NAME ______________________________ DATE ____________ PERIOD _____

P-1 Assessment

Replace each ◯ with <, >, or = to make a true sentence.

1. 47 ◯ 74

2. 97 ◯ 204

3. 1,263 ◯ 1,362

4. 6,789 ◯ 7,689

5. 7,858 ◯ 7,858

6. 43,321 ◯ 34,213

Order the given numbers from least to greatest.

7. 10 1 7 4

8. 604 406 416 164

9. 347 743 437 734

10. 5,369 9,365 6,953

11. 1,645 1,456 1,546

12. 56,302 52,617 6,540 546,000

13. **WEATHER** The high temperatures in San Francisco from January 8 to January 11 were 63, 57, 54, and 50 degrees Fahrenheit. Arrange these numbers in order from greatest to least.

14. **BOOKS** Anna's textbooks vary in size. Her math book has 733 pages, her reading book has 658 pages, and her science book has 686 pages. Arrange these numbers in order from least to greatest.

15. **SAVINGS** Mr. Jacoby's children are comparing the amount of money in each child's savings account. Victoria has $58, David has $103, and Chelanda has $85. Who saved the least amount of money?

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P-2 Assessment

Whole Numbers

What is the place value of each underlined digit?

1. $4\underline{4}$	**2.** $1{,}\underline{4}52$	**3.** $\underline{3}3$
4. $10\underline{0}$	**5.** $\underline{9}18$	**6.** $\underline{2}4{,}765$
7. $4{,}1\underline{3}6$	**8.** $6\underline{8}5$	**9.** $62\underline{4}$
10. $\underline{1}{,}832$	**11.** $3{,}8\underline{5}6$	**12.** $1\underline{1}{,}760$

Write each number in expanded form.

13. 934	**14.** 10,002	**15.** 15
16. 156	**17.** 2,496	**18.** 706
19. 8,345	**20.** 9,753	**21.** 92,340

P-3 Assessment

Addition and Subtraction

Add.

1. 72 + 65

2. 62 + 83

3. 39 + 37

4. 66 + 85

5. 768 + 67

6. 495 + 48

7. $470 + 583

8. 237 + 579

9. 1570 − 2823

10. 5126 + 2899

11. 3973 + 1689

12. 1482 + 3497

Subtract.

13. 87 − 53

14. 56 − 40

15. 854 − 630

16. 695 − 132

17. 34 − 8

18. 70 − 28

19. $78 − 59

20. 480 − 63

21. The table shows the number of field goals made by Henry High School's top three basketball team members during last year's season. How many more field goals did Brad make than Denny?

Name	3-Point Field Goals
Brad	216
Chris	201
Denny	195

22. How many field goals did the three boys make all together?

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P-4 Assessment

Multiplication and Division

Multiply.

1. 700×25

2. 602×4

3. 218×63

4. $\$189 \times 42$

5. $\$125 \times 11$

6. 264×40

7. 3265×72

8. 6019×94

Divide.

9. $5\overline{)3255}$

10. $70\overline{)359}$

11. $47\overline{)517}$

12. $18\overline{)901}$

13. **WEATHER** The temperature dropped 32°F in 4 hours. Suppose the temperature dropped by an equal amount each hour. What integer describes the change?

14. **BASKETBALL** A team scored a total of 27 points for three-point field goals in the season. How many 3-point field goals did they make?

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P-5 Assessment

Exponents

Write each power as a product of the same factor.

1. 7^3 **2.** 2^7 **3.** 9^2 **4.** 15^4

Evaluate each expression.

5. 3^5 **6.** 7^3 **7.** 8^4 **8.** 5^3

Write each product in exponential form.

9. $2 \cdot 2 \cdot 2 \cdot 2$

10. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

11. $10 \cdot 10 \cdot 10$

12. $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

13. $12 \cdot 12 \cdot 12$

14. $5 \cdot 5 \cdot 5 \cdot 5$

15. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

16. $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

17. **MATH** Write 625 using exponents in as many ways as you can.

18. **PREFIXES** Many prefixes are used in mathematics and science. The prefix giga in gigameter represents 1,000,000,000 meters. Write this prefix as a power of ten.

19. **LIBRARY** The school library contains 9^4 books. How many library books are in the school library?

20. **HOT DOGS** The concession stand at the county fair sold 6^3 hot dogs on the first day. How many hot dogs did they sell?

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P-6 Assessment

Prime Factorization

Determine whether each number is *prime* or *composite*.

1. 36 **2.** 71 **3.** 18

4. 27 **5.** 37 **6.** 61

7. 32 **8.** 21 **9.** 40

Find the prime factorization of each number.

10. 425 **11.** 82 **12.** 93

13. 142 **14.** 45 **15.** 56

16. 63 **17.** 236 **18.** 12

19. 110 **20.** 46 **21.** 84

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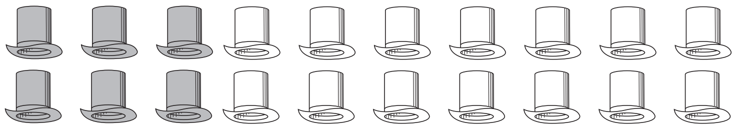
P-7 Assessment

Fractions

Write the fraction.

1. What fraction of the hats are shaded?

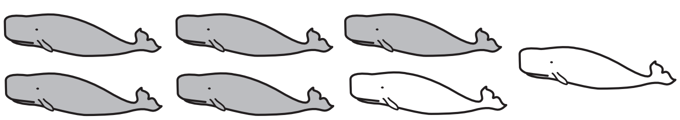


2. What fraction of the stars are shaded?

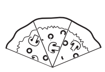
3. What fraction of the toy cars are shaded?

4. What fraction of the wiffle balls are shaded?

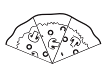
5. What fraction of the whales are shaded?



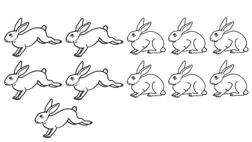
6. What fraction of the pizza is eaten?



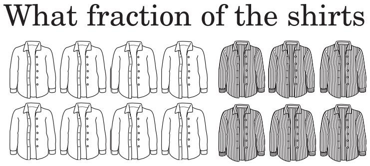
7. What fraction of the pizza is leftover?



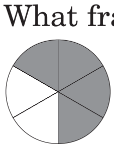
8. What fraction of the rabbits are hopping?



9. What fraction of the shirts are striped?



10. What fraction of the circle is unshaded?



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P-8 Assessment

Decimals

Write each fraction as a decimal.

1. $\frac{4}{5}$

2. $\frac{7}{20}$

3. $\frac{13}{250}$

4. $\frac{7}{8}$

5. $\frac{3}{16}$

6. $\frac{11}{32}$

7. $\frac{8}{40}$

8. $\frac{8}{80}$

9. $\frac{4}{32}$

Replace each ● with <, >, or = to make a true sentence.

10. $\frac{1}{4}$ ● 0.2

11. $\frac{13}{20}$ ● 0.63

12. 0.5 ● $\frac{3}{5}$

13. DISTANCE River Road Trail is $\frac{4}{5}$ miles long. Prairie Road Trail is 0.9 miles long. How much longer is Prairie Road Trail than River Road Trail?

14. ANIMALS The table shows lengths of different pond insects. Using decimals, name the insect having the smallest length and the insect having the greatest length.

Pond Insects				
Insect	Deer Fly	Spongilla Fly	Springtail	Water Treader
Length (in.)	$\frac{2}{5}$	0.3	$\frac{3}{20}$	0.5

Source: *Golden Nature Guide to Pond Life*

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P-9 Assessment

Least Common Multiple

Identify the first three common multiples of each set of numbers.

1. 4 and 5

2. 1 and 9

3. 3 and 4

4. 4, 6, and 8

Find the LCM of each set of numbers.

5. 3 and 5

6. 8 and 12

7. 3, 5, and 6

8. 6, 12, and 15

9. **PATTERNS** List the next four common multiples after the LCM of 3 and 8.

10. **E-MAIL** Alberto gets newsletters by e-mail. He gets one for sports every 5 days, one for model railroads every 10 days, and one for music every 8 days. If he got all three today, how many more days will it be until he gets all three newsletters on the same day?

11. **ROSES** Dante is planting his rose garden. He knows he can plant all of his roses by planting 12 or 15 rose bushes in every row. What is the least number of rose bushes Dante could have?

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P-10 Assessment

Estimation

Use compatable to estimate each quotient.

1. $4{,}782 + 632 =$	**2.** $\begin{array}{r} 578 \\ -\ \ 65 \\ \hline \end{array}$	**3.** $351 \times 78 =$
4. $23\overline{)5{,}789}$	**5.** $\begin{array}{r} 961 \\ +\ 325 \\ \hline \end{array}$	**6.** $1{,}845 - 763 =$
7. $\begin{array}{r} 8{,}602 \\ \times\ \ \ \ 28 \\ \hline \end{array}$	**8.** $4{,}192 \div 5 =$	**9.** $2{,}892 + 96 =$
10. $\begin{array}{r} 3{,}891 \\ -\ 1{,}436 \\ \hline \end{array}$	**11.** $637 \times 7 =$	**12.** $64\overline{)8{,}956}$
13. $783 \div 4 =$	**14.** $33\overline{)8{,}541}$	**15.** $5{,}547 \div 16 =$
16. $2\overline{)621}$	**17.** $947 \div 31 =$	**18.** $78\overline{)1{,}846}$

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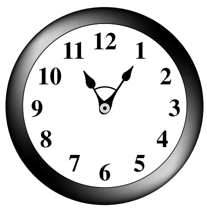
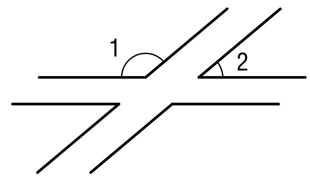
P-11 Assessment

Angles

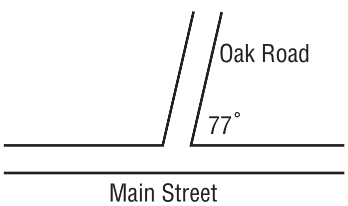
1. CLOCKS The time shown on the clock is 11:05. Starting at this time, approximately what time will it be when the hands form an obtuse angle?



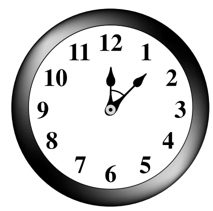
2. AIRPORT The runways at a local airport are sketched in the figure. Classify ∠1 and ∠2 as *acute*, *obtuse*, or *right*.



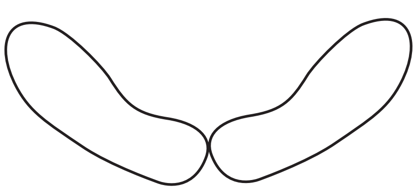
3. STREETS Main Street intersects Oak Road. If a right-hand turn onto Oak Road requires a 77° turn, what type of angle is it?



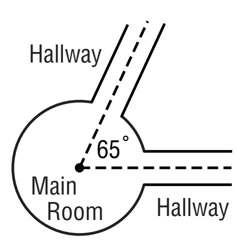
4. CLOCKS The time shown on the clock is 12:07. After 10 minutes have gone by, will the angle formed by the hour and minute hands be *acute*, *obtuse*, or *right*?



5. BALLET When a ballet dancer's feet are in first position, the heels are touching, and the feet are turned out. A dancer with excellent technique can position his or her feet so that they are nearly in a straight line. Isabella is practicing her technique. Classify the angle her feet form as *acute*, *obtuse*, or *right*.



6. ARCHITECTURE The plans for a new aquarium call for several hallways of exhibits leading out of a circular main room. Because of the size of the tanks that will be used, the angle formed between two adjacent hallways can be no smaller than 65°. What type of angle is it?



P-12 Assessment

Probability

A box of crayons contains 3 shades of red, 5 shades of blue, and 2 shades of green. If a child chooses a crayon at random, find the probability of choosing each of the following.

1. a red or blue crayon

2. a red or green crayon

Businesses use statistical surveys to predict customers' future buying habits. A department store surveyed 200 customers on a Saturday in December to find out how much each customer spent on their visit to the store. Use the results at the right to answer Exercises 3–5.

Amount Spent	Number of Customers
Less than $2	14
$2–$4.99	36
$5–$9.99	42
$10–$19.99	32
$20–$49.99	32
$50–$99.99	22
$100 or more	22

3. What is the probability that a customer will spend less than $2.00?

4. What is the probability that a customer will spend less than $10.00?

5. What is the probability that a customer will spend between $20.00 and $100.00?

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1-1 Extension Activity

The Great State Mystery

The United States of America has not always had 50 states. The states gradually joined the Union, starting with the first state in 1787 to the most recent state in 1959. The tables lists 15 states and their populations based on the 2000 Census. Use the 6 clues given and a problem solving process to complete the table below.

Delaware	783,600	**Iowa**	2,926,324	**New York**	18,976,457
Georgia	8,186,453	**Louisiana**	4,468,976	**Ohio**	11,353,140
Hawaii	1,211,537	**Mississippi**	2,844,658	**Texas**	20,851,820
Illinois	12,419,293	**New Jersey**	8,414,350	**Wisconsin**	5,363,675
Indiana	6,080,485	**New Mexico**	1,819,046	**Virginia**	7,078,515

1. The first state to enter the Union has the least population of the states listed.
2. The states beginning with the letter 'I' were the 19th, 21st, and 29th states admitted to the Union. Iowa entered the Union 30 years after Indiana.
3. New Jersey and Georgia were among the original thirteen colonies. Their entry number is the same as the digit in the hundreds place of their population.
4. Hawaii, Texas, and Wisconsin were the 28th, 30th, and 50th states admitted to the Union, but not in that order. To find their order, put them in order from greatest to least population.
5. The state with the second largest population entered the Union 15 years before Ohio and 24 years before the state with a population in the 4 millions.
6. The day of the month that Mississippi was admitted into the Union can be found by dividing its order of entry by 2.

Order of Entry	State Name	Date of Entry
1		December 7, 1787
3		December 18, 1787
4		January 2, 1788
10		June 25, 1788
11		June 26, 1788
17	Ohio	March 1,
18		April 30, 1812
	Indiana	December 11, 1816
20	Mississippi	December , 1817
21		December 3, 1818
		December 29, 1845
29		December 28, 1846
		May 29, 1848
47	New Mexico	January 6, 1912
		August 21, 1959

1-2 Extension Activity

Write Complex Fractions and Simplify

You can use what you know about writing expressions to write more complicated expressions. Then use your knowledge of fraction rules to simplify those expressions. A complex fraction is a fraction whose numerator or denominator is also a fraction or a mixed number.

Write an expression for the quotient of *one-fourth and two-thirds*.

$$\frac{\frac{1}{4}}{\frac{2}{3}}$$

Now rewrite the middle fraction bar as division.

$$\frac{1}{4} \div \frac{2}{3}$$

Use your division rules to multiply by the reciprocal of $\frac{2}{3}$.

$$\frac{1}{4} \cdot \frac{3}{2}$$

The result is $\frac{3}{8}$.

Write an expression for these complex fractions and simplify.

1. the quotient of four-fifths and 100

2. the quotient of 13 and two-thirds

3. the quotient of three and a one-half and three-fourths

1-3 Extension Activity

Nested Expressions

Nested Expressions

Sometimes more than one set of parentheses are used to group the quantities in an expression. These expressions are said to have "nested" parentheses. The expression below has "nested" parentheses.

$$(4 + (3 \cdot (2 + 3)) + 8) \div 9$$

Expressions with several sets of grouping symbols are clearer if braces such as { } or brackets such as [ ] are used. Here is the same example written with brackets and braces.

$$\{4 + [3 \cdot (2 + 3)] + 8\} \div 9$$

To evaluate expressions of this type, work from the inside out.

$$\begin{aligned} \{4 + [3 \cdot (2 + 3)] + 8\} \div 9 &= \{4 + [3 \cdot 5] + 8\} \div 9 \\ &= [4 + 15 + 8] \div 9 \\ &= 27 \div 9 \\ &= 3 \end{aligned}$$

Evaluate each expression.

1. $3 + [(24 \div 8) \cdot 7] - 20$

2. $[(16 - 7 + 5) \div 2] - 7$

3. $[2 \cdot (23 - 6) + 14] \div 6$

4. $50 - [3 \cdot (15 - 5)] + 25$

5. $12 + \{28 - [2 \cdot (11 - 7)] + 3\}$

6. $\{75 + 3 \cdot [(17 - 9) \div 2]\} \cdot 2$

7. $20 + \{3 \cdot [6 + (56 \div 8)]\}$

8. $\{4 + [5 \cdot (12 - 5)] + 15\} \cdot 10$

9. $\{15 \cdot [(38 - 26) \div 4]\} - 15$

10. $\{[34 + (6 \cdot 5)] \div 8\} + 40$

1-4 Extension Activity

Division by Zero?

Some interesting things happen when you try to divide by zero. For example, look at these two equations.

$$\frac{5}{0} = x \qquad \frac{0}{0} = y$$

Because multiplication "undoes" division, you can write two equivalent equations for the ones above.

$$0 \cdot x = 5 \qquad 0 \cdot y = 0$$

There is no number that will make the left equation true. This equation has no solution. For the right equation, *every* number will make it true. The solution set for this equation is "all numbers."

Because division by zero leads to impossible situations, it is not a "legal" step in solving a problem. People say that division by zero is undefined, or not possible, or simply not allowed.

Explain what is wrong with each of these "proofs."

1. **Step 1** $0 \cdot 1 = 0$ and $0 \cdot 2 = 0$

   Step 2 Therefore, $\frac{0}{0} = 1$ and $\frac{0}{0} = 2$.

   Step 3 Therefore, $1 = 2$.

   But, $1 = 2$ is a contradiction.

2. **Step 1** Assume $a \neq b$.

   Step 2 $0 \cdot a = 0$ and $0 \cdot b = 0$

   Step 3 Therefore, $\frac{0}{0} = a$ and $\frac{0}{0} = b$.

   Step 4 Therefore, $a = b$.

   But, $a = b$ contradicts $a \neq b$.

Describe the solution set for each equation.

3. $4x = 0$

4. $x \cdot 0 = 0$

5. $x \cdot 0 = x$

6. $\frac{0}{x} = 0$

7. $\frac{0}{x} = x$

8. $\frac{0}{x} = \frac{0}{y}$

1-5 Extension Activity

Name That Property

Name That Property

You know that the Commutative Property applies to the operations of addition and multiplication. You also know that the Associative Property applies to operations of addition and multiplication. What about the other operations? Does the Commutative Property apply to division? Does the Associative Property apply to subtraction? Does the Distributive Property apply to subtraction or division?

Look at these examples to determine if the properties also apply to subtraction or division.

Commutative Property

Subtraction	**Division**
Try this:	*Try this:*
$5 - 4 \stackrel{?}{=} 4 - 5$	$8 \div 2 \stackrel{?}{=} 2 \div 8$

1. Does the Commutative Property apply to division and subtraction? Explain.

Associative Property

Subtraction	**Division**
Try this:	*Try this:*
$7 - (3 - 2) \stackrel{?}{=} (7 - 3) - 2$	$8 \div (4 \div 2) \stackrel{?}{=} (8 \div 4) \div 2$

2. Does the Associative Property apply to subtraction and division? Explain.

Distributive Property

Subtraction	**Division**
Try this:	*Try this:*
$3(8 - 2) \stackrel{?}{=} 3 \cdot 8 - 3 \cdot 2$	$3(8 \div 2) \stackrel{?}{=} 3 \cdot 8 \div 3 \cdot 2$
$3(6) \stackrel{?}{=} 24 - 6$	$3(4) \stackrel{?}{=} 24 \div 6$
$18 = 18$ ✓	$12 \neq 4$

3. Does the Distributive Property apply to multiplication over subtraction? Does it apply to multiplication over division? Explain.

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1-6 Extension Activity

Operations Puzzles

Now that you have learned how to evaluate an expression using the order of operations, can you work backward? In this activity, the value of the expression will be given to you. It is your job to decide what the operations or the numbers must be in order to arrive at that value.

Fill in each □ with +, −, ×, or ÷ to make a true statement.

1. 48 □ 3 □ 12 = 12

2. 30 □ 15 □ 3 = 6

3. 24 □ 12 □ 6 □ 3 = 4

4. 24 □ 12 □ 6 □ 3 = 18

5. 4 □ 16 □ 2 □ 8 = 24

6. 45 □ 3 □ 3 □ 9 = 3

7. 36 □ 2 □ 3 □ 12 □ 2 = 0

8. 72 □ 12 □ 4 □ 8 □ 3 = 0

Fill in each □ with one of the given numbers to make a true statement. Each number may be used only once.

9. 6, 12, 24

□ ÷ □ × □ = 12

10. 4, 9, 36

□ − □ ÷ □ = 0

11. 6, 8, 12, 24

□ ÷ □ + □ − □ = 4

12. 2, 5, 10, 50

□ − □ ÷ □ + □ = 50

13. 2, 4, 6, 8, 10

□ ÷ □ × □ + □ − □ = 0

14. 1, 3, 5, 7, 9

□ ÷ □ + □ − □ ÷ □ = 1

15. CHALLENGE Fill in each □ with one of the digits from 1 through 9 to make a true statement. Each digit may be used only once.

□ ÷ □ × □ + □ × □ × □ ÷ □ + □ × □ = 100

1-7 Extension Activity

Other Properties

For Exercises 1–5, describe how each pair of numerical expressions is different. Then determine whether the two expressions are equal to each other. If the expressions are equal, name the property that says they are equal.

1. $2 + 5, 5 + 2$

2. $(6 - 4) - 1, 6 - (4 - 1)$

3. $2(5 - 3), 2 \cdot 5 - 2 \cdot 3$

4. $5 \cdot (4 \cdot 7), (5 \cdot 4) \cdot 7$

5. $10 \div 2, 2 \div 10$

6. The Identity Property says that adding __________ to a number results in the number and multiplying __________ by a number is the number.

Remember What You Learned

7. Why are the Distributive Property, Commutative Property, Associative Property, and Identity Property called properties?

Use a dictionary to find the meanings of *distribute* and *commute* that apply to mathematics. Then write an explanation of why the Distributive Property and Commutative Property are named that way.

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1-8 Extension Activity

Algebraic Proof

Recall that properties are statements that are true for any numbers. These properties are used to prove theorems. Use the properties you have learned to complete each proof.

Abbreviations for some properties you may need to use are listed below.

Commutative Property—Addition (CPA)
Commutative Property—Multiplication (CPM)
Associative Property—Addition (APA)
Associative Property—Multiplication (APM)
Additive Identity Property (AIP)
Multiplicative Identity Property (MIP)
Inverse Property of Addition (IPA)
Inverse Property of Multiplication (IPM)
Multiplicative Property of Zero (MPZ)
Distributive Property (DP)

Write the reason for each statement.

1. Prove: $-(y - x) = x - y$

Statement	**Reason**
$-(y - x) = -1(y - x)$	**MIP**
$= -1y - (-1x)$	**a.** ____________
$= -y - (-x)$	**b.** ____________
$= -y + x$	**c.** ____________
$= x + (-y)$	**d.** ____________
$= x - y$	**e.** ____________

2. Prove: $3x - 4 - x = 2x - 4$

Statement	**Reason**
$3x - 4 - x = 3x + (-4) + (-x)$	**a.** ____________
$= 3x + (-x) + (-4)$	**b.** ____________
$= 3x + (-1x) + (-4)$	**c.** ____________
$= [3 + (-1)]x + (-4)$	**d.** ____________
$= 2x + (-4)$	**e.** ____________
$= 2x - 4$	**f.** ____________

3. Prove: $-2x + 6 + 2x = 6$

Statement	**Reason**
$-2x + 6 + 2x = -2x + 2x + 6$	**a.** ____________
$= (-2 + 2)x + 6$	**b.** ____________
$= 0x + 6$	**c.** ____________
$= 0 + 6$	**d.** ____________
$= 6$	**e.** ____________

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2-1 Extension Activity

Equations

Equations as Models

When you write an equation that represents the information in a problem, the equation serves as a model for the problem. One equation can be a model for several different problems.

Each of Exercises 1–8 can be modeled by one of these equations.

$n + 2 = 10$ $\quad$ $n - 2 = 10$ $\quad$ $2n = 10$ $\quad$ $\frac{n}{2} = 10$

Choose the correct equation. Then solve the problem.

1. Chum earned $10 for working two hours. How much did he earn per hour?

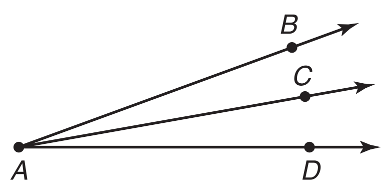
2. Ana needs $2 more to buy a $10 scarf. How much money does she already have?

3. Kathy and her brother won a contest and shared the prize equally. Each received $10. What was the amount of the prize?

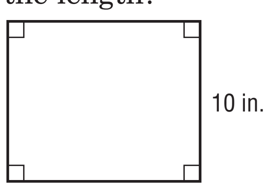
4. Jameel loaned two tapes to a friend. He has ten tapes left. How many tapes did Jameel originally have?

5. In the figure below, the length of $\overline{AC}$ is 10 cm. The length of $\overline{BC}$ is 2 cm. What is the length of $\overline{AB}$?

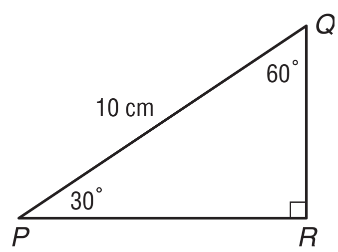
6. Ray $\overline{AC}$ bisects $\angle BAD$. The measure of $\angle BAC$ is 10°. What is the measure of $\angle BAD$?



7. The width of the rectangle below is 2 inches less than the length. What is the length?



8. In the triangle below, the length of $\overline{PQ}$ is twice the length of $\overline{QR}$. What is the length of $\overline{QR}$?



9. **CHALLENGE** On a separate sheet of paper, write a problem that can be modeled by the equation $3a + 5 = 29$.

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2-2 Extension Activity

Football Statistics

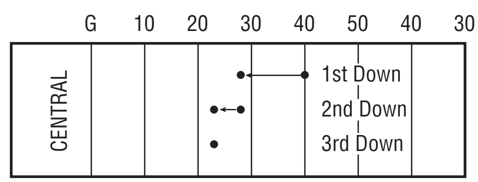
In football, one of the key offensive positions is running back. The job of the running back is to gain as many yards as possible with the ball. The line where the play begins is called the line of scrimmage. If the running back gets beyond the line of scrimmage when he is given the ball, he gains yards on the carry. However, if he is tackled behind the line of scrimmage, he loses yards. When he gains yards, the integer describing the carry is positive. However, when yards are lost, the integer describing the carry is negative.

Example 1 Sam is a running back for Central High School. He gains 3 yards on his first carry. The integer describing the carry is 3.

Example 2 Then Sam is then tackled 7 yards behind the line of scrimmage on his second carry. The integer describing the carry is −7.

Exercises

Sam carried the ball from the quarterback 15 times in their game against Southwest High School, and three consecutive times in one series of plays. Sam's three consecutive carries are illustrated at the right.

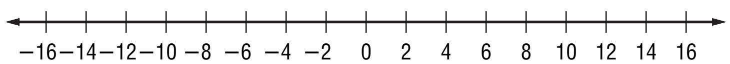


1. On 1st down, Sam was tackled 12 yards behind the line of scrimmage. What integer describes the carry?

2. On 2nd down, he was tackled 5 yards behind the line of scrimmage. What integer describes the number of yards lost on the play?

3. On 3rd down, Sam was tackled on the line of scrimmage. What integer represents the yardage gained on the play?

4. Graph the integers from each of the three consecutive carries on the number line below.



5. If you were the coach, would you play Sam in next week's game against North High School based on the five carries shown in the Examples and Exercises 1–3? Why or why not?

6. The table to the right shows the yards gained or lost each of the first five times that the Southwest High School running back carried the ball. Using this information, write the relationship between the yardage gained or lost on each carry by the Southwest High School running back compared to the Central High School's running back, Sam.

Carry	Yards Gained
1	2
2	5
3	−6
4	−2
5	26

2-3 Extension Activity

Mental Math: Compensation

To add or subtract in your head, work with multiples of 10 (20, 30, 40, . . .) or 100 (200, 300, 400, . . .) and then adjust your answer.

To add 52, first add 50, then add 2 more.

To subtract 74, first subtract 70, then subtract 4 more.

To subtract 38, first subtract 40, then add 2.

To add 296, first add 300, then subtract 4.

Write the second step you would use to do each of the following.

1. Add 83.
1) Add 80.
2)

2. Add 304.
1) Add 300.
2)

3. Subtract 62.
1) Subtract 60.
2)

4. Add 27.
1) Add 30.
2)

5. Subtract 79.
1) Subtract 80.
2)

6. Subtract 103.
1) Subtract 100.
2)

7. Add 499.
1) Add 500.
2)

8. Add 294.
1) Add 300.
2)

9. Subtract 590.
1) Subtract 600.
2)

Use the method above to add 59 to each of the following.

10. 40

11. 72

12. 53

13. 15

Use the method above to subtract 18 from each of the following.

14. 96

15. 45

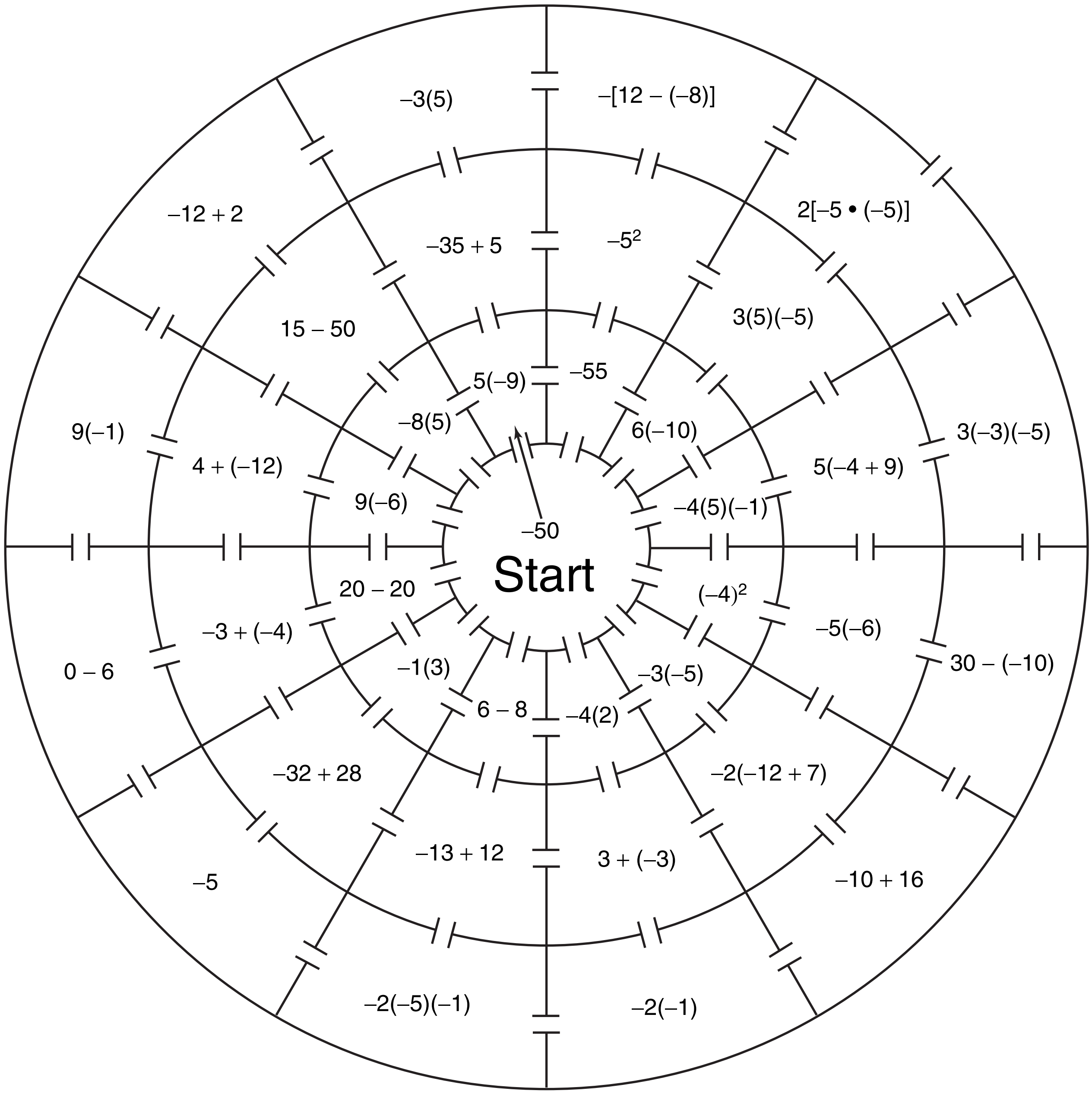
16. 71

17. 67

2-4 Extension Activity

Integer Maze

Find your way through the maze by moving to the expression in an adjacent section with the next highest value.



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2-5 Extension Activity

Divisibility Rules for 7 and 11

Example 1 **Determine whether 4032 is divisible by 7.**

403~~2~~	Cross out the ones digit.
$-\ \ 4$	Subtract twice the value of the ones digit from the rest of the number.
39~~9~~	If the difference is a number that you know is divisible by 7, stop. If not, repeat.
-18	
21	

Since 21 is divisible by 7, 4032 is divisible by 7.

Example 2 **Determine whether 5159 is divisible by 11.**

Method 1

515~~9~~	Cross out the ones digit.
$-\ \ 9$	Subtract the value of the ones digit from the rest of the number.
50~~6~~	If the difference is a number that you know is divisible by 11, stop. If not, repeat.
$-\ \ 6$	
44	

Since 44 is divisible by 11, 5159 is divisible by 11.

Method 2

5159	
$5 + 5 = 10$	Add the odd-numbered digits (first and third).
$1 + 9 = \underline{10}$	Add the even-numbered digits (second and fourth).
0	Subtract the sums. If the difference is divisible by 11, the number is divisible by 11.

Since 0 is divisible by 11, 5159 is divisible by 11.

Example 3 **Determine whether 62,382 is divisible by 11.**

$6 + 3 + 2 = 11$	Add the odd-numbered digits.
$2 + 8 = \underline{10}$	Add the even-numbered digits.
1	Subtract the sums.

Since 1 is not divisible by 11, 62,382 is not divisible by 11.

Exercises

Determine whether each number is divisible by 7 or 11.

1. 266 **2.** 4312 **3.** 8976

4. 936 **5.** 13,293 **6.** 7085

7. 2957 **8.** 3124 **9.** 6545

2-6 Extension Activity

Cyclic Numbers

Look closely at the products below. Do you see a pattern?

$1 \times 142{,}857 =$ 142,857

$2 \times 142{,}857 =$ 285,714

$3 \times 142{,}857 =$ 428,571

$4 \times 142{,}857 =$ 571,428

$5 \times 142{,}857 =$ 714,285

$6 \times 142{,}857 =$ 857,142

The same six digits repeat in all of the products. Numbers like 142,857 are called *cyclic numbers*.

1. Cyclic numbers are related to prime numbers. A prime number is a number that has exactly two factors, 1 and itself. You can use a calculator and the decimal equivalents of fractions of the form $\frac{1}{p}$, where p is a prime number, to find cyclic numbers. Use a calculator to find the decimal equivalent of each fraction below.

a. $\frac{1}{2}$ **b.** $\frac{1}{3}$ **c.** $\frac{1}{5}$ **d.** $\frac{1}{7}$

2. Study the decimal equivalents you found. Do you observe a pattern in any of the digits?

3. The cyclic number 142,857 has six digits. The next largest cyclic number has sixteen digits. What fraction do you think might help you find the next cyclic number? Explain.

4. Explain why the next largest cyclic number cannot be determined using a scientific calculator.

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2-7 Extension Activity

Lesson 2–7

Puzzling Equations

Solve each equation. Notice that the first equation is completed.

1. $\frac{m}{12} = 13$

2. $17v = -578$

3. $\frac{c}{75} = 18$

4. $-252d = -5796$

5. $64 \cdot w = 5568$

6. $g \div 29 = 61$

7. $p(85) = -7225$

8. $39x = 663$

9. $\frac{k}{18} = 30$

10. $\frac{z}{-94} = -32$

11. $-112q = 1456$

12. $201y = -1608$

13. $\frac{a}{14} = -17$

14. $-8045 = -5k$

15. $m \div (-105) = 8$

1. _______ = I

2. _______ = A

3. _______ = E

4. _______ = J

5. _______ = A

6. _______ = M

7. _______ = R

8. _______ = S

9. _______ = Y

10. _______ = R

11. _______ = O

12. _______ = N

13. _______ = R

14. _______ = S

15. _______ = H

Use the letter beside each of your answers to decode the answer to this question.

What woman led a 125-mile march from Pennsylvania to Long Island in 1903 to bring the practice of child labor to the attention of President Theodore Roosevelt?

1769	−34	3008	540
M	A	R	Y

−840	87	−238	−85	156	1609
H	A	R	R	I	S

23	−13	−8	1350	17
J	O	N	E	S

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3-1 Extension Activity

Developing Fraction Sense

If someone asked you to name a fraction between $\frac{4}{7}$ and $\frac{6}{7}$, you probably would give the answer $\frac{5}{7}$ pretty quickly. But what if you were asked to name a fraction between $\frac{4}{7}$ and $\frac{5}{7}$? At the right, you can see how to approach the problem using "fraction sense." So, one fraction between $\frac{4}{7}$ and $\frac{5}{7}$ is $\frac{9}{14}$.

$\frac{4}{7} = \frac{\bullet}{14} \rightarrow \frac{4}{7} = \frac{8}{14}$

$\frac{5}{7} = \frac{\bullet}{14} \rightarrow \frac{5}{7} = \frac{10}{14}$

Use your fraction sense to solve each problem.

1. Name a fraction between $\frac{1}{3}$ and $\frac{2}{3}$.

2. Name a fraction between $\frac{3}{5}$ and $\frac{4}{5}$.

3. Name five fractions between $\frac{1}{2}$ and 1.

4. Name five fractions between 0 and $\frac{1}{4}$.

5. Name a fraction between $\frac{1}{4}$ and $\frac{1}{2}$ whose denominator is 16.

6. Name a fraction between $\frac{2}{3}$ and $\frac{3}{4}$ whose denominator is 10.

7. Name a fraction between 0 and $\frac{1}{6}$ whose numerator is 1.

8. Name a fraction between 0 and $\frac{1}{10}$ whose numerator is *not* 1.

9. Name a fraction that is halfway between $\frac{2}{9}$ and $\frac{5}{9}$.

10. Name a fraction between $\frac{1}{4}$ and $\frac{3}{4}$ that is closer to $\frac{1}{4}$ than $\frac{3}{4}$.

11. Name a fraction between 0 and $\frac{1}{2}$ that is less than $\frac{3}{10}$.

12. Name a fraction between $\frac{1}{2}$ and 1 that is less than $\frac{3}{5}$.

13. Name a fraction between $\frac{1}{2}$ and $\frac{3}{4}$ that is greater than $\frac{4}{5}$.

14. How many fractions are there between $\frac{1}{4}$ and $\frac{1}{2}$?

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3-2 Extension Activity

Recipes

It is common to see mixed fractions in recipes. A recipe for a pizza crust may ask for $1\frac{1}{2}$ cups of flour. You could measure this amount in two ways. You could fill a one-cup measuring cup with flour and a one-half-cup measuring cup with flour or you could fill a half-cup measuring cup three times, because $1\frac{1}{2}$ is the same as $\frac{3}{2}$.

In the following recipes, some mixed numbers have been changed to improper fractions and other fractions may not be written in simplest form. Rewrite each recipe as you would expect to find it in a cookbook.

Quick Pizza Crust	
$\frac{3}{2}$ cups flour	
$\frac{2}{4}$ cup water	
$\frac{9}{4}$ teaspoons yeast	
$\frac{2}{2}$ teaspoon salt	
$\frac{4}{4}$ teaspoon sugar	
$\frac{8}{8}$ tablespoon oil	

Apple Crunch	
$\frac{3}{2}$ cups white sugar	
$\frac{3}{2}$ cups brown sugar	
$\frac{4}{2}$ cups of flour	
$\frac{4}{2}$ cups oatmeal	
$\frac{8}{3}$ sticks margarine	
$\frac{2}{2}$ teaspoon salt	

Granola	
$\frac{4}{3}$ cups sesame seeds	
$\frac{4}{2}$ cups coconut	
$\frac{3}{2}$ cups sunflower seeds	
$\frac{8}{2}$ cups rolled oats	
$\frac{2}{2}$ cup honey	
$\frac{4}{4}$ tablespoon brown sugar	

Chocolate Treats	
$\frac{4}{6}$ cup butter	
$\frac{9}{4}$ cups brown sugar	
$\frac{6}{2}$ eggs	
$\frac{11}{4}$ cups flour	
$\frac{5}{2}$ teaspoons baking powder	
$\frac{6}{3}$ cups chocolate chips	

3-3 Extension Activity

GCFs By Successive Division

Here is a different way to find the greatest common factor (GCF) of two numbers. This method works well for large numbers.

Find the GCF of 848 and 1,325.

Step 1 Divide the smaller number into the larger.

```
       1 R477
848)1,325
      848
      477
```

Step 2 Divide the remainder into the divisor.
Repeat this step until you get a remainder of 0.

```
      1 R371          1 R106           3 R53         2 R0
477)848         371)477          106)371         53)106
    477             371              318            106
    371             106               53              0
```

Step 3 The last divisor is the GCF of the two original numbers.
The GCF of 848 and 1,325 is 53.

Use the method above to find the GCF for each pair of numbers.

1. 187; 578

2. 161; 943

3. 215; 1,849

4. 453; 484

5. 432; 588

6. 279; 403

7. 1,325; 3,498

8. 9,840; 1,751

9. 3,484; 5,963

10. 1,802; 106

11. 45,787; 69,875

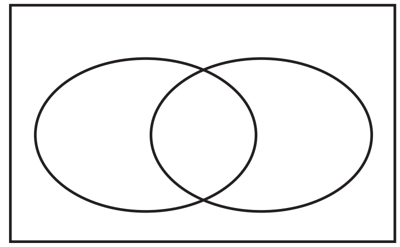
12. 35,811; 102,070

3-4 Extension Activity

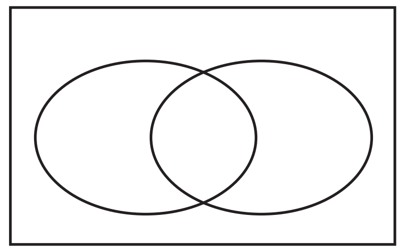
VENN DIAGRAMS

Use a Venn diagram to solve each problem.

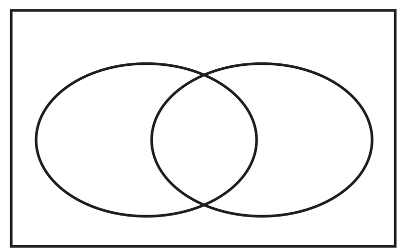
1. **PHONE SERVICE** Of the 5,750 residents of Homer, Alaska, 2,330 pay for landline phone service and 4,180 pay for cell phone service. One thousand seven hundred fifty pay for both landline and cell phone service. How many residents of Homer do not pay for any type of phone service?



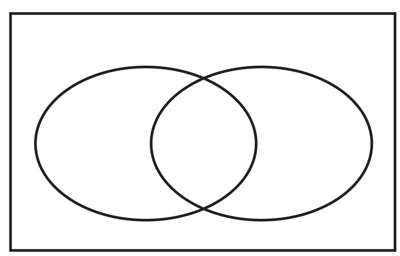
2. **BIOLOGY** Of the 2,890 ducks living in a particular wetland area, scientists find that 1,260 have deformed beaks, while 1,320 have deformed feet. Six hundred ninety of the birds have both deformed feet and beaks. How many of the ducks living in the wetland area have no deformities?



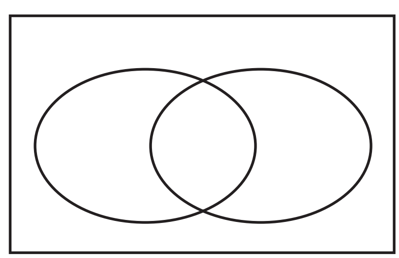
3. **FLU SYMPTOMS** The local health agency treated 890 people during the flu season. Three hundred fifty of the patients had flu symptoms, 530 had cold symptoms, and 140 had both cold and flu symptoms. How many of the patients treated by the health agency had no cold or flu symptoms?



4. **HOLIDAY DECORATIONS** During the holiday season, 13 homes on a certain street displayed lights and 8 displayed lawn ornaments. Five of the homes displayed both lights and lawn ornaments. If there are 32 homes on the street, how many had no decorations at all?



5. **LUNCHTIME** At the local high school, 240 students reported they have eaten the cafeteria's hot lunch, 135 said they have eaten the cold lunch, and 82 said they have eaten both the hot and cold lunch. If there are 418 students in the school, how many bring lunch from home?



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3-5 Extension Activity

Changing Measures of Length

12 inches (in.)	=	1 foot (ft)
3 feet	=	1 yard (yd)
$5\frac{1}{2}$ yards	=	1 rod (rd)
320 rods	=	1 mile (mi)

Fractions and mixed numbers are frequently used with customary measures.

The problems on this page will give you a chance to practice using multiplication of fractions as you change measures of lengths to different equivalent forms.

Use a fraction or a mixed number to complete each statement. Refer to the table above as needed.

1. 12 ft 6 in. = ☐ ft

2. 1 rod = ☐ ft

3. $\frac{5}{8}$ yd = ☐ in.

4. 10 ft = ☐ yd

5. 7 yd 2 ft = ☐ yd

6. 1,540 yd = ☐ mi

7. 1,000 rd = ☐ mi

8. 27 in. = ☐ yd

Use a whole number to complete each statement. Refer to the table above as needed.

9. $10\frac{1}{2}$ ft = 10 ft ☐ in.

10. $12\frac{1}{2}$ yd = ☐ in.

11. 1 mi = ☐ ft

12. 1 mi = ☐ yd

13. $\frac{1}{10}$ mi = ☐ yd

14. $\frac{3}{4}$ ft = ☐ in.

15. 10 rd = ☐ ft

16. $\frac{3}{8}$ mi = ☐ ft

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3-6 Extension Activity

Continued Fractions

The expression at the right is an example of a *continued fraction*. The example shows how to change an improper fraction into a continued fraction.

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{9}}}$$

Example **Write $\frac{72}{17}$ as a continued fraction.**

$$\frac{72}{17} = 4 + \frac{4}{17}$$

$$= 4 + \cfrac{1}{\frac{17}{4}}$$

$$= 4 + \cfrac{1}{4 + \frac{1}{4}}$$

Notice that each fraction must have a numerator of 1 before the process is complete.

Exercises

Change each improper fraction to a continued fraction.

1. $\frac{13}{10}$

2. $\frac{17}{11}$

3. $\frac{25}{13}$

4. $\frac{17}{6}$

Write each continued fraction as an improper fraction.

5. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{2}}}$

6. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{3}}}$

7. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{5}}}$

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3-7 Extension Activity

Fraction Puzzles

In the puzzles below, the sum of the fractions in each row is the same as the sum of the fractions in each column. Use your knowledge of adding and subtracting fractions to find the missing fractions. Hint: Remember to check for like denominators before adding.

$\frac{3}{20}$	$\frac{9}{20}$		
	$\frac{2}{20}$		$\frac{2}{20}$
$\frac{2}{20}$	$\frac{4}{20}$		$\frac{7}{20}$
	$\frac{3}{20}$	$\frac{6}{20}$	

$\frac{9}{15}$		$\frac{3}{15}$	$\frac{2}{15}$
$\frac{4}{15}$		$\frac{0}{15}$	
$\frac{2}{15}$		$\frac{7}{15}$	
$\frac{1}{15}$	$\frac{2}{15}$		$\frac{7}{15}$

$\frac{6}{25}$	$\frac{3}{25}$	$\frac{11}{25}$	
			$\frac{2}{25}$
$\frac{2}{25}$			$\frac{6}{25}$
$\frac{3}{25}$	$\frac{4}{25}$	$\frac{1}{25}$	$\frac{12}{25}$

$\frac{8}{16}$	$\frac{1}{16}$		$\frac{1}{8}$
	$\frac{7}{16}$		$\frac{1}{8}$
$\frac{3}{16}$			$\frac{1}{8}$
$\frac{0}{8}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$

CHALLENGE Create your own fraction puzzle using a box of 5 rows and 5 columns.

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3-8 Extension Activity

Unit Fractions

A **unit fraction** is a fraction with a numerator of 1 and a denominator that is any counting number greater than 1.

unit fractions: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{10}$

A curious fact about unit fractions is that each one can be expressed as a sum of two distinct unit fractions. (*Distinct* means that the two new fractions are different from one another.)

$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$ $\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$ $\frac{1}{10} = \frac{1}{11} + \frac{1}{110}$

Did you know?

The *Rhind Papyrus* indicates that fractions were used in ancient Egypt nearly 4,000 years ago. If a fraction was not a unit fraction, the Egyptians wrote it as a sum of unit fractions. The only exception to this rule seems to be the fraction $\frac{2}{3}$.

1. The three sums shown above follow a pattern. What is it?

2. Use the pattern you described in Exercise 1. Express each unit fraction as a sum of two distinct unit fractions.

a. $\frac{1}{4}$ **b.** $\frac{1}{5}$ **c.** $\frac{1}{12}$ **d.** $\frac{1}{100}$

Does it surprise you to know that other fractions, such as $\frac{5}{6}$, can be expressed as sums of unit fractions? One way to do this is by using equivalent fractions. Here's how.

$\frac{5}{6} = \frac{10}{12}$ → $\frac{10}{12} = \frac{6}{12} + \frac{4}{12} = \frac{1}{2} + \frac{1}{3}$ → $\frac{5}{6} = \frac{1}{2} + \frac{1}{3}$

3. Express each fraction as a sum of two distinct unit fractions.

a. $\frac{2}{3}$ **b.** $\frac{4}{15}$ **c.** $\frac{5}{9}$ **d.** $\frac{2}{5}$

4. Express $\frac{4}{5}$ as the sum of *three* distinct unit fractions.

5. **CHALLENGE** Show two different ways to express $\frac{1}{2}$ as the sum of three distinct unit fractions.

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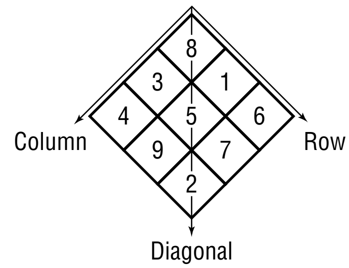
3-9 Extension Activity

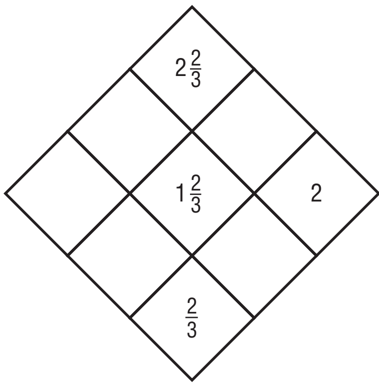
Magic Squares

A **magic square** is an arrangement of numbers such that the rows, columns, and diagonals all have the same sum. In this magic square, the *magic sum* is 15.

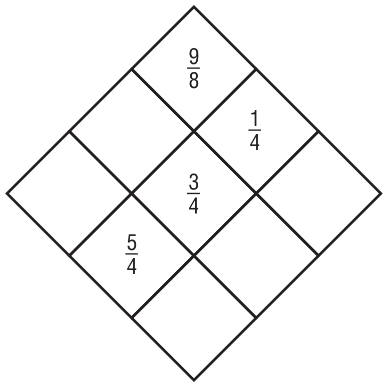


Find the magic sum for each square in Exercises 1–5. Then fill in the empty cells.

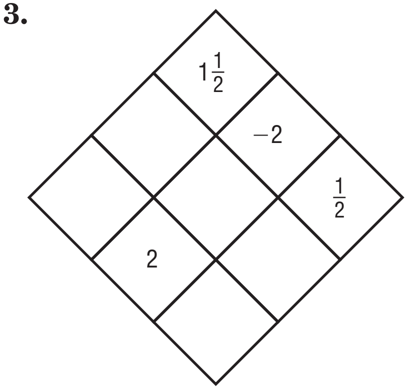
1.



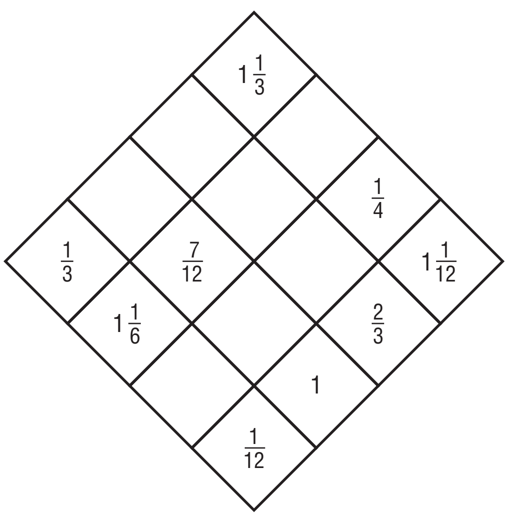
2.



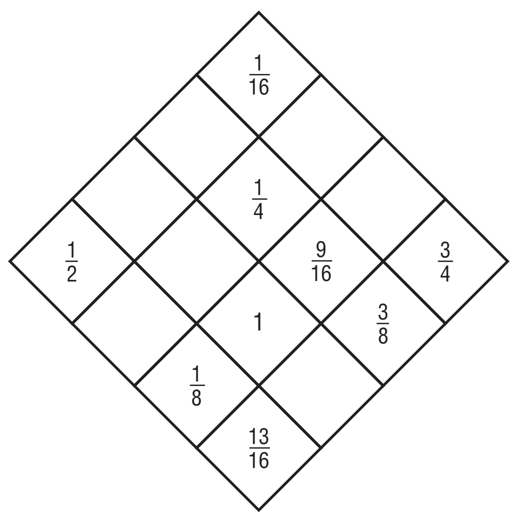
3.



4.



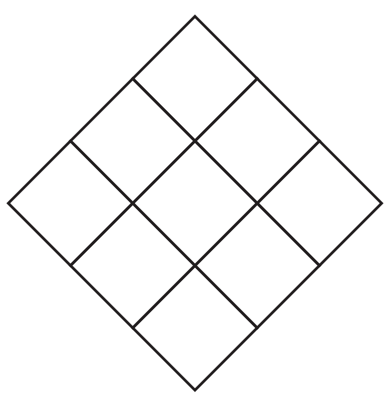
5.



6. Arrange these numbers to make a magic square.

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$

$\frac{1}{6}$ $\frac{1}{12}$ $\frac{5}{12}$ $\frac{7}{12}$



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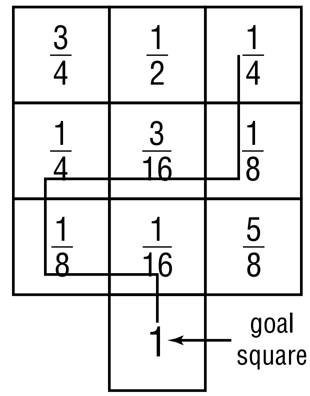
3-10 Extension Activity

Trail Blazers

Each puzzle on this page is called a **trail blazer**. To solve it, you must find a trail that begins at any one of the small squares and ends at the goal square, following these rules.



1. The sum of all the fractions on the trail must equal the number in the goal square.
2. The trail can only go horizontally or vertically.
3. The trail cannot retrace or cross itself.

When you are solving a trail blazer, try to eliminate possibilities. For instance, in the puzzle at the right, you know that you cannot include $\frac{3}{4}$ using $\frac{3}{4} + \frac{1}{4} = 1$ because you can't reach the goal box. $\frac{3}{4} + \frac{1}{2} = 1\frac{1}{4}$ will not work either as the goal for the entire trail is only 1.

1.

$\frac{1}{12}$	$\frac{2}{3}$	$\frac{5}{6}$
$\frac{1}{2}$	$\frac{5}{12}$	$\frac{1}{6}$
$\frac{7}{12}$	$\frac{1}{3}$	$\frac{1}{12}$
	1	

2.

$\frac{4}{5}$	$\frac{1}{5}$	$\frac{3}{5}$
$\frac{7}{10}$	$\frac{1}{2}$	$\frac{2}{5}$
$\frac{9}{10}$	$\frac{3}{10}$	$\frac{1}{10}$
	$2\frac{1}{2}$	

3.

$\frac{7}{8}$	$\frac{2}{3}$	$\frac{3}{8}$
$\frac{5}{12}$	$\frac{3}{4}$	$\frac{5}{8}$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{3}$
	$3\frac{1}{2}$	

4.

$\frac{3}{8}$	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
$\frac{1}{8}$	$\frac{7}{8}$	$\frac{1}{4}$	$\frac{7}{8}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{1}{8}$	$\frac{1}{2}$
$\frac{7}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$
$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$
		6		

5.

$\frac{1}{20}$	$\frac{17}{20}$	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{1}{10}$
$\frac{4}{5}$	$\frac{1}{2}$	$\frac{11}{20}$	$\frac{3}{20}$	$\frac{1}{5}$
$\frac{7}{10}$	$\frac{1}{20}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{19}{20}$
$\frac{1}{5}$	$\frac{13}{20}$	$\frac{3}{10}$	$\frac{3}{5}$	$\frac{1}{2}$
$\frac{9}{20}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{7}{20}$	$\frac{1}{4}$
		4		

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4-1 Extension Activity

Writing Repeating Decimals as Fractions

All fractions can be written as decimals that either terminate or repeat. You have learned how to use a power of 10 to write a terminating decimal as a fraction. Below, you will study a strategy to write a repeating decimal as a fraction.

For Exercises 1–4, write each fraction as a decimal. Use bar notation if the decimal is a repeating decimal.

1. $\frac{1}{9}$

2. $\frac{2}{9}$

3. $\frac{3}{9}$

4. $\frac{5}{9}$

5. Describe the relationship between the numerator of each fraction and its decimal equivalent.

For Exercises 6–9, write each fraction as a decimal. Use bar notation if the decimal is a repeating decimal.

6. $\frac{7}{99}$

7. $\frac{24}{99}$

8. $\frac{37}{99}$

9. $\frac{82}{99}$

10. Describe the relationship between the numerator of each fraction and its decimal equivalent.

11. Use the relationship from Exercise 10 to write the decimal $0.\overline{52}$ as a fraction. Check your work using long division.

12. Using your observations from Exercises 5 and 11, make a prediction about the decimal equivalent of $\frac{127}{999}$. Check to see if your prediction was correct by using long division.

13. How are the decimal equivalents of $\frac{4}{9}$, $\frac{4}{99}$ and $\frac{4}{999}$ different? Explain.

For Exercises 14–19, write each decimal as a fraction. Check your answers with a calculator.

14. 0.47474747…

15. $0.\overline{22}$

16. $0.\overline{530}$

17. 0.010010010…

18. $0.\overline{3266}$

19. $0.\overline{00328}$

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4-2 Extension Activity

Currency

The currency used in the United States is the US dollar. Each dollar is divided into 100 cents. Most countries have their own currencies. On January 1, 2002, 12 countries in Europe converted to a common monetary unit that is called the *euro*.

The symbol, €, is used to indicate the euro.

The exchange rate between dollars and euros changes every day.

$1.00 is worth about 0.85€.

EXERCISES Add or subtract to solve each problem.

1. Henry bought a pair of shoes for €34.75 and a pair of pants for €21.49. How much money did he spend?

2. Louis receives €10.50 a week for doing his chores. His sister is younger and has fewer chores. She receives €5.25. How much money do Louis and his sister receive together in one week?

3. A gallon of Brand A of vanilla ice cream costs €5.49. A gallon of Brand B vanilla ice cream costs €4.87. How much money will Luca save if he buys Brand A instead of Brand B?

4. Michael passed up a pair of jeans that cost €29.50 and decided to buy a pair that were only €15.86. How much money did he save by buying the less expensive jeans?

5. Jesse's favorite magazine costs €1.75 at the store. If he buys a subscription, each issue is only 0.37€. How much money will Jesse save on each issue if he buys a subscription?

6. Layla wants to buy a CD for €11.99 and a book for €6.29. She has €15.00. How much more money does she need to buy the CD and book?

7. **CHALLENGE** Lynne's lunch came to €4.00. Her drink was €1.50. How much did she spend total? What would be the equivalent dollar amount?

8. **CHALLENGE** At the grocery store, Jaden purchased a box of cereal for $3.55 and a gallon of milk for $2.89. He gave the cashier $10.00. How much change did he receive? What would be the equivalent euro amount?

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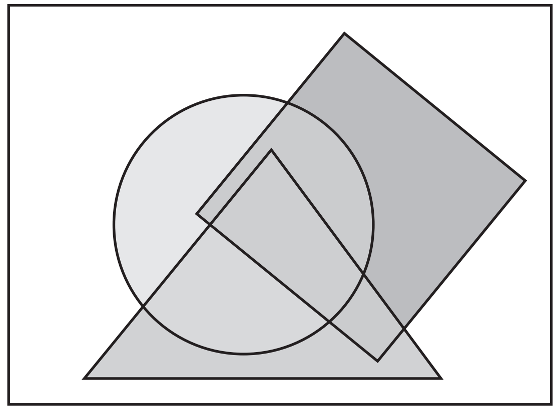
4-3 Extension Activity

A Logic Puzzle

Here is a puzzle that will help you brush up on your logical thinking skills.

The product 3.3 · 8.1 is in both the circle and the triangle, but not in the square. Place the product in the diagram at the right.



```
   8.1
×  3.3
  2 4 3
2 4 3
2 6.7 3
```

Write 26.73 in the correct region of the diagram.

Use the given information to place the product in the diagram above.

1. The product 14.19 · 1.3 is in both the triangle and the square, but not in the circle.

2. The product 0.08 · 2.7 is in the triangle, but not in the circle or the square.

3. The product 1.24 · 0.16 is not in the circle, the square, or the triangle.

4. The product 2.2 · 0.815 is in both the square and the circle, but not in the triangle.

5. The product 0.02 · 0.03 is in the circle, but not the triangle or the square.

6. The product 21.7 · 0.95 is in the circle, the square, and the triangle.

7. The product 2.5 · 12.8 is in the square, but not the circle or triangle.

8. If you did all the calculations correctly, the sum of all the numbers in the diagram should be a "nice" number. What is the sum?

4-4 Extension Activity

The Better Buy

Stores often offer items for sale as individual items or in a bundle. For example, a office supply store may sell a single pencil for \$0.18 and a pack of 10 pencils for \$1.70. To know which is a better deal, you can find the unit cost. The unit cost is the cost for one item in a bunch. To find the unit cost, divide the total cost by the number of items. The unit cost of the 10-pencil pack is \$0.17. The cost for one pencil is less in the 10-pencil pack than for a single pencil.

Look at these products. Which is the better buy?

1. Raphael wants to buy a tree for his yard. Which tree is the better buy: a 6-foot 5-inch tree for \$69.96 or a 7-foot tree for \$84.96?

2. Which memory card is the better buy: a 512 MB memory card for \$44.99 or a 1 GB memory card for \$89.99? (Note: 1 GB = 1,000 MB)

3. Which memory chip is the better buy: a 512 MB for \$31.99 or a 256 MB for \$49.99?

4. Find the unit cost per ounce of cologne. Which bottle is the better buy?
2.5 oz. for \$4.49 or 4.25 oz. for \$5.49

5. Order these ladders from least to most expensive per foot of height:
Type 1 is 16 ft for \$149 Type 2 is 24 ft for \$219 Type 3 is 28 ft for \$257

6. Order these \$6.99 packages of photo prints from least to most expensive per square inch.
a) one 8" × 10" b) two 5" × 7" c) three 4" × 6" d) four $3\frac{1}{2}$" × 5" e) eight 2" × 3"

7. What is the cost per fluid ounce of water if a 24-pack of 16.8-fluid-ounce bottles costs \$4.99?

8. A 4-roll pack of 36-inch wide wrapping paper costs \$6.00 and has a total area of 120 square feet. What is the length of a roll of paper? What is the unit cost of 1 roll of wrapping paper? What is the cost per square foot?

4–5 Extension Activity

Writing Equations to Describe Sequences

A **sequence** can be extended by finding the pattern, describing it, and then applying the description to produce successive **terms**. To describe the pattern in words, we could write, "Add four to the previous term to find the next term." Determine the pattern rule for the sequence below. What are the next three terms?

Position	1	2	3	4	5	6	7	8
Term	**4**	**8**	**12**	**16**	**20**			

Pattern +4 +4 +4 +4

Describe the pattern in words and write the next three terms in each of the following sequences.

A. 2, 4, 6, 8, 10, 12, ___, ___, ___

B. −3, −6, −9, −12, −15, −18, ___, ___, ___

C. 3, 5, 7, 9, 11, 13, ___, ___, ___

D. 3, 9, 27, 81, 243, ___, ___, ___

E. 6,230, 623, 62.3, 6.23, 0.623, ___, ___, ___

F. 1, 4, 9, 16, 25, 36, ___, ___, ___

The **rule** of a sequence can be generalized into an equation so that it is possible to find the 10th term, 100th term, or nth term without writing out of the terms in between. The rule of the sequence shows the relationship between a term and its position number.

Look again at the beginning example. The rule is *multiply the position number by four*. If we call the position numbers n, the algebraic expression for the rule is $4n$. For each term $t = 4n$.

Write an equation rule for each of the sequences in exercises 1–6. Be careful that your rule gives the correct first term.

1. Sequence A

2. Sequence B

3. Sequence C

4. Sequence D

5. Sequence E

6. Sequence F

Write an equation rule for each of the sequences below. Then use the equation to find the 100th term.

7. 4, 7, 10, 13, 16, …

8. 2, 5, 10, 17, 26, …

9. 0, 2, 4, 6, 8, …

10. 0.75, 1.5, 2.25, 3, 3.75, …

11. 11, 12, 13, 14, 15, …

12. Write your own sequence rule and find the first 5 terms.

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4-6 Extension Activity

Equations with Like Terms

Some equations contain two or more expressions that are called *like terms*. For example, in the equation $3a + 2a + 4 = 14$, the expressions $3a$ and $2a$ are like terms. When you see like terms, you can combine them into one expression.

$3a + 2a = 5a$

When you solve an equation containing like terms, combine them first before continuing to solve the equation. To solve $3a + 2a + 4 = 14$, proceed as follows.

$(3a + 2a) + 4 = 14$ *Combine like terms.*

$$5a + 4 = 14$$
$$5a + 4 - 4 = 14 - 4$$
$$5a = 10$$
$$\frac{5a}{5} = \frac{10}{5}$$
$$a = 2$$

Solve each equation. Then locate the solution on the number line below. Place the letter corresponding to the answer on the line at the right of the exercise.

1. $3x + 4x + 3 = -39$ _____

2. $-3x - 2 + 5x = 12$ _____

3. $-5 - 4x + 7x = 1$ _____

4. $-\frac{1}{2}x + 6x - 2 = 20$ _____

5. $-2.4x + 1.2 + 1.2x = 4.8$ _____

6. $\frac{1}{3}(6 - x) = -1$ _____

7. $1 = -\frac{1}{4}x + 5 + \frac{3}{4}x$ _____

8. $7x + (-2x) + x = 42$ _____

9. $\frac{2}{5}(5x + 5x) = -20$ _____

−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8	9
W	I	H	F	L	J	O	P	N	V	G	C	D	T	M	K	A	B	R

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5-1 Extension Activity

A-Mazing Exponents

Solve the following puzzle by finding the correct path through the boxes. The solution is a famous quote from United States history.

Starting with Box 1, draw an arrow to another box connected to Box 1 with the expression of the least value. The arrow cannot go to a box that has already been used. The first arrow has been drawn to get you started.

When you have finished drawing your path through the boxes, write the box numbers on the lines below. Put the numbers in the order in which they are connected. Then, use the chart at the right to convert each box number to a letter.

1 5^3	2 13^2	3 $4^3 + 3^4$	4 17^2	5 $4^5 - 9^3$
6 6^3	7 2^7	8 $2^4 \cdot 3^2$	9 $2^5 \cdot 3^2$	10 18^2
11 $4^4 + 16^2$	12 3^5	13 4^4	14 7^3	15 $5^3 + 3^5$
16 $3^6 - 6^3$	17 $8^3 - 1^8$	18 $16^2 + 6^3$	19 19^2	20 $2^8 + 11^2$
21 $2^9 + 9^2$	22 23^2	23 $3^6 - 3^5$	24 21^2	25 $2^3 \cdot 7^2$

1	G
2	M
3	E
4	E
5	R
6	E
7	I
8	V
9	B
10	T
11	D
12	L
13	I
14	Y
15	R
16	E
17	E
18	E
19	O
20	G
21	T
22	A
23	M
24	V
25	I

Box Number	1	7											
Letter	G	I											

Box Number													
Letter													H

5-2 Extension Activity

Proving Definitions of Exponents

Recall the rules for multiplying and dividing powers with the same base. Use these rules, along with other properties you have learned, to justify each definition. Abbreviations for some properties you may wish to use are listed below.

Associative Property of Multiplication (APM)
Multiplicative Identity Property (MIP)
Inverse Property of Multiplication (IPM)
Additive Identity Property (AIP)
Inverse Property of Addition (IPA)

Write the reason for each statement.

1. Prove: $a^0 = 1$

Statement	**Reason**
Let m be an integer, and let a be any nonzero number.	**a.** Given
$a^m \cdot a^0 = a^{m+0}$	**b.** ____________
$a^m \cdot a^0 = a^m$	**c.** ____________
$\frac{1}{a^m} \cdot (a^m \cdot a^0) = \frac{1}{a^m} \cdot a^m$	**d.** ____________
$\left(\frac{1}{a^m} \cdot a^m\right) \cdot a^0 = \frac{1}{a^m} \cdot a^m$	**e.** ____________
$1 \cdot a^0 = 1$	**f.** ____________
$a^0 = 1$	**g.** ____________

2. Prove: $a^{-n} = \frac{1}{a^n}$

Statement	**Reason**
Let n be an integer, and let a be any nonzero number.	**a.** Given
$a^{-n} \cdot a^n = a^{-n+n}$	**b.** ____________
$a^{-n} \cdot a^n = a^0$	**c.** ____________
$a^{-n} \cdot a^n = 1$	**d.** ____________
$(a^{-n} \cdot a^n) \cdot \frac{1}{a^n} = 1 \cdot \frac{1}{a^n}$	**e.** ____________
$a^{-n} \cdot \left(a^n \cdot \frac{1}{a^n}\right) = 1 \cdot \frac{1}{a^n}$	**f.** ____________
$a^{-n} \cdot 1 = 1 \cdot \frac{1}{a^n}$	**g.** ____________
$a^{-n} = \frac{1}{a^n}$	**h.** ____________

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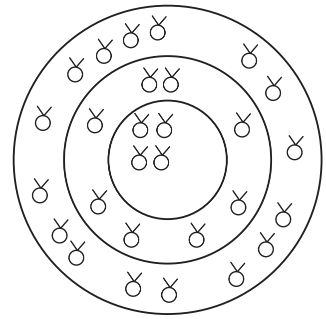
5-3 Extension Activity

Using Patterns

Use what you know about solving a simpler problem to look for a pattern. Then use the pattern to solve each problem.

1. **YARN** A knitting shop is having a huge yarn sale. One skein sells for $1.00, 2 skeins sell for $1.50, and 3 skeins sell for $2.00. If this pattern continues, how many skeins of yarn can you buy for $5.00?

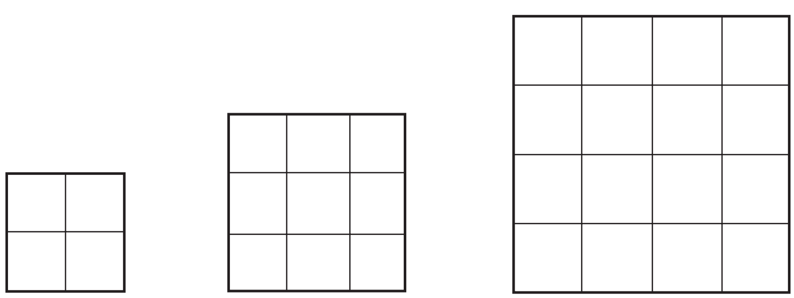
2. **BIOLOGY** Biologists place sensors in 8 concentric circles to track the movement of grizzly bears throughout Yellowstone National Park. Four sensors are placed in the inner circle. Eight sensors are placed in the next circle. Sixteen sensors are placed in the third circle, and so on. If the pattern continues, how many sensors are needed in all?



3. **HONOR STUDENTS** A local high school displays pictures of the honor students from each school year on the office wall. The top row has 9 pictures displayed. The next 3 rows have 7, 10, and 8 pictures displayed. The pattern continues to the bottom row, which has 14 pictures in it. How many rows of pictures are there on the office wall?

4. **CHEERLEADING** The football cheerleaders will arrange themselves in rows to form a pattern on the football field at halftime. In the first five rows there are 12, 10, 11, 9, and 10 girls in each row. They will form a total of twelve rows. If the pattern continues, how many girls will be in the back row?

5. **GEOMETRY** Find the perimeters of the next two figures in the pattern. The length of each side of each small square is 3 feet.



6. **HOT TUBS** A hot tub holds 630 gallons of water when it is full. A hose fills the tub at a rate of 6 gallons every five minutes. How long will it take to fill the hot tub?

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5-4 Extension Activity

Roots

The symbol $\sqrt{}$ indicates a square root. By placing a number in the upper left, the symbol can be changed to indicate higher roots.

Examples

$\sqrt[3]{8} = 2$ because $2^3 = 8$

$\sqrt[4]{81} = 3$ because $3^4 = 81$

$\sqrt[5]{100{,}000} = 10$ because $10^5 = 100{,}000$

Exercises

Find each of the following.

1. $\sqrt[3]{125}$

2. $\sqrt[4]{16}$

3. $\sqrt[8]{1}$

4. $\sqrt[3]{27}$

5. $\sqrt[5]{32}$

6. $\sqrt[3]{64}$

7. $\sqrt[3]{1000}$

8. $\sqrt[3]{216}$

9. $\sqrt[6]{1{,}000{,}000}$

10. $\sqrt[3]{1{,}000{,}000}$

11. $\sqrt[4]{256}$

12. $\sqrt[3]{729}$

13. $\sqrt[6]{64}$

14. $\sqrt[4]{625}$

15. $\sqrt[5]{243}$

5-5 Extension Activity

Exponents

Numbers can be expressed in several ways. Some numbers are expressed as sums. Some numbers are expressed as products of factors, while other numbers are expressed as powers.

Two ways to express 27 are $3 \cdot 3 \cdot 3$ and 3^3.

The number 1 million can be expressed in the following ways.

1,000,000	$1{,}000 \cdot 1{,}000$	$100 \cdot 100 \cdot 100$	$10^2 \cdot 10^2 \cdot 10^2$
$1{,}000{,}000^1$	$1{,}000^2$	100^3	10^6

Write names for each number below using the given exponents.

1. 16; exponents: 2 and 4
2. 81; exponents: 2 and 4
3. 64; exponents: 2 and 6
4. 256; exponents: 2 and 8
5. 625; exponents: 2 and 4
6. 729; exponents: 2 and 6
7. 2,401; exponents: 2 and 4
8. 4,096; exponents: 2 and 12
9. 6,561; exponents: 2 and 8
10. 390,625; exponents: 2 and 8

Numbers that can be named as powers with like bases can be multiplied by adding the exponents.

$$8 \cdot 8 = 2^3 \cdot 2^3$$
$$= 2^{3+3}$$
$$= 2^6$$

Write the product of each pair of factors in exponential form.

11. $9 \cdot 9$
12. $4 \cdot 4$
13. $16 \cdot 8$
14. $125 \cdot 25$
15. $27 \cdot 9$
16. $81 \cdot 27$
17. $49 \cdot 49$
18. $121 \cdot 121$

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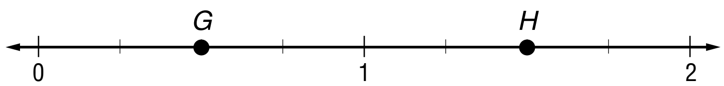
5-6 Extension Activity

A Famous Line-Up

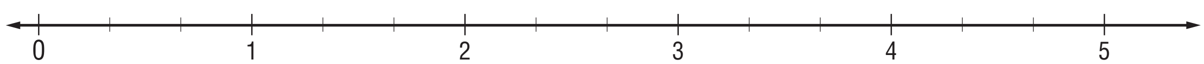
A number line can be used to graph a mixed number or an improper fraction.



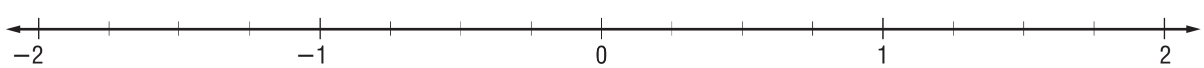
The number line above shows the graph of two points. Point G is at $\frac{1}{2}$ and point H is at $\frac{3}{2}$.

Graph each set of points on the number line. When you are finished, the letters will spell the last names of some famous people.

1. point R at $\frac{10}{3}$, point A at $1\frac{1}{3}$, point N at $4\frac{2}{3}$, point G at $\frac{6}{3}$, point G at $\frac{1}{3}$, point I at $\frac{13}{3}$, and point A at $2\frac{2}{3}$



2. point R at 1, point E at $-\frac{3}{4}$, point S at -2, point D at $\frac{3}{2}$, point A at $\frac{1}{2}$, point H at $-\frac{5}{4}$, and point P at $-\frac{1}{4}$



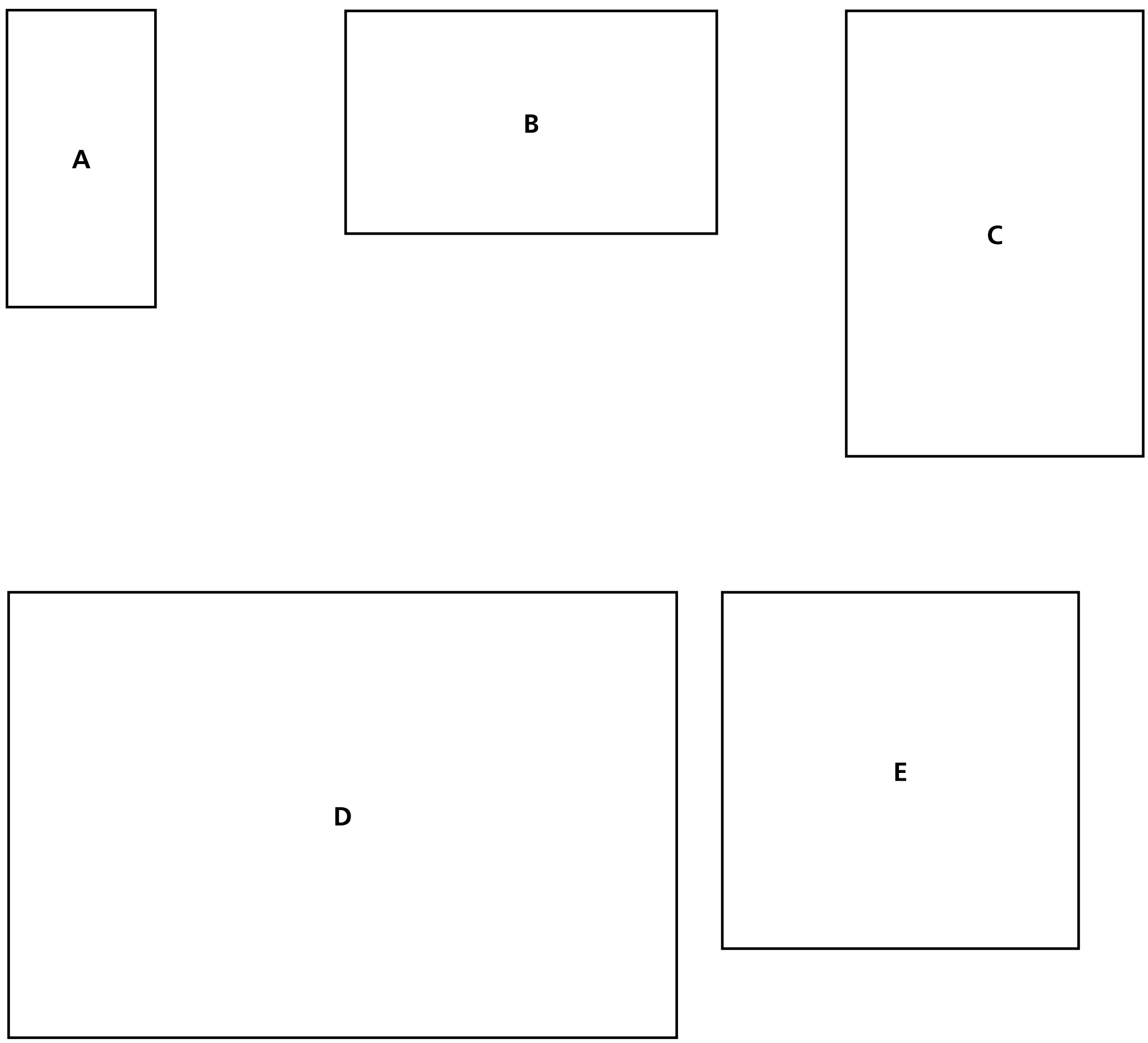
3. point G at $-2\frac{1}{6}$, point M at $-\frac{1}{6}$, point S at $-\frac{5}{6}$, point S at $-1\frac{1}{3}$, point R at $-\frac{11}{6}$, point O at $-\frac{1}{3}$, and point I at $-\frac{5}{3}$

4. Why are these three people famous?

6-1 Extension Activity

Ratios and Rectangles

1. Use a centimeter ruler to measure the width and the length of each rectangle. Then express the ratio of the width to the length as a fraction in simplest form.



2. Similar figures have the same shape, but not necessarily the same size. Two rectangles are similar if the ratio of the width to the length is the same for each. Which rectangles in Exercise 1 are similar?

3. For centuries artists and architects have used a shape called the **golden rectangle** because people seem to find it most pleasant to look at. In a golden rectangle, the ratio of the width to the length is a little less than $\frac{5}{8}$. Which rectangle in Exercise 1 is most nearly a golden rectangle?

6-2 Extension Activity

Shaded Regions

The fractions or percents listed below each represent one of the shaded regions.

Match each fraction or percent with the shaded region it represents.

1. $\frac{1}{2}$

2. $\frac{25}{64}$

3. $\frac{11}{16}$

4. 25%

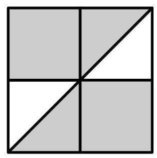
5. $\frac{3}{4}$

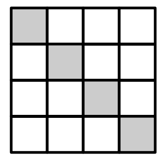
6. $62\frac{1}{2}\%$

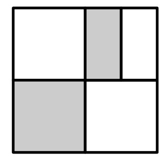
7. $\frac{29}{64}$

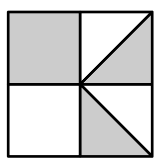
8. 37.5%

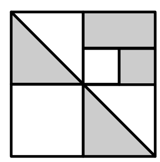
9. $\frac{7}{16}$

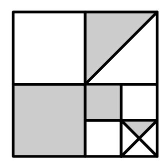
a. 

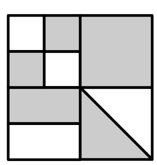
b. 

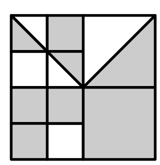
c. 

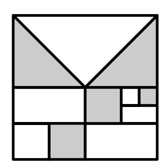
d. 

e. 

f. 

g. 

h. 

i. 

6-3 Extension Activity

Cross Products Proof

Recall the Cross Products Property: If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. Use the statements below to justify this property.

Write the reason for each statement.

1. Prove: $ad = bc$

Statement	Reason
$\frac{a}{b} = \frac{c}{d}$	**a.** Given
$bd \cdot \frac{a}{b} = bd \cdot \frac{c}{d}$	**b.** ______________
$bd \cdot \left(a \cdot \frac{1}{b}\right) = bd \cdot \left(c \cdot \frac{1}{d}\right)$	**c.** Rewrite division as multiplication.
$\left(b \cdot \frac{1}{b}\right)ad = \left(d \cdot \frac{1}{d}\right)cb$	**d.** ______________
$1ad = 1cb$	**e.** ______________
$ad = bc$	**f.** ______________

Solve each proportion problem.

2. AGE The ratio of Brandi's age to Jason's age is 4:5. In eight years the ratio will be 6:7. How old are Brandi and Jason now?

3. AGE The ratio of Drew's age to Stacey's age is 3:4. Four years ago the ratio was 2:3. How old were Drew and Stacey four years ago?

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6-4 Extension Activity

Made in the Shade

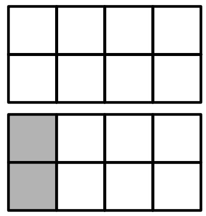
To shade 25% of the figure below, ask yourself how many of the eight squares need to be shaded. Then use the percent proportion to find the answer.

$$\frac{x}{8} = \frac{25}{100}$$

$$100x = 8 \times 25$$

$$\frac{100x}{100} = \frac{200}{100}$$

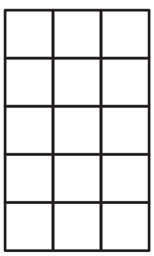
$$x = 2$$



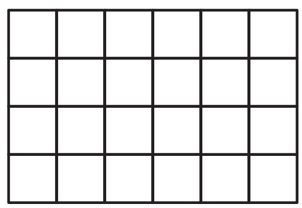
If you shade two squares, you have shaded 25% of the figure.

Shade the indicated percent of each diagram.

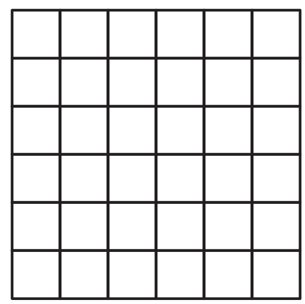
1. Shade 40%.



2. Shade 37.5%.

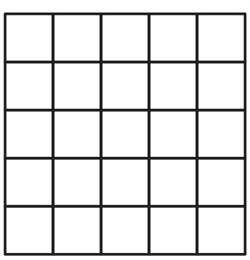


3. Shade $16\frac{2}{3}$%.

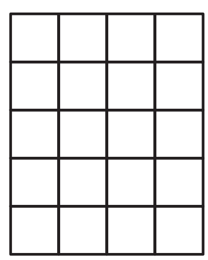


Shade the indicated percent of each diagram. You will need to divide the squares in each diagram into smaller squares.

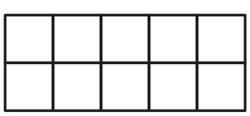
4. Shade 30%.



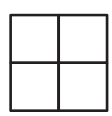
5. Shade 62.5%.



6. Shade 27.5%.



7. Shade 28.125%.



6-5 Extension Activity

Compound Interest

Interest may be paid, or compounded, annually (each year), semiannually (twice per year), quarterly (four times per year), monthly (once per month), or daily.

Example FINANCE **George had $100 in an account for $1\frac{1}{2}$ years that paid 8% interest compounded semiannually. What was the total amount in his account at the end of $1\frac{1}{2}$ years?**

At the end of $\frac{1}{2}$ year: Interest: $\$100 \times 0.08 \times \frac{1}{2} = \$4.00$
New Principal: $\$100 + \$4 = \$104$

At the end of 1 year: Interest: $\$104 \times 0.08 \times \frac{1}{2} = \$4.16$
New Principal: $\$104 + \$4.16 = \$108.16$

At the end of $1\frac{1}{2}$ years: Interest: $\$108.16 \times 0.08 \times \frac{1}{2} = \$4.33$
New Principal: $\$108.16 + \$4.33 = \$112.49$

Exercises

Find the total amount for each of the following.

	Principal	Rate	Time	Compounded	Total Amount
1.	$200	6%	$1\frac{1}{2}$ years	semiannually	
2.	$300	5%	2 years	semiannually	
3.	$100	6%	1 year	quarterly	
4.	$500	8%	$\frac{3}{4}$ year	quarterly	
5.	$500	10%	4 months	monthly	
6.	$800	8%	$\frac{1}{4}$ year	monthly	
7.	$1000	8%	4 years	annually	
8.	$700	6%	$1\frac{1}{2}$ years	semiannually	

6-6 Extension Activity

Finding a Formula and Inverse Variation

If you know the general form of direct variation equations and the value of the constant of variation, you can write an equation that describes the data points of a direct variation. Look at the table at the right which shows a relationship where c varies as r.

r	c
1	6.28
2	12.56
3	18.84
4	25.12
5	31.4

1. What is the constant of variation?

2. What familiar relationship do you recognize in the direct variation?

3. Write an equation for finding the circumference of a circle with radius, r.

4. If r is doubled, what happens to the value of c?

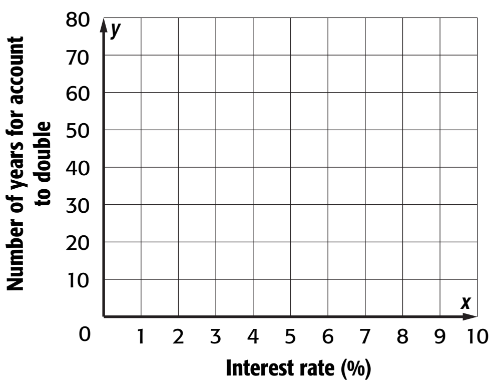
Inverse Variation For the variables x and y, with any constant k, y is said to vary inversely (or indirectly) as x when $y = \frac{k}{x}$.

For Exercises 4–6, refer to the following information.

INVESTMENTS Economists use inverse variation to approximate how fast the balance of an account will double when it is invested at a given compound interest rate. The number of years y it takes for an investment to double varies inversely as the annual interest rate r, expressed as a percent (not a decimal value).

5. If you invest \$1000 at a 6% compound interest rate, it will take 12 years to double your money. Find the constant of variation, k, and write the equation for the variation.

6. Find and graph at least six data points within the domain $0 < r \le 10$. What do you notice about the shape of the graph?



7. If r is doubled, what happens to the value of y?

8. Why do you think investors often refer to this relationship as “The rule of 72?”

6-7 Extension Activity

Business Planning

In order to have a successful business, the manager must plan ahead and decide how certain actions will affect the business. The first step is to predict the financial impact of business decisions. Julie has decided that she wants to start a brownie business to make extra money over the summer. Before she can ask her parents for money to start her business, she needs to have some information about how many batches of brownies she can make in a day and for how much she must sell the brownies to make a profit.

1. Julie can bake 3 batches of brownies in 2 hours. Her goal is to bake 12 batches of brownies each day. Use the table to find how many hours Julie will need to bake to reach her goal.

Batches of Brownies	3			12
Hours	2			

2. Each batch of brownies will be sold for $2.00. How much money will Julie make if she sells 6 batches of brownies?

Batches of Brownies	1				6
Cost	$2				

3. If Julie works for 10 hours a day, how many batches of brownies can she bake?

Batches of Brownies	3	
Hours	2	10

4. Julie hires a friend to help. Together, they can bake 24 batches of brownies in 8 hours. How long does it take for the two of them to bake 6 batches of brownies?

Batches of Brownies	6			24
Hours				8

5. If Julie and her friend can bake 24 batches of brownies in 8 hours, and they both work 40 hours in one week, how many batches of brownies can they bake that week? If Julie still charges $2.00 a batch, how much money will they make that week?

Hours	8				40
Batches of Brownies	24				

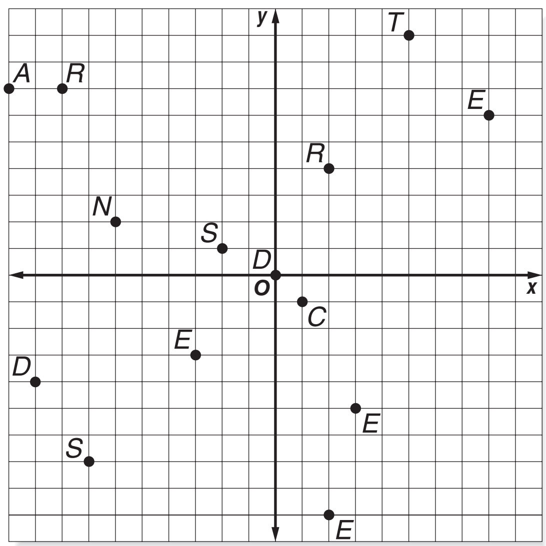
Batches of Brownies	1	
Cost	$2	

7-1 Extension Activity

The Cartesian Plane

Another name for the coordinate plane is the **Cartesian plane**. Its name comes from a French mathematician and philosopher who lived in the 1600s. He invented the coordinate plane. Although it is likely not true, a story is told that this mathematician first came up with the idea of the coordinate plane while lying in bed looking at the ceiling. His ceiling was made of tiles. As he watched a fly crawling on the ceiling, he realized he could describe the fly's location using the tiles on the ceiling. From that, he created the coordinate plane and a system by which to describe locations on the coordinate plane.

Identify the letter that corresponds to the ordered pairs listed below. The letters spell the name of the Frenchman who invented the coordinate plane.



First Name

___ ___ ___ ___

(2, 4) (3, −5) (−6, 2) (−3, −3)

Last Name

___ ___ ___ ___ ___ ___ ___ ___ ___

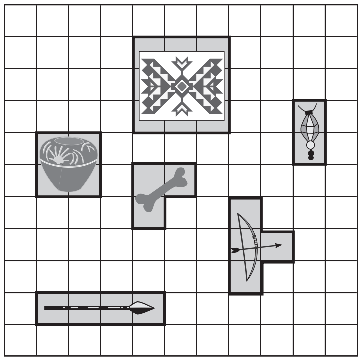
(−9, −4) (8, 6) (−7, −7) (1, −1) (−10, 7) (−8, 7) (5, 9) (2, −9) (−2, 1)

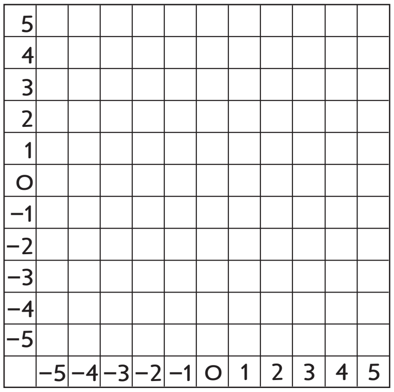
7-2 Extension Activity

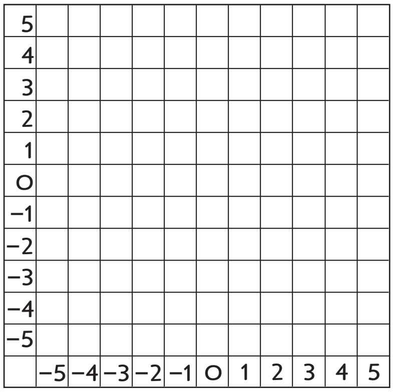
Relic Hunter

The game of Relic Hunter is based on methods used to record the precise locations of artifacts discovered at archaeological digs. Archaeologists use string to position a grid over a dig site. An artifact's location is named by the row and column in the grid.

Relic Hunter is played with two players who each secretly place six artifacts on one of the coordinate grids below. Artifacts may not overlap. Each player should not be able to see where the other player's artifacts are hidden. A player must look for the artifacts by guessing an ordered pair. The other player then finds that location on the secret grid and tells the first player whether part of the artifact is located in that section and what the artifact is. Each player's empty coordinate grid should be used to mark the locations of guesses and of found artifacts. The winner is the player who first uncovers all of the opponent's artifacts.







For Exercises 1–6, list each ordered pair that could contain the rest of the artifact. Then play the game with a partner. Use one coordinate grid to keep track of the points where you hide your artifacts and another coordinate grid to keep track of the points you have guessed.

1. You uncover parts of the spear at points (−2, 3) and (−2, 4).

2. You uncover part of the animal bone at points (2, 1) and (2, 2).

3. You uncover a part of the amulet at point (−4, −2).

4. You uncover a part of the clay pot at point (0, −1).

5. You uncover a part of the bow at point (5, −5).

6. Part of the amulet is located at point (−5, −2), and there is nothing at point (−2, −4). You uncover a part of the mosaic panel at point (−4, −4). What other points could contain the mosaic panel?

7-3 Extension Activity

Periodic Relationships

You have studied scatter plots that demonstrate positive, negative, or no relationship. A **periodic relationship** is another way that two variables can be related. Periodic relationships contain patterns that repeat over time. For example, average monthly temperatures vary on a year basis. The table at right shows the average daily high temperature for each month in Los Angeles and Boston.

Temperature (°F)		
Month	**Los Angeles**	**Boston**
1	64	36
2	63	39
3	68	41
4	69	57
5	76	68
6	80	80
7	80	78
8	82	81
9	81	73
10	76	65
11	71	56
12	66	46

Source: www.wrh.noaa.gov

1. Draw a scatter plot of the data for each city on the axes below. Use a different symbol for each city (for example, an x for Los Angeles temperatures and an • for Boston temperatures).

2. Describe the trend in the data for the monthly average temperature in Boston.

3. Draw a curved line on the graph that demonstrates the trend in the data.

4. What will happen between month 12 and month 24? Describe what you think will happen for each city, and draw curved lines on the graph above to demonstrate the trends.

Lesson 7–3

7-4 Extension Activity

Comparison Shopping

Rates are useful and meaningful when expressed as a unit rate. For example, which is the better buy—one orange for $0.50 or 8 oranges for $3.49?

To find the unit rate for 8 oranges, divide $3.49 by 8. The result is $0.44 per orange. If a shopper needs to buy at least 8 oranges, then 8 oranges for $3.49 is the better buy.

For each exercise below, rates are given in Column A and Column B. In the blank next to each exercise number, write the letter of the column that contains the better buy.

		Column A	Column B
_______	**1.**	1 apple for $0.19	3 apples for $0.59
_______	**2.**	20 pounds of pet food for $14.99	50 pounds of pet food for $37.99
_______	**3.**	A car that travels 308 miles on 11 gallons of gasoline	A car that travels 406 miles on 14 gallons of gasoline
_______	**4.**	10 floppy discs for $8.99	25 floppy discs for $19.75
_______	**5.**	1-gallon can of paint for $13.99	5-gallon can of paint for $67.45
_______	**6.**	84 ounces of liquid detergent for $10.64	48 ounces of liquid detergent for $6.19
_______	**7.**	5000 square feet of lawn food for $10.99	12,500 square feet of lawn food for $29.99
_______	**8.**	2 compact discs for $26.50	3 compact discs for $40.00
_______	**9.**	8 pencils for $0.99	12 pencils for $1.49
_______	**10.**	1000 sheets of computer paper for $8.95	5000 sheets of computer paper for $41.99

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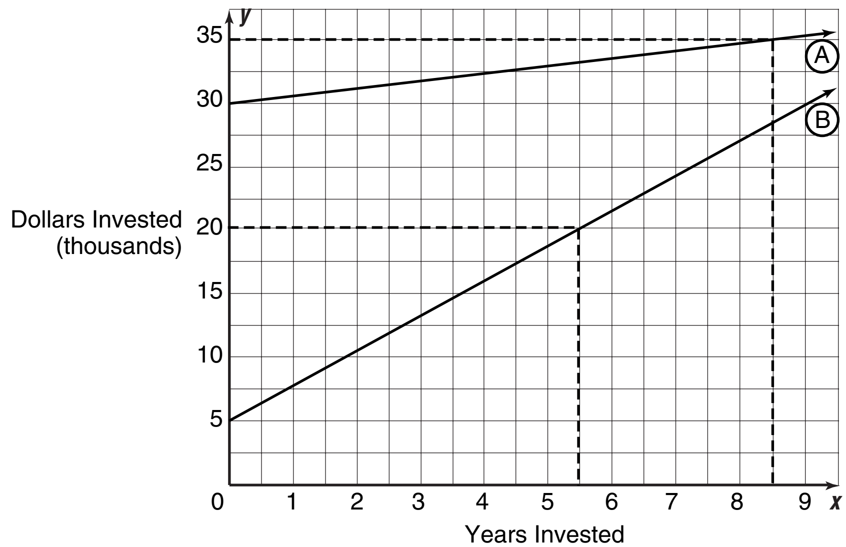
7-5 Extension Activity

Investments

The graph below represents two different investments. Line *A* represents an initial investment of $30,000 at a bank paying passbook-savings interest. Line *B* represents an initial investment of $5000 in a profitable mutual fund with dividends reinvested and capital gains accepted in shares. By deriving the equation, $y = mx + b$, for *A* and *B*, a projection of the future can be made.



Solve.

1. The y-intercept, b, is the initial investment. Find b for each of the following.

a. line *A** **b.** line *B*

2. The slope of the line, m, is the rate of return. Find m for each of the following.

a. line *A* **b.** line *B*

3. What are the equations of each of the following lines?

a. line *A* **b.** line *B*

Answer each of the following, assuming that the growth of each investment continues in the same pattern.

4. What will be the value of the mutual fund after the 11th year?

5. What will be the value of the bank account after the 11th year?

6. When will the mutual fund and the bank account be of equal value?

7. In the long term, which investment has the greater payoff?

Lesson 7–5

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7-6 Extension Activity

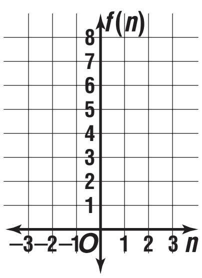
Lesson 7–6

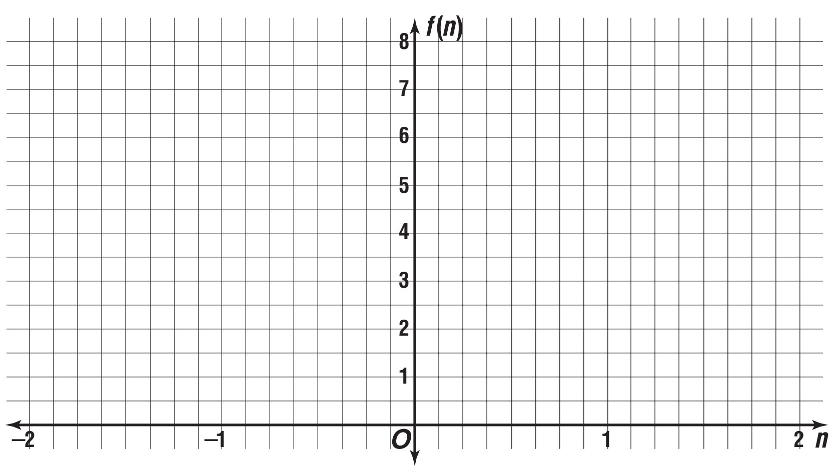
Graphing Functions

Depending on the scale of a graph, the graphed shape of a function can be made to appear very different from one graph to another.

Given the function $f(n) = 2n^2$, find the values of $f(n)$ for each value in the table. Write the ordered pairs, then draw the graph of the function on each grid.

n	$2n^2$	$f(n)$	$(n, f(n))$
-2	$2(-2)^2$	8	$(-2, 8)$
$-\frac{3}{2}$			
-1			
$-\frac{1}{2}$			
0			
$\frac{1}{2}$			
1			
$\frac{3}{2}$			
2			



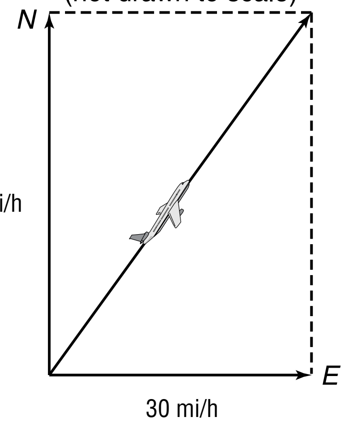


1. Examine each graph of the function $f(n) = 2n^2$. What do you notice about the graphs?

2. Explain why the graphs are different.

7-7 Extension Activity

Pythagoras in the Air



In the diagram at the right, an airplane heads north at 180 mi/h. But, the wind is blowing towards the east at 30 mi/h. So, the airplane is really traveling east of north. The middle arrow in the diagram shows the actual direction of the airplane.

The actual speed of the plane can be found using the Pythagorean Theorem.

$$\sqrt{30^2 + 180^2} = \sqrt{900 + 32{,}400}$$
$$= \sqrt{33{,}300}$$
$$\approx 182.5$$

The plane's actual speed is about 182.5 mi/h.

Find the actual speed of each airplane. Round answers to the nearest tenth. (You might wish to draw a diagram to help you solve the problem.)

1. An airplane travels at 240 mi/h east. A wind is blowing at 20 mi/h toward the south.

2. An airplane travels at 620 mi/h west. A wind is blowing at 35 mi/h toward the south.

3. An airplane travels at 450 mi/h south. A wind is blowing at 40 mi/h toward the east.

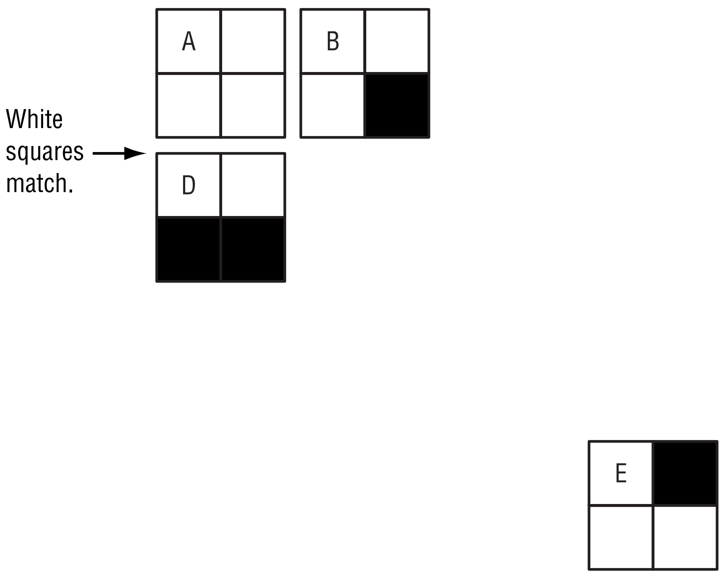
4. An airplane travels at 1,200 mi/h east. A wind is blowing at 30 mi/h toward the north.

8-1 Extension Activities

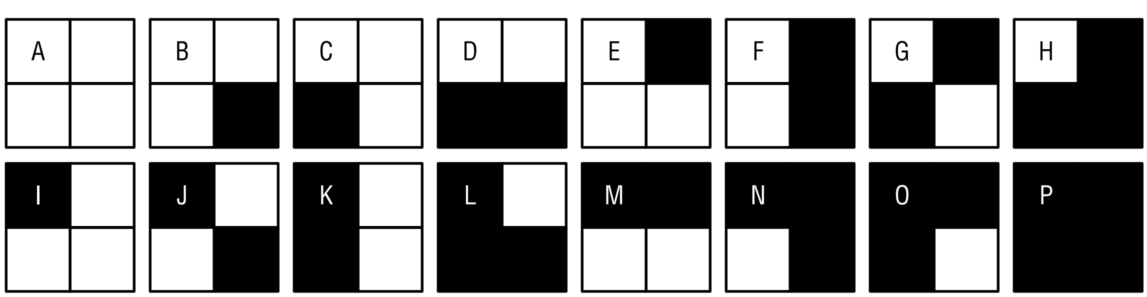
The Colormatch Square

To work this puzzle, cut out the 16 tiles at the bottom of this page. The goal of the puzzle is to create a square so that the sides of any pair of adjacent tiles match. You are not allowed to rotate any of the tiles.

1. Complete the solution to the colormatch square puzzle below.



2. Find at least one other solution in which the A tile is in the upper left corner.



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8-2 Extension Activities

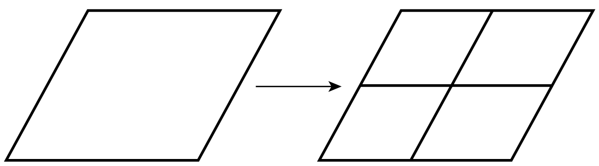
Rep-Tiles

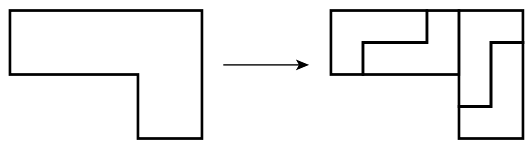
The word **rep-tiles** stands for <u>rep</u>eating <u>tiles</u>. A geometric figure is a rep-tile if it can be divided into smaller parts according to these rules.

1. All the smaller parts must be *congruent* to each other.

2. All the smaller parts must be *similar* to the original tile.

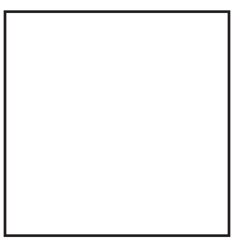
Here are two examples of figures that are rep-tiles.



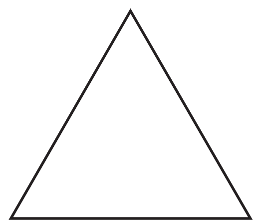


Divide each rep-tile into four congruent parts.

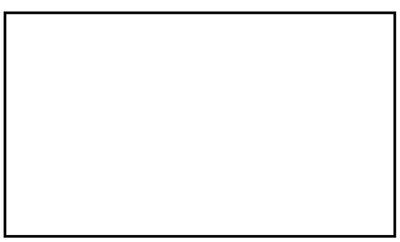
1.



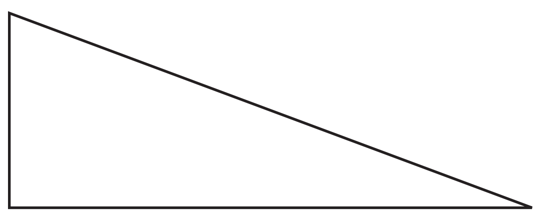
2.



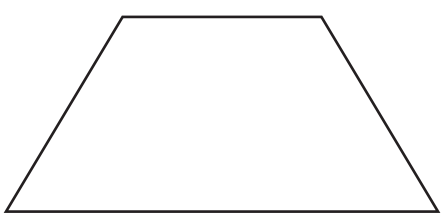
3.



4.



5.



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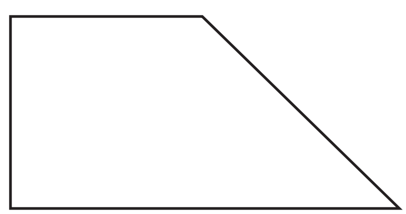
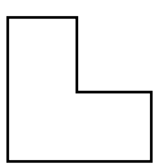


7. CHALLENGE Show how to use four figures like the one at the right to make a rep-tile.



8-3 Extension Activities

The Midpoint Formula

On a line segment, the point that is halfway between the endpoints in called the **midpoint**. To find the midpoint of a segment on the coordinate plane, you can use the Midpoint Formula.

Midpoint Formula	The coordinates of the midpoint of a segment with endpoints (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

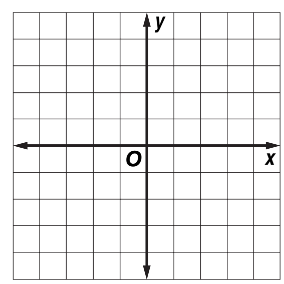
Example **Find the midpoint of $\overline{AB}$ for A(3, 2) and B(2, 0).**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + 2}{2}, \frac{2 + 0}{2}\right) \qquad x_1 = 3,\ y_1 = 2,\ x_2 = 2,\ y_2 = 0$$
$$= (2.5, 1)$$

Exercises

Use the coordinate plane at the right for Exercises 1 and 2.

1. Graph the following points and connect them to form a triangle.
   A(4,0) B(0,−2) C (1, 2)



2. Calculate the midpoint of each line segment using the Midpoint Formula. Graph each midpoint and connect the midpoints to form a triangle.

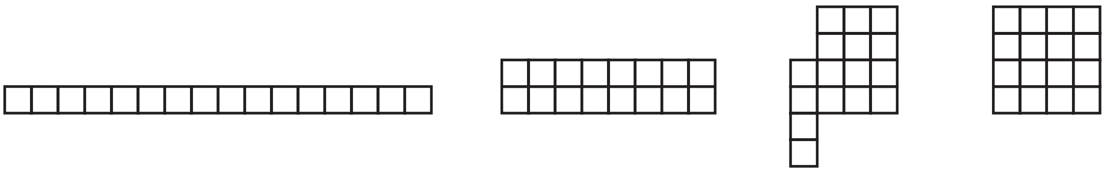
3. Using the Distance Formula, find the perimeter of both the larger and smaller triangles you graphed in Exercises 1 and 2.

4. Based on your finding in Exercise 3, make a conjecture about the relationship between the perimeter of a triangle and the perimeter of a triangle that could be created by connecting the midpoints of each side.

8-4 Extension Activities

Perimeter

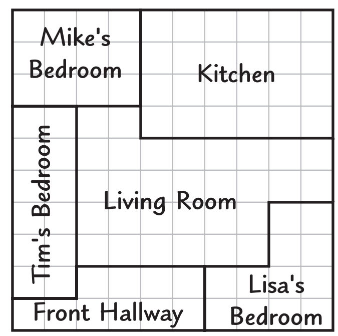
Two shapes can have the same area and different perimeters. Each of these shapes has an area of 16 square units, but their perimeters are different.



$P = 34$ $P = 20$ $P = 20$ $P = 16$

Among rectangles that have an area of 16 square feet, rectangles that are long and thin have the greatest perimeter. Rectangles with the least perimeter are more closely shaped to a square.

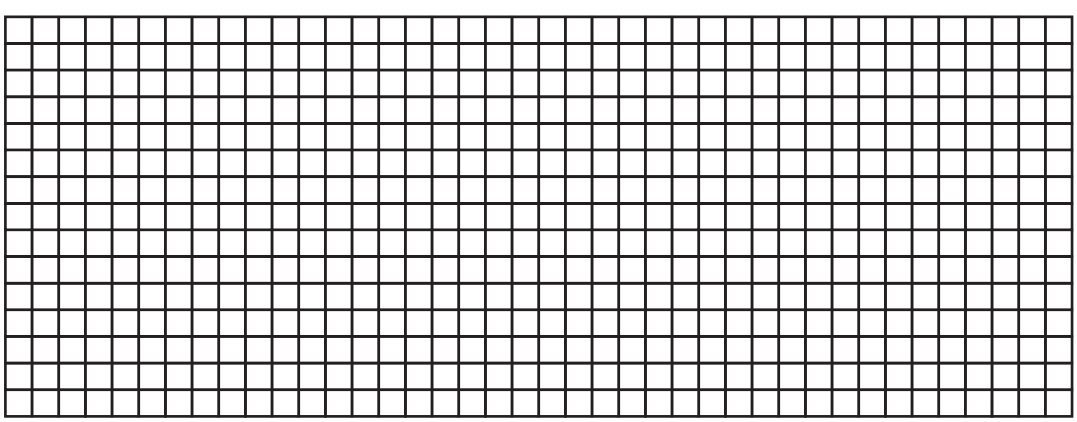
The grid shows the basic floor plan of the Smith's house. The side of each grid represents 3 feet. The three bedrooms all have the same area.



1. Which of the rectangular bedrooms has the greatest perimeter? What is another dimension that will create a rectangle with the same area?

2. Lisa's bedroom has an irregular shape. How does the area of her bedroom compare to the other two bedrooms? How does the perimeter of her bedroom compare to the other two bedrooms?

3. The Smith's are moving to a new house. Design two different floor plans for them from which they may choose. Your floor plans must have five rooms including three bedrooms. Each bedroom must have an area of 162 square feet (18 squares) but not the same perimeters. You may add any other features to the house that you want.



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8-5 Extension Activities

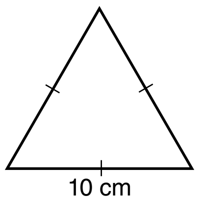
Area of an Equilateral Triangle

The area of an equilateral triangle is the product of one fourth of the square of a side times the square root of 3 (which is approximately 1.732).

$$A = \frac{1}{4}s^2(\sqrt{3})$$

$$\text{or } A \approx \frac{s^2}{4}(1.732)$$

Example



$$A \approx \frac{10^2}{4}(1.732)$$

$$\approx \frac{100}{4}(1.732)$$

$$\approx 43.3$$

The area of the triangle is approximately 43.3 cm^2.

Find the area of each equilateral triangle. Round each answer to the nearest tenth.

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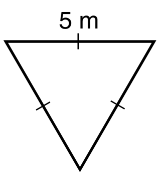

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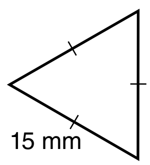

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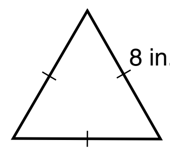

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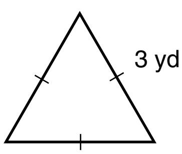

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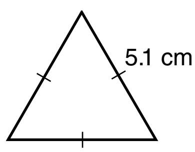

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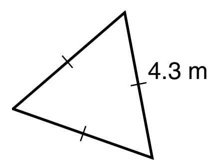

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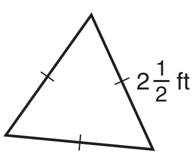

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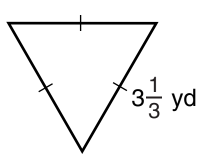

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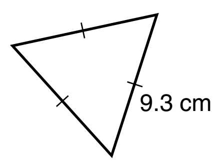

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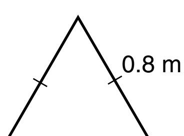

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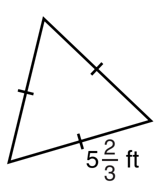

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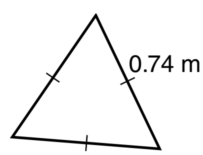

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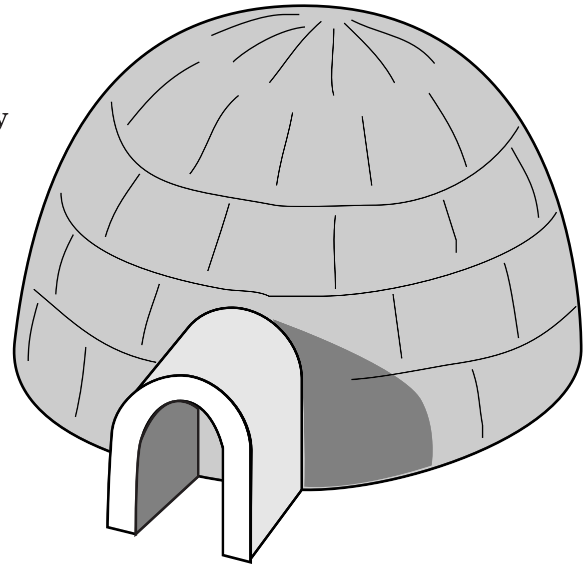
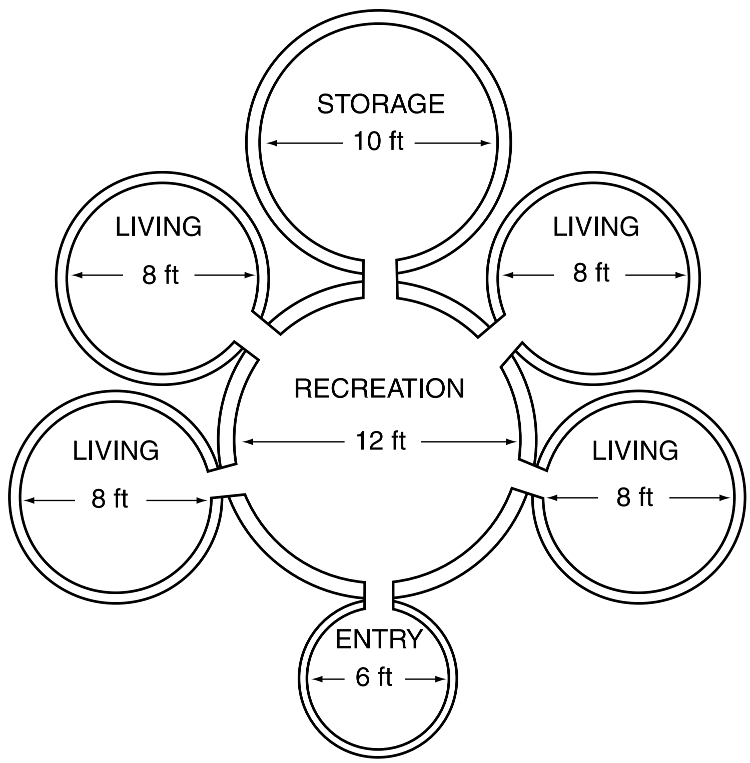
8-6 Extension Activities

Inuit Architecture

The Inuit are a Native American people who live primarily in the arctic regions of Alaska, Canada, Siberia, and Greenland. The Inuit word *iglu* means "winter house," and it originally referred to any permanent structure used for shelter in the winter months. In the nineteenth century, however, the term came to mean a domed structure built of snow blocks, as shown in the figure at the right.



An iglu could shelter a family of five or six people. Sometimes several families built a *cluster* of iglus that were connected by passageways and shared storage and recreation chambers. The figure below is a drawing of such a cluster. Use the drawing to answer each of the following questions. When appropriate, round answers to the nearest whole number.



1. What is the circumference of the entry chamber?

2. What is the circumference of one of the living chambers?

3. Estimate the distance from the front of the entry chamber to the back of the storage chamber.

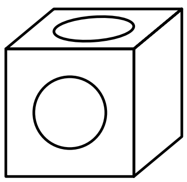
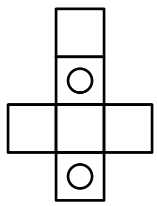
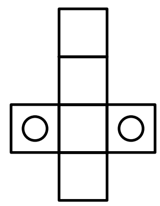
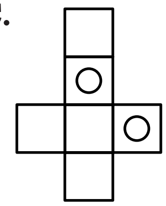
4. An iglu is a *hemisphere*, or half a sphere. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius. Estimate the volume of the storage chamber.

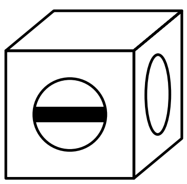
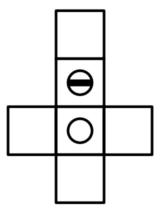
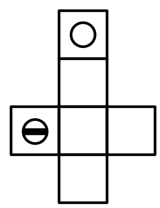
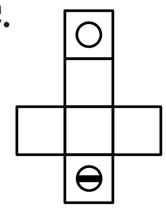
8-7 Extension Activities

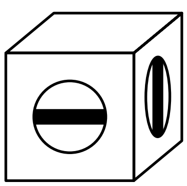
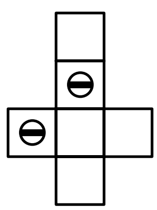
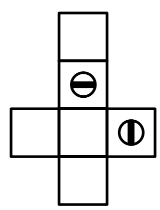
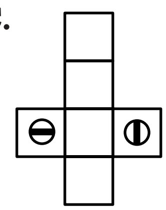
Puzzling Patterns

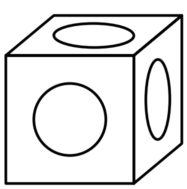
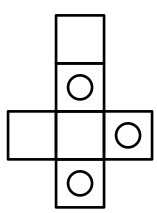
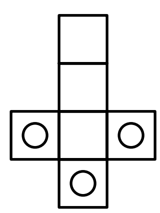
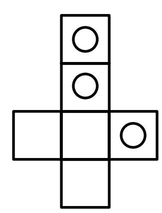
In these visual puzzles, the challenge is to choose the one pattern that could be folded up into the box shown. You are not allowed to make any extra cuts in the patterns. The trick is that the six faces of the box must be arranged in the correct order.

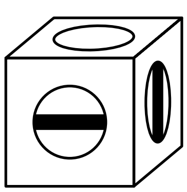
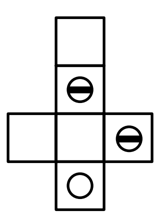
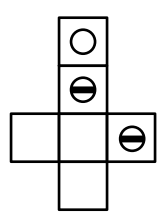
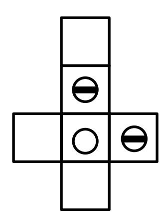
Circle the letter of the pattern that could be used to make each box.

1.  **A.**  **B.**  **C.** 

2.  **A.**  **B.**  **C.** 

3.  **A.**  **B.**  **C.** 

4.  **A.**  **B.**  **C.** 

5.  **A.**  **B.**  **C.** 

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8-8 Extension Activities

Euler's Formula

Leonard Euler (oi'ler), 1707–1783, was a Swiss mathematician who is often called the father of topology. He studied perfect numbers and produced a proof to show that there is an infinite number of primes. He also developed the following formula, relating the numbers of vertices, faces, and edges of a *polyhedron*.

Euler's formula: $V + F = E + 2$

V = number of vertices

F = number of faces

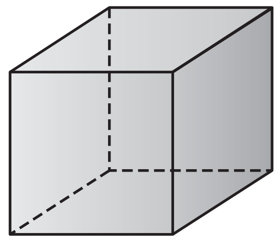
E = number of edges

For a cube, the following is true.

$V + F = E + 2$

$8 + 6 = 12 + 2$

$14 = 14$ ✓



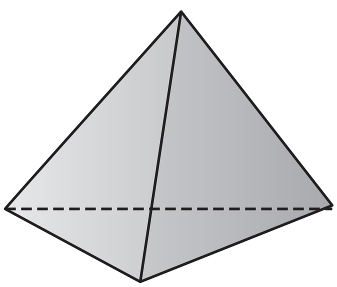
$V = 8$
$F = 6$
$E = 12$

Cube

Another name for a cube is **hexahedron**.

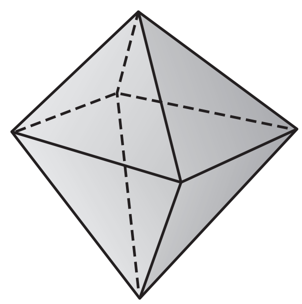
Use Euler's formula to find the number of faces of each polyhedron.

1. $V = 4, E = 6$



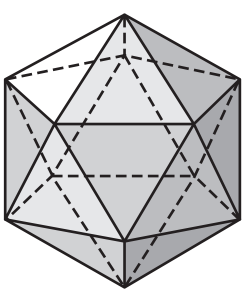
Tetrahedron

2. $V = 6, E = 12$



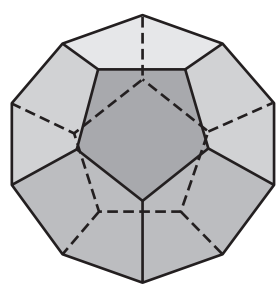
Octahedron

3. $V = 12, E = 30$



Icosahedron

4. $V = 20, E = 30$



Dodecahedron

The suffix *hedron* comes from the Greek language meaning "face." Find the meaning of each of the following prefixes.

5. hexa **6.** tetra **7.** octa **8.** icosa **9.** dodeca