1-1 Study Guide and Intervention

Using a Problem-Solving Plan

FOUR-STEP PROBLEM-SOLVING PLAN

When solving problems, it is helpful to have an organized plan to solve the problem. The following four steps can be used to solve any math problem.

1 **Explore**—get a general understanding of the problem

2 **Plan**—make a plan to solve the problem and estimate the solution

3 **Solve**—use your plan to solve the problem

4 **Check**—check the reasonableness of your solution

Example HEALTH **According to a recent study, 1 out of every 10 people is left-handed. If there are 172 people in the eighth grade, predict the number of students who are left-handed.**

Explore We know that 1 out of 10 people is left-handed. We also know that there are 172 people in the eighth grade. We need to predict how many of the students are left-handed.

Plan Make a table to organize the information and look for a pattern.

Number of people	10	20	30	40	50
Number who are left-handed	1	2	3	4	5

Solve By extending the pattern, we can predict that 17 students will be left-handed.

Check For every 10 students in the class, 1 is left-handed. There are 17 groups of 10 in a class of 172 and $17 \times 1 = 17$. The answer is correct.

Exercises

1. **SODA POP** James needs to buy one can of orange soda for every three cans of cola. If James buys 24 cans of cola, how many cans of orange soda should he buy?

2. **VIDEOS** Bob's Video Venue has a membership fee of $5.00 and tape rentals are $1.50 each. Video Heaven has no membership fee and tape rentals are $2.00 each. How many tapes must be rented in order for Bob's Video Venue to be more economical?

3. **COOKIES** A cookie shop offers 6 varieties of cookies and bakes 5 dozen of each kind every day, Monday–Friday. How many cookies are baked in four weeks?

4. **PATTERNS** Find the next term in 2, 6, 18, 54, 162, . . .

5. **TYPING** Jeremy needs to type a 500-word report for science class. He knows he can type about 19 words per minute. About how long will it take Jeremy to type his report?

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1-2 Study Guide and Intervention

Numbers and Expressions

Use the **order of operations** to evaluate expressions.

Step 1 Evaluate the expressions inside grouping symbols.

Step 2 Multiply and/or divide in order from left to right.

Step 3 Add and/or subtract in order from left to right.

Example 1

$\mathbf{6 \cdot 5 - 10 \div 2}$

$6 \cdot 5 - 10 \div 2$	Multiply 6 and 5.
$= 30 - 10 \div 2$	Divide 10 by 2.
$= 30 - 5$	Subtract 5 from 30.
$= 25$	

Example 2

$\mathbf{4(3 + 6) + 2 \cdot 11}$

$4(3 + 6) + 2 \cdot 11$	Evaluate (3 + 6).
$= 4(9) + 2 \cdot 11$	Multiply 4 and 9, and 2 and 11.
$= 36 + 22$	Add 36 and 22.
$= 58$	

Translate verbal phrases into numerical expressions.

Example 3 **Write and evaluate a numerical expression for the product of seventeen and three.**

Words the **product** of seventeen and three

Expression 17×3

Exercises

Find the value of each expression.

1. $6 + 3 \cdot 9$

2. $7 + 7 \cdot 3$

3. $14 - 6 + 8$

4. $26 - 4 + 9$

5. $10 \div 5 \cdot 3$

6. $22 \div 11 \cdot 6$

7. $2(6 + 2) - 4 \cdot 3$

8. $5(6 + 1) - 3 \cdot 3$

9. $2[(13 - 4) + 2(2)]$

10. $4[(10 - 6) + 6(2)]$

11. $\dfrac{(67 + 13)}{(34 - 29)}$

12. $6(4 - 2) + 8$

13. $3[(2 + 7) \div 9] - 3$

14. $(8 \cdot 7) \div 14 - 1$

15. $\dfrac{4(18)}{2(9)}$

16. $(9 \cdot 8) - (100 \div 5)$

Write a numerical expression for each verbal phrase.

17. eleven less than twenty

18. twenty-five increased by six

19. sixty-four divided by eight

20. the product of seven and twelve

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1-3 Study Guide and Intervention

Variables and Expressions

An **algebraic expression** is a combination of variables, numbers, and at least one operation. To evaluate an algebraic expression, replace the variable(s) with numbers and follow the order of operations.

Example 1 ALGEBRA **Evaluate each expression if $r = 6$ and $s = 2$.**

a. $8s - 2r$

$8s - 2r = 8 \cdot 2 - 2 \cdot 6$	Replace *r* with 6 and *s* with 2.
$= 16 - 12$	Multiply.
$= 4$	Subtract.

b. $3(r + s)$

$3(r + s) = 3(2 + 6)$	Replace *r* with 6 and *s* with 2.
$= 3 \cdot 8$	Evaluate the parentheses.
$= 24$	Multiply.

Example 2 FOOTBALL **Teams earn three points for field goals and six points for touchdowns.**

a. Assuming no other points, write an expression for a team's total points.

Words three points for field goals and six points for touchdowns

Variables Let f = number of field goals and t = number of touchdowns.

Expression $3f + 6t$

The total points for the team is $3f + 6t$.

b. Find the total score if a team scored two field goals and three touchdowns.

$3f + 6t = 3 \cdot 2 + 6 \cdot 3$	Replace *f* with 2 and *t* with 3.
$= 6 + 18$	Multiply.
$= 24$	Add.

The team scored a total of 24 points.

Exercises

ALGEBRA **Evaluate each expression if $x = 10$, $y = 5$, and $z = 1$.**

1. $x + y - z$

2. $\frac{x}{y}$

3. $2x + 4z$

4. $xy + z$

5. $\frac{6y}{10z}$

6. $x(2 + z)$

7. $x - 2y$

8. $\frac{(x + y)}{z}$

Translate each phrase into an algebraic expression.

9. eight inches taller than Mycala's height

10. twelve more than four times a number

11. the difference of sixty and a number

12. three times the number of tickets sold

1-4 Study Guide and Intervention

Properties

In algebra, there are certain statements called **properties** that are true for any numbers.

Property	Explanations	Example
Commutative Property of Addition	$a + b = b + a$	$6 + 3 = 3 + 6$ $9 = 9$
Commutative Property of Multiplication	$a \cdot b = b \cdot a$	$4 \cdot 5 = 5 \cdot 4$ $20 = 20$
Associative Property of Addition	$(a + b) + c =$ $a + (b + c)$	$(3 + 4) + 7 = 3 + (4 + 7)$ $14 = 14$
Associative Property of Multiplication	$(a \cdot b) \cdot c =$ $a \cdot (b \cdot c)$	$(2 \cdot 5) \cdot 8 = 2 \cdot (5 \cdot 8)$ $80 = 80$
Additive Identity	$a + 0 = 0 + a = a$	$10 + 0 = 0 + 10 = 10$
Multiplicative Identity	$a \cdot 1 = 1 \cdot a = a$	$5 \cdot 1 = 1 \cdot 5 = 5$
Multiplicative Property of Zero	$a \cdot 0 = 0 \cdot a = 0$	$15 \cdot 0 = 0 \cdot 15 = 0$

Example **Simplify $3 \cdot (x \cdot 5)$.**

$3 \cdot (x \cdot 5) = 3 \cdot (5 \cdot x)$ Commutative Property of Multiplication

$= (3 \cdot 5) \cdot x$ Associative Property of Multiplication

$= 15 \cdot x$ Multiply 3 and 5.

Exercises

Name the property shown by each statement.

1. $75 + 25 = 25 + 75$

2. $2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$

3. $14 \cdot 1 = 14$

4. $p \cdot 0 = 0$

5. $6 + (5 + m) = (6 + 5) + m$

6. $2(6) = 6(2)$

Simplify each expression.

7. $24 + (x + 6)$

8. $3 \cdot (4a)$

9. $9 + (12 + c)$

10. $13d \cdot 0$

1-5 Study Guide and Intervention

Variables and Equations

An equation that contains a variable is called an **open sentence**. When the variable is replaced with a number, you can determine whether the sentence is true or false. A value that makes the sentence true is called a **solution**.

Example 1 ALGEBRA **Find the solution of $27 - p = 14$. Is it 11, 13, or 15?**

Value for p	27 − p = 14	True or False?
11	$27 - 11 \stackrel{?}{=} 15$	false
13	$27 - 13 \stackrel{?}{=} 14$	true
15	$27 - 15 \stackrel{?}{=} 14$	false

Verbal sentences can be translated into equations and then solved.

Example 2 ALGEBRA **The sum of a number and six is twenty-one. Find the number.**

Let n = the number.

Words The sum of a number and six is twenty-one.

Variables Let n = the number.

Equation $n + 6 = 21$ Write the equation.

$15 + 6 = 21$ Think: What number added to 6 is 21?

$n = 15$ The solution is 15.

Exercises

ALGEBRA **Find the solution of each equation from the list given.**

1. $b + 11 = 29$; 16, 18, 20

2. $h + 7 = 42$; 35, 37, 39

3. $37 - x = 24$; 9, 11, 13

4. $26 - m = 18$; 6, 8, 10

5. $v - 6 = 5$; 7, 9, 11

6. $6r = 48$; 6, 8, 10

7. $\frac{63}{a} = 9$; 7, 9, 11

8. $k - 16 = 15$; 31, 33, 35

9. $121 = 11p$; 9, 11, 13

10. $\frac{x}{5} = 15$; 70, 75, 80

11. $2n + 1 = 7$; 3, 4, 5

12. $11 = 3y - 25$; 10, 11, 12

ALGEBRA **Define the variable. Then write the equation and solve.**

13. The product of seven and a number is fifty-six.

14. The quotient of eighty-two and a number is two.

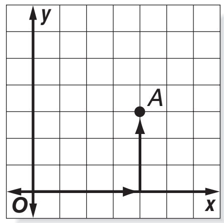
15. The difference between a number and four is twelve.

1-6 Study Guide and Intervention

Ordered Pairs and Relations

In mathematics, a **coordinate system** is used to locate points. The horizontal number line is called the ***x*-axis** and the vertical number line is called the ***y*-axis**. The point where the two axes intersect is the **origin** (0, 0). An **ordered pair** of numbers is used to locate points in the coordinate plane. The point (4, 3) has an ***x*-coordinate** of 4 and a ***y*-coordinate** of 3.

Example 1 **Graph *A*(4, 3) on the coordinate system.**



Step 1 Start at the origin.

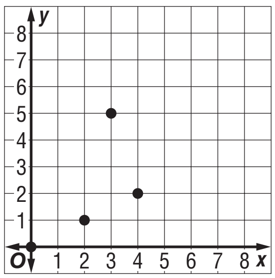
Step 2 Since the *x*-coordinate is 4, move 4 units to the right.

Step 3 Since the *y*-coordinate is 3, move 3 units up. Draw a dot.

A set of ordered pairs is called a **relation**. The set of *x*-coordinates is called the **domain**. The set of *y*-coordinates is called the **range**.

Example 2 **Express the relation {(0, 0), (2, 1), (4, 2), (3, 5)} as a table and as a graph. Then determine the domain and range.**

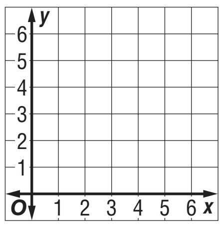
x	*y*
0	0
2	1
4	2
3	5



The domain is {0, 2, 4, 3}, and the range is {0, 1, 2, 5}.

Exercises

Graph each point on the coordinate system.



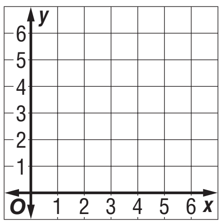
1. *A*(4,1) **2.** *B*(2,0)

3. *C*(1,3) **4.** *D*(5,2)

5. *E*(0,3) **6.** *F*(6,4)

7. Express the relation {(4,6), (0,3), (1,4)} as a table and a graph. Then determine the domain and range.

x	*y*



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1-7 Study Guide and Intervention

Scatter Plots

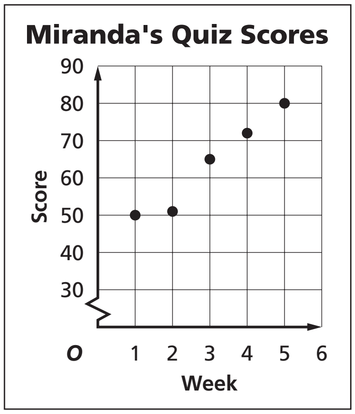
A **scatter plot** is a graph that shows the relationship between two sets of data. In a scatter plot, two sets of data are graphed as ordered pairs on a coordinate system. A scatter plot may show a pattern or relationship of the data. The relation may be positive or negative, or there may be no relationship.

Lesson 1–7

Example SCHOOL **The table shows Miranda's math quiz scores for the last five weeks. Make a scatter plot of the data.**

Since the points are showing an upward trend from left to right, the data suggest a positive relationship.

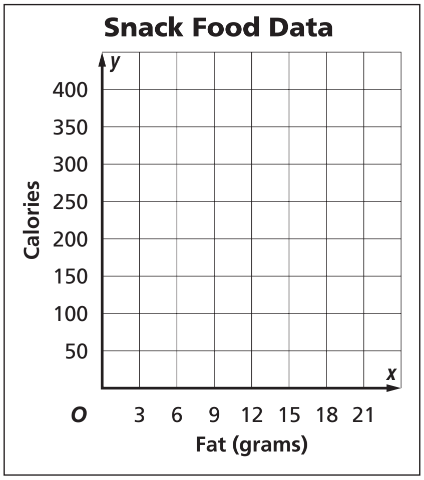
Week	Score
1	50
2	51
3	65
4	72
5	80



Exercises

FOOD **For Exercises 1–3, use the table below which shows the fat grams and calories for several snack foods.**

Food	Fat grams per serving	Calories per serving
doughnut	13	306
corn chips	13	200
pudding	3	150
cake	13	230
snack crackers	6	140
ice cream (light)	5	130
yogurt	2	70
cheese pizza	18	410



1. Make a scatter plot of the data in the table.

2. What do the x-coordinates represent? y-coordinates?

3. Is there a relationship between fat and calories? Explain.

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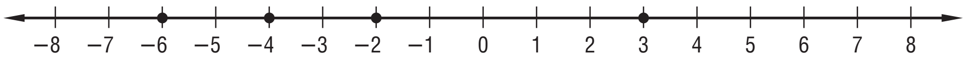
2-1 Study Guide and Intervention

Integers and Absolute Value

The set of **integers** can be written {…, −3, −2, −1, 0, 1, 2, 3, …} where … means continues indefinitely. Two integers can be compared using an **inequality**, which is a mathematical sentence containing $<$ or $>$.

Example 1 **Use the integers graphed on the number line below for each question.**



Replace each ● with < or > to make a true sentence.

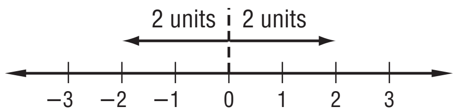
a. −6 ● −2

-2 is greater since it lies to the right of -6.
So write $-6 < -2$.

b. 3 ● −4

3 is greater since it lies to the right of -4.
So write $3 > -4$.

Numbers on opposite sides of zero and the same distance from zero have the same **absolute value.**



The symbol for absolute value is two vertical bars on either side of the number. $|2| = 2$ and $|-2| = 2$

Example 2 **Evaluate each expression.**

a. $|-4|$

$|-4| = 4$

b. $|-3| + |6|$

$|-3| + |6| = 3 + 6$ $\quad$ $|-3| = 3$, $|6| = 6$

$= 9$ $\quad$ Simplify.

Exercises

Replace each ● with <, >, or = to make a true sentence.

1. 4 ● −4 **2.** 8 ● 12 **3.** −7 ● −5 **4.** 2 ● 5

5. −1 ● 1 **6.** 4 ● −3 **7.** 6 ● 8 **8.** −2 ● 12

9. 9 ● −1 **10.** −6 ● −6 **11.** 5 ● −3 **12.** −10 ● 2

Evaluate each expression.

13. $|-6|$ **14.** $|15|$ **15.** $|-12|$ **16.** $|21|$

17. $|4| - |2|$ **18.** $|-8| + |-3|$ **19.** $|-10| - |-6|$ **20.** $|12| + |-4|$

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2-2 Study Guide and Intervention

Adding Integers

Adding Integers with the Same Sign	Add their absolute values. The sum is: • positive if both integers are positive. • negative if both integers are negative.

Example 1 **Find the sum $-3 + (-4)$.**

$-3 + (-4) = -7$ Add $|-3|$ and $|-4|$. The sum is negative.

Adding Integers with Different Signs	Subtract their absolute values. The sum is: • positive if the positive integer's absolute value is greater. • negative if the negative integer's absolute value is greater.

Example 2 **Find each sum.**

a. $-5 + 4$

$-5 + 4 = |-5| - |4|$ Subtract $|4|$ from $|-5|$.

$= 5 - 4$ or 1 Simplify.

$= -1$ The sum is negative because $|-5| > |4|$.

b. $6 + (-2)$

$6 + (-2) = |6| - |-2|$ Subtract $|-2|$ from $|6|$.

$= 6 - 2$ or 4 Simplify.

$= 4$ The sum is positive because $|6| > |-2|$.

Exercises

Find each sum.

1. $6 + (-3)$

2. $-3 + (-5)$

3. $7 + (-3)$

4. $-4 + (-4)$

5. $-8 + 5$

6. $-12 + (-10)$

7. $6 + (-13)$

8. $-14 + 4$

9. $6 + (-6)$

10. $-15 + (-5)$

11. $-9 + 8$

12. $20 + (-8)$

13. $-19 + (-11)$

14. $17 + (-9)$

15. $-16 + (-5)$

16. $-12 + 14$

17. $9 + (-25)$

18. $-36 + 19$

19. $7 + (-18)$

20. $-12 + (-15)$

21. $10 + (-14)$

22. $-33 + 19$

23. $-20 + (-5)$

24. $-12 + (-10)$

25. $-15 + 4$

26. $-34 + 29$

27. $46 + (-32)$

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2-3 Study Guide and Intervention

Subtracting Integers

Subtracting Integers	To subtract an integer, add its additive inverse.

Example 1 **Find each difference.**

a. 9 − 17

$9 - 17 = 9 + (-17)$ To subtract 17, add −17.

$= -8$ Simplify.

b. −7 − 3

$-7 - 3 = -7 + (-3)$ To subtract 3, add −3.

$= -10$ Simplify.

Example 2 **Find each difference.**

a. 4 − (−5)

$4 - (-5) = 4 + 5$ To subtract −5, add +5.

$= 9$ Simplify.

b. −6 − (−2)

$-6 - (-2) = -6 + 2$ To subtract −2, add +2.

$= -4$ Simplify.

Exercises

Find each difference.

1. $9 - 16$

2. $7 - 19$

3. $12 - 21$

4. $-5 - 3$

5. $-8 - 9$

6. $-13 - 17$

7. $7 - (-4)$

8. $9 - (-9)$

9. $-11 - (-2)$

10. $-6 - (-9)$

11. $-6 - 4$

12. $-16 - (-20)$

13. $-14 - 4$

14. $8 - (-6)$

15. $-10 - (-6)$

16. $13 - (-17)$

17. $24 - (-16)$

18. $17 - (-9)$

19. $-24 - 8$

20. $18 - (-9)$

21. $26 - 49$

22. $-45 - (-26)$

23. $-15 - (-25)$

24. $29 - (-6)$

2-4 Study Guide and Intervention

Multiplying Integers

Multiplying Integers with Different Signs	The product of two integers with different signs is negative.

Example 1 **Find each product.**

a. $\mathbf{4(-3)}$

$4(-3) = -12$

b. $\mathbf{-8(5)}$

$-8(5) = -40$

Multiplying Integers with the Same Sign	The product of two integers with the same sign is positive.

Example 2 **Find each product.**

a. $\mathbf{6(6)}$

$6(6) = 36$

b. $\mathbf{-7(-4)}$

$-7(-4) = 28$

Example 3 **Find $\mathbf{6(-3)(-2)}$.**

$6(-3)(-2) = [6(-3)](-2)$	Use the Associative Property.
$= -18(-2)$	$6(-3) = -18$
$= 36$	$-18(-2) = 36$

Exercises

Find each product.

1. $-5(7)$

2. $6(-9)$

3. $-10 \cdot 4$

4. $-12 \cdot -2$

5. $5(-11)$

6. $-15(-4)$

7. $-14(2)$

8. $6(14)$

9. $-18 \cdot 2$

10. $-9(10)$

11. $12(-6)$

12. $-11(-11)$

13. $-4(-4)(5)$

14. $6(-7)(2)$

15. $-10(-4)(-6)$

16. $-7(-3)(2)$

17. $-9(4)(2)$

18. $6(-4)(-12)$

19. $11(3)(-2)$

20. $-5(-6)(7)$

21. $-3(-4)(-8)$

22. $22(3)(-3)$

23. $-8(10)(-2)$

24. $-6(5)(-9)$

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2-5 Study Guide and Intervention

Dividing Integers

Dividing Integers with the Same Sign	The quotient of two integers with the same sign is positive.

Example 1 **Find each quotient.**

a. $\mathbf{14 \div 2}$

$14 \div 2 = 7$ The dividend and the divisor have the same sign. The quotient is positive.

b. $\mathbf{\frac{-25}{-5}}$

$$\frac{-25}{-5} = -25 \div (-5)$$ The dividend and divisor have the same sign.

$$= 5$$ The quotient is positive.

Dividing Integers with Different Signs	The quotient of two integers with different signs is negative.

Example 2 **Find each quotient.**

a. $\mathbf{36 \div (-4)}$ The signs are different.

$36 \div (-4) = -9$ The quotient is negative.

b. $\mathbf{\frac{-42}{6}}$ The signs are different.

$$\frac{-42}{6} = -42 \div 6$$ The quotient is negative.

$$= -7$$ Simplify.

Exercises

Find each quotient.

1. $32 \div (-4)$

2. $-18 \div (-2)$

3. $-24 \div 6$

4. $-36 \div (-2)$

5. $50 \div (-5)$

6. $-81 \div (-9)$

7. $-72 \div (-2)$

8. $-45 \div 3$

9. $-60 \div (-12)$

10. $99 \div (-11)$

11. $-200 \div (-4)$

12. $38 \div (-2)$

13. $-144 \div 12$

14. $100 \div (-5)$

15. $-200 \div (-20)$

16. $\frac{-28}{2}$

17. $\frac{36}{-4}$

18. $\frac{-150}{-25}$

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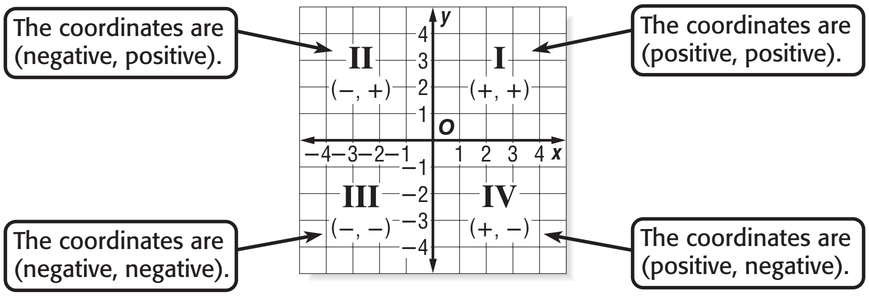
2-6 Study Guide and Intervention

The Coordinate System



Example **Graph and label each point on a coordinate plane. Name the quadrant in which each point lies.**

a. *M*(–2, 5)

Start at the origin. Move 2 units left.

Then move 5 units up and draw a dot.

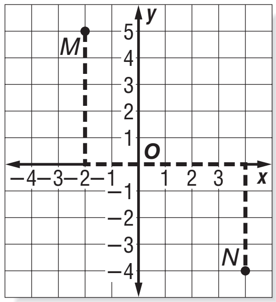
Point *M*(–2, 5) is in Quadrant II.

b. *N*(4, –4)

Start at the origin. Move 4 units right.

Then move 4 units down and draw a dot.

Point *N*(4, –4) is in Quadrant IV.



Exercises

Graph and label each point on the coordinate plane. Name the quadrant in which each point is located.

1. *A*(2, 6)

2. *B*(–1, 4)

3. *C*(0, –5)

4. *D*(–4, –3)

5. *E*(2, 0)

6. *F*(3, –2)

7. *G*(–4, 4)

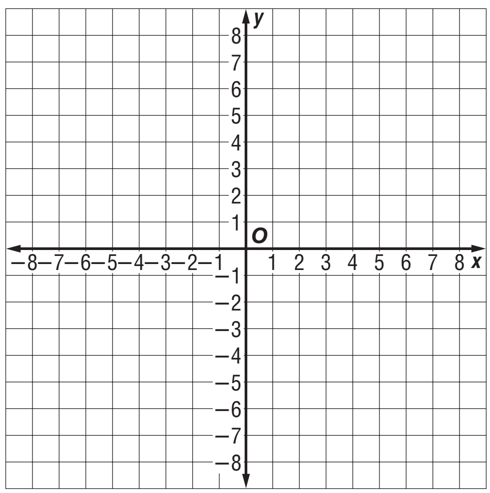
8. *H*(2, –5)

9. *I*(6, 3)

10. *J*(–5, –8)

11. *K*(3, –5)

12. *L*(–7, –3)



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3-1 Study Guide and Intervention

The Distributive Property

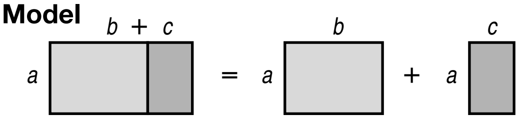
The expressions 2(1 + 5) and 2 · 1 + 2 · 5 are equivalent expressions because they have the same value, 12. The **Distributive Property** combines addition and multiplication.

Symbols

$a(b + c) = ab + ac$

$(b + c)a = ba + ca$

Model



The Distributive Property also combines subtraction and multiplication. Rewrite subtraction expressions as addition expressions.

Example 1 **Use the Distributive Property to write each expression as an equivalent expression. Then evaluate the expression.**

a. 2(6 + 3)

$$2(6 + 3) = 2 \cdot 6 + 2 \cdot 3$$
$$= 12 + 6$$
$$= 18$$

b. 5(9 − 3)

$$5(9 - 3) = 5\,[9 + (-3)]$$
$$= 5 \cdot 9 + 5 \cdot (-3)$$
$$= 45 + (-15)$$
$$= 30$$

The Distributive Property can also be used with algebraic expressions containing variables.

Example 2 **Use the Distributive Property to write each expression as an equivalent algebraic expression.**

a. 7(*m* + 5)

$$7(m + 5) = 7m + 7 \cdot 5$$
$$= 7m + 35$$

b. −3(*n* − 8)

$$-3(n - 8) = -3[n + (-8)]$$
$$= -3 \cdot n + (-3)(-8)$$
$$= -3n + 24$$

Exercises

Use the Distributive Property to write each expression as an equivalent expression. Then evaluate the expression, if possible.

1. $3(8 + 2)$

2. $2(9 + 11)$

3. $5(19 - 6)$

4. $-6(3 + 14)$

5. $(17 - 4)3$

6. $(5 + 3)7$

7. $3(d + 4)$

8. $(w - 5)4$

9. $-2(c + 7)$

3-2 Study Guide and Intervention

Simplifying Algebraic Expressions

term: a number, a variable, or a product of numbers and variables

coefficient: the numerical part of a term that also contains a variable

constant: term without a variable

like terms: terms that contain the same variables

Example 1 **Identify the terms, like terms, coefficients, and constants in the expression $4m - 5m + n - 7$.**

$4m - 5m + n - 7 = 4m + (-5m) + n + (-7)$ Definition of subtraction

$= 4m + (-5m) + 1n + (-7)$ Identity Property

terms $4m$, $-5m$, $1n$, -7; like terms: $4m$, $-5m$; coefficients: 4, -5, 1; constants: -7

When an algebraic expression has no like terms and no parentheses, we say that it is in **simplest form.**

Example 2 **Simplify $6x - 5 - 2x + 7$.**

$6x - 5 - 2x + 7 = 6x + (-5) + (-2x) + 7$ Definition of subtraction

$= 6x + (-2x) + (-5) + 7$ Commutative Property

$= [6 + (-2)]x + (-5) + 7$ Distributive Property

$= 4x + 2$ Simplify

Exercises

Identify the terms, like terms, coefficients, and constants in each expression.

1. $2 + 6a + 4a$

2. $m + 4m + 2m + 5$

3. $3c + 4d - c + 2$

4. $5h - 3g + 2g - h$

5. $3w + 4u - 6$

6. $4r - 5s + 5s - 2r$

Simplify each expression.

7. $9m + 3m$

8. $5x - x$

9. $8y + 2y + 3y$

10. $4 + m - 3m$

11. $13a + 7a + 2a$

12. $3y + 1 + 5 + 4y$

13. $8d - 4 - d + 5$

14. $10 - 4s + 2s - 3$

3-3 Study Guide and Intervention

Solving Equations by Adding or Subtracting

Step 1	Identify the variable.
Step 2	To isolate the variable, add the same number to or subtract the same number from each side of the equation.
Step 3	Check the solution.

Example 1 **Solve $x + 2 = 6$.**

$$x + 2 = 6$$

$$x + 2 - 2 = 6 - 2 \quad \text{Subtract 2 from each side.}$$

$$x = 4$$

Check: $x + 2 = 6$

$4 + 2 = 6$

$6 = 6$ ✔

The solution is 4.

Example 2 **Solve $x - 9 = -13$.**

$$x - 9 = -13$$

$$x - 9 + 9 = -13 + 9 \quad \text{Add 9 to each side.}$$

$$x = -4$$

Check: $x - 9 = -13$

$-4 - 9 = -13$

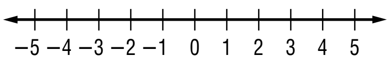
$-13 = -13$ ✔

The solution is -4.

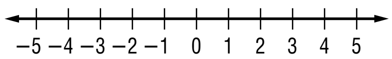
Exercises

Solve each equation. Graph the solution of each equation on the number line.

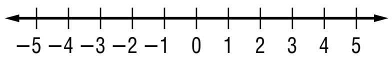
1. $x + 5 = 2$



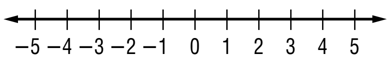
2. $11 + w = 10$



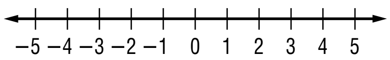
3. $k + 3 = -1$



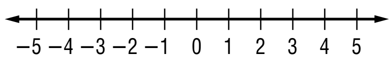
4. $m - 2 = 3$



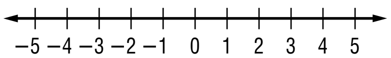
5. $a - 7 = -5$



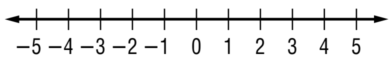
6. $b - 13 = -13$



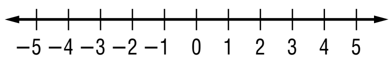
7. $-3 + h = -7$



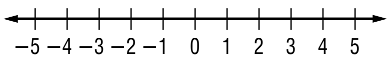
8. $-12 = y - 9$



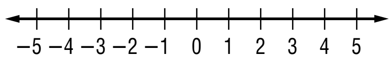
9. $2 + r = -3$



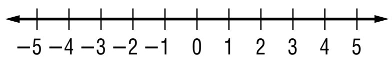
10. $9 + b = 9$



11. $7 + k = 10$



12. $g - 9 = -5$



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3-4 Study Guide and Intervention

Solving Equations by Multiplying or Dividing

Step 1	Identify the variable.
Step 2	To isolate the variable, multiply or divide each side of the equation by the same nonzero number.
Step 3	Check the solution.

Example 1 **Solve $-7x = 42$.**

$$-7x = 42$$

$$\frac{-7x}{-7} = \frac{42}{-7} \quad \text{Divide each side by } -7.$$

$$x = -6 \quad \text{Check your solution.}$$

The solution is -6.

Example 2 **Solve $\frac{y}{2} = -2$.**

$$\frac{y}{2} = -2$$

$$\left(\frac{y}{2}\right)2 = (-2)2 \quad \text{Multiply each side by 2.}$$

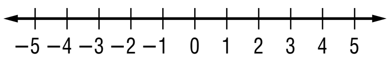
$$y = -4 \quad \text{Check your solution.}$$

The solution is -4.

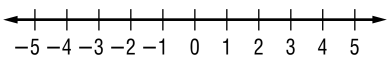
Exercises

Solve each equation. Graph the solution of each equation on the number line.

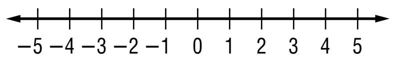
1. $-3a = 15$



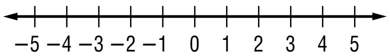
2. $-t = 5$



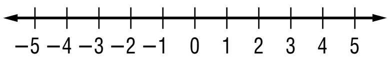
3. $-1 = \frac{n}{4}$



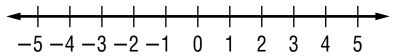
4. $7r = 28$



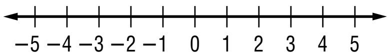
5. $0 = \frac{h}{7}$



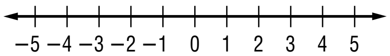
6. $24 = -8m$



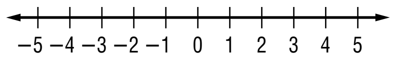
7. $-11b = 44$



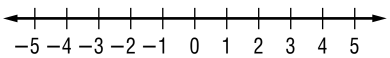
8. $\frac{a}{-2} = -1$



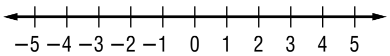
9. $12d = -48$



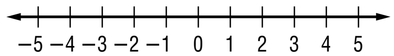
10. $-10p = 10$



11. $\frac{r}{-5} = -1$



12. $-11w = -33$



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3-5 Study Guide and Intervention

Solving Two-Step Equations

A two-step equation contains two operations. To solve two-step equations, use inverse operations to undo each operation in reverse order. First, undo addition/subtraction. Then, undo multiplication/division.

Example **Solve $\frac{c}{2} - 13 = 7$.**

$$\frac{c}{2} - 13 = 7$$

$$\frac{c}{2} - 13 + 13 = 7 + 13 \quad \text{Add 13 to each side.}$$

$$\frac{c}{2} = 20$$

$$\left(\frac{c}{2}\right)2 = (20)2 \quad \text{Multiply each side by 2.}$$

$$c = 40$$

Check:

$$\frac{c}{2} - 13 = 7$$

$$\frac{40}{2} - 13 = 7$$

$$20 - 13 = 7$$

$$7 = 7 \checkmark$$

The solution is 40.

For some problems, it may be necessary to combine like terms before solving.

Exercises

Solve each equation. Check your solution.

1. $5t + 2 = 7$

2. $2x + 5 = 9$

3. $6u - 8 = 28$

4. $8m - 7 = 17$

5. $16 = 2w + 6$

6. $50 = 6d + 8$

7. $21 = 42 + 7k$

8. $4a - 10 = 42$

9. $7c - 4 = -32$

10. $12 - 3m = 18$

11. $28 = 2h - 18$

12. $-10 = -5x - 25$

13. $\frac{m}{4} + 6 = 70$

14. $5 + \frac{p}{2} = 45$

15. $18 = \frac{g}{3} + 6$

16. $4 + \frac{n}{5} = 29$

17. $\frac{m}{7} - 9 = 5$

18. $\frac{k}{9} - 3 = -11$

19. $13 + \frac{a}{4} = -3$

20. $-3 + \frac{c}{2} = 12$

21. $\frac{v}{-3} + 8 = 22$

22. $8x - 16 + 8x = 16$

23. $12a - 14a = 8$

24. $7c - 8 - 2c = 17$

25. $6 = -y + 42 - 2y$

26. $16 + 8r - 4r + 4 = 24$

3-6 Study Guide and Intervention

Writing Two-Step Equations

You can use two-step equations to represent situations in which you start with a given amount and then increase it at a certain rate.

Example **PRINTING A laser printer prints 9 pages per minute. Liza refilled the paper tray after it had printed 92 pages. In how many more minutes will there be a total of 245 pages printed?**

EXPLORE You know the number of pages printed and the total number of pages to be printed. You need to find the number of minutes required to print the remaining pages.

PLAN Let m = the number of minutes. Write and solve an equation. The remaining pages to print is $9m$.

remaining pages + pages printed = total pages

$$9m \quad + \quad 92 \quad = \quad 245$$

SOLVE

$$9m + 92 = 245$$
$$9m + 92 - 92 = 245 - 92$$
$$9m = 153$$
$$9m = \frac{153}{9}$$
$$m = 17$$

CHECK The remaining 153 pages will print in 17 minutes. Since 245 – 153 = 92, the answer is correct.

Exercises

Solve each problem by writing and solving an equation.

1. **METEOROLOGY** During one day in 1918, the temperature in Granville, North Dakota, began at $-33°$ and rose for 12 hours. The high temperature was about 51°. About how many degrees per hour did the temperature rise?

2. **SAVINGS** John has $825 in his savings account. He has decided to deposit $65 per month until he has a total of $1800. In how many months will this occur?

3. **SKYDIVING** A skydiver jumps from an airplane at an altitude of 12,000 feet. After 42 seconds, she reaches 4608 feet and opens her parachute. What was her average velocity during her descent?

4. **FLOODING** The water level of a creek has risen 4 inches above its flood stage. If it continues to rise steadily at 2 inches per hour, how long will it take for the creek to be 12 inches above its flood stage?

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3-7 Study Guide and Intervention

Sequences and Equations

A sequence is an ordered list of numbers. An arithmetic sequence is a sequence in which the difference between any two consecutive terms, called the common difference, is the same. In the sequence 2, 9, 16, 23, 30, . . . , the common difference is 7.

Example 1 **Describe each sequence using words and symbols.**

a. 3, 6, 9, 12, ...

Place the terms into a table.

Term Number (n)	1	2	3	4
Term (t)	3	6	9	12

The terms have a common difference of 3. If n = term number and t = term, $t = 3n$ describes the sequence.

b. 10, 14, 18, 22, ...

Place the terms into a table.

Term Number (n)	1	2	3	4
Term (t)	10	14	18	22

The terms have a common difference of 4. But notice that the first term is 6 more than 4 times the term number. If n = term number and t = term, $t = 4n + 6$ describes the sequence.

Example 2 **Find the 14th term of the following sequence: 5, 7, 9, 11, ...**

First write an equation that describes the sequence. Place the terms into a table.

Term Number (n)	1	2	3	4
Term (t)	5	7	9	11

So the common difference is 2 times the difference in the term numbers. This suggests that $t = 2n$. However, the first term of the pattern is 3 more than 2 times the term number. So, $t = 2n + 3$ describes the sequence.

To find the 14th term of the sequence, let $n = 14$ and solve for t.

$$t = 2(14) + 3 \text{ or } 31$$

So the 14th term is 31.

Exercises

Describe each sequence using words and symbols.

1. 4, 5, 6, 7, …

2. 6, 7, 8, 9, …

3. 6, 12, 18, 24, …

4. 8, 16, 24, 32, …

Write an equation that describes each sequence. Then find the indicated term.

5. 5, 13, 21, 29, …; 11th term

6. 6, 10, 14, 18, …; 14th term

7. 8, 11, 14, 17, …; 10th term

8. 1, 5, 9, 13, …; 20th term

Lesson 3–7

3-8 Study Guide and Intervention

Using Formulas

The formula $d = rt$ relates distance d, rate r, and time t, traveled.

Example 1 **Find the distance traveled if you drive at 40 miles per hour for 3 hours.**

$d = rt$

$d = 40 \times 3$ Replace r with 40 and t with 3.

$d = 120$ The distance traveled is 120 miles.

The formula $P = 2(\ell + w)$ relates perimeter P, length ℓ, and width w for a rectangle.
The formula $A = \ell w$ relates area A, length ℓ, and width w for a rectangle.

Example 2 **Find the perimeter and area of a rectangle with length 7 feet and width 2 feet.**

$P = 2(\ell + w)$

$P = 2(7 + 2)$

$P = 2(9)$

$P = 18$

The perimeter is 18 feet.

$A = \ell \cdot w$

$A = 7 \cdot 2$

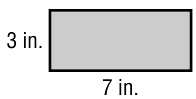
$A = 14$

The area is 14 square feet.

Exercises

1. **TRAIN TRAVEL** How far does a train travel in 12 hours at 48 miles per hour?
2. **TRAVEL** How long does it take a car traveling 40 miles per hour to go 200 miles?
3. **BICYCLING** What is the rate, in miles per hour, of a bicyclist who travels 56 miles in 4 hours?
4. **RACING** How long will it take a driver to finish a 980-mile rally race at 70 miles per hour?

Find the perimeter and area of each rectangle.

5.


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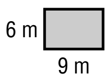

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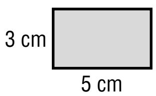

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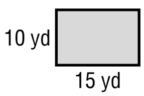

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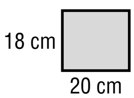

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4-1 Study Guide and Intervention

Powers and Exponents

A number that is expressed using an exponent is called a **power**. The **base** is the number that is multiplied. The **exponent** tells how many times the base is used as a factor. So, 4^3 has a base of 4 and an exponent of 3, and $4^3 = 4 \cdot 4 \cdot 4 = 64$.

Example 1 **Write each expression using exponents.**

a. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

The base is 10. It is a factor 5 times, so the exponent is 5.

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5$

b. $(p + 2)(p + 2)(p + 2)$

The base is $p + 2$. It is a factor 3 times, so the exponent is 3.

$(p + 2)(p + 2)(p + 2) = (p + 2)^3$

Expressions involving powers are evaluated using order of operations. Powers are repeated multiplications. They are evaluated after any grouping symbols and before other multiplication or division operations.

Example 2 **Evaluate $x^2 - 4$ if $x = -6$.**

$x^2 - 4$	$= (-6)^2 - 4$	Replace x with -6.
	$= (-6)(-6) - 4$	-6 is a factor 2 times.
	$= 36 - 4$	Multiply.
	$= 32$	Subtract.

Exercises

Write each expression using exponents.

1. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

2. $(-7)(-7)(-7)$

3. $d \cdot d \cdot d \cdot d$

4. $x \cdot x \cdot y \cdot y$

5. $(z - 4)(z - 4)$

6. $3(-t)(-t)(-t)$

Evaluate each expression if $g = 3$, $h = -1$, and $m = 9$.

7. g^5

8. $5g^2$

9. $g^2 - m$

10. hm^2

11. $g^3 + 2h$

12. $m + hg^3$

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4-2 Study Guide and Intervention

Prime Factorization

A **prime number** is a whole number that has exactly two factors, 1 and itself. A **composite number** is a whole number that has more than two factors. Zero and 1 are neither prime nor composite.

Example 1 **Determine whether 29 is prime or composite.**

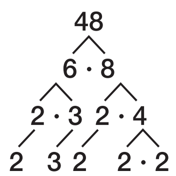
Find the factors of 29.

$29 = 1 \cdot 29$

The only factors of 29 are 1 and 29, therefore 29 is a prime number.

Any composite number can be written as a product of prime numbers. A factor tree can be used to find the prime factorization.

Example 2 **Find the prime factorization of 48.**



48 is the number to be factored.

Find any pair of whole number factors of 48.

Continue to factor any number that is not prime.

The factor tree is complete when there is a row of prime numbers.

The prime factorization of 48 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ or $2^4 \cdot 3$.

In algebra, monomials can be factored as a product of prime numbers and variables with no exponent greater than 1. So, $8x^2$ factors as $2 \cdot 2 \cdot 2 \cdot x \cdot x$.

Exercises

Determine whether each number is *prime* or *composite*.

1. 27

2. 151

3. 77

4. 25

Write the prime factorization for each number. Use exponents for repeated factors.

5. 16

6. 45

7. 78

8. 70

Factor each monomial.

9. $6m^3$

10. $-20xy^2$

11. $a^2b^2c^3$

12. $25h$

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4-3 Study Guide and Intervention

Greatest Common Factor (GCF)

The greatest number that is a factor of two or more numbers is the **greatest common factor (GCF)**. Two ways to find the GCF are shown below.

Example 1 **Find the GCF of 24 and 32.**

Method 1 List the factors.

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 — Look for factors common to both lists, 1, 2, 4, and 8.

factors of 32: 1, 2, 4, 8, 16, 32

The greatest common factor of 24 and 32 is 8.

Method 2 Use prime factorization.

$24 = 2 \cdot 2 \cdot 2 \cdot 3$ — Find the common prime factors of 24 and 32.

$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Multiply the common prime factors. The greatest common factor of 24 and 32 is $2 \cdot 2 \cdot 2$ or 8.

In algebra, greatest common factors are used to factor expressions.

Example 2 **Factor $5x + 10$.**

First, find the GCF of $5x$ and 10.

$5x = 5 \cdot x$

$10 = 2 \cdot 5$ — The GCF is 5.

Now write each term as a product of the GCF and its remaining factors.

$5x + 10 = 5(x) + 5(2)$

$= 5(x + 2)$ — Distributive Property

So, $5x + 10 = 5(x + 2)$.

Exercises

Find the GCF of each set of numbers.

1. 30, 42 **2.** 15, 33 **3.** 44, 110 **4.** 16, 48

Factor each expression.

5. $4g + 16$ **6.** $2d - 6$ **7.** $8a + 24$

8. $f^2 + 2f$ **9.** $6 - 3j$ **10.** $16n^2 - 40n$

Lesson 4–3

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4-4 Study Guide and Intervention

Simplifying Algebraic Fractions

A fraction is in **simplest form** when the GCF of the numerator and the denominator is 1. One way to write a fraction in simplest form is to write the prime factorization of the numerator and the denominator. Then divide the numerator and denominator by the GCF.

Example 1 **Write $\frac{18}{24}$ in simplest form.**

Write the prime factorization of the numerator and the denominator.

$$\frac{18}{24} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot 3}{\underset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{3}}}$$ Divide the numerator and denominator by the GCF, 2 · 3.

$$= \frac{3}{2 \cdot 2} \text{ or } \frac{3}{4}$$ Simplify.

Algebraic fractions can also be written in simplest form. Again, you can write the prime factorization of the numerator and the denominator, then divide by the GCF.

Example 2 **Simplify $\frac{24ab^2}{36a^2}$.**

$$\frac{24ab^2}{36a^2} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{a}} \cdot b \cdot b}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 3 \cdot \underset{1}{\cancel{a}} \cdot a}$$ Divide the numerator and denominator by the GCF, 2 · 2 · 3 · *a*.

$$= \frac{2 \cdot b \cdot b}{3 \cdot a} \text{ or } \frac{2b^2}{3a}$$ Simplify.

Exercises

Simplify each fraction. If the fraction is already in simplest form, write *simplified*.

1. $\frac{12}{20}$ **2.** $\frac{16}{36}$ **3.** $\frac{75}{100}$

4. $\frac{6}{15}$ **5.** $\frac{8}{24}$ **6.** $\frac{3}{8}$

7. $\frac{c}{c^3}$ **8.** $\frac{r^4}{r^2}$ **9.** $\frac{14b}{21b}$

10. $\frac{24w}{26w}$ **11.** $\frac{5s}{12t}$ **12.** $\frac{d}{3d^2}$

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4-5 Study Guide and Intervention

Multiplying and Dividing Monomials

When multiplying powers with the same base, add the exponents.

Symbols	**Example**
$a^m \cdot a^n = a^{m+n}$	$4^2 \cdot 4^5 = 4^{2+5}$ or 4^7

When dividing powers with the same base, subtract the exponents.

Symbols	**Example**
$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$	$\frac{5^6}{5^2} = 5^{6-2}$ or 5^4

Example 1 **Find $2a^2(3a)$. Express your answer using exponents.**

$2a^2(3a) = (2 \cdot 3)(a^2 \cdot a)$ — Use the Commutative and Associative Properties.

$= (6)(a^{2+1})$ — The common base is a.

$= 6a^3$ — Add the exponents.

Example 2 **Find $\frac{(-8)^4}{(-8)^2}$. Express your answer using exponents.**

$\frac{(-8)^4}{(-8)^2} = (-8)^{4-2}$ — The common base is -8.

$= (-8)^2$ — Subtract the exponents.

Exercises

Find each product or quotient. Express your answer using exponents.

1. $4^7 \cdot 4^6$

2. $v^5 \cdot v^4$

3. $(f^3)(f^9)$

4. $22^5 \cdot 22^5$

5. $7h(5h^3)$

6. $-10x^2(7x^3)$

7. $\frac{7^5}{7^2}$

8. $\frac{1^8}{1^6}$

9. $\frac{(-12)^3}{(-12)^3}$

10. $3^8 \cdot 3^3$

11. $\frac{c^{20}}{c^{13}}$

12. $\frac{(-p)^{18}}{(-p)^{12}}$

13. $-7u^6(-6u^5)$

14. $\frac{2w^3}{2w}$

15. $-5m^3(4m^6)$

16. the product of two cubed and two squared

17. the quotient of six to the eighth power and six squared

4-6 Study Guide and Intervention

Negative Exponents

Extending the pattern below shows that $4^{-1} = \frac{1}{4}$ or $\frac{1}{4^1}$.

$$4^2 = 16$$
$$\div 4$$
$$4^1 = 4$$
$$\div 4$$
$$4^0 = 1$$
$$\div 4$$
$$4^{-1} = \frac{1}{4}$$

This suggests the following definition.

$a^{-n} = \frac{1}{a^n}$, for $a \neq 0$ and any integer n.

Example 1 **Write each expression using a positive exponent.**

a. $\mathbf{3^{-4}}$

$3^{-4} = \frac{1}{3^4}$

b. $\boldsymbol{y^{-2}}$

$y^{-2} = \frac{1}{y^2}$

We can evaluate algebraic expressions with negative exponents using the definition of negative exponents.

Example 2 **Evaluate $\boldsymbol{b^{-2}}$ if $\boldsymbol{b = 3}$.**

$b^{-2} = 3^{-2}$	Replace b with 3.
$= \frac{1}{3^2}$	Definition of negative exponent
$= \frac{1}{9}$	Find 3^2.

Exercises

Write each expression using a positive exponent.

1. 6^{-4} **2.** $(-7)^{-8}$ **3.** b^{-6} **4.** n^{-1}

Write each fraction as an expression using a negative exponent other than −1.

5. $\frac{1}{2^2}$ **6.** $\frac{1}{13^4}$ **7.** $\frac{1}{25}$ **8.** $\frac{1}{49}$

Evaluate each expression if $\boldsymbol{m = -4}$, $\boldsymbol{n = 1}$, and $\boldsymbol{p = 6}$.

9. p^{-2} **10.** m^{-3} **11.** $(np)^{-1}$ **12.** 3^m

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4-7 Study Guide and Intervention

Scientific Notation

Lesson 4–7

Numbers like 5,000,000 and 0.0005 are in **standard form** because they do not contain exponents. However, when you deal with very large numbers or very small numbers such as these, it can helpful to use **scientific notation**. A number is expressed in scientific notation when it is written as a product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

By definition, a number in scientific notation is written as $a \cdot 10^n$, where $1 \leq a < 10$ and n is an integer.

Example 1 **Express each number in standard form.**

a. 6.32×10^5

$6.32 \times 10^5 = 632{,}000$ Move the decimal point 5 places to the right.

b. 7.8×10^{-6}

$7.8 \times 10^{-6} = 0.0000078$ Move the decimal point 6 places to the left.

Example 2 **Express each number in scientific notation.**

a. 62,000,000

To write in scientific notation, place the decimal point after the first nonzero digit, then find the power of 10.

$62{,}000{,}000 = 6.2 \times 10^7$ The decimal point moves 7 places. The power of 10 is 7.

b. 0.00025

$0.00025 = 2.5 \times 10^{-4}$ Place the decimal point after the first nonzero digit. The power of 10 is -4.

Exercises

1. 4.12×10^6 **2.** 5.8×10^2 **3.** 9.01×10^{-3}

4. 6.72×10^{-7} **5.** 8.72×10^4 **6.** 4.44×10^{-5}

Express each number in scientific notation.

7. 12,000,000,000 **8.** 5000 **9.** 0.00475

10. 0.00007463 **11.** 235,000 **12.** 0.000377

Choose the greater number in each pair.

13. 4.9×10^4, 9.9×10^{-4} **14.** 2.004×10^3, 2.005×10^{-2}

15. 3.2×10^2, 700 **16.** 0.002, 3.6×10^{-4}

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5-1 Study Guide and Intervention

Writing Fractions as Decimals

Fractions can be written as decimals by dividing the numerator by the denominator. If the division ends, or terminates, when the remainder is 0, it is a terminating decimal. If the decimal number repeats without end, it is a repeating decimal.

Example 1 **Write each fraction as a decimal.**

a. $\frac{7}{8}$

$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ \underline{6\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

0.875 is a terminating decimal.

b. $\frac{4}{9}$

$$\begin{array}{r} 0.444 \\ 9\overline{)4.000} \\ \underline{3\,6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

0.4. . . or $0.\overline{4}$ is a repeating decimal.

It may be easier to compare numbers if they are written as decimals.

Example 2 **Replace ● with < , > , or = to make $\frac{3}{8}$ ● 0.28 a true sentence.**

$\frac{3}{8} = 0.375$, so $\frac{3}{8}$ ● 0.28 can be written as 0.375 ● 0.28. Since $0.375 > 0.28$, $\frac{3}{8} > 0.28$.

Exercises

Write each fraction or mixed number as a decimal. Use a bar to show a repeating decimal.

1. $\frac{7}{20}$

2. $\frac{2}{11}$

3. $\frac{5}{9}$

4. $\frac{5}{6}$

5. $\frac{6}{25}$

6. $\frac{5}{20}$

7. $8\frac{3}{5}$

8. $3\frac{7}{25}$

9. $\frac{4}{15}$

10. $\frac{12}{32}$

11. $-6\frac{9}{10}$

12. $1\frac{5}{11}$

Replace each ● with <, >, or = to make a true sentence.

13. $\frac{5}{8}$ ● $\frac{6}{9}$

14. $1\frac{4}{5}$ ● 1.8

15. $-2\frac{7}{8}$ ● $-2\frac{4}{5}$

16. 5.09 ● $5\frac{1}{2}$

17. 9.3 ● $\frac{28}{3}$

18. $\frac{28}{20}$ ● $\frac{31}{23}$

5-2 Study Guide and Intervention

Rational Numbers

A number that can be written as a fraction is called a rational number. Mixed numbers, integers, and many decimals can be written as fractions. Only decimals that terminate or repeat can be written as fractions. Other decimal numbers such as $\pi = 3.141592654...$ are infinite and nonrepeating. They are called irrational numbers.

Example **Write each number as a fraction.**

a. $3\frac{2}{5}$

$3\frac{2}{5} = \frac{17}{5}$ Write the mixed number as an improper fraction.

b. -7

$-7 = -\frac{7}{1}$ The denominator is 1.

c. 0.14

0.14 is 14 hundredths.

$0.14 = \frac{14}{100}$ or $\frac{7}{50}$ Simplify.

d. $0.\overline{5}$

$0.\overline{5} = 0.555...$

$N = 0.555...$ Let N represent the number.

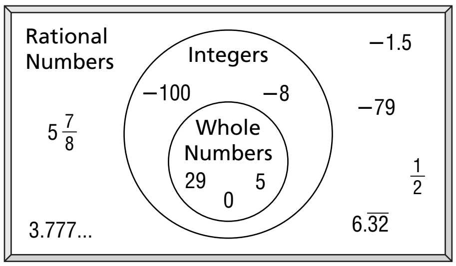
$10N = 5.555...$ Multiply each side by 10 because one digit repeats.

$$\begin{array}{r} 10N = 5.555... \\ \underline{-(N = 0.555...)} \\ 9N = 5 \end{array}$$ Subtract N from $10N$.

$\frac{9N}{9} = \frac{5}{9}$ Divide each side by 9.

$N = \frac{5}{9}$ Simplify.

The diagram at the right illustrates the relationships among whole numbers, integers, and rational numbers.



Exercises

Write each number as a fraction.

1. $1\frac{1}{5}$

2. -2

3. 0.7

4. 0.32

5. $-0.\overline{1}$

6. $0.\overline{49}$

Identify all sets to which each number belongs (W = whole numbers, I = integers, Q = rational numbers).

7. -12

8. 8.5

9. 582

Lesson 5–2

5-3 Study Guide and Intervention

Multiplying Rational Numbers

To multiply fractions, multiply the numerators and multiply the denominators. So $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, where $b, d \neq 0$. The fractions may be simplified either before or after multiplying.

Example **Find each product. Write in simplest form.**

a. $\frac{8}{15} \cdot \frac{5}{7}$

$\frac{8}{15} \cdot \frac{5}{7} = \frac{8}{\cancel{15}_{3}} \cdot \frac{\cancel{5}^{1}}{7}$ Divide 5 and 15 by their GCF, 5.

$= \frac{8 \cdot 1}{3 \cdot 7}$ Multiply.

$= \frac{8}{21}$ Simplify.

b. $7\frac{1}{2} \cdot 2\frac{2}{3}$

$7\frac{1}{2} \cdot 2\frac{2}{3} = \frac{15}{2} \cdot \frac{8}{3}$ Rename mixed numbers as improper fractions.

$= \frac{\cancel{15}^{5}}{\cancel{2}_{1}} \cdot \frac{\cancel{8}^{4}}{\cancel{3}_{1}}$ Divide 15 and 3 by 3, and 8 and 2 by 2.

$= \frac{5 \cdot 4}{1 \cdot 1}$ Multiply.

$= \frac{20}{1}$ or 20 Simplify.

Exercises

Find each product. Write in simplest form.

1. $\frac{1}{2} \cdot \frac{3}{5}$

2. $-\frac{8}{9} \cdot \frac{5}{16}$

3. $\frac{4}{5} \cdot \frac{5}{8}$

4. $\frac{3}{10} \cdot \left(-\frac{1}{4}\right)$

5. $\frac{7}{9} \cdot \frac{11}{20}$

6. $\frac{2}{5} \cdot (-5)$

7. $-4\frac{4}{5} \cdot 1\frac{1}{6}$

8. $1\frac{5}{7} \cdot 10\frac{1}{2}$

9. $-2\frac{1}{8} \cdot \left(-4\frac{4}{7}\right)$

10. $\frac{5x}{y} \cdot \frac{y^3}{z^2}$

Lesson 5–3

5-4 Study Guide and Intervention

Dividing Rational Numbers

Two numbers whose product is 1 are called multiplicative inverses or reciprocals. For any fraction $\frac{a}{b}$, where $a, b \neq 0$, $\frac{b}{a}$ is the multiplicative inverse and $\frac{a}{b} \cdot \frac{b}{a} = 1$. This means that $\frac{2}{3}$ and $\frac{3}{2}$ are multiplicative inverses because $\frac{2}{3} \cdot \frac{3}{2} = 1$.

To divide by a fraction, multiply by its multiplicative inverse: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$, where $b, c, d \neq 0$.

Example **Find each quotient. Write in simplest form.**

a. $\mathbf{\frac{3}{4} \div \frac{5}{8}}$

$\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \cdot \frac{8}{5}$ Multiply by the multiplicative inverse of $\frac{5}{8}$, $\frac{8}{5}$.

$= \frac{3}{\cancel{4}_1} \cdot \frac{\cancel{8}^2}{5}$ Divide 4 and 8 by their GCF, 4.

$= \frac{6}{5}$ or $1\frac{1}{5}$ Simplify.

b. $\mathbf{6\frac{2}{5} \div 2\frac{1}{5}}$

$6\frac{2}{5} \div 2\frac{1}{5} = \frac{32}{5} \div \frac{11}{5}$ Rename mixed numbers as improper fractions.

$= \frac{32}{5} \cdot \frac{5}{11}$ Multiply by the multiplicative inverse of $\frac{11}{5}$, $\frac{5}{11}$.

$= \frac{32}{\cancel{5}_1} \cdot \frac{\cancel{5}^1}{11}$ Divide out common factors.

$= \frac{32}{11}$ or $2\frac{10}{11}$ Simplify.

Exercises

Find each quotient. Write in simplest form.

1. $\frac{5}{16} \div \frac{5}{8}$

2. $\frac{7}{9} \div \frac{2}{3}$

3. $\frac{16}{21} \div \left(-\frac{2}{7}\right)$

4. $-\frac{4}{5} \div \frac{3}{10}$

5. $1\frac{1}{4} \div 2\frac{3}{8}$

6. $-8\frac{4}{7} \div 2\frac{1}{7}$

7. $\frac{18}{21} \div 3$

8. $-4\frac{5}{8} \div \left(-3\frac{1}{3}\right)$

9. $\frac{2x}{y} \div \frac{3}{y}$

10. $\frac{c}{4d} \div \frac{3}{8d}$

11. $\frac{4a}{b} \div \frac{2ac}{b}$

12. $\frac{m}{9} \div \frac{mn^2}{3}$

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5-5 Study Guide and Intervention

Adding and Subtracting Like Fractions

To add fractions with like denominators, add the numerators and write the sum over the denominator. So, $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$, where $c \neq 0$.

Example 1 **Find $1\frac{2}{9} + 3\frac{4}{9}$. Write the sum in simplest form.**

$1\frac{2}{9} + 3\frac{4}{9} = (1 + 3) + \left(\frac{2}{9} + \frac{4}{9}\right)$ Add the whole numbers and fractions separately or write as improper fractions.

$= 4 + \frac{2+4}{9}$ Add the numerators.

$= 4\frac{6}{9}$ or $4\frac{2}{3}$ Simplify.

To subtract fractions with like denominators, subtract the numerators and write the difference over the denominator. So, $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$, where $c \neq 0$.

Example 2 **Find $7\frac{1}{3} - 5\frac{2}{3}$. Write the difference in simplest form.**

$7\frac{1}{3} - 5\frac{2}{3} = \frac{22}{3} - \frac{17}{3}$ Write mixed numbers as improper fractions.

$= \frac{22-17}{3}$ Subtract the numerators.

$= \frac{5}{3}$ or $1\frac{2}{3}$ Simplify.

Exercises

Find each sum or difference. Write in simplest form.

1. $\frac{11}{12} + \frac{9}{12}$

2. $\frac{13}{15} + \frac{9}{15}$

3. $\frac{19}{20} - \frac{17}{20}$

4. $\frac{23}{25} - \frac{8}{25}$

5. $3\frac{7}{8} + \left(-4\frac{5}{8}\right)$

6. $9 + 4\frac{3}{7}$

7. $9\frac{2}{5} - 6\frac{3}{5}$

8. $4\frac{11}{12} - \left(-3\frac{7}{12}\right)$

5-6 Study Guide and Intervention

Least Common Multiple

The least common multiple (LCM) is the least nonzero number that is a multiple of two or more given numbers. When finding the LCM of small numbers, list several multiples of each number until a common multiple is found.

Example 1 **Find the LCM of 3 and 5.**

multiples of 3: 0, 3, 6, 9, 12, 15, 18, . . .

multiples of 5: 0, 5, 10, 15, 20, . . . The least nonzero number that is a multiple of both numbers is 15.

When finding the LCM of large numbers, write the prime factorization of each and multiply the greatest power of each prime factor used.

Example 2 **Find the LCM of 24 and 40.**

$24 = 2^3 \cdot 3$

$40 = 2^3 \cdot 5$ Find the prime factorization of each number.

$\text{LCM} = 2^3 \cdot 3^1 \cdot 5^1 = 120$ Write the product of the greatest power of each prime factor.

The least common denominator (LCD) of two or more fractions is the LCM of the denominators.

Example 3 **Find the LCD of $\frac{7}{8}$ and $\frac{2}{6}$.**

Find the LCM of the denominators, 6 and 8.

$6 = 2 \cdot 3$

$8 = 2^3$

$\text{LCM} = 2^3 \cdot 3 = 24$

The LCD is 24.

Exercises

Find the least common multiple (LCM) of each pair of numbers or monomials.

1. 3, 6 **2.** 4, 9 **3.** 10, 15

4. 6, 14 **5.** 16, 20 **6.** 21, 49

7. $5x, 6x$ **8.** $15b^2, 25b^3$ **9.** $18x, 12x^4$

Find the least common denominator (LCD) of each pair of fractions.

10. $\frac{10}{11}, \frac{29}{44}$ **11.** $\frac{3}{4}, \frac{9}{10}$ **12.** $\frac{5}{16}, \frac{13}{20}$

13. $\frac{3}{t}, \frac{2}{4st}$ **14.** $\frac{2}{ab}, \frac{4}{b^2}$ **15.** $\frac{6}{x^2}, \frac{1}{xy^3}$

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5-7 Study Guide and Intervention

Adding and Subtracting Unlike Fractions

To add fractions with unlike denominators, rename the fractions with a common denominator. Then add and simplify.

Example 1 **Find $\frac{4}{7} + \frac{1}{3}$.**

$\frac{4}{7} + \frac{1}{3} = \frac{4}{7} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{7}{7}$ Use 7 · 3 or 21 as the common denominator.

$= \frac{12}{21} + \frac{7}{21}$ Rename each fraction with the common denominator.

$= \frac{19}{21}$ Add the numerators.

To subtract fractions with unlike denominators, rename the fractions with a common denominator. Then subtract and simplify.

Example 2 **Find $9\frac{2}{9} - 8\frac{5}{6}$.**

$9\frac{2}{9} - 8\frac{5}{6} = \frac{83}{9} - \frac{53}{6}$ Write the mixed numbers as improper fractions.

$= \frac{83}{9} \cdot \frac{2}{2} - \frac{53}{6} \cdot \frac{3}{3}$ Rename fractions using the LCD, 18.

$= \frac{166}{18} - \frac{159}{18}$ Simplify.

$= \frac{7}{18}$ Subtract the numerators.

Exercises

Find each sum or difference. Write in simplest form.

1. $\frac{8}{9} + \frac{2}{5}$ **2.** $\frac{7}{15} - \frac{3}{10}$ **3.** $-\frac{2}{3} + \frac{1}{4}$

4. $-\frac{6}{11} - \frac{6}{11}$ **5.** $\frac{7}{8} + \frac{1}{4}$ **6.** $\frac{13}{15} - \frac{2}{5}$

7. $3\frac{1}{5} + 2\frac{3}{4}$ **8.** $7\frac{5}{6} + \left(-3\frac{1}{3}\right)$ **9.** $\frac{3}{8} - \frac{1}{12}$

10. $4\frac{3}{10} - \left(-2\frac{4}{5}\right)$ **11.** $6\frac{3}{4} + 3\frac{1}{2}$ **12.** $7\frac{4}{9} + 9\frac{1}{6}$

13. $4\frac{1}{6} - 3\frac{1}{8}$ **14.** $5\frac{8}{9} - \left(-2\frac{1}{3}\right)$ **15.** $5\frac{1}{10} - 3\frac{2}{3}$

Lesson 5–7

5-8 Study Guide and Intervention

Solving Equations With Rational Numbers

To solve rational number equations, use the same skills applied to solving equations involving integers.

Example 1 **Solve $4.2 = p - 1.7$.**

$4.2 = p - 1.7$	Write the equation.
$4.2 + 1.7 = p - 1.7 + 1.7$	Add 1.7 to each side.
$5.9 = p$	Simplify.

Example 2 **Solve $y + \frac{1}{2} = \frac{8}{9}$.**

$y + \frac{1}{2} = \frac{8}{9}$	Write the equation.
$y + \frac{1}{2} - \frac{1}{2} = \frac{8}{9} - \frac{1}{2}$	Subtract $\frac{1}{2}$ from each side.
$y = \frac{8}{9} - \frac{1}{2}$	Simplify.
$y = \frac{16}{18} - \frac{9}{18}$ or $\frac{7}{18}$	Rename fractions using LCD and subtract.

Example 3 **Solve $-6z = 4.2$.**

$-6z = 4.2$	Write the equation.
$\frac{-6z}{-6} = \frac{4.2}{-6}$	Divide each side by –6.
$z = -0.7$	Simplify.

Example 4 **Solve $8 = \frac{3}{4}x$.**

$8 = \frac{3}{4}x$	Write the equation.
$\frac{4}{3}(8) = \frac{4}{3}\left(\frac{3}{4}x\right)$	Multiply each side by $\frac{4}{3}$.
$\frac{32}{3} = x$	Simplify.
$10\frac{2}{3} = x$	Simplify.

Exercises

Solve each equation. Check your solution.

1. $p + 7.4 = 9.8$

2. $a - 5.8 = -1.3$

3. $\frac{1}{3} + h = \frac{5}{6}$

4. $9v = 8.1$

5. $\frac{5}{6}q = \frac{15}{42}$

6. $1\frac{1}{2} = c - 3\frac{2}{5}$

7. $m + \frac{1}{2} = 9$

8. $\frac{7}{8}d = 56$

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5-9 Study Guide and Intervention

Measures of Central Tendency

When working with numerical data, it is often helpful to use one or more numbers to represent the whole set. These numbers are called the measures of central tendency. You will study the mean, median, and mode.

Statistic	Definition
mean	sum of the data divided by the number of items in the data set
median	middle number of the ordered data, or the mean of the middle two numbers
mode	number or numbers that occur most often

Example **Jason recorded the number of hours he spent watching television each day for a week. Find the mean, median, and mode for the number of hours.**

Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
2	3.5	3	0	2.5	6	4

$$\text{mean} = \frac{\text{sum of hours}}{\text{number of days}}$$

$$= \frac{2 + 3.5 + 3 + \ldots + 4}{7}$$

$= 3$ The mean is 3 hours.

To find the median, order the numbers from least to greatest and locate the number in the middle.

0 2 2.5 3 3.5 4 6 The median is 3 hours.

There is no mode because each number occurs once in the set.

Exercises

Find the mean, median, and mode for each set of data.

1. Maria's test scores

92, 86, 90, 74, 95, 100, 90, 50

2. Rainfall last week in inches

0, 0.3, 0, 0.1, 0, 0.5, 0.2

3. Resting heart rates of 8 males

84, 59, 72, 63, 75, 68, 72, 63

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6-1 Study Guide and Intervention

Ratios and Rates

Ratios written as 7 to 12, 7:12, and $\frac{7}{12}$ are different ways to write the same ratio. Ratios should be written in simplest form.

Example 1 **Express the ratio *6 feet to 15 inches* as a fraction in simplest form.**

$\frac{6 \text{ feet}}{15 \text{ inches}}$ Write the ratio as a fraction.

$= \frac{72 \text{ inches}}{15 \text{ inches}}$ Convert feet to inches.

$= \frac{\cancel{72}^{24} \cancel{\text{inches}}}{\cancel{15}_{5} \cancel{\text{inches}}}$ Divide the numerator and denominator by the GCF, 3.

$= \frac{24}{5}$

Written as a fraction in simplest form, the ratio is 24 to 5.

Example 2 **Express the ratio *\$10 for 8 fish* as a unit rate. Round to the nearest tenth, if necessary.**

$\frac{10 \text{ dollars}}{8 \text{ fish}}$ Write the ratio as a fraction.

$\frac{10 \text{ dollars}}{8 \text{ fish}} \overset{\div 8}{=} \frac{1.25 \text{ dollars}}{1 \text{ fish}}$ ($\div 8$) Divide the numerator and denominator by 8 to get a denominator of 1.

The unit rate is \$1.25 per fish.

Exercises

Express each ratio as a fraction in simplest form.

1. 4 weeks to plan 2 events

2. 9 inches to 2 feet

3. 8 teaspoons to 12 forks

4. 16 cups to 10 servings

5. 7 shelves to 84 books

6. 6 teachers to 165 students

Express each ratio as a unit rate. Round to the nearest tenth or nearest cent, if necessary.

7. \$58 for 5 tickets

8. \$4.19 for 4 cans of soup

9. \$274.90 for 6 people

10. 565 miles in 12 hours

6-2 Study Guide and Intervention

Proportional and Nonproportional Relationships

PROPORTIONS Two quantities are **proportional** if they have a constant ratio or rate. Proportional relationships can also be described using equations of the form $y = kx$, where k is the constant ratio. The constant ratio is called the **constant of proportionality**.

Example 1 **Determine whether the numbers in the table are proportional.**

Time (minutes)	1	2	3	4
Distance (yards)	300	600	900	1200

Write the rate of time to distance for each minute in simplest form.

$\frac{1}{300} = \frac{1}{300}$ $\frac{2}{600} = \frac{1}{300}$ $\frac{3}{900} = \frac{1}{300}$ $\frac{4}{1200} = \frac{1}{300}$

Since all rates are equal, the time is proportional to the distance.

Example 2 **GEOMETRY The perimeter of a square with a side of 3 inches is 12 inches. A square's perimeter is proportional to the length of one of its sides. Write an equation relating the perimeter of a square to the length of one of its sides. What would be the perimeter of a square with 9-inch sides?**

Find the constant of proportionality between circumference and diameter.

$$\frac{\text{perimeter}}{\text{length of sides}} = \frac{12}{3} \text{ or } 4$$

$P = 4s$ Write the equation.
$P = 4(9)$ Repace d with the diameter.
$P = 36$ Multiply.

The perimeter of a square with a side of 9 inches is about 36 inches.

Exercises

Determine whether the set of numbers in the table are proportional.

1.

Cookies	6	9	12	15
Cupcakes	4	6	8	10

2.

Population (100,000)	1.3	2.1	3.3	5.2
Years	1	2	3	4

3. **SCHOOL** A school is repainting some of its classrooms. Each classroom is repainted with 5.5 gallons of paint. Write and solve an equation to determine the gallons of paint the school must purchase if they repaint 18 classrooms.

NAME ____ DATE ____________ PERIOD _____

6-3 Study Guide and Intervention

Using Proportions

A **proportion** is an equation stating that two ratios are equal. You can use cross products to solve a proportion.

Example 1 **Solve the proportion $\frac{14.1}{c} = \frac{3}{4}$.**

$$\frac{14.1}{c} = \frac{3}{4}$$

$14.1 \cdot 4 = c \cdot 3$ Cross products.

$56.4 = 3c$ Multiply.

$\frac{56.4}{3} = \frac{3c}{3}$ Divide.

$18.8 = c$

The solution is 18.8.

Exercises

ALGEBRA Solve each proportion.

1. $\frac{x}{9} = \frac{16}{12}$

2. $\frac{32}{28} = \frac{w}{7}$

3. $\frac{5}{u} = \frac{60}{132}$

4. $\frac{36}{21} = \frac{24}{s}$

5. $\frac{a}{64} = \frac{225}{480}$

6. $\frac{42}{w} = \frac{56}{8}$

7. $\frac{1}{10} = \frac{m}{12}$

8. $\frac{5}{3} = \frac{85}{h}$

9. $\frac{24}{g} = \frac{2}{30}$

10. $\frac{f}{21} = \frac{57}{63}$

11. $\frac{22}{z} = \frac{121}{16.5}$

12. $\frac{2}{3} = \frac{k}{12.6}$

13. $\frac{r}{9} = \frac{5}{20}$

14. $\frac{d}{21} = \frac{1.5}{3.5}$

15. $\frac{46}{57.5} = \frac{360}{q}$

16. $\frac{4.2}{4.8} = \frac{d}{80}$

17. $\frac{1}{c} = \frac{4.5}{11.7}$

18. $\frac{0.3}{n} = \frac{4.75}{14.25}$

19. $\frac{9.1}{14.7} = \frac{1.3}{p}$

20. $\frac{0.4}{3} = \frac{y}{98.25}$

21. $\frac{v}{33.44} = \frac{1}{3.2}$

6-4 Study Guide and Intervention

Scale Drawings and Models

A **scale** gives the relationship between the measurements on the drawing or model and the measurements of the real object.

Example **A map shows a scale of 1 inch = 6 miles. The distance between two places on the map is 4.25 inches. What is the actual distance?**

Let x represent the actual distance. Write and solve a proportion.

$$\begin{array}{rcl} \text{map width} \rightarrow & \dfrac{1 \text{ inch}}{6 \text{ miles}} = \dfrac{4.25 \text{ inches}}{x \text{ miles}} & \leftarrow \text{map width} \\ \text{actual width} \rightarrow & & \leftarrow \text{actual width} \end{array}$$

$1 \cdot x = 6 \cdot 4.25$ Find the cross products.

$x = 25.5$ Simplify.

The actual distance is 25.5 miles.

Exercises

On a set of architectural drawings for an office building, the scale is 0.25 inch = 5 feet. Find the actual length of each room.

	Room	Drawing Distance	Actual Distance
1.	Lobby	1.6 inches	
2.	CEO Office	1.35 inches	
3.	Copy Room	0.55 inch	
4.	CEO Secretary's Office	0.6 inch	
5.	Vice President's Office	0.9 inch	
6.	Library	1.525 inches	
7.	Storage Area	2.1125 inches	
8.	Personal Manager's Office	1.7375 inches	
9.	Manager's Office	0.625 inch	
10.	Mail Room	2.2625 inches	
11.	Boiler Room	3.725 inches	
12.	Conference Room A	2.62 inches	
13.	Conference Room B	0.925 inch	
14.	Cafeteria	2.3 inches	
15.	Kitchen	2 inches	

6-5 Study Guide and Intervention

Fractions, Decimals, and Percents

A **percent** is a part to whole ratio that compares a number to 100.

Example 1 **Express the percent as a fraction.**

65%

$65\% = \frac{65}{100}$ Write as a fraction with denominator of 100.

$= \frac{13}{20}$ Simplify.

Example 2 **Express the percent as a decimal.**

150%

$150\% = 150\%$ Divide by 100 and remove the %.

$= 1.5$

Example 3 **Express the fraction as a percent.**

$\mathbf{\frac{3}{20}}$

$\frac{3}{20} = \frac{15}{100}$ or 15% Write equivalent fraction with denominator of 100.

Example 4 **Express the decimal as a percent.**

3.17

$3.17 = 3.17$ Multiply by 100 and add the %.

$= 317\%$

Exercises

Express each percent as a fraction or mixed number in simplest form and as a decimal.

1. 12% **2.** 5% **3.** 17%

4. 72% **5.** 150% **6.** 2%

7. 98% **8.** 825% **9.** 0.6%

Express each decimal or fraction as a percent. Round to the nearest tenth percent, if necessary.

10. 0.3 **11.** 0.21 **12.** 0.09

13. 3.255 **14.** $\frac{3}{5}$ **15.** $\frac{3}{8}$

16. $\frac{7}{9}$ **17.** $\frac{5}{7}$ **18.** $4\frac{3}{4}$

6-6 Study Guide and Intervention

Using the Percent Proportion

In a **percent proportion**, one of the numbers, called the **part**, is being compared to the whole quantity, called the **base**. The other ratio is the percent, written as a fraction, whose base is 100.

Example 1 **Find each percent.**

a. Twelve is what percent of 16?

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{12}{16} = \frac{p}{100}$ Replace the variables.

$12 \cdot 100 = p \cdot 16$ Find the cross products.

$1200 = 16p$ Simplify.

$75 = p$ Divide.

So, twelve is 75% of 16.

b. What percent of 8 is 7?

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{7}{8} = \frac{p}{100}$

$p \cdot 8 = 100 \cdot 7$

$700 = 8p$

$87.5 = p$

So, 87.5% of 8 is 7.

Example 2 **Find the part or the base.**

a. What number is 1.4% of 15?

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{a}{15} = \frac{1.4}{100}$ Replace the variables.

$a \cdot 100 = 15 \cdot 1.4$ Find the cross products.

$100a = 21$ Simplify.

$a = 0.21$ Divide.

So, 0.21 is 1.4% of 15.

b. 225 is 36% of what number?

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{225}{b} = \frac{36}{100}$

$225 \cdot 36 = 100 \cdot b$

$22{,}500 = 36b$

$625 = b$

So, 225 is 36% of 625.

Exercises

Use the percent proportion to solve each problem. Round to the nearest tenth.

1. 48 is what percent of 52?

2. 295 is what percent of 400?

3. What percent of 22 is 56?

4. What percent of 4 is 15?

5. What is 99% of 840?

6. What is 4.5% of 38?

7. What is 16% of 36.2?

8. 85 is 80% of what number?

9. 60 is 29% of what number?

10. 4.5 is 90% of what number?

6-7 Study Guide and Intervention

Finding Percents Mentally

When working with common percents like 10%, 25%, 40%, and 50%, it may be helpful to use the fraction form of the percent.

Percent-Fraction Equivalents				
$20\% = \frac{1}{5}$	$10\% = \frac{1}{10}$	$25\% = \frac{1}{4}$	$12\frac{1}{2}\% = \frac{1}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$40\% = \frac{2}{5}$	$30\% = \frac{3}{10}$	$50\% = \frac{1}{2}$	$37\frac{1}{2}\% = \frac{3}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$
$60\% = \frac{3}{5}$	$70\% = \frac{7}{10}$	$75\% = \frac{3}{4}$	$62\frac{1}{2}\% = \frac{5}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$
$80\% = \frac{4}{5}$	$90\% = \frac{9}{10}$	$100\% = 1$	$87\frac{1}{2}\% = \frac{7}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$

Example **Find 20% of 35 mentally.**

$20\% \text{ of } 35 = \frac{1}{5} \text{ of } 35$ Think: $20\% = \frac{1}{5}$.

$= 7$ Think: $\frac{1}{5}$ of 35 is 7. So, 20% of 35 is 7.

Exercises

Find the percent of each number mentally.

1. 50% of 6
2. 25% of 100
3. 60% of 25
4. 75% of 28
5. $66\frac{2}{3}\%$ of 33
6. 150% of 2
7. 125% of 4
8. 175% of 4
9. 10% of 110
10. 80% of 20
11. 20% of 80
12. 20% of 800
13. 30% of 250
14. 60% of 250
15. 75% of 1000
16. 10% of 900
17. 20% of 900
18. 40% of 900
19. 25% of 360
20. 50% of 360
21. 75% of 360
22. $62\frac{1}{2}\%$ of 32
23. $37\frac{1}{2}\%$ of 32
24. 200% of 21
25. $66\frac{2}{3}\%$ of 54
26. 150% of 2222
27. $12\frac{1}{2}\%$ of 720
28. 30% of 30
29. $66\frac{2}{3}\%$ of 150
30. 80% of 1500

6-8 Study Guide and Intervention

Using Percent Equations

A **percent equation** is an equivalent form of a percent proportion. In a percent equation, the percent is written as a decimal.

Example **Solve each problem using the percent equation.**

a. Find 22% of 95.

$n = 0.22(95)$

$n = 20.9$

So, 22% of 95 is 20.9.

b. 15 is what percent of 75?

$15 = n(75)$

$0.2 = n$

So, 15 is 20% of 75.

c. 90 is 20% of what number?

$90 = 0.2n$

$450 = n$

So, 90 is 20% of 450.

Exercises

Solve each problem using the percent equation.

1. Find 76% of 25.

2. Find 9% of 410.

3. Find 40% of 7.

4. Find 26% of 505.

5. Find 3.5% of 280.

6. Find 18.5% of 60.

7. Find 107% of 1080.

8. 256 is what percent of 800?

9. 36 is what percent of 240?

10. 2089.5 is what percent of 2100?

11. 15.4 is what percent of 55?

12. 7 is what percent of 350?

13. 13.2 is what percent of 80?

14. 14.4 is what percent of 120?

15. 36 is 9% of what number?

16. 2925 is 39% of what number?

17. 576 is 90% of what number?

18. 24.2 is 55% of what number?

19. 25 is 125% of what number?

20. 0.6 is 7.5% of what number?

6-9 Study Guide and Intervention

Percent of Change

A **percent of change** tells how much an amount has increased or decreased in relation to the original amount. There are two methods you can use to find percent of change.

Example **Find the percent of change from 75 yards to 54 yards.**

Step 1 Subtract to find the amount of change.

$54 - 75 = -21$ new measurement – original measurement

Step 2 Write a ratio that compares the amount of change to the original measurement. Express the ratio as a percent.

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original measurement}}$$

$$= \frac{-21}{75} \quad \text{Substitution}$$

$$= -0.28 \text{ or } -28\% \quad \text{Write the decimal as a percent.}$$

Exercises

State whether each change is a *percent of increase* or a *percent of decrease*. Then find the percent of change. Round to the nearest tenth, if necessary.

1. from 22 inches to 16 inches

2. from 8 years to 10 years

3. from $815 to $925

4. from 15 meters to 12 meters

5. from 55 people to 217 people

6. from 45 mi per gal to 24 mi per gal

7. from 28 cm to 32 cm

8. from 128 points to 144 points

9. from $8 to $2.50

10. from 800 roses to 639 roses

11. from 8 tons to 4.2 tons

12. from 5 qt to 18 qt

13. from $85.75 to $90.15

14. from 198 lb to 112 lb

6-10 Study Guide and Intervention

Using Sampling to Predict

SAMPLING TECHNIQUES A **sample** is a randomly selected smaller group chosen from the larger group, or **population**. An **unbiased sample** is representative of the larger population, selected without preference, and large enough to provide accurate data. A **biased sample** is not representative of the larger population. Depending on the sample method used, you can use a sample to predict the characteristics of larger populations.

Unbiased Samples	simplified random sample, stratified random sample, systematic random sample
Biased Samples	convenience sample, voluntary response sample

Example 1 **POLITICS To determine the popularity of a political candidate, 5 people are randomly polled from 10 different age groups of eligible voters. Identify the sample as biased or unbiased and describe its type.**

Since all eligible voters are equally likely to be polled, it is an unbiased sample. Since eligible voters are randomly polled from similar, non-overlapping groups, the sample is a stratified random sample.

Example 2 **SHOPPING To determine the number of first-time visitors to a mall, every 15th shopper to enter the mall was polled. There were 3000 total shoppers in the mall, and, of the shoppers polled, 26 shoppers were in the mall for the first time. Is this sampling method valid? If so, about how many of the 3000 shoppers were in the mall for the first time?**

Yes, this is a valid sampling method. This is a systematic random sample because the shoppers were selected according to a specific interval. Since every 15th shopper was sampled, there were a total of $3000 \div 15$ or 200 shoppers sampled and 26 were in the mall for the first time. This means $\frac{26}{200}$ or 13% of the shoppers were in the mall for the first time. So a prediction of the total number of shoppers in the mall for the first time is 13% of 3000 or 390.

Exercises

1. **STUDYING** To determine the average number of hours that students study, members of the math club are polled. Identify the sample as biased or unbiased and describe its type.

2. **PRINTING** To determine the consistency of a printer, 100 printed sheets are randomly checked and 4 sheets are defective. What type of sampling method is this? About how many defective sheets would be expected if 2400 sheets were printed?

7-1 Study Guide and Intervention

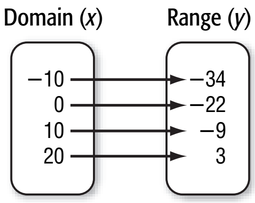
Functions

Function	A special relation in which each member of the domain is paired with exactly one member in the range.
Vertical Line Test	Move a pencil or straightedge from left to right across the graph of a relation. • If it passes through no more than one point on the graph, the graph represents a function. • If it passes through more than one point on the graph, the graph does not represent a function.

Since functions are relations, they can be represented using ordered pairs, tables, or graphs.

Example **Determine whether each relation is a function. Explain.**

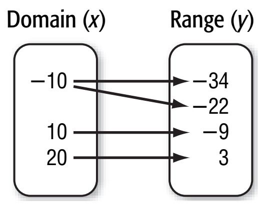
a. **{(−10, −34), (0, −22), (10, −9), (20, 3)}**



Because each element in the domain is paired with only one value in the range, this is a function.

b.

x	−10	−10	10	20
y	−34	−22	−9	3



Because −10 in the domain is paired with −34 and −22 in the range, this is not a function.

Exercises

Determine whether each relation is a function. Explain.

1. {(−5, 2), (3, −3), (1, 7), (3, 0)}

2. {(2, 7), (−5, 20), (−10, 20), (−2, 10), (1, 20)}

3.

x	1	−3	8	−8	20
y	2	6	6	5	11

4.

x	8	1	−5	1	−10
y	−2	3	7	7	13

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7-2 Study Guide and Intervention

Representing Linear Functions

A function can be represented with an equation. An equation such as $y = 1.50x$ is called a linear equation. A linear equation in two variables is an equation in which the variables appear in separate terms and neither variable contains an exponent other than 1.

Linear Equations $y = x + 1$, $y = -2x$, $y = \frac{1}{3}x$

Nonlinear Equations $y = x^2 + 1$, $y = -2x^3$, $y = \frac{3}{x}$, $xy = 4$

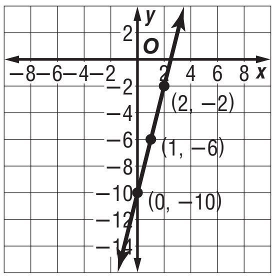
Solutions of a linear equation are ordered pairs that make the equation true. One way to find solutions is to make a table.

Example 1 **Complete the table. Use the results to write four solutions of $y = 4x - 10$. Write the solution as ordered pairs.**

x	$y = 4x - 10$	y	(x, y)
-1	$y = 4(-1) - 10$	-14	$(-1, -14)$
0	$y = 4(0) - 10$	-10	$(0, -10)$
1	$y = 4(1) - 10$	-6	$(1, -6)$
2	$y = 4(2) - 10$	-2	$(2, -2)$

Example 2 **A linear equation can also be represented by a graph. The coordinates of all points on a line are solutions to the equation. Graph $y = 4x - 10$ by plotting ordered pairs.**

Plot the points found in Example 1. Connect the points using a straight line.



Exercises

Find four solutions of each equation. Write the solutions as ordered pairs.

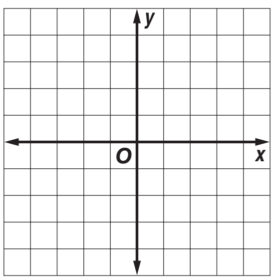
1. $y = 2x + 4$

2. $y = -3x - 7$

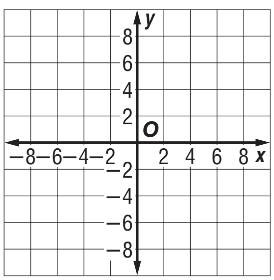
3. $4x + y = 5$

Graph each equation by plotting ordered pairs.

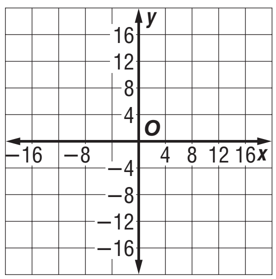
4. $y = -4x$



5. $y = x + 6$



6. $-2x + y = 8$



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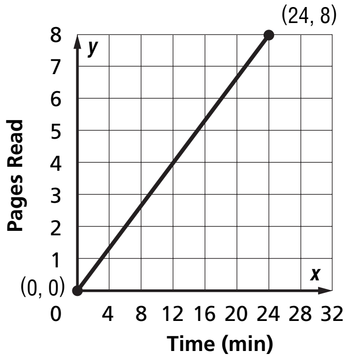
7-3 Study Guide and Intervention

Rate of Change

A **rate of change** is a rate that describes how one quantity changes in relation to another quantity.

Example 1 **Find the rate of change for the linear function represented in the table.**



$$\text{rate of change} = \frac{\text{change in pages read}}{\text{change in time}}$$
$$= \frac{8 \text{ pages} - 0 \text{ pages}}{24 \text{ minutes} - 0 \text{ minutes}}$$
$$= \frac{8 \text{ pages}}{24 \text{ minutes}}$$
$$= \frac{1}{3} \text{ page per minute}$$

So, the rate of change is $\frac{1}{3}$ page/minute, or an increase of $\frac{1}{3}$ page per 1 minute increase in time.

Example 2 **Find the rate of change for the linear function represented in the table.**

$$\text{rate of change} = \text{slope}$$
$$= \frac{\text{change in } y}{\text{change in } x}$$
$$= \frac{2}{1} \text{ or } 2$$

For each time increase of 1h, the temperature increases by 2°C.

Time (h)	*x*	0	1	2	3
Temperature (˚C)	*y*	0	2	4	6

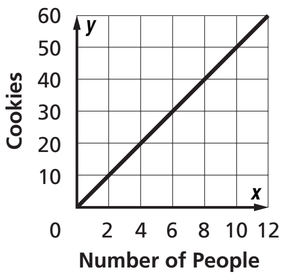
Exercises

Find the rate of change for each linear function.

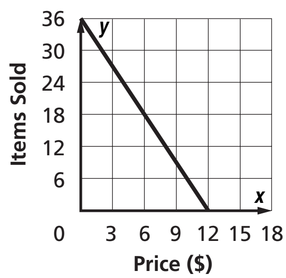
1.

Time (h)	*x*	0	2	4	6
Distance Flown (mi)	*y*	0	1000	2000	3000

2. **Cookies Needed**



3. **Sales**



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7-4 Study Guide and Intervention

Constant Rate of Change and Direct Variation

Relationships that have straight-line graphs are called **linear relationships**. A linear relationship has a **constant rate of change**. A special type of linear equation that describes a constant rate of change is called a **direct variation**. The graph of a direct variation always passes through the origin and can be expressed as $y = kx$, where k is called the **constant of variation**.

Example **SCUBA DIVING As scuba divers descend below the surface of the ocean, the pressure they feel from water increases at a constant rate. This is shown in the table.**

Depth (ft)	Water Pressure (lb/in^2)
x	y
20	8.9
30	13.35
40	17.8
50	22.25

a. Find the constant rate of change for this linear function and interpret its meaning.

$$\text{rate of change} = \frac{\text{change in water pressure}}{\text{depth}}$$

$$= \frac{13.35 - 8.9}{30 - 20} = \frac{4.45}{10} = 0.445$$

The water pressure increases by 0.445 lb/in^2 per foot increase in depth.

b. Determine whether a proportional linear relationship exists between the two quantities shown in the table. Explain your reasoning.

Find $\frac{\text{water pressure}}{\text{depth}}$ for each pair of values.

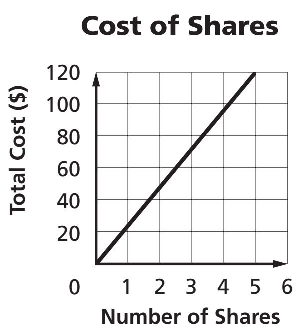
$\frac{8.9}{20} = 0.445$ $\frac{13.35}{30} = 0.445$ $\frac{17.8}{40} = 0.445$ $\frac{22.5}{50} = 0.445$

c. Write an equation that relates the depth and the amount of water pressure.

The linear relationship between depth and amount of water pressure is proportional, so we can use the direct variation equation $y = kx$, where the constant of variation is the equivalent ration computed in b. So $k = 0.445$, and the equation is $y = 0.445x$.

Exercises

STOCKS For exercises 1–3 refer to the graph at the right.



1. Find a constant rate of change for the linear function.

2. Determine whether a proportional linear relationship exists between the two quantities.

3. Write an equation that relates the number of shares and the total cost.

Lesson 7-4

7-5 Study Guide and Intervention

Slope

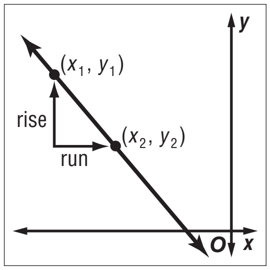
Slope describes the steepness of a line.

slope = $\frac{\text{rise}}{\text{run}}$ ← vertical change / ← horizontal change

Note that the slope is the same for any two points on a straight line.

Symbols $m_1 = \frac{y_2 - y_1}{x_2 - x_2}$, where $x_2 \neq x_1$

Model



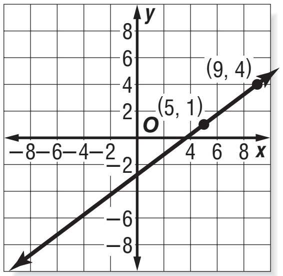
Example 1 **Find the slope of the line.**

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \frac{4 - 1}{9 - 5}$ $(x_1, y_1) = (5, 1)$, $(x_2, y_2) = (9, 4)$

$m = \frac{3}{4}$

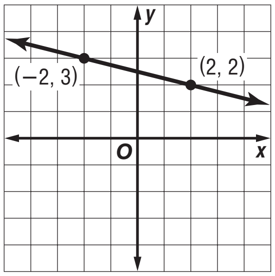
The slope is $\frac{3}{4}$.



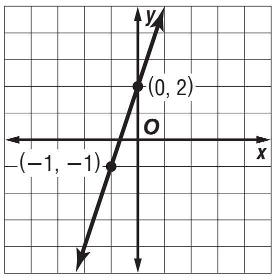
Exercises

Find the slope of each line.

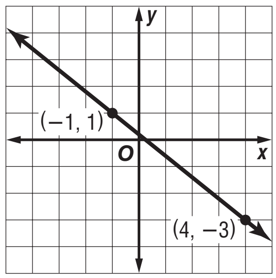
1.



2.



3.



Find the slope of the line that passes through each pair of points.

4. $A(2, 2)$, $B(-5, 4)$

5. $L(5, 5)$, $M(4, 2)$

6. $R(7, -4)$, $S(7, 3)$

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7-6 Study Guide and Intervention

Slope-Intercept Form

Lesson 7-6

The **slope-intercept form** of a line makes it easy to graph the line:	
$y = mx + b$	**Example:** $y = 3x + 2$
slope $= m$	slope $= 3$
y-intercept $= b$	y-intercept $= 2$

Example 1 **Graph $y = -4x - 3$ using the slope and y-intercept.**

Step 1 Find the slope and y-intercept.
slope $= -4$ $\quad$ y-intercept $= -3$

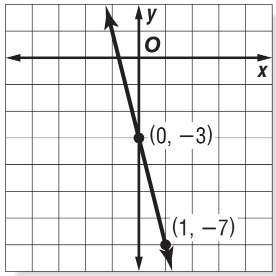
Step 2 Graph the y-intercept point at $(0, -3)$.

Step 3 Write the slope -4 as $\frac{-4}{1}$. Use the slope to locate a second point on the line.

$m = \frac{-4}{1}$ change in y: down 4 units / change in x: right 1 unit

Step 4 Draw a line through the two points.

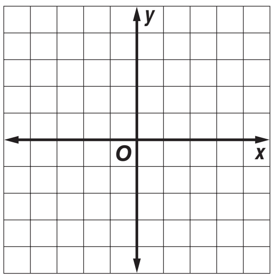
Step 5 Check by locating another point on the line and substituting the coordinates into the original equation.



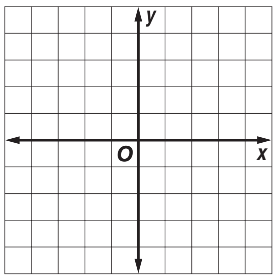
Exercises

Given the slope and y-intercept, graph each line.

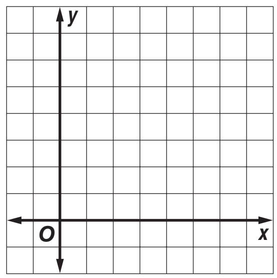
1. slope $= 4$, y-intercept $= -1$



2. slope $= 6$, y-intercept $= 4$

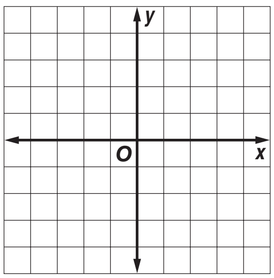


3. slope $= -\frac{1}{4}$, y-intercept $= 5$

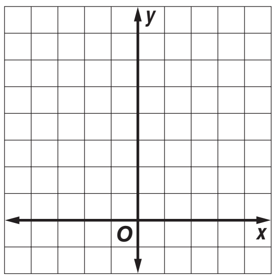


Graph each equation using the slope and y-intercept.

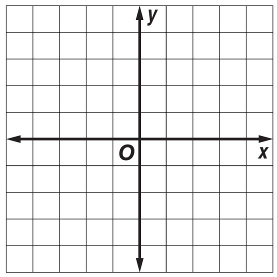
4. $y = 3x - 2$



5. $y = \frac{2}{3}x + 3$



6. $y = 5x - 3$



7-7 Study Guide and Intervention

Writing Linear Functions

If you know the slope and y-intercept, you can write the equation of a line by substituting these values in $y = mx + b$.

Example 1 **Write an equation in slope-intercept form for each line.**

a. slope = $-\frac{1}{4}$, y-intercept = −3

$y = mx + b$ Slope-intercept form

$y = -\frac{1}{4}x + (-3)$ Replace m with $-\frac{1}{4}$ and b with −3.

$y = -\frac{1}{4}x - 3$ Simplify.

b. slope = 0, y-intercept = −9

$y = mx + b$ Slope-intercept form

$y = 0x + (-9)$ Replace m with 0 and b with −9.

$y = -9$ Simplify.

Example 2 **Write an equation in slope-intercept form for the line passing through (–4, 4) and (2, 7).**

Step 1 Find the slope m.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \frac{7 - 4}{2 - (-4)}$ or $\frac{1}{2}$ $(x_1, y_1) = (-4, 4)$, $(x_2, y_2) = (2, 7)$

Step 2 Find the y-intercept b. Use the slope and the coordinates of either point.

$y = mx + b$ Slope-intercept form

$4 = \frac{1}{2}(-4) + b$ Replace (x, y) with (−4, 4) and m with $\frac{1}{2}$.

$6 = b$ Simplify.

An equation is $y = \frac{1}{2}x + 6$.

Exercises

Write an equation in slope-intercept form for each line.

1. slope = 1, y-intercept = 2

2. slope = $-\frac{3}{4}$, y-intercept = −5

3. slope = 0, y-intercept = −3

Write an equation in slope-intercept form for the line passing through each pair of points.

4. (6, 2) and (3, 1)

5. (8, 8) and (−4, 5)

6. (7, −3) and (−5, −3)

Lesson 7-7

7-8 Study Guide and Intervention

Prediction Equations

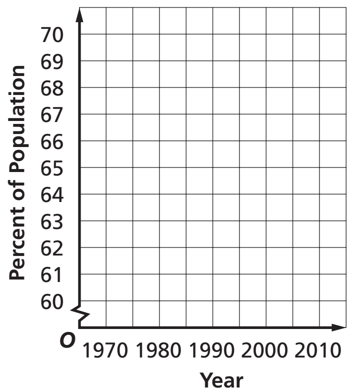
A **best-fit** line is a line that is very close to most of the data points. You can use best-fit lines to make predictions from real-world data.

Example 1 **Make a scatter plot and draw a best-fit line for the data in the table.**

Year	Percent of Population	Year	Percent of Population
1970	60.4	2000	67.1
1980	63.8	2001	66.8
1985	64.8	2002	66.6
1990	66.5	2003	66.2

Source: U.S. Census Bureau

U.S. Civilian Labor Force



Source: U.S. Census Bureau

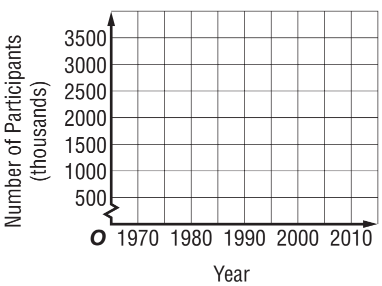
Example 2 **Use the best-fit line in example 1 to predict the percent of the population in the U.S. labor force in 2010.**

Use the extended line to find the y value for an x value of 2010—about 70. A prediction for the percent of the U.S. population in the labor force in 2010 is approximately 70 percent.

Exercises

Use the table that shows the number of girls who participated in high school athletic programs in the United States from 1973 to 1998.

1. Make a scatter plot and draw a best-fit line.



Year	1973	1978	1983	1988	1993	1998	2003
Number of Participants (thousands)	817	2083	1780	1850	1997	2570	2856

Source: U.S. Census Bureau

2. Use the best-fit line to predict the number of female participants in 2008.

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8-1 Study Guide and Intervention

Solving Equations with Variables on Each Side

To solve equations with variables on each side, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variable on one side. Then solve the equation.

Example **Solve the equation $12x - 3 = 4x + 13$. Then check your solution.**

$12x - 3 = 4x + 13$	Write the equation.
$12x - 4x - 3 = 4x - 4x + 13$	Subtract 4*x* from each side.
$8x - 3 = 13$	Simplify.
$8x - 3 + 3 = 13 + 3$	Add 3 to each side.
$8x = 16$	Simplify.
$x = 2$	Mentally divide each side by 8.

To check your solution, replace x with 2 in the original equation.

CHECK	$12x - 3 = 4x + 13$	Write the equation.
	$12(2) - 3 \stackrel{?}{=} 4(2) + 13$	Replace *x* with 2.
	$24 - 3 \stackrel{?}{=} 8 + 13$	Simplify.
	$21 = 21$ ✓	The statement is true.

Exercises

Solve each equation. Check your solution.

1. $2x + 1 = x + 11$

2. $a + 2 = 5 + 4a$

3. $7y + 25 = 2y$

4. $n + 11 = 2n$

5. $7 - 4c = 3c - 7$

6. $4 - 3b = 6b - 5$

7. $9d - 9 = 3d - 3$

8. $f - 4 = 6f + 26$

9. $-2s + 3 = 5s + 24$

10. $5a - 3 = 8a + 6$

11. $8n - 12 = -12n + 8$

12. $7y + 8 = -2y - 64$

13. $1 + 3x = 7x - 7$

14. $6a - 3 = 4 + 7a$

15. $3b - 1 = 14 + 2b$

16. $12c + 18 = 4 + 5c$

17. $9y + 3 = 5y - 13$

18. $3n - 2 = 5n + 12$

8-2 Study Guide and Intervention

Solving Equations with Grouping Symbols

Equations with grouping symbols can be solved by first using the Distributive Property to remove the grouping symbols.

Example 1 **Solve the equation $2(6m - 1) = 8m$. Check your solution.**

$2(6m - 1) = 8m$	Write the equation.
$12m - 2 = 8m$	Apply the Distributive Property.
$12m - 12m - 2 = 8m - 12m$	Subtract 12*m* from each side.
$-2 = -4m$	Simplify.
$\frac{-2}{-4} = \frac{-4m}{-4}$	Divide each side by –4.
$\frac{1}{2} = m$	Simplify.

CHECK $2(6m - 1) = 8m$

$2\left[6\left(\frac{1}{2}\right) - 1\right] \stackrel{?}{=} 8\left(\frac{1}{2}\right)$	Replace *m* with $\frac{1}{2}$.
$2(3 - 1) \stackrel{?}{=} 4$	Simplify.
$4 = 4$ ✓	The solution checks.

Some equations have no solution. The solution set is the **null** or **empty set**, which is represented by ∅. Other equations have every number as a solution. Such an equation is called an **identity.**

Example 2 **Solve each equation.**

a. $2(x - 1) = 4 + 2x$

$2x - 2 = 4 + 2x$

$2x - 2x - 2 = 4 + 2x - 2x$

$-2 = 4$

The solution set is Ø.

b. $-2(x - 1) = 2 - 2x$

$-2x + 2 = 2 - 2x$

$-2x + 2 - 2 = 2 - 2 - 2x$

$-2x = -2x$

$x = x$

The solution set is all real numbers.

Exercises

Solve each equation. Check your solution.

1. $8(g - 3) = 24$

2. $5(x + 3) = 25$

3. $7(2c - 5) = 7$

4. $2(3d + 7) = 5 + 6d$

5. $2(s + 11) = 5(s + 2)$

6. $7y - 1 = 2(y + 3) - 2$

7. $2(f + 3) - 2 = 8 + 2f$

8. $2(x - 2) + 3 = 2x - 1$

9. $1 + 2(b + 6) = 5(b - 1)$

10. $2x - 5 = 3(x + 3)$

8-3 Study Guide and Intervention

Inequalities

A mathematical sentence that contains any of the symbols listed below is called an **inequality.**

$<$	$>$	$\leq$	$\geq$
• is less than • is fewer than	• is greater than • is more than • exceeds	• is less than or equal to • is no more than • is at most	• is greater than or equal to • is no less than • is at least

Example 1 **Write an inequality for each sentence.**

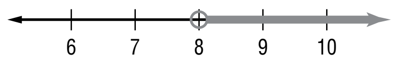
a. Less than 70 students attended the last dance.

b. At the store, the camera cost at least as much as the mail-order price of $229.

Graph an inequality on a number line to help visualize the values that make the inequality true.

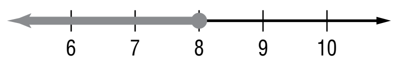
Example 2 **Graph each inequality on a number line.**

a. $x > 8$



The open circle means the number 8 is *not* included in the graph.

b. $x \leq 8$



The closed circle means the number 8 *is* included in the graph.

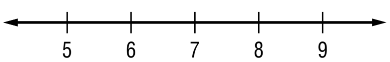
Exercises

Write an inequality for each sentence.

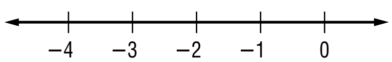
1. Your age is greater than 12 years.
2. A number decreased by 25 is no more than 65.
3. More than $1000 was collected for the charity.
4. At least 80 runners showed up for the charity race.

Graph each inequality on a number line.

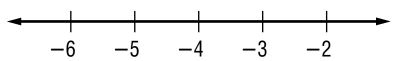
5. $x > 7$



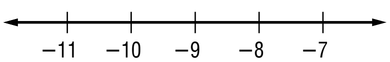
6. $a \leq -2$



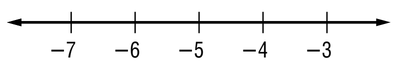
7. $d < -4$



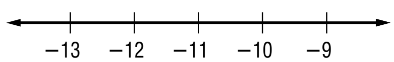
8. $w > -9$



9. $t \geq -5$



10. $n < -11$



8-4 Study Guide and Intervention

Solving Inequalities by Adding or Subtracting

Use the Addition and Subtraction Properties of Inequalities to solve inequalities. When you add or subtract a number from each side of an inequality, the inequality remains true.

Example **Solve $12 + y > 20$. Check your solution.**

$12 + y > 20$	Write the inequality.
$12 - 12 + y > 20 - 12$	Subtract 12 from each side.
$y > 8$	Simplify.

To check your solution, try any number greater than 8.

CHECK	$12 + y \overset{?}{>} 20$	Write the inequality.
	$12 + 9 \overset{?}{>} 20$	Replace *y* with 9.
	$21 > 20$ ✓	This statement is true.

Any number greater than 8 will make the statement true. Therefore, the solution is $y > 8$.

Exercises

Solve each inequality. Check your solution.

1. $-12 < 8 + b$

2. $t - 5 > -4$

3. $p + 5 < -13$

4. $5 > -6 + y$

5. $21 < n - (-18)$

6. $s - 4 \leq 3$

7. $14 > w + (-2)$

8. $j + 6 \geq -4$

9. $z + (-4) < -2.5$

10. $b - \frac{1}{4} < 2\frac{1}{4}$

11. $g - 2\frac{1}{3} \geq 3\frac{1}{6}$

12. $-2 + z < 5$

13. $-10 \leq x - 5$

14. $-23 \geq a + (-6)$

15. $20 < m - 6$

16. $1\frac{1}{2} + b > 7$

17. $k + 5 \geq -7$

18. $-\frac{2}{3} \leq w - 2$

Lesson 8-4

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8-5 Study Guide and Intervention

Solving Inequalities by Multiplying or Dividing

Use the Multiplication and Division Properties of Inequalities to solve inequalities. When you multiply or divide each side of an inequality by a positive number, the inequality remains true. The direction of the inequality sign does not change.

Example 1 **Solve $8x \geq 72$. Check your solution.**

$8x \geq 72$	Write the inequality.
$\frac{8x}{8} \geq \frac{72}{8}$	Divide each side by 8.
$x \geq 9$	Simplify.

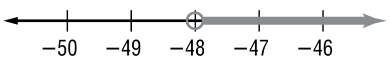
The solution is $x \geq 9$. You can check this solution by substituting 9 or a number greater than 9 into the inequality.

For an inequality to remain true when multiplying or dividing each side of the inequality by a negative number, however, you must reverse the direction of the inequality symbol.

Example 2 **Solve $\frac{y}{-12} < 4$ and check your solution. Then graph the solution on a number line.**

$\frac{y}{-12} < 4$	Write the inequality.
$-12\left(\frac{y}{-12}\right) > -12(4)$	Multiply each side by –12 and reverse the symbol.
$y > -48$	Check the result.

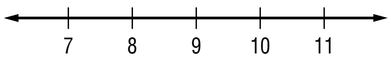
Graph the solution, $y > -48$.



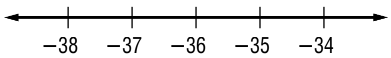
Exercises

Solve each inequality and check your solution. Then graph the solution on a number line.

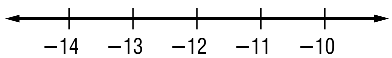
1. $81 < 9d$



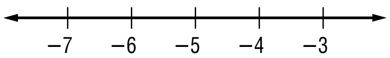
2. $\frac{p}{3} < -12$



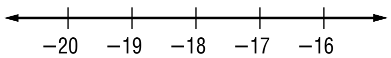
3. $\frac{h}{-4} \geq 3$



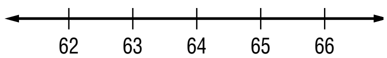
4. $-20t \leq 100$



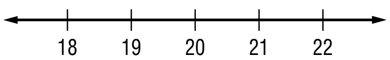
5. $-\frac{2}{3}x > 12$



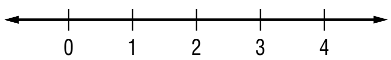
6. $-16 \leq -\frac{1}{4}b$



7. $-8 < \frac{c}{-2.5}$



8. $\frac{n}{3} > 0.5$



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8-6 Study Guide and Intervention

Solving Multi-Step Inequalities

Lesson 8-6

Some inequalities require more than one step to solve. For such inequalities, undo the operations in reverse order, just as in solving multi-step equations.

Remember to reverse the inequality symbol when multiplying or dividing each side of the inequality by a negative number.

Example **Solve $12 - 2x > 24 + 2x$ and check your solution. Graph the solution on a number line.**

$12 - 2x > 24 + 2x$	Write the inequality.
$12 - 2x - 2x > 24 + 2x - 2x$	Subtract $2x$ from each side.
$12 - 4x > 24$	Simplify.
$12 - 12 - 4x > 24 - 12$	Subtract 12 from each side.
$-4x > 12$	Simplify.
$\frac{-4x}{-4} < \frac{12}{-4}$	Divide each side by –4 and reverse the symbol.
$x < -3$	Simplify.

CHECK

$12 - 2x > 24 + 2x$	Try –4, a number less than –3.
$12 - 2(-4) > 24 + 2(-4)$	Replace x with –4.
$12 + 8 > 24 - 8$	Simplify.
$20 > 16$ ✓	The solution checks.

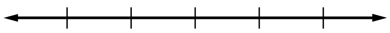
Graph the solution $x < -3$.

If the inequality contains parentheses, use the Distributive Property to begin simplifying the inequality.

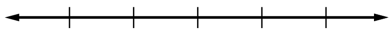
Exercises

Solve each inequality and check your solution. Graph the solution on a number line.

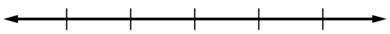
1. $5c + 9 < -11$



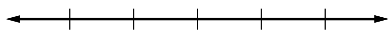
2. $8 - 4p > 20$



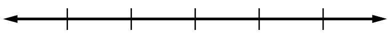
3. $c + 5 \leq 4c - 1$



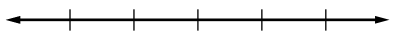
4. $18 - 2n \geq 6$



5. $3(d + 2) < -15$



6. $\frac{b}{3} + 9 > 8$



NAME ______________________________ DATE ____________ PERIOD _____

9-1 Study Guide and Intervention

Squares and Square Roots

Squares and Square Roots

- A **perfect square** is the square of an integer.
- A **square root** of a number is one of two equal factors of the number.
- Every positive number has a positive square root and a negative square root.
- The square root of a negative number such as –25, is not real because the square of a number cannot be negative.

Example 1 **Find each square root.**

a. $\sqrt{144}$ $\sqrt{144}$ indicates the positive square root of 144.

Since $12^2 = 144$, $\sqrt{144} = 12$

b. $-\sqrt{121}$ $-\sqrt{121}$ indicates the negative square of 121.

Since $11^2 = 121$, $-\sqrt{121} = -11$

c. $\pm\sqrt{4}$ $\pm\sqrt{4}$ indicates both square roots of 4.

Since $2^2 = 4$, $\sqrt{4} = 2$ and $-\sqrt{4} = -2$

Example 2 **Use a calculator to find $\sqrt{34}$ to the nearest tenth.**

2nd [$\sqrt{}$] 34 ENTER 5.830951895 Use a calculator.

$\sqrt{34} \approx 5.8$ Round to the nearest tenth.

Exercises

Find each square root.

1. $-\sqrt{100}$ **2.** $\sqrt{256}$ **3.** $\sqrt{4}$

4. $\sqrt{-4}$ **5.** $-\sqrt{144}$ **6.** $-\sqrt{225}$

7. $\pm\sqrt{289}$ **8.** $-\sqrt{16}$ **9.** $\sqrt{484}$

10. $\sqrt{-25}$ **11.** $\pm\sqrt{361}$ **12.** $\sqrt{1521}$

Use a calculator to find each square root to the nearest tenth.

13. $\sqrt{11}$ **14.** $\sqrt{19}$ **15.** $-\sqrt{47}$

16. $-\sqrt{57}$ **17.** $\sqrt{172}$ **18.** $\sqrt{521}$

19. $\sqrt{427}$ **20.** $-\sqrt{310}$ **21.** $-\sqrt{490}$

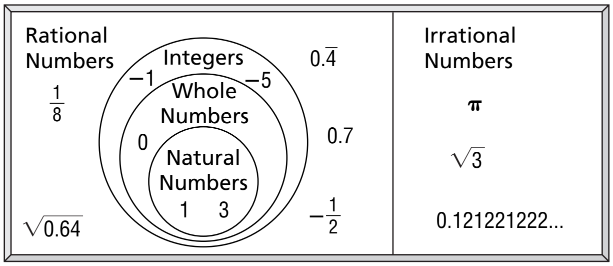
NAME ____ DATE ____________ PERIOD _____

9-2 Study Guide and Intervention

The Real Number System

The set of real numbers consists of all natural numbers, whole numbers, integers, rational numbers, and irrational numbers.

- Rational numbers can be written as fractions.
- Irrational numbers are decimals that do not repeat or terminate.



Example 1 **Name all of the sets of numbers to which each real number belongs.**

a. 7

This number is a natural number, a whole number, an integer, and a rational number.

b. $0.\overline{6}$

This repeating decimal is a rational number because it is equivalent to $\frac{2}{3}$.

c. $\sqrt{71}$

It is not the square root of a perfect square so it is irrational.

d. $\frac{-81}{3}$

Since $-\frac{81}{3} = -27$, this number is an integer and a rational number.

e. $-\sqrt{169}$

Because $-\sqrt{169} = -13$, this number is an integer and a rational number.

Exercises

Name all of the sets of numbers to which each real number belongs. Let N = natural numbers, W = whole numbers, Z = integers, Q = rational numbers, and I = irrational numbers.

1. 21

2. $\frac{3}{7}$

3. $\frac{8}{12}$

4. -5

5. 17

6. 0

7. 0.257

8. 0.9

9. 78

10. $\frac{1}{4}$

11. π

12. $-\sqrt{169}$

13. 5.7

14. -8.5

15. $-\sqrt{289}$

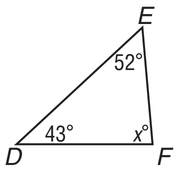
9-3 Study Guide and Intervention

Triangles

Angles of a Triangle The sum of the measures of the angles of a triangle is 180°.

Example 1 **Find the value of x in $\triangle DEF$.**

$m\angle D + m\angle E + m\angle F = 180$	The sum of the measures is 180°
$43 + 52 + x = 180$	$m\angle D = 43°$ and $m\angle E = 52°$
$95 + x = 180$	Simplify.
$x + 95 - 95 = 180 - 95$	Subtract 95 from each side.
$x = 85$	



Classifying Angles Angles are classified by their degree measure. Acute angles measure between 0° and 90°. An obtuse angle measures between 90° and 180°. A straight angle measures 180°.

Classifying Triangles

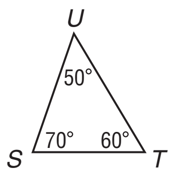
- Triangles can be classified by their angles. Acute triangles have all acute angles. Obtuse triangles have one obtuse angle. Right triangles have one right angle.
- Triangles can classified by their sides. Scalene triangles have no congruent sides. Isosceles triangles have at least two sides congruent. Equilateral triangles have all sides congruent.

Example 2 **Classify the triangle by its angles and by its sides.**

$m\angle TUS < 90°$, $m\angle STU < 90°$, and $m\angle UST < 90°$, so $\triangle STU$ has all acute angles.

$\triangle STU$ has no two sides that are congruent.

So, $\triangle STU$ is an acute scalene triangle.



Exercises

Find the value of x in each triangle. Then classify each triangle as *acute, right,* or *obtuse.*

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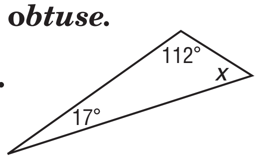

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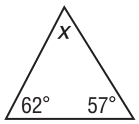

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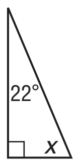
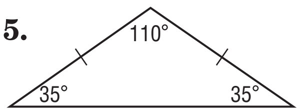
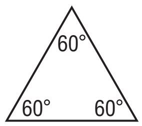


Classify each triangle by its angles and by its sides.

4.


5.


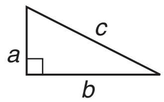
6.


9-4 Study Guide and Intervention

The Pythagorean Theorem

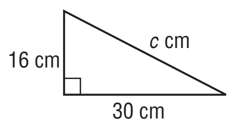
Pythagorean Theorem

- **Words** If a triangle is a right triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.
- **Model**



- **Symbols** $c^2 = a^2 + b^2$

Example **Find the length of the hypotenuse of the right triangle.**



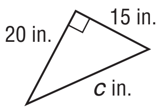
$c^2 = a^2 + b^2$	Pythagorean Theorem
$c^2 = 16^2 + 30^2$	Replace a with 16 and b with 30.
$c^2 = 256 + 900$	Evaluate 16^2 and 30^2.
$c^2 = 1156$	Add 256 and 900.
$\sqrt{c^2} = \pm\sqrt{1156}$	Take the square root of each side. Only use the positive root
$c = 34$	Simplify.

The length of the hypotenuse is 34 cm.

Exercises

Find the length of the hypotenuse in each right triangle. Round to the nearest tenth if necessary.

1.



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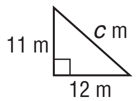

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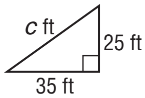

If *c* is the measure of the hypotenuse, find each missing measure. Round to the nearest tenth, if necessary.

4. $a = 18, b = 80, c = ?$

5. $a = ?, b = 70, c = 74$

6. $a = 14, b = ?, c = 22$

7. $a = ?, b = 48, c = 57$

The lengths of three sides of a triangle are given. Determine whether each triangle is a right triangle.

8. $a = 16, b = 30, c = 34$

9. $a = 25, b = 31, c = 37$

10. $a = 21, b = 29, c = 42$

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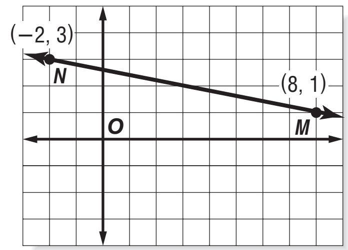
9-5 Study Guide and Intervention

The Distance Formula

Distance Formula On a coordinate plane, the distance d between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example **Find the distance between $M(8, 1)$ and $N(-2, 3)$. Round to the nearest tenth, if necessary.**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$MN = \sqrt{(8 - (-2))^2 + (1 - 3)^2}$	$(x_2, y_1) = (2, 3)$, $(x_2, y_2) = (8, 1)$
$MN = \sqrt{(10)^2 + (-2)^2}$	Simplify.
$MN = \sqrt{100 + 4}$	Evaluate 10^2 and $(-2)^2$.
$MN = \sqrt{104}$	Add 100 and 4.
$MN \approx 10.2$	Take the square root.



The distance between points M and N is about 10.2 units.

Exercises

Find the distance between each pair of points. Round the nearest tenth, if necessary.

1. A (3, 1), B (2,5)

2. C (–2, –4), D (3, 7)

3. E (5, –3), F (4,2)

4. G (–6, 5), H (–4, –3)

5. I (–4, –3), J (4,4)

6. K (5, 0), L (–2, 1)

7. M (2,1), N (6,5)

8. O (0,0), P (–5, 6)

9. Q (3, 5), R (4, 2)

10. S (–6, –4), T (–5, 6)

11. U (2, 1), V (4, 4)

12. W (5, 1), X (–2, –1)

13. Y (–5, –3), Z (2, 5)

14. A (8, –1), B (3, –1)

15. C (0, 0), D (2, 4)

16. E (–5, 3), F (4, 7)

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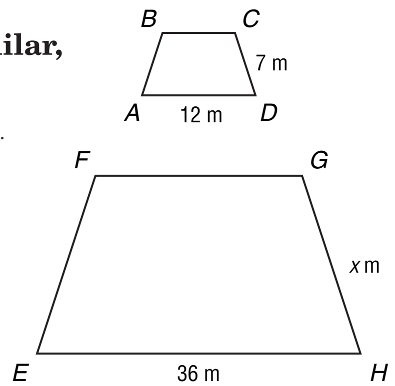
9-6 Study Guide and Intervention

Similar Polygons and Indirect Measurement

Corresponding Parts of Similar Polygons	Similar polygons are polygons that have the same shape but not necessarily the same size. If two polygons are similar, then the corresponding angles have the same measure, and the corresponding sides are proportional.

Example 1 **If the polygons *ABCD* and *EFGH* are similar, what is the value of *x*?**



$\frac{AD}{EH} = \frac{CD}{GH}$ The corresponding sides are proportional. Write a proportion.

$\frac{12}{36} = \frac{7}{x}$ Replace *AD* with 12, *EH* with 36, *CD* with 7, and *GH* with *x*.

$12 \cdot x = 36 \cdot 7$ Find the cross products.

$12x = 252$ Simplify.

$x = 21$ Divide each side by 12.

Indirect Measurement	The properties of similar triangles can be used to find measurements which are difficult to measure directly. This is called indirect measurement.

Example 2 **The Chrysler Building in New York casts a 793.5 foot shadow the same time a 5.8 foot tourist casts a 4.4 foot shadow. How tall is the Chrysler Building to the nearest tenth?**

tourist's height / Chrysler Building's height $\frac{5.8}{h} = \frac{4.4}{793.5}$ tourist's shadow / Chrysler Building's shadow

$5.8 \cdot 793.5 = h \cdot 4.4$ Find the cross products.

$4602.3 = 4.4h$ Multiply.

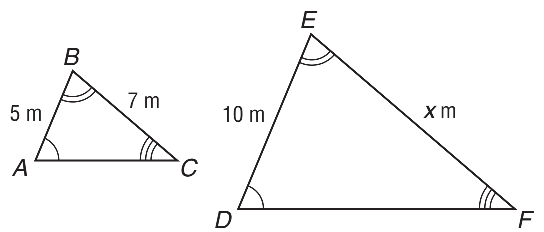
$1046.0 = h$ Divide each side by 4.4.

The height of the Chrysler Building is 1046 feet.

Exercises

The polgons are similar. Write a proportion and find the value of *x*.

1.



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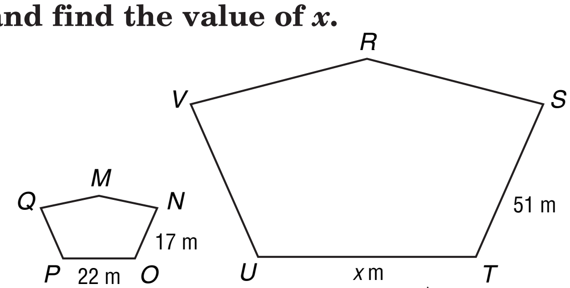
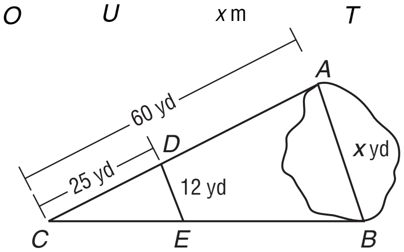


3. SURVEYOR A surveyor needs to find the distance *AB* across a pond. He constructs $\triangle CDE$ similar to $\triangle CAB$ and measures the distances as shown on this figure.

Find *AB*.



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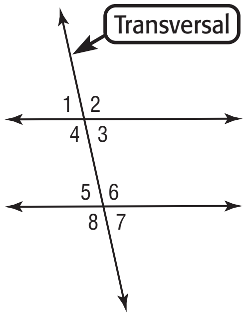
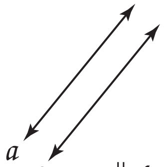
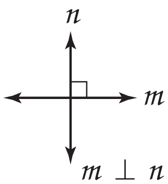
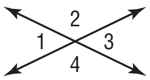
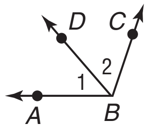
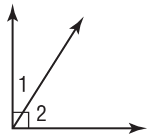
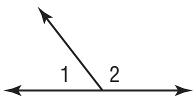
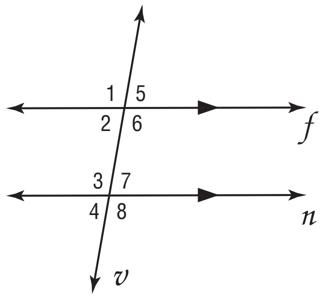
10-1 Study Guide and Intervention

Line and Angle Relationships

Names of Special Angles		
Interior angles lie inside the parallel lines.	$\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$	
Exterior angles lie outside the parallel lines.	$\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$	
Alternate interior angles are on opposite sides of the transversal and inside the parallel lines.	$\angle 3$, and $\angle 5$, $\angle 4$ and $\angle 6$	
Alternate exterior angles are on opposite sides of the transversal and outside the parallel lines.	$\angle 1$ and $\angle 7$, $\angle 2$, and $\angle 8$	
Corresponding angles are in the same position on the parallel lines in relation to the transversal.	$\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$	

Line and Angle Relationships					
Parallel Lines	**Perpendicular Lines**	**Vertical Angles**	**Adjacent Angles**	**Compelementary Angles**	**Supplementary Angles**
 $a \parallel b$	 $m \perp n$	 $\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$	 $m\angle ABC =$ $m\angle 1 + m\angle 2$	 $m\angle 1 + m\angle 2 = 90°$	 $m\angle 1 + m\angle 2 = 180°$

Example **In the figure, $f \parallel n$ and v is a transversal. If $m\angle 3 = 100°$, find $m\angle 1$ and $m\angle 6$.**

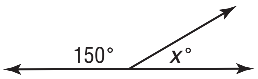


Since $\angle 1$ and $\angle 3$ are corresponding angles, they are congruent. So, $m\angle 1 = 100°$. Since $\angle 3$ and $\angle 6$ are alternate interior angles, they are congruent. So, $m\angle 6 = 100°$.

Exercises

Find the value of x in each figure.

1.



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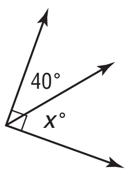

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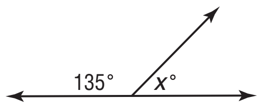

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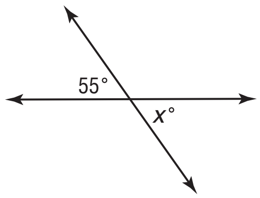
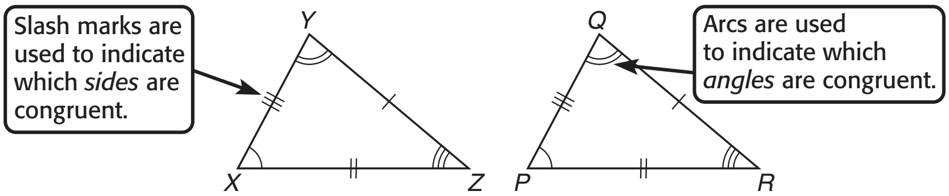


5. ALGEBRA Angles A and B are complementary. If $m\angle A = 3x - 8$ and $m\angle B = 5x + 10$, what is the measure of each angle?

6. ALGEBRA Angles Q and R are supplementary. If $m\angle Q = 4x + 9$ and $m\angle R = 8x + 3$, what is the measure of each angle?

10-2 Study Guide and Intervention

Congruent Triangles

Corresponding Parts of Congruent Triangles	
Words	Two triangles are **congruent** if they have the same size and shape. If two triangles are congruent, their corresponding sides are congruent and their corresponding angles are congruent.
Model	
Symbols	Congruent Angles: $\angle X \cong \angle P$, $\angle Y \cong \angle Q$, $\angle Z \cong \angle R$ Congruent Sides: $\overline{XY} \cong \overline{PQ}$, $\overline{YZ} \cong \overline{QR}$, $\overline{XZ} \cong \overline{PR}$

Example **Name the corresponding parts in the congruent triangles shown. Then write a congruence statement.**

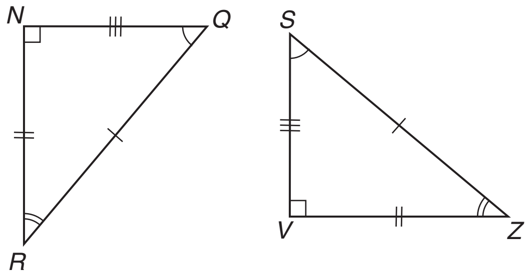
Corresponding Angles

$\angle Q \cong \angle S, \angle R \cong \angle Z, \angle N \cong \angle V$

Corresponding Sides

$\overline{SZ} \cong \overline{QR}, \overline{ZV} \cong \overline{RN}, \overline{VS} \cong \overline{NQ}$

$\triangle NQR \cong \triangle VSZ$



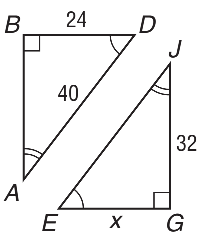
Exercises

Complete each congruence statement if $\triangle DFH \cong \triangle PWZ$.

1. $\angle F \cong$ _____ **2.** $\angle P \cong$ _____ **3.** $\overline{DH} \cong$ _____ **4.** $\overline{ZW} \cong$ _____

Find the value of x for each pair of congruent triangles.

5.



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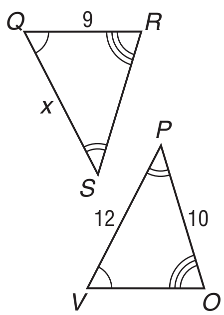

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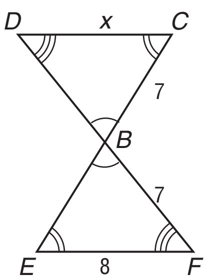

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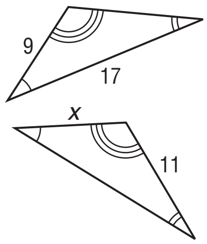


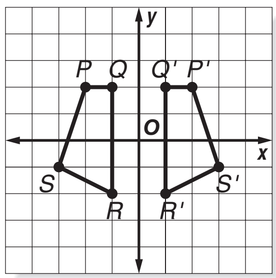
10-3 Study Guide and Intervention

Transformations on the Coordinate Plane

Transformation	Movement	How To's
Translation	You slide a figure from one position to another without turning it. Every point of the original figure is moved the same distance and in the same direction.	To describe the translation using an ordered pair, add the coordinates of the ordered pair to the coordinates of the original point.
Reflection	You flip a figure over a **line of symmetry**. The figures are mirror images of each other. Every corresponding point on the figure after a reflection is called its **image.**	• To reflect a point over the x-axis, use the same x-coordinate and multiply the y-coordinate by -1. • To reflect a point over the y-axis, use the same y-coordinate and multiply the x-coordinate by -1.
Dilation	You enlarge or reduce a figure by a scale factor with respect to a fixed point called the center. The resulting image is similar to the original figure.	• To dilate a figure when the center of dilation is the origin, multiply each coordinate by the scale factor. • To dilate a figure for any other center of dilation is not the origin, subtract the coordinates for the center of dilation from the coordinates of each point, multiply by the scale factor, and then add the coordinates for the center of dilation.

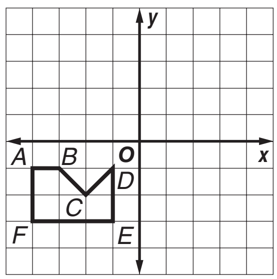
Example **The vertices of figure $PQRS$ are $P(-2, 2)$, $Q(-1, 2)$, $R(-1, -2)$, and $S(-3, -1)$. Graph the figure and its image after a reflection over the y-axis. Use the same y-coordinate and multiply the x-coordinate by –1.**

vertex				reflection
$P(-2, 2)$	$\rightarrow$	$(-2 \cdot -1, 2)$	$\rightarrow$	$P'(2, 2)$
$Q(-1, 2)$	$\rightarrow$	$(-1 \cdot -1, 2)$	$\rightarrow$	$Q'(1, 2)$
$R(-1, -2)$	$\rightarrow$	$(-1 \cdot -1, -2)$	$\rightarrow$	$R'(1, -2)$
$S(-3, -1)$	$\rightarrow$	$(-3 \cdot -1, -1)$	$\rightarrow$	$S'(3, -1)$



Exercises

For Exercises 1–3, use the graph shown.

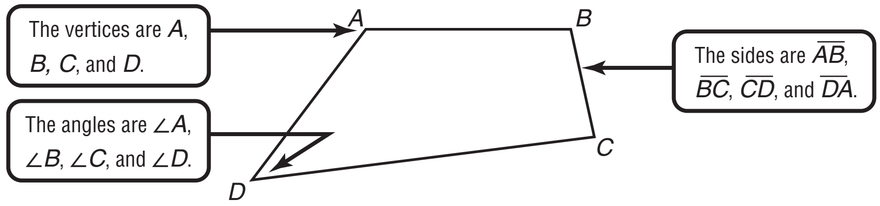


1. Graph the image of the figure after a translation of (5, 0).
2. Graph the image of the figure after a translation of (0, 5).
3. Find the vertices of the figure after a translation of (5, 5).

10-4 Study Guide and Intervention

Quadrilaterals

A **quadrilateral** is a polygon with four sides and four vertices. The segments of a quadrilateral intersect only at their endpoints.



A quadrilateral can be separated into two triangles. Since the sum of the measures of the angles of a triangle is 180°, the sum of the measures of the angles of a quadrilateral is 2(180°) or 360°.

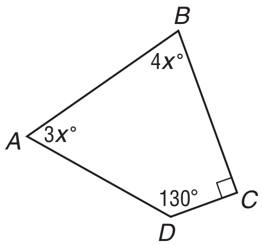
Example ALGEBRA **Find the value of x. Then find each missing angle measure.**

Words The sum of the measures of the angles is 360°.

Variable Let $m\angle A$, $m\angle B$, $m\angle C$, and $m\angle D$ represent the measures of the angles.

Equation

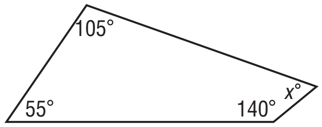
$m\angle A + m\angle B + m\angle C + m\angle D = 360$	Angles of a quadrilateral
$3x + 4x + 90 + 130 = 360$	Substitution
$7x + 220 = 360$	Combine like terms.
$7x + 220 - 220 = 360 - 220$	Subtract 220 from each side.
$7x = 140$	Simplify.
$x = 20$	Divide each side by 7.

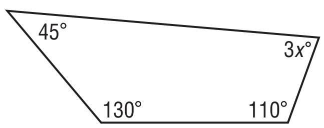


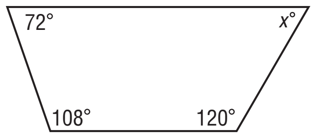
The value of x is 20. So, $m\angle A = 3(20)$ or 60° and $m\angle B = 4(20)$ or 80°.

Exercises

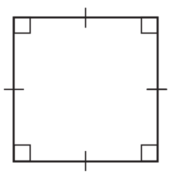
ALGEBRA **Find the value of x. Then find the missing angle measures.**

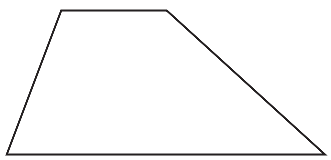
1. 

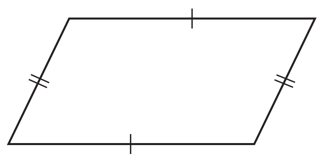
2. 

3. 

Classify each quadrilateral using the name that *best* describes it.

4. 

5. 

6. 

10-5 Study Guide and Intervention

Polygons

A **polygon** is a simple, closed figure formed by three or more coplanar line segments. The line segments, called *sides,* meet only at their endpoints. The points of intersection are called *vertices.* Polygons can be classified by the number of sides they have.

A **diagonal** is a line segment in a polygon that joins two nonconsecutive vertices, forming triangles. You can use the property of the sum of the measures of the angles of a triangle to find the sum of the measures of the interior angles of any polygon. An **interior angle** is an angle inside a polygon.

Number of Sides	Name of Polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon

If a polygon has n sides, then $n - 2$ triangles are formed. The sum of the degree measures of the interior angles of the polygon is $(n - 2)180$.

A regular polygon is a polygon that is *equilateral* (all sides are congruent) and *equiangular* (all angles are congruent). Since the angles of a regular polygon are congruent, their measures are equal.

Example **Find the measure of one interior angle of a regular 20-gon.**

Step 1 A 20-gon has 20 sides. Therefore, $n = 20$.

$(n - 2)180 = (20 - 2)180$ Replace n with 20.

$= 18(180)$ or 3240 Simplify.

The sum of the measures of the interior angles is 3240°.

Step 2 Divide the sum by 20 to find the measure of one angle.

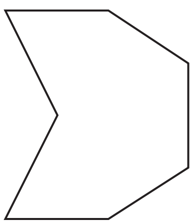
$3240 \div 20 = 162$

So, the measure of one interior angle in a regular 20-gon is 162°.

Exercises

Classify each polygon. Then determine whether it appears to be *regular* or *not regular*.

1.



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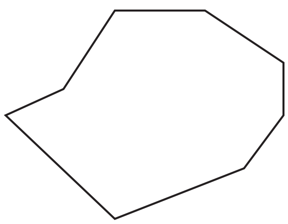
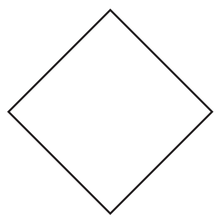


3.



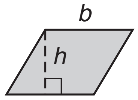
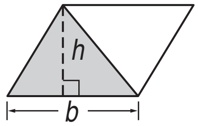
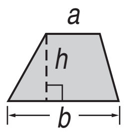
Find the sum of the measures of the interior angles of each polygon.

4. quadrilateral **5.** nonagon **6.** heptagon **7.** 12-gon

Lesson 10–5

10-6 Study Guide and Intervention

Area: Parallelograms, Triangles, and Trapezoids

Shape	Words	Area Formula	Model
Parallelogram	The area of a parallelogram can be found by multiplying the measures of the base and the height.	$A = bh$	
Triangle	A diagonal of a parallelogram separates the parallelogram into two congruent triangles. The area of each triangle is one-half the area of the parallelogram.	$A = \frac{1}{2}bh$	
Trapezoid	A trapezoid has two bases. The height of a trapezoid is the distance between the bases. A trapezoid can be separated into two triangles.	$A = \frac{1}{2}h(a + b)$	

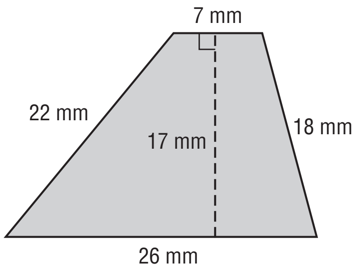
Example **Find the area of the trapezoid.**

$A = \frac{1}{2}h(a + b)$ Area of a trapezoid

$A = \frac{1}{2} \cdot 17(7 + 26)$ Replace h with 17, a with 7, and b with 26.

$A = \frac{1}{2} \cdot 17 \cdot 33$ 7 + 26 = 33

$A = \frac{561}{2}$ or $280\frac{1}{2}$

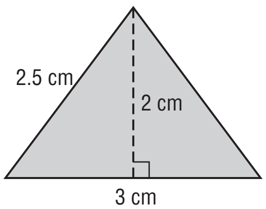


The area of the trapezoid is $280\frac{1}{2}$ mm^2.

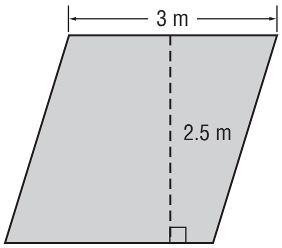
Exercises

Find the area of each figure.

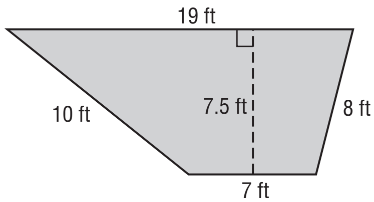
1.



2.



3.



Find the area of each figure described.

4. trapezoid: height, 12 yd; bases, 6 yd, 8 yd

5. parallelogram: base, 4.5 cm; height, 8 cm

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10-7 Study Guide and Intervention

Circumference and Area: Circles

Circles A **circle** is the set of all points in a plane that are the same distance from a given point.	The **circumference** of a circle is equal to its diameter times π, or 2 times its radius times π.	$C = \pi d$ or $C = 2\pi r$
	The area of a circle is equal to π times the square or its radius.	$A = \pi r^2$

Example 1

Find the circumference of the circle to the nearest tenth.

a.

$C = 2\pi r$ Circumference of a circle

$C = 2 \cdot \pi \cdot 7$ Replace r with 7.

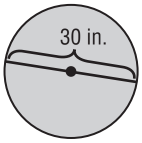
$C \approx 44.0$ Simplify. Use a calculator.

The circumference is about 44.0 kilometers.

Example 2

Find the area of the circle. Round to the nearest tenth.

b.



$A = \pi r^2$ Area of a circle

$A = \pi \cdot (15)^2$ Replace r with 15.

$A = \pi \cdot 225$ Evaluate $(15)^2$.

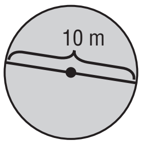
$A \approx 706.9$ Use a calculator.

The area is about 706.9 square inches.

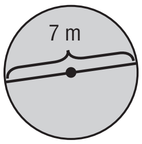
Exercises

Find the circumference and area of each circle. Round to the nearest tenth.

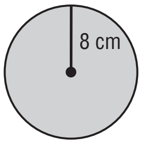
1.



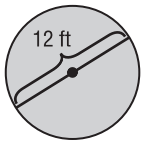
2.



3.



4.



Match each circle described in the column on the left with its corresponding measurement in the column on the right.

5. radius: 5 units

6. diameter: 9 units

7. diameter: 12 units

8. diameter: 16 units

a. area: 63.6 units^2

b. circumference: 50.3 units

c. circumference: 31.4 units

d. area: 113.1 units^2

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10-8 Study Guide and Intervention

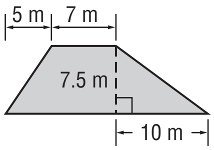
Area: Composite Figures

To find the area of a composite figure, separate the composite figure into figures whose area you know how to find. Use the area formulas you have learned in this chapter.

Triangle	Trapezoid	Parallelogram	Circle
$A = \frac{1}{2}bh$	$A = \frac{1}{2}h(a + b)$	$A = bh$	$A = \pi r^2$

Example **Find the area of each figure. Round to the nearest tenth, if necessary.**

a.



Area of Parallelogram

$A = bh$

$A = 7(7.5)$ or 52.5

Area of Triangle

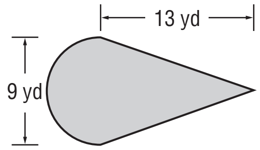
$A = \frac{1}{2}bh$

$A = \frac{1}{2}(15 \cdot 7.5)$

$A = 56.25$

The area of the figure is $52.5 + 56.25$ or about 108.8 m^2.

b.



Area of Semicircle

$A = \frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi(4.5)^2$

$A = 31.8$

Area of Triangle

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(9 \cdot 13)$

$A = 58.5$

The area of the figure is $31.8 + 58.5$ or about 90.3 yd^2.

Exercises

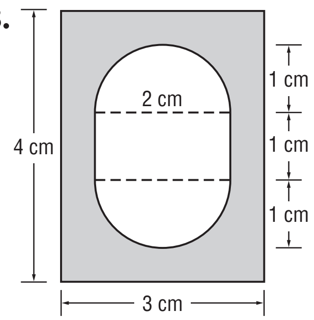
Find the area of each figure. Round to the nearest tenth, if necessary.

1. What is the area of a figure formed using a rectangle with a base of 10 yards and a height of 4 yards and two semicircles, one with a radius of 5 yards and the other a radius of 2 yards?

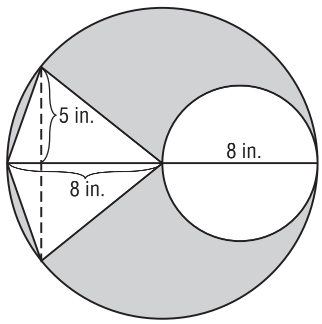
2. Find the area of a figure formed using a square and three triangles all with sides of 9 centimeters. Each triangle has a height of 6 centimeters.

Find the area of each shaded region. Round to the nearest tenth. (*Hint:* Find the total area and subtract the non-shaded area.)

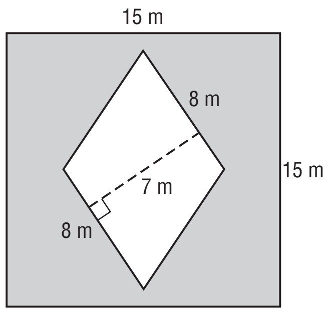
3.



4.



5.



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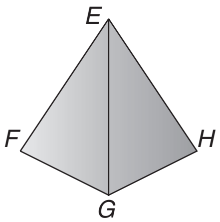
11-1 Study Guide and Intervention

Three-Dimensional Figures

Identifying Three-Dimensional Figures A prism is a polyhedron with two parallel bases. A pyramid is a polyhedron with one base. Prisms and pyramids are named by the shape of their bases, such as triangular or rectangular.

Example 1 **Identify the solid. Name the bases, faces, edges, and vertices.**



triangular pyramid

Any of the faces can be considered a base.

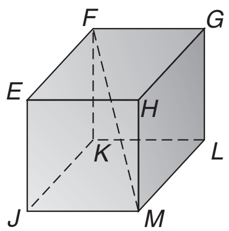
faces: *EFG, EGH, EFH, FGH*

edges: $\overline{EF}, \overline{EG}, \overline{EH}, \overline{FG}, \overline{FH}, \overline{GH}$

vertices: *E, F, G, H*

Diagonals and Skew Lines Skew lines are lines that lie in different planes and do not intersect. A diagonal of a figure joins two vertices that have no faces in common.

Example 2 **Identify a diagonal and name all segments that are skew to it.**



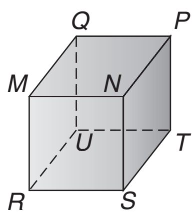
diagonal: $\overline{FM}$

skew segments: $\overline{EH}, \overline{HG}, \overline{JK}, \overline{KL}, \overline{EJ}, \overline{GL}$

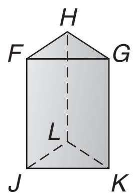
Exercises

Identify each solid. Name the bases, faces, edges, and vertices.

1.



2.



For Exercises 3–4, use the rectangular prism in Example 2.

3. Identify a diagonal that could be drawn from point *E*.

4. Name all segments that are skew to the new diagonal.

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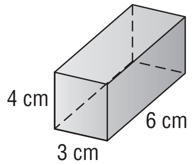
11-2 Study Guide and Intervention

Volume: Prisms and Cylinders

Volume of Prisms To find the volume *V* of a prism, use the formula $V = Bh$, where *B* is the area of the base, and *h* is the height of the solid.

Example 1 **Find the volume of each prism.**

a.



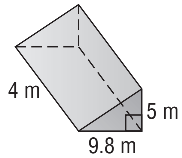
$V = Bh$

$V = (3 \cdot 6)4$

$V = 72$

The volume is 72 cm^3.

b.



$V = Bh$

$V = \left(\frac{1}{2} \cdot 9.8 \cdot 5\right)4$

$V = 98$

The volume is 98 m^3.

Volume of Cylinders The volume *V* of a cylinder with radius *r* is the area of the base, πr^2, times the height *h*, or $V = \pi r^2 h$.

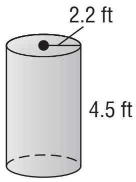
Example 2 **Find the volume of each cylinder.**

$V = \pi r^2 h$

$V = \pi \cdot 2.2^2 \cdot 4.5$

$V \approx 68.4$

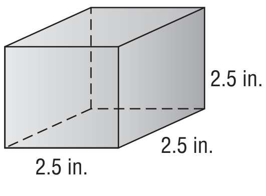
The volume is about 68.4 ft^3.



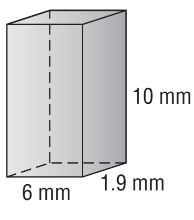
Exercises

Find the volume of each solid. If necessary, round to the nearest tenth.

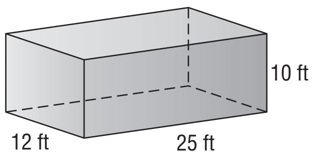
1.



2.



3.



4. rectangular prism: length 9 mm, width 8.2 mm, height 5 mm

5. triangular prism: base of triangle 5.8 ft, altitude of triangle 5.2 ft, height of prism 6 ft

11-3 Study Guide and Intervention

Volume: Pyramids, Cones, and Spheres

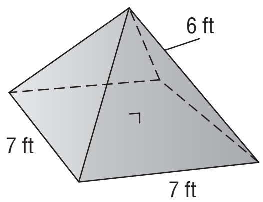
Volume of Pyramids and Cones To find the volume V of a pyramid, use the formula $V = \frac{1}{3}Bh$, where B is the area of the base, and h is the height of the solid. To find the volume V of a cone, use the formula $V = \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height of the solid.

Example 1 **Find the volume of the pyramid.**

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}(7 \cdot 7)6$

$V = 98$



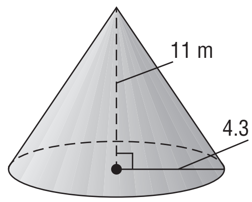
The volume is 98 ft^3.

Example 2 **Find the volume of the cone. Round to the nearest tenth.**

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(4 \cdot 3)^2 \cdot 11$

$V \approx 213.0\ m^3$



The volume is about 213.0 m^3.

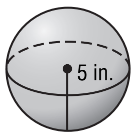
Volume of Spheres To find the volume V of a pyramid, use the formula $V = \frac{4}{3}\pi r^3$, where r is the radius.

Example 3 **Find the volume of the sphere. Round to the nearest tenth.**

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(5)^3$

$V \approx 523.6\ in^3$

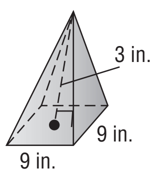


The volume is about 523.6 in^3.

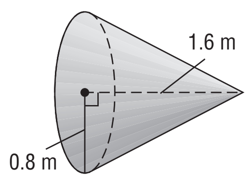
Exercises

Find the volume of each solid. If necessary, round to the nearest tenth.

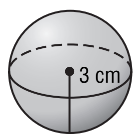
1.



2.



3.



4. square pyramid: length 1.2 cm, height 5 cm

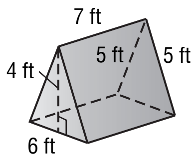
5. cone: diameter 4 yd, height 7 yd

11-4 Study Guide and Intervention

Surface Area: Prisms and Cylinders

Lateral Area of Prisms and Cylinders To find the lateral area of a prism with a height h and base with a perimeter, use the formula $L = Ph$. To find the lateral area L of a cylinder with radius r and height h, use the formula $L = 2\pi rh$.

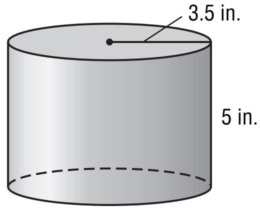
Example 1 **Find the lateral area of the triangular prism.**



$L = Ph$
$L = (5 + 5 + 6)(7)$
$L = 112$

The lateral area is 112 m^2.

Example 2 **Find the lateral area of the cylinder.**

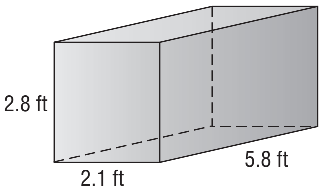


$L = 2\pi rh$
$L = 2\pi(3.5)(5)$
$L \approx 110$

The lateral area is 110 in^2.

Surface Area of Prisms and Cylinders To find the surface area S of a rectangular prism with length ℓ, width w, and height h, use the formula $S = 2\ell w + 2\ell h + 2wh$. To find the surface area of a triangular prism, add the area of each face. To find the surface area S of a cylinder with radius r and height h, use the formula $S = 2\pi r^2 + 2\pi rh$.

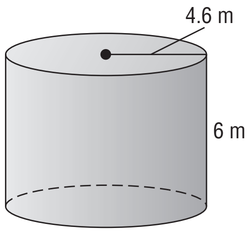
Example 3 **Find the surface area of the rectangular prism.**



$S = 2\ell w + 2\ell h + 2wh$
$S = 2(2.8)(2.1) + 2(2.8)(5.8) + 2(2.1)(5.8)$
$S = 68.6$

The surface area is 68.6 ft^2.

Example 4 **Find the surface area of the cylinder. Round to the nearest tenth.**



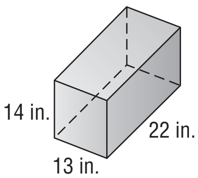
$S = 2\pi r^2 + 2\pi rh$
$S = 2\pi(4.6)^2 + 2\pi(4.6)(6)$
$S \approx 306.4$

The surface area is about 306.4 m^2.

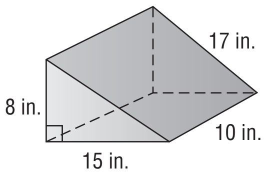
Exercises

Find the lateral area and surface area of each solid. If necessary, round to the nearest tenth.

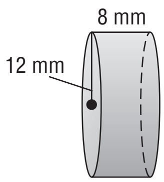
1.



2.



3.



4. cube: side length 8.3 cm

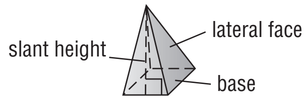
5. cylinder: diameter 20 yd, height 22 yd

Lesson 11-4

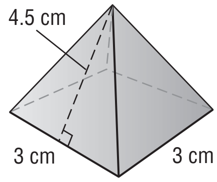
11-5 Study Guide and Intervention

Surface Area: Pyramids and Cones

Surface Area of Pyramids The sum of the areas of the lateral faces is the lateral area of a pyramid. The surface area of a pyramid is the sum of the lateral area and the base area.



Example 1 **Find the surface area of the square pyramid.**



Area of base

$A = s^2$

$A = 3^2$ or 9

Area of each lateral face

$A = \frac{1}{2}bh$

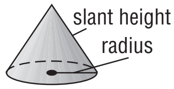
$A = \frac{1}{2}(3)(4.5)$ or 6.75

Lateral area

$4(6.75) = 27$

Total surface area $= 27 + 9$ or 36 cm^2.

Surface Area of Cones The surface area S of a cone with radius r and slant height ℓ can be found by using the formula $S = \pi r\ell + \pi r^2$.



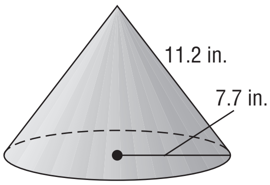
Example 2 **Find the surface area of the cone. Round to the nearest tenth.**

$S = \pi r\ell + \pi r^2$

$S = \pi(7.7)(11.2) + \pi(7.7)^2$

$S \approx 457.2 \text{ in}^2$

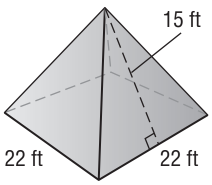
The surface area is about 457.2 in^2.



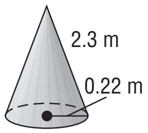
Exercises

Find the surface area of each solid. If necessary, round to the nearest tenth.

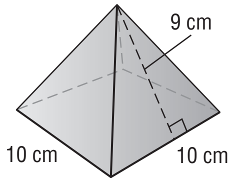
1.



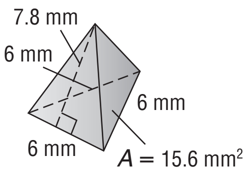
2.



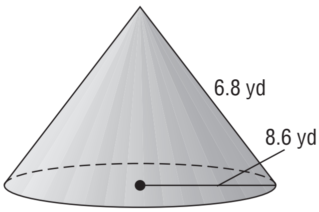
3.



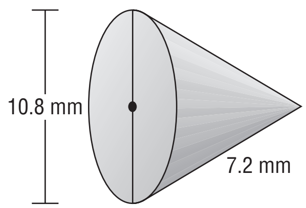
4.



5.



6.



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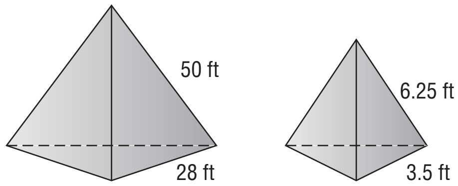
11-6 Study Guide and Intervention

Similar Solids

Identify Similar Solids Solids are similar if they have the same shape and their corresponding linear measures are proportional.

Example 1 **Determine whether each pair of solids is similar.**

a.



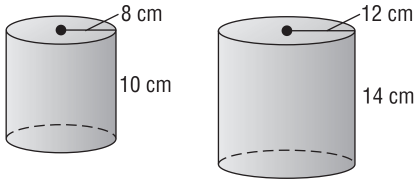
$\frac{50}{6.25} \stackrel{?}{=} \frac{28}{3.5}$ Write a proportion comparing corresponding edge lengths.

$50(3.5) \stackrel{?}{=} 6.25(28)$ Find the cross products.

$175 = 175$ ✓ Simplify.

The corresponding measures are proportional, so the pyramids are similar.

b.



$\frac{8}{12} \stackrel{?}{=} \frac{10}{14}$ Write a proportion comparing radii and heights.

$8(14) \stackrel{?}{=} 12(10)$ Find the cross products.

$112 \neq 120$ Simplify.

The radii and heights are not proportional, so the cylinders are not similar.

Use Similar Solids You can find missing measures if you know solids are similar.

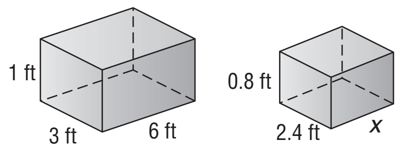
Example 2 **Find the missing measure for the pair of similar solids.**

$\frac{1}{0.8} = \frac{6}{x}$ Write a proportion.

$1x = 0.8(6)$ Find the cross products.

$x = 4.8$ Simplify.

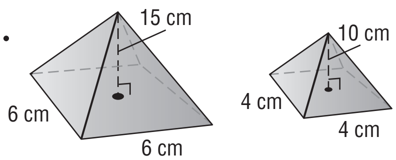
The missing length is 4.8 ft.



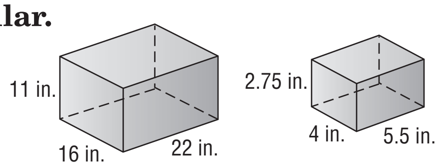
Exercises

Determine whether each pair of solids is similar.

1.

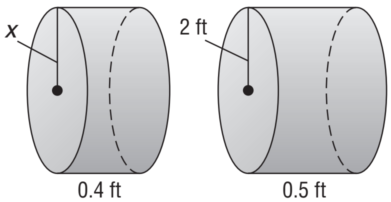


2.

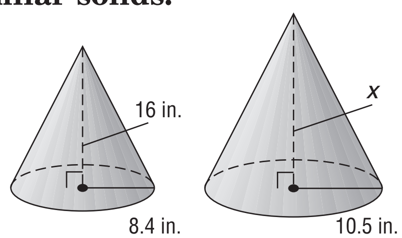


Find the missing measure for each pair of similar solids.

3.



4.



12-1 Study Guide and Intervention

Stem-and-Leaf Plots

Stem-and-Leaf Plot	**Words** One way to organize and display data is to use a **stem-and-leaf plot**. In a stem-and-leaf plot, numerical data are listed in ascending or descending order.
	Model

Stem	Leaf
2	0 1 1 2 3 5 5 6
3	1 2 2 3 7 9
4	0 3 4 8 8

3 | 7 = 37

The greatest place value of the data is used for the **stems**.

The next greatest place value forms the **leaves**.

Example ZOOS **Display the data shown at the right in a stem-and-leaf plot.**

Step 1 The least and the greatest numbers are 55 and 95. The greatest place value digit in each number is in the tens. Draw a vertical line and write the stems from 5 to 9 to the left of the line.

Step 2 Write the leaves to the right of the line, with the corresponding stem. For example, for 85, write 5 to the right of 8.

Stem	Leaf
5	8 5
6	4
7	5
8	5 0 0
9	5 0 2

Step 3 Rearrange the leaves so they are ordered from least to greatest. Then include a key or an explanation.

Stem	Leaf
5	5 8
6	4
7	5
8	0 0 5
9	0 2 5

8 | 5 = 85 acres

Size of U. S. Zoos

Zoo	Size (acres)
Audubon (New Orleans)	58
Cincinnati	85
Dallas	95
Denver	80
Houston	55
Los Angeles	80
Oregon	64
St. Louis	90
San Francisco	75
Woodland Park (Seattle)	92

Exercises

Display each set of data in a stem-and-leaf plot.

1. {27, 35, 39, 27, 24, 33, 18, 19}

2. {94, 83, 88, 77, 95, 99, 88, 87}

ROLLER COASTERS **For Exercises 3 and 4, use the stem-and-leaf plot shown.**

The Fastest Roller Coasters

Stem	Leaf
8	3 5
9	2 5
10	0

8 | 3 = 83 mph

3. What is the speed of the fastest roller coaster? The slowest?

4. What is the median speed?

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12-2 Study Guide and Intervention

Measures of Variation

The **range** and the **interquartile range** describe how a set of data varies.

Term	Definition
range	The difference between the greatest and the least values of the set
median	The value that separates the data set in half
lower quartile	The median of the lower half of a set of data
upper quartile	The median of the upper half of a set of data
interquartile range	The difference between the upper quartile and the lower quartile
outlier	Data that is more than 1.5 times the value of the interquartile range beyond the quartiles.

Example **Find the range, interquartile range, and any outliers for each set of data.**

a. {3, 12, 17, 2, 21, 14, 14, 8}

Step 1 List the data from least to greatest. The range is 21 – 2 or 19. Then find the median.

2 3 8 12 14 14 17 21

median $= \frac{14 + 12}{2}$ or 13

Step 2 Find the upper and lower quartiles.

2 3 8 12 14 14 17 21

LQ $= \frac{3 + 8}{2}$ or 5.5 median UQ $= \frac{14 + 17}{2}$ or 15.5

The interquartile range is 15.5 – 5.5 or 10. There are no outliers.

b.

Stem	Leaf
2	2 6 9
3	1 1 3 4 9
4	0 2 5 5 7 7 8
5	3 4 6 6

3 | 4 = 34

The stem-and-leaf plot displays the data in order. The greatest value is 56. The least value is 22. So, the range is 56 – 22 or 34.

The median is 42. The LQ is 31 and the UQ is 48. So, the interquartile range is 48 – 31 or 17.

There are no outliers.

Exercises

WEATHER **For Exercises 1 and 2, use the data in the stem-and-leaf plot at the right.**

1. Find the range, median, upper quartile, lower quartile, interquartile range, and any outliers for each set of data.

2. Write a sentence that compares the data.

Average Extreme July Temperatures in World Cities

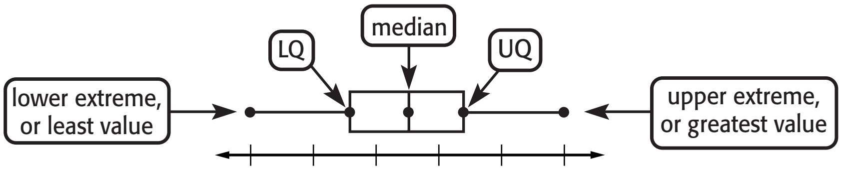
Low Temps.		High Temps.
9 1 1 0	5	
4	6	4 7 9
9 8 6 5 5 4 3 0 0	7	9
0	8	1 1 3 3 4 8
	9	0 1 2 5
	10	7

0 | 8 = 80°F *7 | 9 = 79°F*

Lesson 12-2

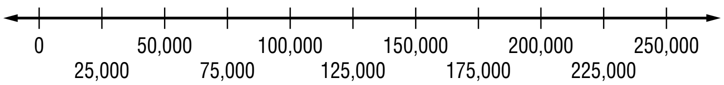
12-3 Study Guide and Intervention

Box-and-Whisker Plots

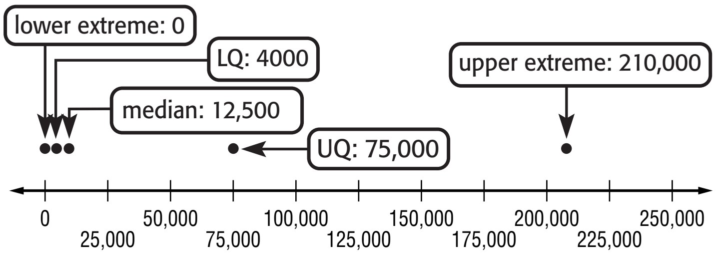
Box-and-Whisker Plot	**Words**	A **box and whisker plot** divides a set of data into four parts using the median and quartiles. Each of these parts contains 25% of the data.
	Model	

Example 1 FOOD **The heat levels of popular chile peppers are shown in the table. Display the data in a box-and-whisker plot.**

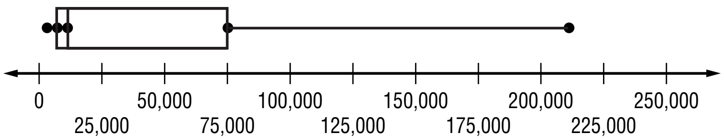
Step 1 Find the least and greatest number. Then draw a number line that covers the range of the data.



Step 2 Mark the median, the extremes, and the quartiles. Mark these points above the number line.



Step 3 Draw a box and the whiskers.



Heat Level of Chile Peppers	
Name	**Heat Level***
Aji escabeche	17,000
Bell	0
Cayenne	8,000
Habañero	210,000
Jalapeño	25,000
Mulato	1,000
New Mexico	4,500
Pasilla	5,500
Serrano	4,000
Tabasco	120,000
Tepín	75,000
Thai hot	60,000

Source: Chile Pepper Institute
*Scoville heat units

Exercises

Draw a box-and-whisker plot for each set of data.

1. {17, 5, 28, 33, 25, 5, 12, 3, 16, 11, 22, 31, 9, 11}

2. {$21, $50, $78, $13, $45, $5, $12, $37, $61, $11, $77, $31, $19, $11, $29, $16}

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12-4 Study Guide and Intervention

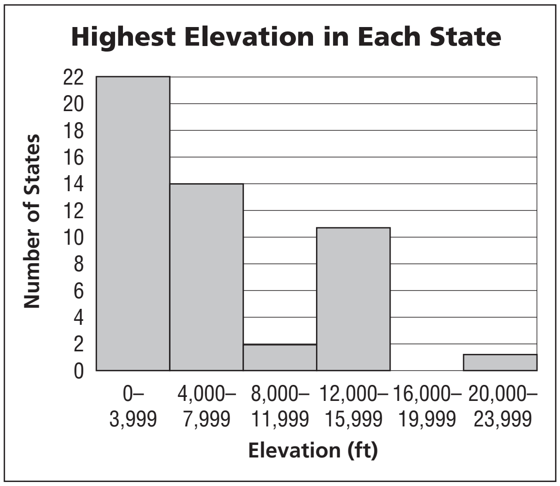
Histograms

Histograms	A histogram uses bars to display numerical data that have been organized into equal intervals. • There is no space between bars. • Because the intervals are equal, all of the bars have the same width. • Intervals with a frequency of 0 have no bar.

Example ELEVATIONS **The frequency table shows the highest elevations in each state. Display the data in a histogram.**

Highest Elevation in Each State		
Elevation (ft)	**Tally**	**Frequency**
0–3999	𝍸 𝍸 𝍸 𝍸 II	22
4000–7999	𝍸 𝍸 IIII	14
8000–11,999	II	2
12,000–15,999	𝍸 𝍸 I	11
16,000–19,999		0
20,000–23,999	I	1

Source: www.peakware.com



Step 1 Draw and label the axes as shown. Include the title.

Step 2 Show the frequency intervals on the horizontal axis and an interval of 2 on the vertical axis.

Step 3 For each elevation interval, draw a bar whose height is given by the frequency.

Exercises

For Exercises 1–3, use the information shown in the table below.

1. The frequency table shows voter participation in 2000. Display the data in a histogram.

Voter Participation by State (2000)		
Percent voting	**Tally**	**Frequency**
35–39	I	1
40–44		0
45–49	𝍸 I	6
50–54	𝍸 𝍸 II	12
55–59	𝍸 𝍸 III	13
60–64	𝍸 𝍸 II	12
65–69	𝍸 I	6

Source: U.S. Census Bureau

2. How many states had a voter turnout greater than 50 percent?

3. How many states had fewer than 40 percent voting?

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12-5 Study Guide and Intervention

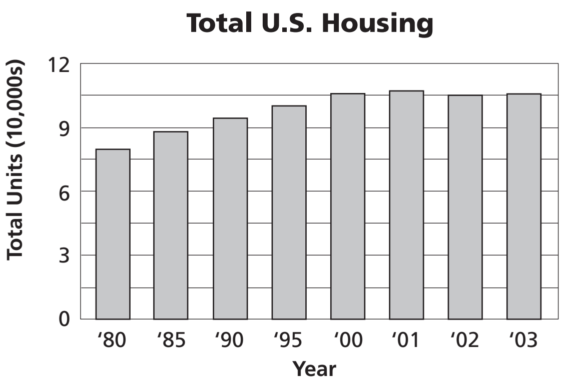
Choosing an Appropriate Display

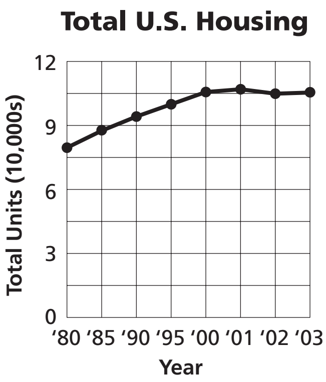
CHOOSE APPROPRIATE DISPLAYS Data can be visually represented in many different ways, including bar graphs, box-and-whisker plots, circle graphs, frequency tables, histograms, line graphs, line plots, stem-and-leaf plots, tables, and Venn diagrams.

Example **HOUSING The table shows the total number of houses occupied in the U.S. from 1980–2003. Choose an appropriate type of display for this situation. Then make the display.**

Year	1980	1985	1990	1995	2000	2001	2002	2003
Total Housing	79,638	87,887	94,224	99,985	105,720	107,010	104,965	105,560

The data can be represented in two ways. First, you can use a bar graph showing the number in each year. Second, you can use a line graph to show the change from 1980 to 2003.





Exercise

Choose an appropriate type of display for the data set. Then make the display.

Player	Points Per Game
Allen Iverson	33.2
LeBron James	30.7
Gilbert Arenas	29.1
Dwyane Wade	27.7
Paul Pierce	27.1

Source: espn.com

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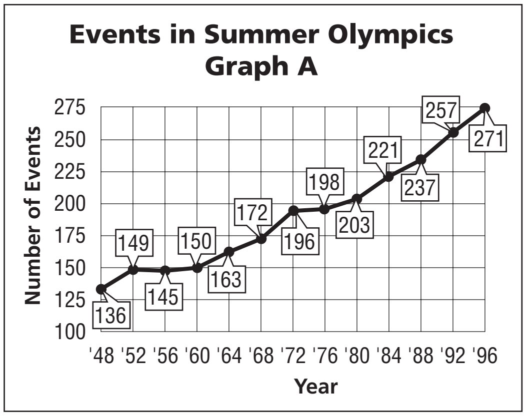
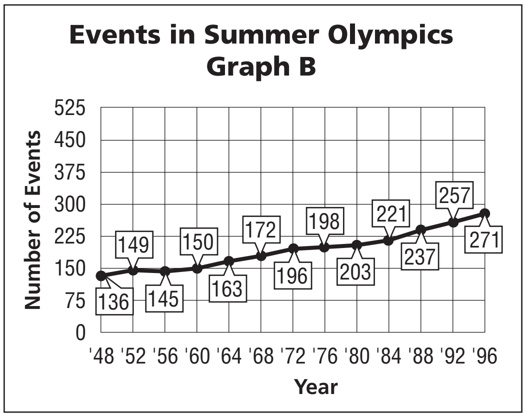
12-6 Study Guide and Intervention

Misleading Graphs

Example **The graphs below show the increase in the number of events held in the Summer Olympics from 1948 to 1996.**





a. **What causes the graphs to differ in their appearance?**

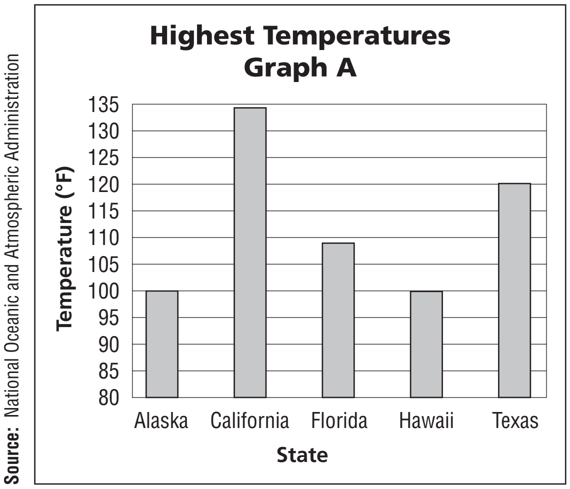
Different vertical scales and the lack of a zero on Graph A's axis result in different visual impressions.

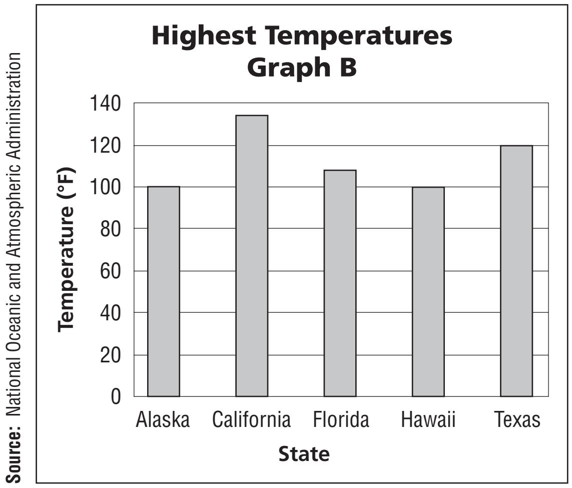
b. **Which graph appears to show a more rapid increase in the number of events held in the Summer Olympics? Explain.**

Graph A; the steeper slope of the line and the higher position of the line relative to the scale make it appear that the number of events is greater and increasing faster.

Exercises

For Exercises 1–3, refer to the graphs below.





1. What is the highest recorded temperature in California? In Alaska?
2. In which graph does the difference between these two temperatures appear to be minimized?
3. How do the graphs create different appearances?

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12-7 Study Guide and Intervention

Simple Probability

Probability is the chance some event will happen.

$P(\text{event}) = \dfrac{(\text{number of favorable outcomes})}{(\text{number of possible outcomes})}$

Example **A bag contains 6 red marbles, 1 blue marble, and 3 yellow marbles. One marble is selected at random. Find the probability of each outcome.**

a. P(yellow)

$P(\text{event}) = \dfrac{(\text{number of favorable outcomes})}{(\text{number of possible outcomes})}$

$= \dfrac{3}{10}$ or 30%

There is a 30% chance of choosing a yellow marble.

b. *P*(blue or yellow)

$P(\text{event}) = \dfrac{(\text{number of favorable outcomes})}{(\text{number of possible outcomes})}$

$= \dfrac{(1 + 3)}{10} = \dfrac{4}{10}$ or 40%

There is a 40% chance of choosing a blue or yellow marble.

c. *P*(red, blue, or yellow)

$P(\text{event}) = \dfrac{(\text{number of favorable outcomes})}{(\text{number of possible outcomes})}$

$= \dfrac{(6 + 1 + 3)}{10} = \dfrac{10}{10}$ or 100%

There is a 100% chance of choosing a red, blue, or yellow marble.

d. *P*(black)

$P(\text{event}) = \dfrac{(\text{number of favorable outcomes})}{(\text{number of possible outcomes})}$

$= \dfrac{0}{10}$ or 0%

There is a 0% chance of choosing a black marble.

Exercises

A bag contains 5 red marbles, 5 blue marbles, 6 green marbles, 8 purple marbles, and 1 white marble. One is selected at random. Find the probability of each outcome. Express each probability as a fraction and as a percent.

1. *P*(white)

2. *P*(red)

3. *P*(green)

4. *P*(purple)

5. *P*(white, blue, or green)

6. *P*(red or blue)

7. *P*(red or purple)

8. *P*(green or purple)

9. *P*(green, purple, or white)

10. *P*(red, blue, green, purple, or white)

11. *P*(red, blue, or purple)

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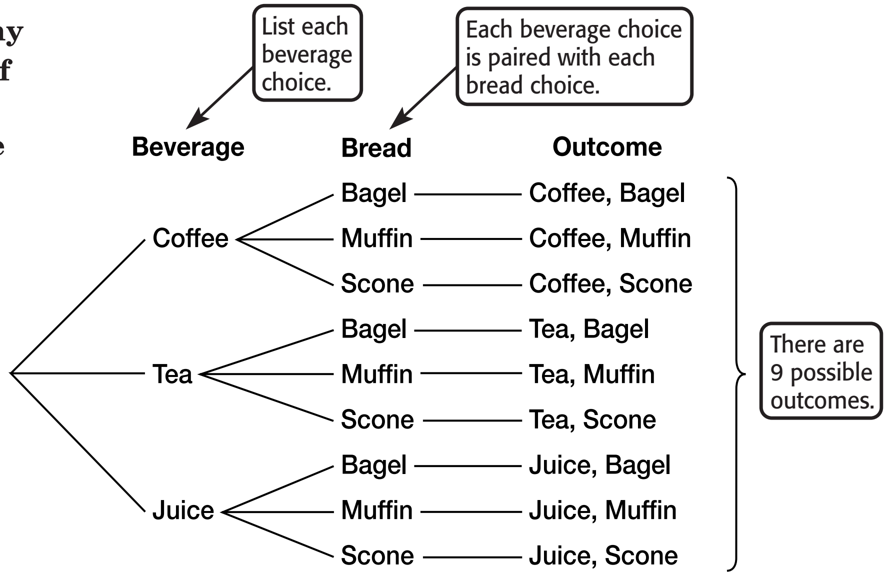
12-8 Study Guide and Intervention

Counting Outcomes

Example 1 **How many different combinations of beverage and bread can be made from 3 beverage choices and 3 bread choices?**

Draw a **tree diagram** to find the number of different combinations.



The **Fundamental Counting Principle** also relates the number of outcomes to the number of choices. When you know the number of outcomes, you can find the probability that an event will occur.

Example 2 **Refer to Example 1. What is the probability of randomly selecting coffee with a scone?**

Use the Fundamental Counting Principle to find the number of outcomes.

$$\overbrace{3}^{\text{beverage choices}} \quad \overbrace{\times}^{\text{times}} \quad \overbrace{3}^{\text{bread choices}} \quad \overbrace{=}^{\text{equals}} \quad \overbrace{9}^{\text{total number of possible outcomes}}$$

Only one of the 9 possible outcomes is coffee and a scone.
So, the probability of randomly selecting coffee with a scone is $\frac{1}{9}$.

Exercises

Find the number of possible outcomes for each situation.

1. one six-sided number cube is rolled, and one card is drawn from a 52-card deck

2. There are 512 juniors and 498 seniors. One junior and one senior are randomly drawn as raffle winners.

Find the probability of each event.

3. A coin is tossed and a card is drawn from a 52-card deck. What is the probability of getting tails and the ten of diamonds?

4. Four coins are tossed. What is the probability of four tails?

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12-9 Study Guide and Intervention

Permutations and Combinations

Permutations	**Words**	An arrangement or listing in which order is important is called a **permutation**.
	Symbols	$P(m, n)$ means m number of choices taken n at a time.
	Example	$P(3, 2) = 3 \cdot 2 = 6$
Combinations	**Words**	An arrangement or listing where order is *not* important is called a **combination**.
	Symbols	$C(m, n) = \frac{P(m, n)}{n!}$
	Example	$C(6, 2) = \frac{P(6, 2)}{2!} = \frac{6 \cdot 5}{2 \cdot 1}$ or 15

Example 1 SPORTS **How many ways can the top five finishers be arranged in a 20-person cross-country race?**

Order is important.
So, this arrangement is a permutation.

20 runners

choose 5

20 choices for 1st place
19 choices for 2nd place
18 choices for 3rd place
17 choices for 4th place
16 choices for 5th place

$$P(20, 5) = \underbrace{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}$$
$$= 1{,}860{,}480 \text{ ways}$$

Example 2 SCHOOL **In a science class with 42 students, how many 3-person lab teams can be formed?**

Order is not important.
So, this arrangement is a combination.

From 42 students, take 3 at a time.

$$C(42, 3) = \frac{P(42, 3)}{3!}$$
$$= \frac{42 \cdot 41 \cdot 40}{3 \cdot 2 \cdot 1} \text{ or } 11{,}480 \text{ lab teams}$$

Exercises

Tell whether each situation is a *permutation* or *combination*. Then solve.

1. How many ways can three people be selected from a group of seven?
2. How many ways can a 6-person kickball team be chosen from 27 students?
3. How many ways can 15 actors fill 6 roles in a play?
4. How many ways can 5 books be borrowed from a collection of 40 books?
5. **JOBS** A telemarketing firm has 35 applicants for 8 identical entry-level positions. How many ways can the firm choose 8 employees?
6. **FOOD** A pizza place sends neighbors a coupon for a 4 topping pizza of any size. If the pizzeria has 15 toppings and 3 sizes to choose from, how many possible pizzas could be purchased using the coupon?

12-10 Study Guide and Intervention

Probability of Composite Events

Probability of Two Independent Events	**Words**	The probability of two independent events is found by multiplying the probability of the first event by the probability of the second event.
	Symbols	$P(A \text{ and } B) = P(A) \cdot P(B)$
Probability of Two Dependent Events	**Words**	If two events, *A* and *B*, are dependent, then the probability of events occurring is the product of the probability of *A* and the probability of *B* after *A* occurs.
	Symbols	$P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

Example 1 **GAMES A card is drawn from a standard deck of 52 cards. The card is replaced and another is drawn. Find the probability if the first card is the 3 of hearts and the second card is the 2 of clubs.**

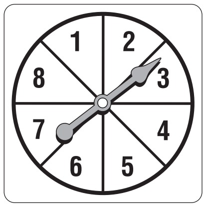
Since the first card is replaced, the events are independent.

$$P(\text{3 of hearts and 2 of clubs}) = P(\text{3 of hearts}) \cdot P(\text{2 of clubs})$$
$$= \frac{1}{52} \cdot \frac{1}{52}$$
$$= \frac{1}{2704}$$

The probability is $\frac{1}{2704}$.

Probability of Mutually Exclusive Events	**Words**	The probability of one or the other of two **mutually exclusive events** can be found by adding the probability of the first event to the probability of the second event.
	Symbols	$P(A \text{ or } B) = P(A) + P(B)$

Example 2 **The spinner at the right is spun. What is the probability that the spinner will stop on 7 or an even number?**



The events are mutually exclusive because the spinner cannot stop on both 7 and an even number at the same time.

$P(7 \text{ or even}) = P(7) + P(\text{even}) = \frac{1}{8} + \frac{1}{2} = \frac{5}{8}$

The probability that the spinner will stop on 7 or an even number is $\frac{5}{8}$.

Exercises

A card is drawn from a standard deck of cards. The card is not replaced and a second card is drawn. Find each probability.

1. $P(4 \text{ and } 8)$

2. $P(\text{queen of hearts and } 10)$

A card is drawn from a standard deck of cards. Find each probability.

3. $P(\text{queen of clubs or a red card})$

4. $P(\text{queen of hearts or } 10)$

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13-1 Study Guide and Intervention

Polynomials

Classify Polynomials Polynomials are classified according to the number of terms they have. A *monomial* has one term, a *binomial* has two terms, and a *trinomial* has three terms. The exponent of a variable in a monomial must be a whole number, and the variable cannot be in the denominator or under a radical sign.

Example 1 **Determine whether each expression is a polynomial. If it is, classify it as a *monomial, binomial,* or *trinomial*.**

a. $2y + \frac{3}{y}$

The expression is not a polynomial because $\frac{3}{y}$ has a variable in the denominator.

b. $\frac{3a}{4} + 6a^3 - 5a^4$

The expression is a polynomial with three terms, so it is a trinomial.

Degree of Polynomials A polynomial also has a degree. The degree of a polynomial is the same as that of the term with the greatest degree. The degree of a term is the sum of the exponents of its variables.

Example 2 **Find the degree of each polynomial.**

a. $x^6 - 3x^4 + 1$

The greatest degree is 6, so the degree of the trinomial is 6.

b. $10b^2c + 8bc - c^2$

$10b^2c$ has degree 2 + 1 or 3. $8bc$ has degree 1 + 1 or 2. c^2 has degree 2.

The greatest degree is 3, so the trinomial has degree 3.

Exercises

Determine whether each expression is a polynomial. If it is, classify it as a *monomial, binomial,* or *trinomial*.

1. $7q + r - 10$

2. $\sqrt{8r}$

3. $x^2 - 4$

4. -89

5. $3v^2 + 4w$

6. $a^5 + b^2 + c$

Find the degree of each polynomial.

7. $28y$

8. $-5h$

9. $2x^3y$

10. $9p^3 - 6p^2$

11. $mn^5 + mn^4 + m^2$

12. $8x^2 + 4xy - y^2$

13-2 Study Guide and Intervention

Adding Polynomials

Add Polynomials Add polynomials by combining like terms, which are monomials that contain the same variables to the same power.

Example **Find $(8x^2 - 7x + 1) + (x^2 + 5)$.**

Method 1 Add vertically.

$$\begin{array}{r} 8x^2 - 7x + 1 \\ (+)\ x^2 \qquad + 5 \\ \hline 9x^2 - 7x + 6 \end{array}$$

Method 2 Add horizontally.

$(8x^2 - 7x + 1) + (x^2 + 5)$

$= (8x^2 + x^2) - 7x + (1 + 5)$

$= 9x^2 - 7x + 6$

Exercises

Find each sum.

1. $\begin{array}{r} 3x - 7 \\ (+)\ x + 1 \\ \hline \end{array}$

2. $\begin{array}{r} 6d + 8 \\ (+)\ -4d + 1 \\ \hline \end{array}$

3. $\begin{array}{r} 4w^2 - 6w + 3 \\ (+)\ w^2 \qquad - 5 \\ \hline \end{array}$

4. $\begin{array}{r} 5a^2 - a \\ (+) \qquad 2a - 5 \\ \hline \end{array}$

5. $(-m + 3) + (7m - 1)$

6. $(9x^2 + 3x - 1) + (4x + 1)$

7. $(2k^2 - k) + (k - 1)$

8. $(5a^2 + 6ab) + (-ab + b^2)$

9. $(4c^2 - 7) + (c^2 - 3c + 6)$

10. $(x^2 + y) + (xy + y)$

11. $(12h - 6) + (h^2 - 8h + 6)$

12. $(10x^2 + x + 5) + (x - 10x^2)$

13. $(6y^2 - y + 1) + (y^2 - 3y - 6)$

14. $(p^3 + 4) + (2p^2 - 2p + 3)$

15. $(3g^2 + 3g + 5) + (5g^2 - 3)$

16. $(5r^2 - 6) + (-r^2 - 4r + 7)$

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13-3 Study Guide and Intervention

Subtracting Polynomials

Subtract Polynomials To subtract polynomials, subtract like terms.

Example **Find $(x^2 + 3x - 6) - (4x^2 - 1)$.**

Method 1 Subtract vertically.

$$\begin{array}{r} x^2 + 3x - 6 \\ (-)\ 4x^2 \qquad - 1 \\ \hline -3x^2 + 3x - 5 \end{array}$$

Method 2 Add the additive inverse of $4x^2 - 1$, which is $(-1)(4x^2 - 1)$ or $-4x^2 + 1$.

$= (x^2 + 3x - 6) - (4x^2 - 1)$

$= (x^2 + 3x - 6) + (-4x^2 + 1)$

$= (x^2 - 4x^2) + (3x) + (-6 + 1)$

$= -3x^2 + 3x - 5$

Exercises

Find each difference.

1. $\begin{array}{r} 4c + 7 \\ (-)\ 3c + 3 \\ \hline \end{array}$

2. $\begin{array}{r} 2m + 5 \\ (-)\ -8m + 1 \\ \hline \end{array}$

3. $\begin{array}{r} 9k^2 - 4k + 5 \\ (-)\quad k^2 \qquad - 5 \\ \hline \end{array}$

4. $\begin{array}{r} 3z^2 - z \\ (-)\qquad 3z - 5 \\ \hline \end{array}$

5. $(-6r + 3) - (7r + 2)$

6. $(8f^2 - 7f - 3) - (2f + 4)$

7. $(5n^2 - 2n) - (3n + 9)$

8. $(a^2 + 5ab) - (-2ab - 3b^2)$

9. $(6g^2 + 8) - (5g^2 - 2g + 6)$

10. $(8x^2 - 3y) - (2xy + 3y)$

11. $(n - 12) - (n^2 + n + 9)$

12. $(h^2 - 2h + 1) - (3h - 7h^2)$

13. $(y^2 + y + 1) - (y^2 - y + 1)$

14. $(6p^2 - 5p - 1) - (2p - 4)$

15. $(4q^2 + q) - (q^2 + 3)$

16. $(6v^2 + 8) - (7v^2 + 2v - 5)$

17. $(u^2 + u - 4) - (5u^2 - 4)$

18. $(9b^2 + 2) - (-b^2 + b + 9)$

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13-4 Study Guide and Intervention

Multiplying a Polynomial by a Monomial

The Distributive Property can be used to multiply a polynomial by a monomial.

Example 1 **Find $7(4x - 8)$.**

$$7(4x - 8) = 7(4x) - 7(8)$$
$$= 28x - 56$$

Example 2 **Find $(x^2 - 5x + 4)(-2x)$.**

$$(x^2 - 5x + 4)(-2x) = x^2(-2x) - 5x(-2x) + 4(-2x)$$
$$= -2x^3 + 10x^2 - 8x$$

Exercises

Find each product.

1. $5(7y + 4)$

2. $(3h + 6)4$

3. $-9(q + 8)$

4. $6(d - 2)$

5. $(4g - 5)(-2)$

6. $-7(4x^2 - 7)$

7. $-2(n^2 - 3n + 9)$

8. $(a^2 - 2ab + b^2)5$

9. $r(r + 9)$

10. $(b^2 - 4)(-b)$

11. $-x(3x + 6)$

12. $(2k - 9)(k^2)$

13. $-m(6m + 1)$

14. $p(7p - 2)$

15. $(8 - 3h)(-h)$

16. $w(4w^2 - 2w + 3)$

17. $ab(2a + b)$

18. $x(7x + y)$

19. $(m^2 - mn - n)m$

20. $2y(5y + 1)$

21. $-10u(u - 5)$

22. $(5r^2 - 2r)(-3r)$

23. $8z(2z + 7)$

24. $5b^2(6b - 2)$

25. $4p^2(6p^2 + 3p)$

26. $(5v^2 - 2v - 4)(-2v)$

27. $8y^3(3y^2 - y + 8)$

28. $3m(2m + 4n)$

29. $(8gh - 3h)(-3gh)$

30. $5a(2a - 3ab + b)$

13-5 Study Guide and Intervention

Linear and Nonlinear Functions

Linear functions have constant rates of change. Their graphs are straight lines and their equations can be written in the form $y = mx + b$. Nonlinear functions do not have constant rates of change and their graphs are not straight lines.

Example 1 **Determine whether each equation represents a *linear* or *nonlinear* function.**

a. $y = 9$

This is linear because it can be written as $y = 0x + 9$.

b. $y = x^2 + 4$

This is nonlinear because the exponent of x is not 1, so the equation cannot be written in the form $y = mx + b$.

Tables can represent functions. A nonlinear function does not increase or decrease at a constant rate.

Example 2 **Determine whether each table represents a *linear* or *nonlinear* function.**

a.

	x	y	
	0	−7	
+ 2	2	1	+ 8
+ 2	4	9	+ 8
+ 2	6	17	+ 8

As x increases by 2, y increases by 8. The rate of change is constant, so this is a linear function.

b.

	x	y	
	0	100	
+ 5	5	75	− 25
+ 5	10	0	− 75
+ 5	15	−125	− 125

As x increases by 5, y decreases by a greater amount each time. The rate of change is not constant, so this is a nonlinear function.

Exercises

Determine whether each equation or table represents a *linear* or *nonlinear* function. Explain.

1. $x + 3y = 9$

2. $y = \frac{8}{x}$

3. $y = 6x(x + 1)$

4. $y = 9 - 5x$

5.

x	y
0	24
2	14
4	4
6	−6

6.

x	y
1	1
2	8
3	27
4	64

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13-6 Study Guide and Intervention

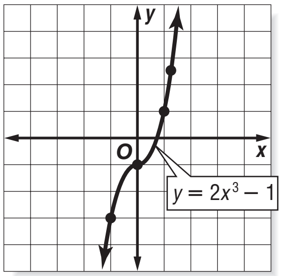
Graphing Quadratic and Cubic Functions

Lesson 13-6

To graph a quadratic or cubic function, make a table of values and then plot the points.

Example **Graph $y = 2x^3 - 1$.**

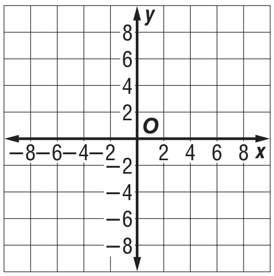
x	y
−1	–3
0	–1
1	1
1.2	2.5



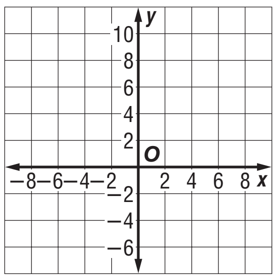
Exercises

Graph each function.

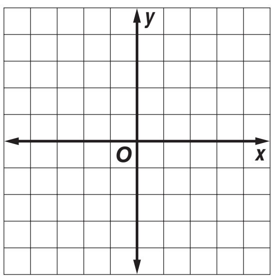
1. $y = x^2 + 2$



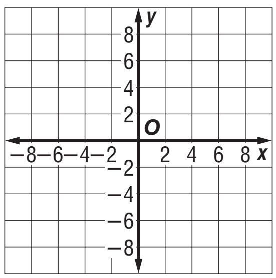
2. $y = x^3 + 2$



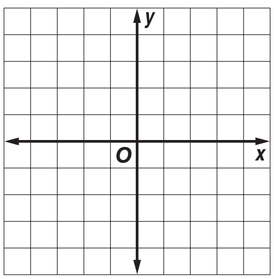
3. $y = -x^2 + 2$



4. $y = -x^3 + 2$



5. $y = x^2 - 2$



6. $y = x^3 - 2$