CHAPTER 5
WHAT-IF ANALYSIS FOR LINEAR PROGRAMMING
SOLUTION TO SOLVED PROBLEMS

5.S1 Sensitivity Analysis at Stickley Furniture

Stickley Furniture is a manufacturer of fine hand-crafted furniture. During the next production period, management is considering producing dining room tables, dining room chairs, and/or bookcases. The time required for each item to go through the two stages of production (assembly and finishing), the amount of wood required (fine cherry wood), and the corresponding unit profits are given in the following table, along with the amount of each resource available in the upcoming production period.

	Tables	*Chairs*	*Bookcases*	*Available*
Assembly (minutes)	*80*	*40*	*50*	*8100*
Finishing (minutes)	*30*	*20*	*30*	*4500*
Wood (pounds)	*80*	*10*	*50*	*9000*
Unit Profit	*$360*	*$125*	*$300*	

After formulating a linear programming model to determine the production levels that would maximize profit, the solved model and the corresponding sensitivity report are shown below.

	B	C	D	E	F	G	H
3		Tables	Chairs	Bookcases			
4	Unit Profit	$360	$125	$300			
5							
6		Resource Required per Unit			Used		Available
7	Assembly (minutes)	80	40	50	8,100	<=	8,100
8	Finishing (minutes)	30	20	30	4,500	<=	4,500
9	Wood (pounds)	80	10	50	8,100	<=	9,000
10							
11		Tables	Chairs	Bookcases			Total Profit
12	Production	20	0	130			$46,200

	A	B	C	D	E	F	G	H
6	Variable Cells							
7				Final	Reduced	Objective	Allowable	Allowable
8		Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9		C12	Production Tables	20	0	360	120	60
10		D12	Production Chairs	0	-88.333	125	88.333	1E+30
11		E12	Production Bookcases	130	0	300	60	75
12								
13	Constraints							
14				Final	Shadow	Constraint	Allowable	Allowable
15		Cell	Name	Value	Price	R.H. Side	Increase	Decrease
16		F7	Assembly (minutes) Used	8,100	2	8100	900	600
17		F8	Finishing (minutes) Used	4,500	6.67	4500	360	1462.5
18		F9	Wood (pounds) Used	8,100	0	9000	1E+30	900

a. *Suppose the profit per table increases by $100. Will this change the optimal production quantities? What can be said about the change in total profit?*

No. The increase of $100 is less than the allowable increase for the profit per table of $120. Therefore, the optimal production quantities will stay the same. The total profit will increase by $100 per table produced, or (20)($100) = $2000, to a total profit of $48,200.

b. *Suppose the profit per chair increases by $100. Will this change the optimal production quantities? What can be said about the change in total profit?*

Yes. The increase of $100 is more than the allowable increase for the profit per chair of $88.33. Therefore, the optimal production quantities will change. The total profit will likely increase, but the amount it will increase cannot be determined without re-solving.

c. *Suppose the profit per table increases by $90 and the profit per bookcase decreases by $50. Will this change the optimal production quantities? What can be said about the change in total profit?*

$90 is 90/120 = 75% of the allowable increase for tables. $50 is 50/75 = 66.7% of the allowable decrease for bookcases. 75% + 66.7% > 100%. Therefore, the optimal solution may or may not change. We can't determine the new profit without re-solving.

d. *Suppose a worker in the assembly department calls in sick, so eight fewer hours now are available in the assembly department. How much would this affect total profit? Would it change the optimal production quantities?*

The decrease of 8 hours = 480 minutes is within the allowable decrease for assembly of 600 minutes. Therefore the shadow price of $2 is valid. The change in profit is equal to the shadow price times the change in the right-hand-side or ($2)(–480 minutes) = –$960. Therefore, the total profit decreases by $960 to $45,240. The current optimal solution used all of the available assembly time, so the production quantities must change with the decrease in available assembly time.

e. *Explain why the shadow price for the wood constraint is zero.*

The current optimal solution uses only 8,100 out of the available 9,000 pounds of wood. Therefore, there are 900 pounds of slack in this constraint. Adding more wood or taking a small amount of wood away will not change the optimal solution or the total profit because the current optimal solution isn't using all of the wood anyway.

f. *A new worker has been hired who is trained to do both assembly and finishing. She will split her time between the two areas, so there now are four additional hours available in both assembly and finishing. How much would this affect total profit? Would this change the optimal production quantities?*

4 hours = 240 minutes is 240/900 = 26.7% of the allowable increase for assembly time. 4 hours = 240 minutes is 240/360 = 66.7% of the allowable increase for finishing. 26.7% + 66.7% < 100%, so the 100% rule says that both shadow prices are valid. The increase in profit is therefore

Change in Profit = (Shadow Price)(Change in Right-Hand-Side)
= ($2)(240) + ($6.67)(240)
= $480 + $1,600
= $2,080.

Therefore, profit will increase by $2,080 to $48,280. The optimal production quantities will change to take advantage of the extra hours and this increased profit.

g. *Based on the sensitivity report, is it wise to have the new worker in part f split her time equally between assembly and finishing, or would some other plan be better?*

The shadow price is higher for finishing than assembly. Thus, the profit will increase if more hours are devoted to finishing rather than assembly. However, after 360 minutes (6 hours) are added to assembly, the allowable increase is exceeded and the shadow price will change. Depending on what the shadow price changes to, it would either be advisable to devote all 8 hours to assembly (if the new shadow price is still greater than $2), or devote 6 hours to assembly and 2 hours to finishing (if the new shadow price is less than $2).

h. *Use a parameter analysis report to determine how the optimal production quantities and total profit will change depending on how the new worker in part f allocates her time between assembly and finishing. In particular, assume 0, 1, 2, ..., or 8 hours are added to assembly, with a corresponding 8, 7, 6, ..., or 0 hours added to finishing. (The original spreadsheet is contained on the CD included with the textbook.)*

The original spreadsheet needs to be modified to account for the extra minutes available in assembly and finishing. The total extra minutes for assembly are put into a data cell in J3. Cell J7 is defined as an optimization parameter by choosing Optimization from the Parameters menu on the Risk Solver Platform tab, and specifying a lower limit of 0 and an upper limit of 480. The extra minutes left over for finishing are then calculated in J8. These extra minutes are accounted for in the total available minutes in H7:H8.

Next, a parameter analysis report is run by choosing Optimization>Parameter Analysis from the Reports menu of the Risk Solver Platform tab. All results cells and all parameter cells are chosen, and 9 major axis points are specified. The parameter analysis report generated below is then created by RSPE. (Note, an extra column was created to also show the extra finishing time, and the labels in the first row were modified.)

From the parameter analysis report, the highest profit is achieved when 1 hour is allocated to assembly and 7 hours to finishing. A little trial and error reveals that the highest profit is achieved with 75 minutes allocated to assembly and 405 minutes allocated to finishing.

	A	B	C	D	E	F	G	H	I	J
1	**Stickley Furniture**									
2										Total Extra
3			Tables	Chairs	Bookcases					480
4		Unit Profit	$360	$125	$300					
5								Total	Original	Extra
6			Resource Required per Unit			Used		Available	Available	Available
7		Assembly (minutes)	80	40	50	8,175	<=	8,100	8,100	0
8		Finishing (minutes)	30	20	30	4,905	<=	4,980	4,500	480
9		Wood (pounds)	80	10	50	8,175	<=	9,000	9,000	
10										
11			Tables	Chairs	Bookcases			Total Profit		
12		Production	0	0	163.5			$49,050		

Extra to Assembly	Extra to Finishing	Tables	Chairs	Bookcases	Total Profit
0	480	0	0	162	$48,600
60	420	0	0	163.2	$48,960
120	360	4	0	158	$48,840
180	300	9.33333	0	150.66667	$48,560
240	240	14.6667	0	143.33333	$48,280
300	180	20	0	136	$48,000
360	120	25.3333	0	128.66667	$47,720
420	60	30.6667	0	121.33333	$47,440
480	0	36	0	114	$47,160