if I get at least a 3.5 grade point average.
    I do not get at least a 3.5 grade point average.
    ∴ I will not backpack through Europe.

Critical Thinking

51. Oscar Wilde once said, "Few parents nowadays pay any regard to what their children say to them. The old-fashioned respect for the young is fast dying out." This statement can be translated to an argument, as shown next.
    If parents respected their children, then parents would listen to them.
    Parents do not listen to their children.
    ∴ Parents do not respect their children.

Using a truth table, determine whether the argument is valid or invalid.

52. Winston Churchill once said, "If you have an important point to make, don't try to be subtle or clever. Use a pile driver. Hit the point once. Then come back and hit it again. Then a third time—a tremendous wack!" This statement can be translated to an argument as shown.

If you have an important point to make, then you should not be subtle or clever.

You are not being subtle or clever.
∴ You will make your point.

Using a truth table, determine whether the argument is valid or invalid.

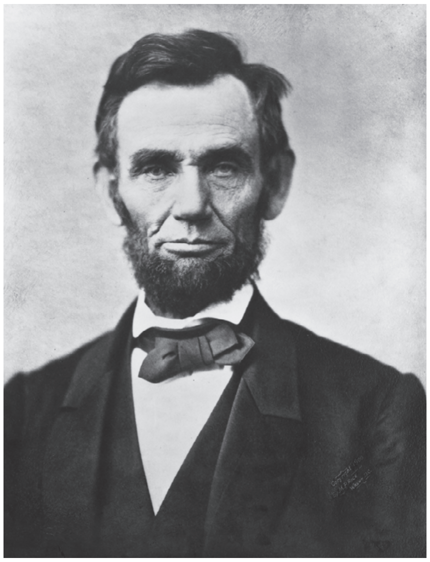
53. Write an argument matching the law of syllogism that involves something about your school. Then explain why the conclusion of your argument is valid.
54. Write an example of the fallacy of the inverse that involves something about your school. Then explain why the conclusion of your argument is invalid.
55. Look up the literal translation of the Latin term *modus ponens* on the Internet, and explain how that applies to the law of detachment.
56. Look up the literal translation of the Latin term *modus tollens* on the Internet, and explain how that applies to the law of contraposition.

Section 3-5 Euler Circles

LEARNING OBJECTIVES

- ❑ 1. Define *syllogism*.
- ❑ 2. Use Euler circles to determine the validity of an argument.

Abraham Lincoln once said, "You can fool some of the people all of the time, and all of the people some of the time, but you cannot fool all of the people all of the time." Lincoln was a really smart guy—he understood the power of logical arguments and the fact that cleverly crafted phrases could be an effective tool in the art of persuasion. What's interesting about this quote from our perspective is the liberal use of the quantifiers *some* and *all*. In this section, we will study a particular type of argument that uses these quantifiers, along with *no* or *none*. A technique developed by Leonhard Euler way back in the 1700s is a useful method for analyzing these arguments and testing their validity.

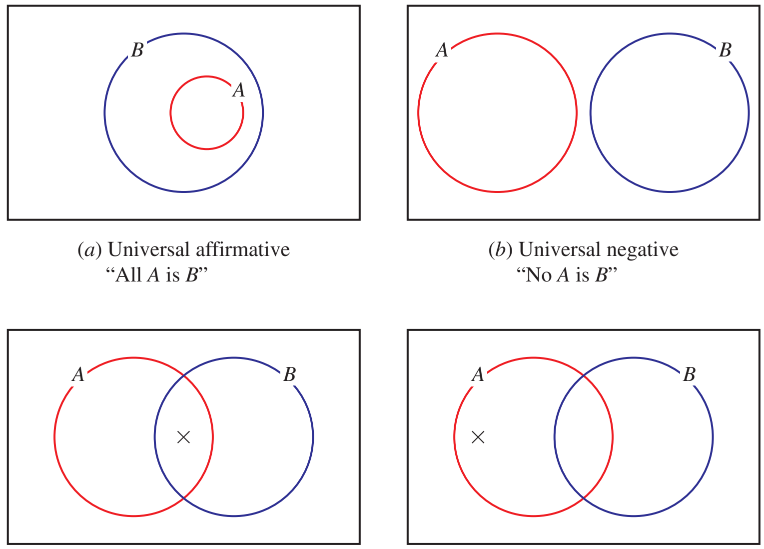


Euler circles are diagrams similar to Venn diagrams. We will use them to study arguments using four types of statements. The statement types are listed in Table 3-4, and the Euler circle that illustrates each is shown in Figure 3-1 on the next page.

Each statement can be represented by a specific diagram. The universal affirmative "All A is B" means that every member of set A is also a member of set B. For example,

TABLE 3-4

Type	General Form	Example
Universal affirmative	All A is B	All chickens have wings.
Universal negative	No A is B	No horses have wings.
Particular affirmative	Some A is B	Some horses are black.
Particular negative	Some A is not B	Some horses are not black.



(*a*) Universal affirmative
"All *A* is *B*"

(*b*) Universal negative
"No *A* is *B*"

(*c*) Particular affirmative
"Some *A* is *B*"

(*d*) Particular negative
"Some *A* is not *B*"

Figure 3-1

Math Note

If I say that "Some horses are black," you cannot assume that "Some horses are not black." If I say, "Some horses are not black," you cannot assume that "Some horses are black."

the statement "All chickens have wings" means that the set of all chickens is a subset of the set of animals that have wings.

The universal negative "No *A* is *B*" means that no member of set *A* is a member of set *B*. In other words, set *A* and set *B* are *disjoint sets*. For example, "No horses have wings" means that the set of all horses and the set of all animals with wings are disjoint (nonintersecting).

The particular affirmative "Some *A* is *B*" means that there is at least one member of set *A* that is also a member of set *B*. For example, the statement "Some horses are black"means that there is at least one horse that is a member of the set of black animals. The × in Figure 3-1(*c*) means that there is at least one black horse.

The particular negative "Some *A* is not *B*" means that there is at least one member of set *A* that is not a member of set *B*. For example, the statement "Some horses are not black" means that there is at least one horse that does not belong to the set of black animals. The diagram for the particular negative is shown in Figure 3-1(*d*). The × is placed in circle *A* but not in circle *B*. The × in this example means that there exists at least one horse that is some color other than black.

☑ 1. Define *syllogism*.

Many of the arguments we studied in Section 3-4 consisted of two premises and a conclusion. This type of argument is called a **syllogism.** We will use Euler circles to test the validity of syllogisms involving the statement types in Table 3-4. Here's a simple example:

Premise	All cats have four legs.
Premise	Some cats are black.
Conclusion	Therefore, some four-legged animals are black.

Remember that we are not concerned with whether the conclusion is true or false, but only whether the conclusion logically follows from the premises. If yes, the argument is valid. If no, the argument is invalid.

Euler Circle Method for Testing the Validity of an Argument

To determine whether an argument is valid, diagram both premises in the same figure. If the conclusion is shown in the figure, the argument is valid.

Many times the premises can be diagrammed in several ways. If there is even one way in which the diagram contradicts the conclusion, the argument is *invalid* since the conclusion does not necessarily follow from the premises.

Examples 1, 2, and 3 show how to determine the validity of an argument by using Euler circles.

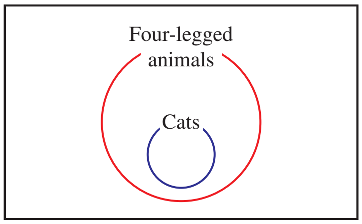
EXAMPLE 1 Using Euler Circles to Determine the Validity of an Argument

Use Euler circles to determine whether the argument is valid.

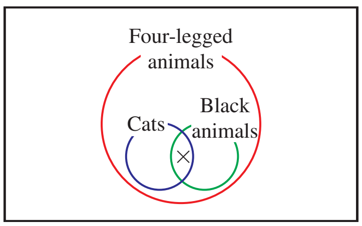
All cats have four legs.
Some cats are black.
___
Therefore, some four-legged animals are black.

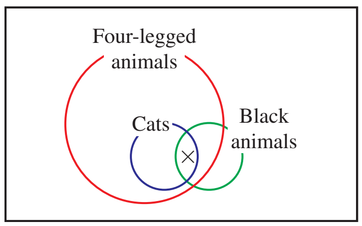
SOLUTION

The first premise, "All cats have four legs," is the universal affirmative; the set of cats diagrammed as a subset of four-legged animals is shown.



The second premise, "Some cats are black," is the particular affirmative and is shown by placing an × in the intersection of the cats' circle and the black animals' circle. The diagram for this premise is drawn on the diagram of the first premise and can be done in two ways, as shown.





The conclusion is that some four-legged animals are black, so the diagram for the conclusion must have an × in the four-legged animals' circle and in the black animals' circle. Notice that both of the diagrams corresponding to the premises do have an × in both circles, so the conclusion matches the premises and the argument is valid. Since there is no other way to diagram the premises, the conclusion is shown to be true without a doubt.

▼ Try This One 1

Use Euler circles to determine whether the argument is valid.

All college students buy textbooks.
Some book dealers buy textbooks.
___
Therefore, some college students are book dealers.

It isn't necessary to use actual subjects such as cats, four-legged animals, etc. in syllogisms. Arguments can use letters to represent the various sets, as shown in Example 2.

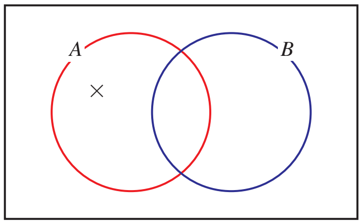
EXAMPLE 2 Using Euler Circles to Determine the Validity of an Argument

Use Euler circles to determine whether the argument is valid or invalid.

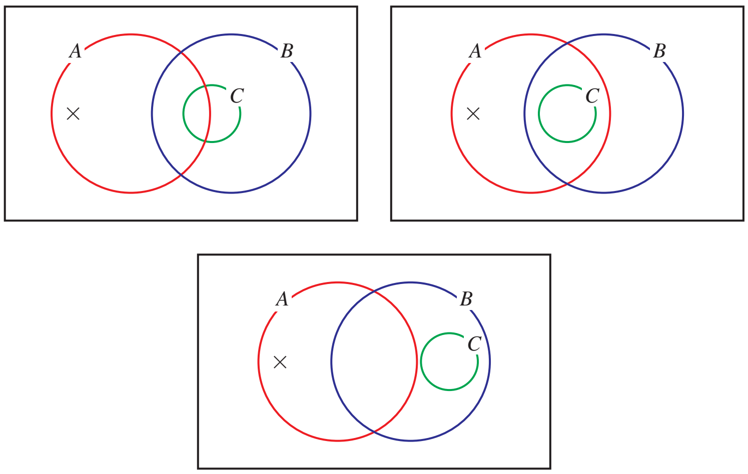
Some A is not B.
All C is B.
———
$\therefore$ Some A is C.

SOLUTION

The first premise, "Some A is not B," is diagrammed as shown.



The second premise, "All C is B," is diagrammed by placing circle C inside circle B. This can be done in several ways, as shown.



The third diagram shows that the argument is invalid. It matches both premises, but there are no members of A that are also in C, so it contradicts the conclusion "Some A is C."

▼ Try This One 2

Use Euler circles to determine whether the argument is valid.

Some A is B.
Some A is not C.
———
$\therefore$ Some B is not C.

Let's try one more specific example.

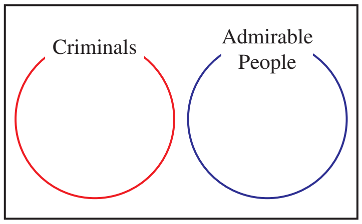
EXAMPLE 3 Using Euler Circles to Determine the Validity of an Argument

Use Euler circles to determine whether the argument is valid.

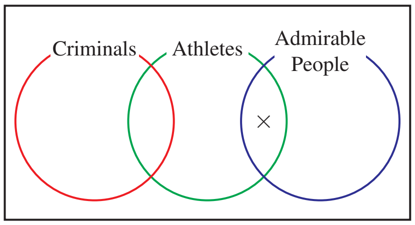
No criminal is admirable.
Some athletes are not criminals.
∴ Some admirable people are athletes.

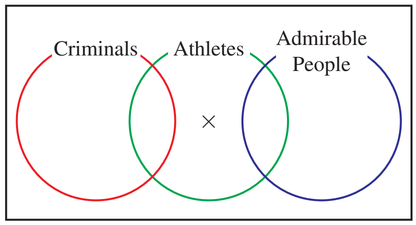
SOLUTION

Diagram the first premise, "No criminal is admirable."



We can add the second premise, "Some athletes are not criminals," in at least two different ways:





In the first diagram, the conclusion appears to be valid: some athletes are admirable. But the second diagram doesn't support that conclusion, so the argument is invalid.

Try This One 3

Use Euler circles to determine whether the argument is valid.

All dogs bark.
No animals that bark are cats.
∴ No dogs are cats.

2. Use Euler circles to determine the validity of an argument.

We have now seen that for syllogisms that involve the quantifiers *all, some,* or *none,* Euler circles are an efficient way to determine the validity of the argument. We diagram both premises on the same figure, and if all possible diagrams display the conclusion, then the conclusion must be valid.

Answers to Try This One

1 Invalid

2 Invalid

3 Valid

EXERCISE SET 3-5

Writing Exercises

1. Name and give an example of each of the four types of statements that can be diagrammed with Euler circles.
2. Explain how to decide whether an argument is valid or invalid after drawing Euler circles.
3. What is a syllogism?
4. How do Euler circles differ from Venn diagrams?

Computational Exercises

For Exercises 5–14, draw an Euler circle diagram for each statement.

5. All computers are calculators.
6. No unicorns are real.
7. Some people do not go to college.
8. Some CD burners are DVD burners.
9. No math courses are easy.
10. Some fad diets do not result in weight loss.
11. Some laws in the United States are laws in Mexico.
12. All members of Mensa are smart.
13. No cheeseburgers are low in fat.
14. Some politicians are crooks.

For Exercises 15–24, determine whether each argument is valid or invalid.

15. All X is Y.
    Some Y is Z.
    ∴ Some X is Z.
16. Some A is not B.
    No B is C.
    ∴ Some A is not C.
17. Some P is Q.
    No Q is R.
    ∴ Some P is not R.
18. All S is T.
    No S is R.
    ∴ Some T is R.
19. No M is N.
    No N is O.
    ∴ Some M is not O.
20. Some U is V.
    Some U is not W.
    ∴ No W is U.
21. Some A is not B.
    No A is C.
    ∴ Some A is not C.
22. All P is Q.
    All Q is R.
    ∴ All P is R.
23. No S is T.
    No T is R.
    ∴ No S is R.
24. Some M is N.
    Some N is O.
    ∴ Some M is O.

Real-World Applications

For Exercises 25–38, use Euler circles to determine if the argument is valid.

25. All phones are communication devices.
    Some communication devices are inexpensive.
    ∴ Some phones are inexpensive.
26. Some students are overachievers.
    No overachiever is lazy.
    ∴ Some students are not lazy.
27. Some animated movies are violent.
    No kids' movies are violent.
    ∴ No kids' movies are animated.
28. Some protesters are angry.
    Some protesters are not civil.
    ∴ Some civil people are not angry.
29. Some math tutors are patient.
    No patient people are demeaning.
    ∴ Some math tutors are not demeaning.
30. Some students are hard-working.
    Some hard-working people are not successful.
    ∴ Some students are not successful.
31. Some movie stars are fake.
    No movie star is talented.
    ∴ No fake people are talented.

32. Some CEOs are women.
    Some women are tech-savvy.
    ∴ Some CEOs are not tech-savvy.
33. Some juices have antioxidants.
    Some fruits have antioxidants.
    ∴ No juices are fruits.
34. Some funny people are sad.
    No serious people are sad.
    ∴ No funny people are serious.
35. Some women have highlighted hair.
    All women watch soap operas.
    All people who watch soap operas are emotional.
    ∴ Some emotional people watch soap operas.
36. All students text message during class.
    Some students in class take notes.
    All students who take notes in class pass the test.
    ∴ Some students pass the test.
37. Some birds can talk.
    Some animals that can talk can also moo.
    All cows can moo.
    ∴ Some cows can talk.
38. All cars use gasoline.
    All things that use gasoline emit carbon dioxide.
    Some cars have four doors.
    ∴ Some things with four doors emit carbon dioxide.

Critical Thinking

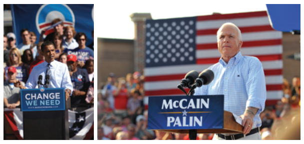
For Exercises 39–42, write a conclusion so that the argument is valid. Use Euler circles.

39. All A is B.
    All B is C.
    ∴
40. No M is P.
    All S is M.
    ∴
41. All calculators can add.
    No adding machines can make breakfast.
    ∴
42. Some people are prejudiced.
    All people have brains.
    ∴

CHAPTER 3 Summary

Section	Important Terms	Important Ideas
3-1	Statement Simple statement Compound statement Connective Conjunction Disjunction Conditional Biconditional Negation	**Formal** symbolic logic uses statements. A statement is a sentence that can be determined to be true or false but not both. A simple statement contains only one idea. A compound statement is formed by joining two or more simple statements with connectives. The four basic connectives are the conjunction (which uses the word *and* and the symbol $\wedge$), the disjunction (which uses the word *or* and the symbol $\vee$), the conditional (which uses the words *if . . . then* and the symbol $\rightarrow$), and the biconditional (which uses the words *if and only if* and the symbol $\leftrightarrow$). The symbol for negation is $\sim$. Statements are usually written using logical symbols and letters of the alphabet to represent simple statements.
3-2	Truth table	**A** truth table can be used to determine when a compound statement is true or false. A truth table can be constructed for any logical statement.
3-3	Tautology Self-contradiction Logically equivalent statements Converse Inverse Contrapositive De Morgan's laws	**A** statement that is always true is called a tautology. A statement that is always false is called a self-contradiction. Two statements that have the same truth values are said to be logically equivalent. De Morgan's laws are used to find the negation of a conjunction or disjunction. From the conditional statement, three other statements can be made: the converse, the inverse, and the contrapositive.
3-4	Argument Premise Conclusion	**Truth** tables can be used to determine the validity of an argument. An argument consists of two or more statements called premises and a statement called the conclusion. An argument is valid if when the premises are true, the conclusion is true. Otherwise, the argument is invalid.
3-5	Syllogism Euler circles Universal affirmative Universal negative Particular affirmative Particular negative	**A** mathematician named Leonhard Euler developed a method using circles to determine the validity of an argument that is particularly effective for syllogisms involving quantifiers. This method uses four types of statements: (1) the universal affirmative, (2) the universal negative, (3) the particular affirmative, and (4) the particular negative. Euler circles are similar to Venn diagrams.

MATH IN The Art of Persuasion REVISITED



All the arguments on the list are logically invalid except the last one. Even though the claim made by the last argument is false—it did not snow in Hawaii in 2008—this is so because the first premise is false. Remember, the validity of an argument is based on whether it follows from the premises, not on its actual truth.

The first is invalid because there are sources of smoke other than fire. The second is invalid because it's possible to be happy for some reason other than having lots of money. The third is invalid because the initial statement says nothing about teams not in the SEC. The fourth is invalid because it is circular reasoning (see Sidelight in Section 3-4). The fifth is invalid because the first statement doesn't say that weapons of mass destruction are the *only* reason to go to war.

Review Exercises

Section 3-1

For Exercises 1–5, decide whether the sentence is a statement.

1. Let's go with the flow.
2. My duvet cover is indigo.
3. The girls at this school are smart.
4. Ignorance is always a choice.
5. Are we there yet?

For Exercises 6–10, decide whether each statement is simple or compound.

6. Andre is interesting and caring.
7. The monitor is blinking.
8. If it is not raining, I will go kiteboarding.
9. The book is stimulating or informative.
10. There is a silver lining behind every cloud.

For Exercises 11–20, write the negation of the statement.

11. It is scary.
12. The cell phone is out of juice.
13. The Popsicle is green.
14. No people who live in glass houses throw stones.
15. Some failing students can learn new study methods.
16. Everyone will pass the test on logic.
17. There is a printer that has no ink.
18. None of these links are broken.
19. At least one of the contestants will be voted off the island.
20. All SUVs are gas guzzlers.

For Exercises 21–25, classify the statement as a conjunction, disjunction, conditional, or biconditional.

21. In the department store the air is scented and stuffy.
22. If you dream it, you can achieve it.
23. I will go to the 7-Eleven or to the IHOP.
24. It is dangerous if and only if there is ice on the sidewalk.
25. I will slip if I walk on the icy sidewalk.

For Exercises 26–35, let p = "It is ambitious" and let q = "It is worthwhile." Write each statement in symbols.

26. It is ambitious and worthwhile.
27. If it is worthwhile, then it is ambitious.
28. It is worthwhile if and only if it is ambitious.
29. It is worthwhile and not ambitious.
30. If it is not ambitious, then it is not worthwhile.
31. It is not true that it is worthwhile and ambitious.
32. It is not true that if it is ambitious, then it is worthwhile.
33. It is not worthwhile if and only if it is not ambitious.
34. It is not true that it is not worthwhile.
35. It is neither ambitious nor worthwhile.

For Exercises 36–40, let p = "It is cool." Let q = "It is cloudy." Write each statement in words.

36. $p \vee \sim q$
37. $q \rightarrow p$
38. $p \leftrightarrow q$
39. $(p \vee q) \rightarrow p$
40. $\sim(\sim p \vee q)$

Section 3-2

For Exercises 41–48, construct a truth table for each statement.

41. $p \leftrightarrow \sim q$
42. $\sim p \rightarrow (\sim q \vee p)$
43. $(p \rightarrow q) \wedge \sim q$
44. $\sim p \vee (\sim q \rightarrow p)$
45. $\sim q \leftrightarrow (p \rightarrow q)$
46. $(p \rightarrow \sim q) \vee r$
47. $(p \vee \sim q) \wedge r$
48. $r \rightarrow (\sim p \vee q)$

For Exercises 49–52, use the truth value of each simple statement to determine the truth value of the compound statement.

49. p: January is the first month of the year.
    q: It snows in January in every state.
    Statement: $p \rightarrow q$

50. p: Gas prices reached record highs in 2008.
    q: Automobile makers started making more fuel-efficient cars.
    Statement: $p \wedge q$

51. p: Barack Obama was a Presidential candidate in 2008.
    q: Lindsay Lohan was a Presidential candidate in 2008.
    r: Obama won the Democratic nomination in 2008.
    Statement: $(p \vee q) \rightarrow r$

52. p: Attending college costs thousands of dollars.
    q: Lack of education does not lead to lower salaries.
    r: The average college graduate will make back more than they paid for school.
    Statement: $(p \wedge \sim q) \leftrightarrow r$

Section 3-3

For Exercises 53–57, determine if the statement is a tautology, self-contradiction, or neither.

53. $p \rightarrow (p \vee q)$
54. $(p \rightarrow q) \rightarrow (p \vee q)$
55. $(p \wedge \sim q) \leftrightarrow (q \wedge \sim p)$
56. $q \rightarrow (p \vee \sim p)$
57. $(\sim q \vee p) \wedge q$

For Exercises 58–60, determine whether the two statements are logically equivalent.

58. $\sim(p \rightarrow q)$; $\sim p \wedge \sim q$
59. $\sim p \vee \sim q$; $\sim(p \leftrightarrow q)$
60. $(\sim p \wedge q) \vee r$; $(\sim p \vee r) \wedge (q \vee r)$

For Exercises 61–64, use De Morgan's laws to write the negation of each statement.

61. The Internet connection is either dial-up or DSL.
62. We will increase sales or our profit margin will go down.
63. The signature is not authentic and the check is not valid.
64. It is not strenuous and I am tired.

For Exercises 65 and 66, assign a letter to each simple statement and write the compound statement in symbols.

65. I will be happy only if I get rich.
66. Having a good career is sufficient for a fulfilling life.

For Exercises 67–69, write the converse, inverse, and contrapositive of the statement.

67. If gas prices go any higher, I will start riding my bike to work.
68. If I don't pass this class, my parents will kill me.
69. The festival will move inside the student center only if it rains.

Section 3-4

For Exercises 70–73, use truth tables to determine whether each argument is valid or invalid.

70. $p \to \sim q$
    $\underline{\sim q \leftrightarrow \sim p}$
    $\therefore p$

71. $\sim q \vee p$
    $\underline{p \wedge q}$
    $\therefore \sim q \leftrightarrow p$

72. $\sim p \vee q$
    $\underline{q \vee \sim r}$
    $\therefore q \to (\sim p \wedge \sim r)$

73. $\sim r \to \sim p$
    $\underline{\sim q \vee \sim r}$
    $\therefore p \leftrightarrow q$

For Exercises 74–77, write the argument in symbols; then use a truth table to determine if the argument is valid.

74. If I go to Barnes & Noble, then I will buy the latest Stephen King novel.
    I did not buy the latest Stephen King novel.
    ∴ I did not go to Barnes and Noble.

75. I'm going to Wal-Mart and McDonald's.
    If I go to McDonald's, I will get the sweet tea.
    ∴ I did not get sweet tea and go to Wal-Mart.

76. If we don't hire two more workers, the union will strike.
    If the union strikes, our profit will not increase.
    We hired two more workers.
    ∴ Our profit will not increase.

77. If I don't study, I won't make honor roll.
    I made the honor roll.
    ∴ I didn't study or I cheated.

For Exercises 78–81, use the commonly used forms of arguments from Section 3-4 to determine if the argument is valid.

78. I will drink a mineral water or a Gatorade.
    I drank a Gatorade.
    ∴ I did not drink mineral water.

79. If it is early, I will get tickets to the comedy club.
    It is not early.
    ∴ I will not get tickets to the comedy club.

80. If I am cold, I will wear a sweater.
    If I wear a sweater, I am warm.
    ∴ If I am cold, I am warm.

81. If pigs fly, then I'm a monkey's uncle.
    Pigs fly or birds don't sing.
    Birds do sing.
    ∴ I'm a monkey's uncle.

Section 3-5

For Exercises 82–86, use Euler circles to determine whether the argument is valid or invalid.

82. No *A* is *B*.
    Some *B* is *C*.
    ∴ No *A* is *C*.

83. Some *A* is not *C*.
    Some *B* is not *C*.
    ∴ Some *A* is not *B*.

84. All money is green.
    All grass is green.
    ∴ Grass is money.

85. No humans have three eyes.
    Some Martians have three eyes.
    ∴ No Martians are humans.

86. Some movies are rated "R."
    All R-rated movies are inappropriate for children.
    ∴ Some movies are inappropriate for children.

Chapter Test

For Exercises 1–4, decide whether the sentence is a statement.

1. I'm going to the karaoke club.
2. $4 + 7 = 10$
3. That woman is really smart.
4. Don't let it get to you.

For Exercises 5–8, write the negation of the statement.

5. The image is uploading to my online bio.
6. All men have goatees.
7. Some students ride a bike to school.
8. No short people can dunk a basketball.

For Exercises 9–14, let p = "It is warm." Let q = "It is sunny." Write each statement in symbols.

9. It is warm and sunny.
10. If it is sunny, then it is warm.
11. It is warm if and only if it is sunny.
12. It is warm or sunny.
13. It is false that it is not warm and sunny.
14. It is not sunny, and it is not warm.

For Exercises 15–19, let p = "It is sunny." Let q = "It is warm." Write each in words.

15. $p \vee \sim q$
16. $q \rightarrow p$
17. $p \leftrightarrow q$
18. $(p \vee q) \rightarrow p$
19. $\sim(\sim p \vee q)$

For Exercises 20–24, construct a truth table for each statement.

20. $p \rightarrow \sim q$
21. $(p \rightarrow \sim q) \wedge r$
22. $(p \wedge \sim q) \vee \sim r$
23. $(\sim q \vee p) \wedge p$
24. $p \rightarrow (\sim q \vee r)$

For Exercises 25–29, determine whether each statement is a tautology, self-contradiction, or neither.

25. $(p \wedge q) \wedge \sim p$
26. $(p \vee q) \rightarrow (p \rightarrow q)$
27. $(p \vee \sim q) \leftrightarrow (p \rightarrow \sim q)$
28. $q \wedge (p \vee \sim p)$
29. $\sim(p \wedge q) \vee p$

For Exercises 30–31, determine if the two statements are logically equivalent.

30. $p; \sim(\sim p)$
31. $(p \vee q) \wedge r; (p \wedge r) \vee (q \wedge r)$
32. Write the converse, inverse, and contrapositive for the statement "If I exercise regularly, then I will be healthy."

For Exercises 33–34 use De Morgan's laws to write the negation of the compound statement.

33. It is not cold and it is snowing.
34. I am hungry or thirsty.

For Exercises 35–38, use truth tables to determine the validity of each argument.

35.
$$\begin{array}{l} p \leftrightarrow \sim q \\ \underline{\sim q \rightarrow \sim p} \\ \therefore p \end{array}$$

36.
$$\begin{array}{l} p \rightarrow q \\ \underline{\sim q \vee \sim r} \\ \therefore q \leftrightarrow (\sim p \wedge \sim r) \end{array}$$

37.
$$\begin{array}{l} \sim q \vee p \\ \underline{p \vee q} \\ \therefore \sim q \rightarrow p \end{array}$$

38.
$$\begin{array}{l} \sim p \rightarrow \sim r \\ \underline{\sim r \vee \sim q} \\ \therefore q \leftrightarrow p \end{array}$$

For Exercises 39–41, determine if the argument is valid or invalid by using the given forms of valid arguments and fallacies.

39. If I finish my paper early, I will have my professor proofread it.
    I have my professor proofread my paper.
    ∴ I finish my paper early.
40. The scratch-off ticket is a winner or a loser.
    The scratch-off ticket is a winner.
    ∴ The scratch-off ticket is not a loser.
41. If Starbuck's is not too busy, I will study there.
    If Starbuck's is too busy, I will study at the library.
    ∴ If I do not study at Starbuck's, I will study at the library.

For Exercises 42–45, use Euler circles to determine whether the argument is valid or invalid.

42. No *B* is *A*.
    Some *A* is *C*.
    ∴ No *B* is *C*.
43. Some *C* is not *A*.
    Some *B* is not *A*.
    ∴ Some *C* is not *B*.
44. No good relationship has bad moments.
    Fred and Suzie have a good relationship.
    ∴ Fred and Suzie never have bad moments.
45. Some computer mice are wireless.
    All computer devices that are wireless are expensive.
    ∴ Some computer mice are expensive.

Projects

1. Truth tables are related to Euler circles. Arguments in the form of Euler circles can be translated into statements by using the basic connectives and the negation as follows:

   Let p be "The object belongs to set A." Let q be "The object belongs to set B."

   All A is B is equivalent to $p \rightarrow q$.
   No A is B is equivalent to $p \rightarrow \sim q$.
   Some A is B is equivalent to $p \wedge q$.
   Some A is not B is equivalent to $p \wedge \sim q$.

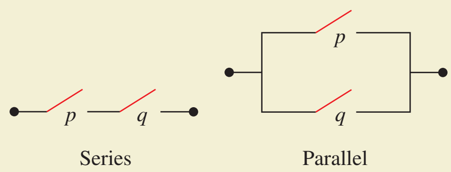
   Determine the validity of the next arguments by using Euler circles; translate the statements into logical statements using the basic connectives; and using truth tables, determine the validity of the arguments. Compare your answers.

   (a) No *A* is *B*.
   Some *C* is *A*.
   ∴ Some *C* is not *B*.

   (b) All *B* is *A*.
   All *C* is *A*.
   ∴ All *C* is *B*.

2. Politicians argue in favor of positions all the time. An informed voter doesn't vote for a candidate because of the candidate's party, gender, race, or how good they look on TV—an informed voter listens to the candidates' positions and evaluates them.

   Do a Google search for the text of a speech by each of the main candidates in the 2008 Presidential election. Then find at least three logical arguments within the text, write the arguments in symbols, and use truth tables or commonly used argument forms to analyze the arguments, and see if they are valid.
3. Electric circuits are designed using truth tables. A circuit consists of switches. Two switches wired in *series* can be represented as $p \wedge q$. Two switches wired in *parallel* can be represented as $p \vee q$.



In a series, circuit electricity will flow only when both switches p and q are closed. In a parallel circuit, electricity will flow when one or the other or both switches are closed. In a truth table, T represents a closed switch and F represents an open switch. So the truth table for $p \wedge q$ shows electricity flowing only when both switches are closed.

Truth table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Circuit

p	q	$p \wedge q$
closed	closed	current
closed	open	no current
open	closed	no current
open	open	no current

Also, when switch p is closed, switch $\sim p$ will be open and vice versa, and p and $\sim p$ are different switches. Using this knowledge, design a circuit for a hall light that has switches at both ends of the hall so that the light can be turned on or off from either switch.