

Chapter 3 - Data Description

Note: Answers may vary due to rounding, TI 83's, or computer programs.

EXERCISE SET 3-2

1.

$$\bar{X} = \frac{\Sigma X}{n} = \frac{93.09}{25} = 3.7236 \approx 3.724$$

MD: 3.57, 3.64, 3.64, 3.65, 3.66, 3.67, 3.67, 3.68, 3.7, 3.7, 3.7, 3.73, **3.73**, 3.74, 3.74, 3.74, 3.75, 3.76, 3.77, 3.78, 3.78, 3.8, 3.8, 3.83, 3.86

Mode: 3.7 and 3.74 MR: $\frac{3.57+3.86}{2} = 3.715$

3.

$$\bar{X} = \frac{\Sigma X}{n} = \frac{136}{9} = 15.1$$

MD: 1, 2, 3, 3, **7**, 11, 18, 30, 61

Mode = 3 MR = $\frac{1+61}{2} = 31$

For the best measure of average, answers will vary.

5.

$$\bar{X} = \frac{\Sigma X}{n} = \frac{3249}{12} = 270.75$$

MD: 75, 88, 102, 117, 136, **189, 229**, 239, 372, 465, 574, 663 MD = $\frac{189+229}{2} = 209$

Mode: no mode MR = $\frac{75+663}{2} = 369$

It would seem that the average number of identity thefts is not higher than 300.

7.

$$\bar{X} = \frac{\Sigma X}{n} = \frac{79.6}{12} = 6.63$$

MD: 5.4, 5.4, 6.2, 6.2, 6.4, **6.4, 6.5**, 7.0, 7.2, 7.2, 7.7, 8.0 MD = $\frac{6.4+6.5}{2} = 6.45$

Mode: no mode MR = $\frac{5.4+8.0}{2} = 6.7$

For the best measure of average, answers will vary.

9.

$$\bar{X} = \frac{\Sigma X}{n} = \frac{238,512}{42} = 5678.9$$

MD: 150, 885, ..., **5315, 5370**, ..., 11070, 11413 MD = $\frac{5315+5370}{2} = 5342.5$

Mode: 4450 MR = $\frac{150+11,413}{2} = 5781.5$

The distribution is skewed to the right.

11.

For Year 1:

$$\bar{X} = \frac{\Sigma X}{n} = \frac{24,911}{27} = 922.6 \text{ MD} = 527$$

Chapter 3 - Data Description

11. continued

Mode: no mode $MR = \frac{69+4192}{2} = 2130.5$

For Year 2:

$$\bar{X} = \frac{\Sigma X}{n} = \frac{24,615}{2} = 911.7 \text{ MD} = 485$$

Mode: 1430 $MR = \frac{70+4040}{2} = 2055$

The mean, median, and midrange of the traffic fatalities for Year 2 are somewhat less than those for the Year 1 fatalities, indicating that the number of fatalities has decreased.

13.

<i>Class Limits</i>	<i>Boundaries</i>	X_m	f	$f \cdot X_m$
202 - 204	201.5 - 204.5	203	2	406
205 - 207	204.5 - 207.5	206	7	1442
208 - 210	207.5 - 210.5	209	16	3344
211 - 213	210.5 - 213.5	212	26	5512
214 - 216	213.5 - 216.5	215	18	3870
217 - 219	216.5 - 219.5	218	<u>4</u>	<u>872</u>
			73	15,446

a. $\bar{X} = \frac{\Sigma f \cdot X_m}{n} = \frac{15,446}{73} = 211.6$

b. modal class: 211 – 213

15.

<i>Limits</i>	<i>Boundaries</i>	X_m	f	$f \cdot X_m$
34 - 96	33.5 - 96.5	65	13	845
97 - 159	96.5 - 159.5	128	2	256
160 - 222	159.5 - 222.5	191	0	0
223 - 285	222.5 - 285.5	254	5	1270
286 - 348	285.5 - 348.5	317	1	317
349 - 411	348.5 - 411.5	380	1	380
412 - 474	411.5 - 474.5	443	0	0
475 - 537	474.5 - 537.5	506	1	506
538 - 600	537.5 - 600.5	569	2	1138
				4712

a. $\bar{X} = \frac{\Sigma f \cdot X_m}{n} = \frac{4712}{25} = 188.48$

b. modal class: 34 – 96

Since most of the data is in the lowest class, the mean is probably not the best measure of average. If the individual data values are available, the median may be a better measure of average. A procedure for finding the approximate median for grouped data is found in Exercise 42 of this section.

Chapter 3 - Data Description

17.

<i>Boundaries</i>	X_m	f	$f \cdot X_m$
52.5 – 63.5	58	6	348
63.5 – 74.5	69	12	828
74.5 – 85.5	80	25	2000
85.5 – 96.5	91	18	1638
96.5 – 107.5	102	14	1428
107.5 – 118.5	113	<u>5</u>	<u>565</u>
	80		6807

a. $\bar{X} = \frac{\sum f \cdot X_m}{n} = \frac{6807}{80} = 85.1$

b. modal class: 74.5 – 85.5

19.

<i>Class Limits</i>	<i>Boundaries</i>	X_m	f	$f \cdot X_m$
13 – 19	12.5 – 19.5	16	2	32
20 – 26	19.5 – 26.5	23	7	161
27 – 33	26.5 – 33.5	30	12	360
34 – 40	33.5 – 40.5	37	5	185
41 – 47	40.5 – 47.5	44	6	264
48 – 54	47.5 – 54.5	51	1	51
55 – 61	54.5 – 61.5	58	0	0
62 – 68	61.5 – 68.5	65	<u>2</u>	<u>130</u>
		35		1183

a. $\bar{X} = \frac{\sum f \cdot X_m}{n} = \frac{1183}{35} = 33.8$

b. modal class: 27 – 33

21.

<i>Boundaries</i>	X_m	f	$f \cdot X_m$
15.5 – 18.5	17	14	238
18.5 – 21.5	20	12	240
21.5 – 24.5	23	18	414
24.5 – 27.5	26	10	260
27.5 – 30.5	29	15	435
30.5 – 33.5	32	<u>6</u>	<u>192</u>
	75		1779

a. $\bar{X} = \frac{\sum f \cdot X_m}{n} = \frac{1779}{75} = 23.7$

b. modal class: 21.5 – 24.5

Chapter 3 - Data Description

23.

<i>Limits</i>	<i>Boundaries</i>	X_m	f	$f \cdot X_m$
27 - 33	26.5 - 33.5	30	7	210
34 - 40	33.5 - 40.5	37	14	518
41 - 47	40.5 - 47.5	44	15	660
48 - 54	47.5 - 54.5	51	11	561
55 - 61	54.5 - 61.5	58	3	174
62 - 68	61.5 - 68.5	65	3	195
69 - 75	68.5 - 75.5	72	<u>2</u>	<u>144</u>
			55	2462

$$\bar{X} = \frac{\sum f \cdot X_m}{n} = \frac{2462}{55} = 44.8$$

modal class: 40.5 – 47.5

25.

<i>Limits</i>	<i>Boundaries</i>	X_m	f	$f \cdot X_m$
31 - 39	30.5 - 39.5	35	4	140
40 - 48	39.5 - 48.5	44	5	220
49 - 57	48.5 - 57.5	53	5	265
58 - 66	57.5 - 66.5	62	12	744
67 - 75	66.5 - 75.5	71	13	923
76 - 84	75.5 - 84.5	80	5	400
85 - 93	84.5 - 93.5	89	<u>3</u>	<u>267</u>
			47	2959

$$\bar{X} = \frac{\sum f \cdot X_m}{n} = \frac{2959}{47} = 62.96 \text{ or } 63.0$$

modal class: 66.5 – 75.5

27.

$$\bar{X} = \frac{\sum w \cdot X}{\sum w} = \frac{3(3.33) + 3(3.00) + 2(2.5) + 2.5(4.4) + 4(1.75)}{3 + 3 + 2 + 2.5 + 4} = \frac{41.99}{14.5} = 2.896$$

29.

$$\bar{X} = \frac{\sum w \cdot X}{\sum w} = \frac{9(427000) + 6(365000) + 12(725000)}{9 + 6 + 12} = \frac{14,733,000}{27} = \$545,666.67$$

31.

$$\bar{X} = \frac{\sum w \cdot X}{\sum w} = \frac{1(62) + 1(83) + 1(97) + 1(90) + 2(82)}{6} = \frac{496}{6} = 82.7$$

33.

- | | |
|-----------|---------|
| a. Median | d. Mode |
| b. Mean | e. Mode |
| c. Mode | f. Mean |

35.

Both could be true since one could be using the mean for the average salary, and the other could be using the mode for the average.

Chapter 3 - Data Description

37.

$$5 \cdot 8.2 = 41$$

$$6 + 10 + 7 + 12 + x = 41$$

$$x = 6$$

39.

$$\text{a. } \frac{2}{\frac{1}{30} + \frac{1}{45}} = 36 \text{ mph}$$

$$\text{b. } \frac{2}{\frac{1}{40} + \frac{1}{25}} = 30.77 \text{ mph}$$

$$\text{c. } \frac{2}{\frac{1}{50} + \frac{1}{10}} = \$16.67$$

41.

$$\sqrt{\frac{8^2 + 6^2 + 3^2 + 5^2 + 4^2}{5}} = \sqrt{30} = 5.48$$

EXERCISE SET 3-3

1.

The square root of the variance is equal to the standard deviation.

3.

$$\sigma^2, \sigma$$

5.

When the sample size is less than 30, the formula for the true standard deviation of the sample will underestimate the population standard deviation.

7.

$$R = 48 - 0 = 48$$

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} = \frac{4061 - \frac{(133)^2}{10}}{10-1} = \frac{2292.1}{9} = 254.68 \approx 254.7$$

$$s = \sqrt{254.7} = 15.96 \approx 16$$

9.

For Temperature:

$$R = 61 - 29 = 32$$

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} = \frac{20,777 - \frac{441^2}{10}}{10-1} = 147.66$$

$$s = \sqrt{147.66} = 12.15$$

For Precipitation:

$$R = 5.1 - 1.1 = 4.0$$

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} = \frac{86.13 - \frac{26.3^2}{10}}{10-1} = 1.88$$

$$s = \sqrt{1.88} = 1.37$$

Temperature is more variable.

Chapter 3 - Data Description

11.

St. Paul, MN:

$$R = 46 - 16 = 30$$

$$s^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1} = \frac{9677 - \frac{313^2}{11}}{11-1} = \frac{770.727}{10} = 77.1$$

$$s = \sqrt{77.1} = 8.8$$

Chicago, IL:

$$R = 100 - 57 = 43$$

$$s^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1} = \frac{59,980 - \frac{796^2}{11}}{11-1} = \frac{2378.545}{10} = 237.85 \approx 237.9$$

$$s = \sqrt{237.9} = 15.4$$

The data for Chicago is more variable since the standard deviation is much larger.

13.

$$R = 22 - 1 = 21$$

$$s^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1} = \frac{1061 - \frac{89^2}{15}}{15-1} = 38.1$$

$$s = \sqrt{38.1} = 6.2$$

Using the range rule of thumb, $s \approx \frac{22-1}{4} = 5.25$. The estimate is close.

15.

For 1995:

$$R = 4192 - 69 = 4123$$

$$s^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1} = \frac{49,784,885 - \frac{24,911^2}{27}}{27-1} = 1,030,817.63$$

$$s = \sqrt{1,030,817.63} = 1015.3$$

For 1996:

$$R = 4040 - 70 = 3970$$

$$s^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1} = \frac{48,956,875 - \frac{24,615^2}{27}}{27-1} = 1,019,853.85$$

$$s = \sqrt{1,019,853.85} = 1009.9$$

The fatalities in 1995 are more variable.

17.

$$R = 11,413 - 150 = 11,263$$

$$s^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1} = \frac{1,659,371,050 - \frac{238,512^2}{42}}{42-1} = \frac{304,895,475.1}{41} = 7,436,475.003$$

$$s = \sqrt{7,436,475.003} = 2726.99 \text{ or } 2727$$

Chapter 3 - Data Description

19.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
16	2	32	512
23	7	161	3703
30	12	360	10,800
37	5	185	6845
44	6	264	11,616
51	1	51	2601
58	0	0	0
65	<u>2</u>	<u>130</u>	<u>8450</u>
	35	1183	44527

$$s^2 = \frac{\sum f \cdot X_m^2 - \frac{(\sum f \cdot X_m)^2}{n}}{n-1} = \frac{44,527 - \frac{1183^2}{35}}{35-1} = \frac{4541.6}{34} = 133.58 \text{ or } 133.6$$

$$s = \sqrt{133.58} = 11.6$$

21.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
65	13	845	54,925
128	2	256	32,768
191	0	0	0
254	5	1270	322,580
317	1	317	100,489
380	1	380	144,400
443	0	0	0
506	1	506	256,036
569	<u>2</u>	<u>1138</u>	<u>647,522</u>
	4712	1,558,720	

$$s^2 = \frac{\sum f \cdot X_m^2 - \frac{(\sum f \cdot X_m)^2}{n}}{n-1} = \frac{1,558,720 - \frac{4712^2}{25}}{25-1} = \frac{670,602.24}{24} = 27,941.76$$

$$s = \sqrt{27941.76} = 167.16 \text{ or } 167.2$$

23.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
58	6	348	20,184
69	12	828	57,132
80	25	2000	160,000
91	18	1638	148,058
102	14	1428	145,656
112	<u>5</u>	<u>565</u>	<u>63,845</u>
	80	6807	595,875

$$s^2 = \frac{\sum f \cdot X_m^2 - \frac{(\sum f \cdot X_m)^2}{n}}{n-1} = \frac{59,5875 - \frac{6807^2}{80}}{80-1} = \frac{16,684.39}{79} = 211.2$$

$$s = \sqrt{211.2} = 14.5$$

Chapter 3 - Data Description

25.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
68	5	340	23,120
79	14	1106	87,374
90	18	1620	145,800
101	25	2525	255,025
112	12	1344	150,528
123	<u>6</u>	<u>738</u>	<u>90,774</u>
	80	7673	752,621

$$s^2 = \frac{\sum f \cdot X_m^2 - \frac{(\sum f \cdot X_m)^2}{n}}{n-1} = \frac{752,621 - \frac{7673^2}{80}}{80-1} = \frac{16,684,3875}{79} = 211.19 \text{ or } 211.2$$

$$s = \sqrt{211.2} = 14.5$$

No, the variability of the lifetimes of the batteries is quite large.

27.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
27	5	135	3645
30	9	270	8100
33	32	1056	34848
36	30	720	25920
39	12	468	18252
62	<u>2</u>	<u>84</u>	<u>3528</u>
	80	2733	94293

$$s^2 = \frac{\sum f \cdot X_m^2 - \frac{(\sum f \cdot X_m)^2}{n}}{n-1} = \frac{94,293 - \frac{2733^2}{80}}{80-1} = \frac{926.89}{79} = 11.7$$

$$s = \sqrt{11.7} = 3.4$$

29.

For East: $\bar{X} = 2660$, $s = 991.9$; C. Var = $\frac{s}{\bar{X}} = \frac{991.9}{2660} = 0.373$ or 37.3%

For West: $\bar{X} = 2261.2$, $s = 1117.9$; C. Var = $\frac{s}{\bar{X}} = \frac{1117.9}{2261.2} = 0.494$ or 49.4%

The data for the West is more variable.

31.

$$\text{C. Var} = \frac{s}{\bar{X}} = \frac{6}{26} = 0.231 = 23.1\%$$

$$\text{C. Var} = \frac{s}{\bar{X}} = \frac{4000}{31,000} = 0.129 = 12.9\%$$

The age is more variable.

33.

a. $1 - \frac{1}{5^2} = 0.96$ or 96%

b. $1 - \frac{1}{4^2} = 0.9375$ or 93.75%

Chapter 3 - Data Description

35.

$$\bar{X} = 5.02 \quad s = 0.09$$

At least 75% of the data values will fall within two standard deviations of the mean; hence, $2(\$0.09) = \0.18 and $\$5.02 - \$0.18 = \$4.84$ and $\$5.02 + \$0.18 = \$5.20$. Hence at least 75% of the data values will fall between \$4.84 and \$5.20.

37.

$$\bar{X} = 95 \quad s = 2$$

At least 88.89% of the data values will fall within 3 standard deviations of the mean, hence $95 - 3(2) = 89$ and $95 + 3(2) = 101$. Therefore at least 88.89% of the data values will fall between 89 mg and 101 mg.

39.

$$\bar{X} = 12 \quad s = 3$$

$$20 - 12 = 8 \text{ and } 8 \div 3 = 2.67$$

$$\text{Hence, } 1 - \frac{1}{k^2} = 1 - \frac{1}{2.67^2} = 1 - 0.14 = 0.86 = 86\%$$

At least 86% of the data values will fall between 4 and 20.

41.

$$26.8 + 1(4.2) = 31$$

By the Empirical Rule, 68% of consumption is within 1 standard deviation of the mean. Then $\frac{1}{2}$ of 32%, or 16%, of consumption would be more than 31 pounds of citrus fruit per year.

43.

$n = 30$ $\bar{X} = 214.97$ $s = 20.76$ At least 75% of the data values will fall between $\bar{X} \pm 2s$.

$$\bar{X} - 2(20.76) = 214.97 - 41.52 = 173.45 \text{ and } \bar{X} + 2(20.76) = 214.97 + 41.52 = 256.49$$

In this case all 30 values fall within this range; hence Chebyshev's Theorem is correct for this example.

45.

$$\text{For } k = 1.5, 1 - \frac{1}{1.5^2} = 1 - 0.44 = 0.56 \text{ or } 56\%$$

$$\text{For } k = 2, 1 - \frac{1}{2^2} = 1 - 0.25 = 0.75 \text{ or } 75\%$$

$$\text{For } k = 2.5, 1 - \frac{1}{2.5^2} = 1 - 0.16 = 0.84 \text{ or } 84\%$$

$$\text{For } k = 3, 1 - \frac{1}{3^2} = 1 - 0.1111 = .8889 \text{ or } 88.89\%$$

$$\text{For } k = 3.5, 1 - \frac{1}{3.5^2} = 1 - 0.08 = 0.92 \text{ or } 92\%$$

47.

$$\bar{X} = 13.3$$

$$\text{Mean Dev} = \frac{|5-13.3|+|9-13.3|+|10-13.3|+|11-13.3|+|11-13.3|}{10}$$

$$+ \frac{|12-13.3|+|15-13.3|+|18-13.3|+|20-13.3|+|22-13.3|}{10} = 4.36$$

49.

For $n = 25$, $\bar{X} = 50$, and $s = 3$:

$$s\sqrt{n-1} = 3\sqrt{25-1} = 14.7 \quad \bar{X} + s\sqrt{n-1} = 50 + 14.7 = 64.7$$

67 must be an incorrect data value, since is beyond the range using the formula $s\sqrt{n-1}$.

EXERCISE SET 3-4

1.

A z score tells how many standard deviations the data value is above or below the mean.

Chapter 3 - Data Description

3.

A percentile is a relative measure while a percent is an absolute measure of the part to the total.

5.

$$Q_1 = P_{25}, Q_2 = P_{50}, Q_3 = P_{75}$$

7.

$$D_1 = P_{10}, D_2 = P_{20}, D_3 = P_{30}, \text{ etc}$$

9.

$$\text{a. } z = \frac{X - \bar{X}}{s} = \frac{136 - 127}{9} = 1$$

$$\text{b. } z = \frac{109 - 127}{9} = -2$$

$$\text{c. } z = \frac{104.5 - 127}{9} = -2.5$$

$$\text{d. } z = \frac{113.5 - 127}{9} = -1.5$$

$$\text{e. } z = \frac{133 - 127}{9} = 0.67$$

11.

$$\text{a. } z = \frac{X - \bar{X}}{s} = \frac{87 - 84}{4} = 0.75$$

$$\text{b. } z = \frac{79 - 84}{4} = -1.25$$

$$\text{c. } z = \frac{93 - 84}{4} = 2.25$$

$$\text{d. } z = \frac{76 - 84}{4} = -2$$

$$\text{e. } z = \frac{82 - 84}{4} = -0.5$$

13.

$$\text{a. } z = \frac{42 - 39}{4} = 0.75$$

$$\text{b. } z = \frac{76 - 71}{3} = 1.67$$

The score for part b is has a higher relative position.

15.

$$\text{a. } z = \frac{3.2 - 4.6}{1.5} = -0.93 \quad \text{b. } z = \frac{630 - 800}{200} = -0.85 \quad \text{c. } z = \frac{43 - 50}{5} = -1.4$$

The score in part b is the highest.

17.

$$\text{a. } 21^{\text{st}} \quad \text{b. } 58^{\text{th}} \quad \text{c. } 77^{\text{th}} \quad \text{d. } 33^{\text{rd}}$$

18.

$$\text{a. } 7 \quad \text{b. } 25 \quad \text{c. } 64 \quad \text{d. } 76 \quad \text{e. } 93$$

19.

$$\text{a. } 235 \quad \text{b. } 255 \quad \text{c. } 261 \quad \text{d. } 275 \quad \text{e. } 283$$

Chapter 3 - Data Description

20.

a. 376 b. 389 c. 432 d. 473 e. 498

21.

a. 17th b. 39th c. 53rd d. 79th e. 91st

23.

$$c = \frac{6(30)}{100} = 1.8 \text{ or } 2 \quad 82$$

25.

$$c = \frac{n \cdot p}{100} = \frac{7(60)}{100} = 4.2 \text{ or } 5 \quad \text{Hence, 47 is the closest value to the 60}^{\text{th}} \text{ percentile.}$$

27.

$$c = \frac{10(40)}{100} = 4 \quad \text{average the 4th and 5th values: } P_{40} = \frac{2.1+2.2}{2} = 2.15$$

29.

$$c = \frac{6(33)}{100} = 1.98 \text{ or } 2 \quad 5, 12, 15, 16, 20, 21$$

↑ P_{33}

The second data value is 12.

31.

a. 5, 12, 16, 25, 32, 38 $Q_1 = 12, Q_2 = 20.5, Q_3 = 32$

$$\text{Midquartile} = \frac{12+32}{2} = 22 \quad \text{Interquartile range: } 32 - 12 = 20$$

b. 53, 62, 78, 94, 96, 99, 103 $Q_1 = 62, Q_2 = 94, Q_3 = 99$

$$\text{Midquartile} = \frac{62+99}{2} = 80.5 \quad \text{Interquartile range: } 99 - 62 = 37$$

EXERCISE SET 3-5

1. Data arranged in order: 6, 8, 12, 19, 27, 32, 54

Minimum: 6

Q_1 : 8

Median: 19

Q_3 : 32

Maximum: 54

Interquartile Range: $32 - 8 = 24$

3. Data arranged in order: 188, 192, 316, 362, 437, 589

Minimum: 188

Q_1 : 192

Median: $\frac{316+362}{2} = 339$

Q_3 : 437

Maximum: 589

Interquartile Range: $437 - 192 = 245$

5. Data arranged in order: 14.6, 15.5, 16.3, 18.2, 19.8

Minimum: 14.6

Q_1 : $\frac{14.6+15.5}{2} = 15.05$

Chapter 3 - Data Description

5. continued

Median: 16.3

$$Q_3: \frac{18.2+19.8}{2} = 19.0$$

Maximum: 19.8

$$\text{Interquartile Range: } 19.0 - 15.05 = 3.95$$

7. Minimum: 3

Q_1 : 5

Median: 8

Q_3 : 9

Maximum: 11

$$\text{Interquartile Range: } 9 - 5 = 4$$

9. Minimum: 55

Q_1 : 65

Median: 70

Q_3 : 90

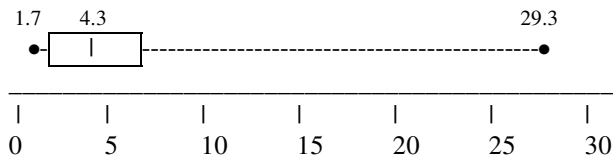
Maximum: 95

$$\text{Interquartile Range: } 90 - 65 = 25$$

11.

$$\text{MD} = \frac{3.9+4.7}{2} = 4.3$$

$$Q_1 = 2.0 \quad Q_3 = 7.6$$



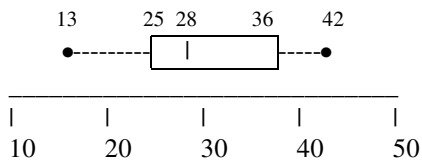
The distribution is positively skewed.

13. Data arranged in order: 13, 25, 25, 26, 28, 34, 35, 37, 42

Minimum: 13 Maximum: 42

MD = 28

$$Q_1 = \frac{25+25}{2} = 25 \quad Q_3 = \frac{35+37}{2} = 36$$



15. Data arranged in order: 3.2, 3.9, 4.4, 8.0, 9.8, 11.7, 13.9, 15.9, 17.6, 21.7, 24.8, 34.1

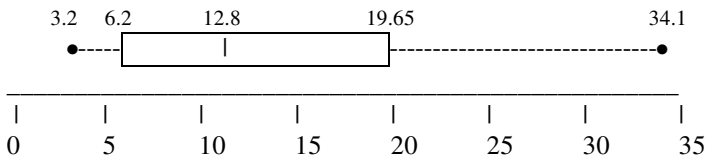
Minimum: 3.2 Maximum: 34.1

$$\text{MD: } \frac{11.7+13.9}{2} = 12.8$$

$$Q_1: \frac{4.4+8.0}{2} = 6.2 \quad Q_3: \frac{17.6+21.7}{2} = 19.65$$

Chapter 3 - Data Description

15. continued



The distribution is positively skewed.

17.

(a)

For April: $\bar{X} = 149.3$

For May: $\bar{X} = 264.3$

For June: $\bar{X} = 224.0$

For July: $\bar{X} = 123.3$

The month with the highest mean number of tornadoes is May.

(b)

For 2001: $\bar{X} = 186.0$

For 2000: $\bar{X} = 165.0$

For 1999: $\bar{X} = 219.75$

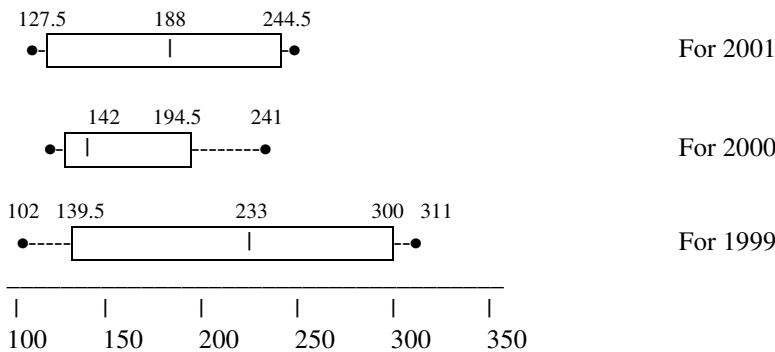
The year with the highest mean number of tornadoes is 1999.

(c) The 5-number summaries for each year are:

For 2001: 120, 127.5, 188, 244.5, 248

For 2000: 135, 135.5, 142, 194.5, 241

For 1999: 102, 139.5, 233, 300, 311



The distribution for 2001 is approximately symmetric while the distributions for 2000 and 1999 are skewed.

The distribution for 2000 is positively skewed and the distribution for 1999 is negatively skewed.

The data for the year 2000 is the least variable and has the smallest median.

REVIEW EXERCISES - CHAPTER 3

1.

a. $\bar{X} = \frac{\sum X}{n} = \frac{1649}{15} = 109.9$

Chapter 3 - Data Description

1. continued

b. 60, 68, 70, 75, 89, 93, 95, **97**, 112, 114, 114, 122, 128, 182, 229

MD = 97

c. Mode = 114

d. MR = $\frac{60+229}{2} = 144.5$

e. Range = $229 - 60 = 169$

f. $s^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1} = \frac{209379 - \frac{1649^2}{15}}{15-1} = 2007.1$

g. $s = \sqrt{2007.1} = 44.8$

3.

Class	X_m	f	$f \cdot X_m$	$f \cdot X_m^2$	cf
1 - 3	2	1	2	4	1
4 - 6	5	4	20	100	5
7 - 9	8	5	40	320	10
10 - 12	11	1	11	121	11
13 - 15	14	<u>1</u>	<u>14</u>	<u>196</u>	12
		12	87	741	

a. $\bar{X} = \frac{\sum f \cdot X_m}{n} = \frac{87}{12} = 7.3$

b. Modal Class = 7 - 9 or 6.5 - 9.5

c. $s^2 = \frac{741 - \frac{87^2}{12}}{11} = \frac{110.25}{11} = 10.0$

f. $s = \sqrt{10.0} = 3.2$

5.

Class Boundaries	X_m	f	$f \cdot X_m$	$f \cdot X_m^2$	cf
12.5 - 27.5	20	6	120	2400	6
27.5 - 42.5	35	3	105	3675	9
42.5 - 57.5	50	5	250	12,500	14
57.5 - 72.5	65	8	520	33,800	22
72.5 - 87.5	80	6	480	38,400	28
87.5 - 102.5	95	<u>2</u>	<u>190</u>	<u>18,050</u>	30
		30	1665	108,825	

a. $\bar{X} = \frac{\sum f \cdot X_m}{n} = \frac{1665}{30} = 55.5$

b. Modal class = 57.5 - 72.5

c. $s^2 = \frac{\sum f \cdot X_m^2 - \frac{(\sum f \cdot X_m)^2}{n}}{n-1} = \frac{108825 - \frac{1665^2}{30}}{30-1} = \frac{16417.5}{29} = 566.1$

d. $s = \sqrt{566.1} = 23.8$

Chapter 3 - Data Description

7.

$$\bar{X} = \frac{\sum w \cdot X}{\sum w} = \frac{12 \cdot 0 + 8 \cdot 1 + 5 \cdot 2 + 5 \cdot 3}{12 + 8 + 5 + 5} = \frac{33}{30} = 1.1$$

9.

$$\bar{X} = \frac{\sum w \cdot X}{\sum w} = \frac{8 \cdot 3 + 1 \cdot 6 + 1 \cdot 30}{8 + 1 + 1} = \frac{60}{10} = 6$$

11.

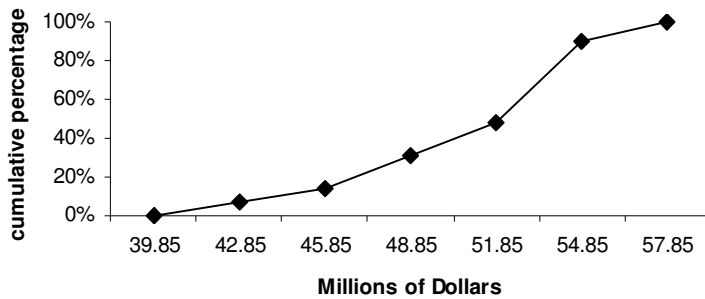
Magazines: C. Var = $\frac{s}{\bar{X}} = \frac{12}{56} = 0.214$

Year: C. Var = $\frac{s}{\bar{X}} = \frac{2.5}{6} = 0.417$

The number of years is more variable.

13.

a.



b. $P_{35} = 49$; $P_{65} = 52$; $P_{85} = 53$ (answers are approximate)

c. 44 = 15th percentile; 48 = 33rd percentile; 54 = 91nd percentile (answers are approximate)

15.

$$\bar{X} = 0.32 \quad s = 0.03 \quad k = 2$$

$$0.32 - 2(0.03) = 0.26 \quad \text{and} \quad 0.32 + 2(0.03) = 0.38$$

At least 75% of the values will fall between \$0.26 and \$0.38.

17.

$$\bar{X} = 54 \quad s = 4 \quad 60 - 54 = 6 \quad k = \frac{6}{4} = 1.5 \quad 1 - \frac{1}{1.5^2} = 1 - 0.44 = 0.56 \quad \text{or} \quad 56\%$$

19.

$$\bar{X} = 32 \quad s = 4 \quad 44 - 32 = 12 \quad k = \frac{12}{4} = 3 \quad 1 - \frac{1}{3^2} = 0.8889 = 88.89\%$$

21.

Before Christmas:

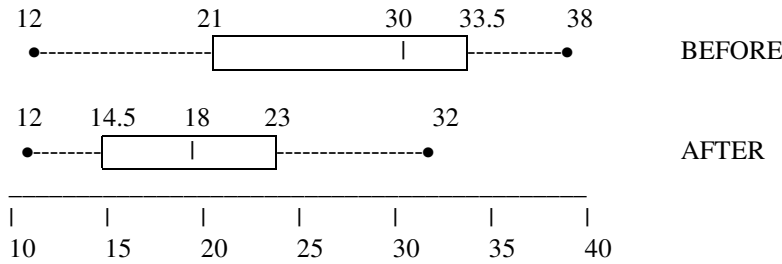
$$MD = 30 \quad Q_1 = 21 \quad Q_3 = 33.5$$

After Christmas:

$$MD = 18 \quad Q_1 = 14.5 \quad Q_3 = 23$$

Chapter 3 - Data Description

21. continued



The employees worked more hours before Christmas. Also, the range and variability of the distribution of hours worked before Christmas is greater than that of hours worked after Christmas.

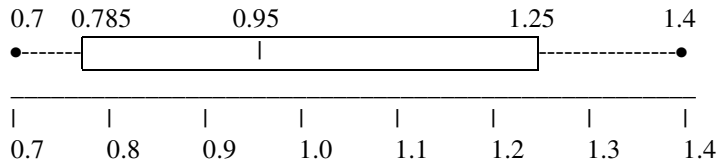
CHAPTER 3 QUIZ

1. True
2. True
3. False
4. False
5. False
6. False
7. False
8. False
9. False
10. c.
11. c.
12. a. and b.
13. b.
14. d.
15. b.
16. statistic
17. parameters, statistics
18. standard deviation
19. σ
20. midrange
21. positively
22. outlier
23. a. 15.3 b. 15.5 c. 15, 16, 17 d. 15 e. 6 f. 3.61 g. 1.9
24. a. 6.4 b. 6 – 8 c. 11.6 d. 3.4
25. a. 51.4 b. 35.5 – 50.5 c. 451.5 d. 21.2
26. a. 8.2 b. 7 – 9 c. 21.6 d. 4.6
27. 1.6
28. 4.46 or 4.5
29. 0.33; 0.162; newspapers
30. 0.3125; 0.229; brands
31. – 0.75; – 1.67; science
32. a. 0.5 b. 1.6 c. 15, c is higher
33. a. 56.25; 43.75; 81.25; 31.25; 93.75; 18.75; 6.25; 68.75 b. 0.9

Chapter 3 - Data Description

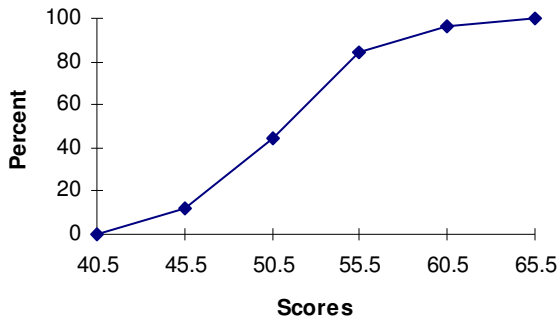
33. continued

c.



34.

a.



b. 47; 53; 65

c. 60th percentile; 6th percentile; 98th percentile

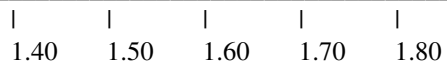
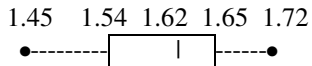
35.

For Pre-buy:

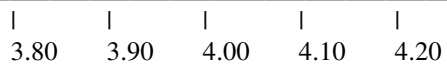
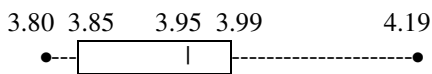
MD = 1.62 $Q_1 = 1.54$ $Q_3 = 1.65$

For No Pre-buy:

MD = 3.95 $Q_1 = 3.85$ $Q_3 = 3.99$



Pre-buy Cost



No Pre-buy Cost

The cost of pre-buy gas is much less than to return the car without filling it with gas. The variability of the return without filling with gas is larger than the variability of the pre-buy gas.

Chapter 3 - Data Description

36.

For above 1129: 16%

For above 799: 97.5%