Important Formulas

Chapter 3   Data Description

Mean for individual data: $\bar{X} = \dfrac{\Sigma X}{n}$

Mean for grouped data: $\bar{X} = \dfrac{\Sigma f \cdot X_m}{n}$

Standard deviation for a sample:

$$s = \sqrt{\frac{\Sigma X^2 - [(\Sigma X)^2/n]}{n-1}}$$

Standard deviation for grouped data:

$$s = \sqrt{\frac{\Sigma f \cdot X_m^2 - [(\Sigma f \cdot X_m)^2/n]}{n-1}}$$

Range rule of thumb: $s \approx \dfrac{\text{range}}{4}$

Chapter 4   Probability and Counting Rules

Addition rule 1 (mutually exclusive events):

$$P(A \text{ or } B) = P(A) + P(B)$$

Addition rule 2 (events not mutually exclusive):

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Multiplication rule 1 (independent events):

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Multiplication rule 2 (dependent events):

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

Conditional probability: $P(B \mid A) = \dfrac{P(A \text{ and } B)}{P(A)}$

Complementary events: $P(\bar{E}) = 1 - P(E)$

Fundamental counting rule: Total number of outcomes of a sequence when each event has a different number of possibilities: $k_1 \cdot k_2 \cdot k_3 \cdots k_n$

Permutation rule: Number of permutations of n objects taking r at a time is $_nP_r = \dfrac{n!}{(n-r)!}$

Combination rule: Number of combinations of r objects selected from n objects is $_nC_r = \dfrac{n!}{(n-r)!r!}$

Chapter 5   Discrete Probability Distributions

Mean for a probability distribution: $\mu = \Sigma[X \cdot P(X)]$

Variance and standard deviation for a probability distribution:

$$\sigma^2 = \Sigma[X^2 \cdot P(X)] - \mu^2$$
$$\sigma = \sqrt{\Sigma[X^2 \cdot P(X)] - \mu^2}$$

Expectation: $E(X) = \Sigma[X \cdot P(X)]$

Binomial probability: $P(X) = \dfrac{n!}{(n-X)!X!} \cdot p^X \cdot q^{n-X}$

Mean for binomial distribution: $\mu = n \cdot p$

Variance and standard deviation for the binomial distribution:

$$\sigma^2 = n \cdot p \cdot q \qquad \sigma = \sqrt{n \cdot p \cdot q}$$

Multinomial probability:

$$P(X) = \frac{n!}{X_1!X_2!X_3! \cdots X_k!} \cdot p_1^{X_1} \cdot p_2^{X_2} \cdot p_3^{X_3} \cdots p_k^{X_k}$$

Poisson probability: $P(X; \lambda) = \dfrac{e^{-\lambda}\lambda^X}{X!}$ where

$X = 0, 1, 2, \ldots$

Hypergeometric probability: $P(X) = \dfrac{_aC_X \cdot {_bC_{n-X}}}{_{a+b}C_n}$

Chapter 6   The Normal Distribution

Standard score $z = \dfrac{X - \mu}{\sigma}$ or $\dfrac{X - \bar{X}}{s}$

Mean of sample means: $\mu_{\bar{X}} = \mu$

Standard error of the mean: $\sigma_{\bar{X}} = \dfrac{\sigma}{\sqrt{n}}$

Central limit theorem formula: $z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

Chapter 7   Confidence Intervals and Sample Size

z confidence interval for means:

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

t confidence interval for means:

$$\bar{X} - t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

Sample size for means: $n = \left(\dfrac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$ where E is the maximum error of estimate

Confidence interval for a proportion:

$$\hat{p} - (z_{\alpha/2})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\alpha/2})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$
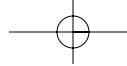
Sample size for a proportion: $n = \hat{p}\hat{q}\left(\dfrac{z_{\alpha/2}}{E}\right)^2$

where $\quad \hat{p} = \dfrac{X}{n} \quad$ and $\quad \hat{q} = 1 - \hat{p}$

Confidence interval for variance:

$$\frac{(n-1)s^2}{\chi^2_{\text{right}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\text{left}}}$$

Confidence interval for standard deviation:

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\text{right}}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{\text{left}}}}$$

Chapter 8 Hypothesis Testing

z test: $z = \dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}}$ for any value n. If $n < 30$,

population must be normally distributed.

$$z = \frac{\overline{X} - \mu}{s/\sqrt{n}} \qquad \text{for } \sigma \text{ unknown and } n \geq 30$$

t test: $t = \dfrac{\overline{X} - \mu}{s/\sqrt{n}} \qquad$ for $n < 30$ (d.f. $= n - 1$)

z test for proportions: $z = \dfrac{\hat{p} - p}{\sqrt{pq/n}}$

Chi-square test for a single variance: $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$

(d.f. $= n - 1$)

Chapter 9 Testing the Difference between Two Means, Two Variances, and Two Proportions

z test for comparing two means (independent samples):

$$z = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

Formula for the confidence interval for difference of two means (large samples):

$$(\overline{X}_1 - \overline{X}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2$$

$$< (\overline{X}_1 - \overline{X}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Note: s_1^2 and s_2^2 can be used when $n_1 \geq 30$ and $n_2 \geq 30$.

F test for comparing two variances: $F = \dfrac{s_1^2}{s_2^2}$

where s_1^2 is the larger variance and
d.f.N. $= n_1 - 1$, d.f.D. $= n_2 - 1$

t test for comparing two means (independent samples, variances not equal):

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

(d.f. $=$ the smaller of $n_1 - 1$ or $n_2 - 1$)

Formula for the confidence interval for difference of two means (small independent samples, variance unequal):

$$(\overline{X}_1 - \overline{X}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2$$

$$< (\overline{X}_1 - \overline{X}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

(d.f. $=$ smaller of $n_1 - 1$ and $n_2 - 1$)

t test for comparing two means (independent samples, variances equal):

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 + n_2 - 2)}}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

(d.f. $= n_1 + n_2 - 2$)

Formula for the confidence interval for difference of two means (small independent samples, variances equal):

$$(\overline{X}_1 - \overline{X}_2) - t_{\alpha/2}\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$< \mu_1 - \mu_2 <$$

$$(\overline{X}_1 - \overline{X}_2) + t_{\alpha/2}\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 + n_2 - 2)}} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

and d.f. $= n_1 + n_2 - 2$.

t test for comparing two means for dependent samples:

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}} \qquad \text{where} \qquad \overline{D} = \frac{\Sigma D}{n} \qquad \text{and}$$

$$s_D = \sqrt{\frac{\Sigma D^2 - [(\Sigma D)^2/n]}{n - 1}} \qquad \text{(d.f. } = n - 1)$$

Formula for confidence interval for the mean of the difference for dependent samples:

$$\overline{D} - t_{\alpha/2}\frac{s_D}{\sqrt{n}} < \mu_D < \overline{D} + t_{\alpha/2}\frac{s_D}{\sqrt{n}}$$

(d.f. $= n - 1$)
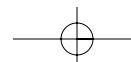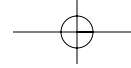
z test for comparing two proportions:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where
$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} \qquad \hat{p}_1 = \frac{X_1}{n_1}$$

$$\bar{q} = 1 - \bar{p} \qquad \hat{p}_2 = \frac{X_2}{n_2}$$

Formula for the confidence interval for the difference of two proportions:

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2$$

$$< (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Chapter 10   Correlation and Regression

Correlation coefficient:

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{[n(\Sigma x^2) - (\Sigma x)^2][n(\Sigma y^2) - (\Sigma y)^2]}}$$

t test for correlation coefficient: $t = r\sqrt{\dfrac{n - 2}{1 - r^2}}$

(d.f. $= n - 2$)

The regression line equation: $y' = a + bx$

where
$$a = \frac{(\Sigma y)(\Sigma x^2) - (\Sigma x)(\Sigma xy)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$b = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

Coefficient of determination: $r^2 = \dfrac{\text{explained variation}}{\text{total variation}}$

Standard error of estimate:

$$s_{\text{est}} = \sqrt{\frac{\Sigma y^2 - a\,\Sigma y - b\,\Sigma xy}{n - 2}}$$

Prediction interval for y:

$$y' - t_{\alpha/2}s_{\text{est}}\sqrt{1 + \frac{1}{n} + \frac{n(x - \bar{X})^2}{n\,\Sigma x^2 - (\Sigma x)^2}}$$

$$< y < y' + t_{\alpha/2}s_{\text{est}}\sqrt{1 + \frac{1}{n} + \frac{n(x - \bar{X})^2}{n\,\Sigma x^2 - (\Sigma x)^2}}$$

(d.f. $= n - 2$)

Formula for the multiple correlation coefficient:

$$R = \sqrt{\frac{r_{yx_1}^2 + r_{yx_2}^2 - 2r_{yx_1}\cdot r_{yx_2}\cdot r_{x_1 x_2}}{1 - r_{x_1 x_2}^2}}$$

Formula for the F test for the multiple correlation coefficient:

$$F = \frac{R^2/k}{(1 - R^2)/(n - k - 1)}$$

(d.f.N. $= n - k$ and d.f.D. $= n - k - 1$)

Formula for the adjusted R^2:

$$R_{\text{adj}}^2 = 1 - \left[\frac{(1 - R^2)(n - 1)}{n - k - 1}\right]$$

Chapter 11   Other Chi-Square Tests

Chi-square test for goodness-of-fit:

$$\chi^2 = \Sigma \frac{(O - E)^2}{E}$$

(d.f. $=$ no. of categories $- 1$)

Chi-square test for independence and homogeneity of proportions:

$$\chi^2 = \Sigma \frac{(O - E)^2}{E}$$

[d.f. $=$ (rows $- 1$)(col. $- 1$)]

Chapter 12   Analysis of Variance

ANOVA test: $F = \dfrac{s_B^2}{s_W^2}$ where $\bar{X}_{\text{GM}} = \dfrac{\Sigma X}{N}$

d.f.N. $= k - 1$ where $N = n_1 + n_2 + \cdots + n_k$
d.f.D. $= N - k$ where $k =$ number of groups

$$s_B^2 = \frac{\Sigma n_i(\bar{X}_i - \bar{X}_{\text{GM}})^2}{k - 1}$$

$$s_W^2 = \frac{\Sigma(n_i - 1)s_i^2}{\Sigma(n_i - 1)}$$

Scheffé test: $F_S = \dfrac{(\bar{X}_i - \bar{X}_j)^2}{s_W^2(1/n_i + 1/n_j)}$ and

$$F' = (k - 1)(\text{C.V.})$$

Tukey test: $q = \dfrac{\bar{X}_i - \bar{X}_j}{\sqrt{s_W^2/n}}$

Formulas for two-way ANOVA:

$$\text{MS}_A = \frac{\text{SS}_A}{a - 1} \qquad F_A = \frac{\text{MS}_A}{\text{MS}_W}$$

$$\text{MS}_B = \frac{\text{SS}_B}{b - 1} \qquad F_B = \frac{\text{MS}_B}{\text{MS}_W}$$

$$\text{MS}_{A\times B} = \frac{\text{SS}_{A\times B}}{(a - 1)(b - 1)} \qquad F_{A\times B} = \frac{\text{MS}_{A\times B}}{\text{MS}_W}$$

$$\text{MS}_W = \frac{\text{SS}_W}{ab(n - 1)}$$
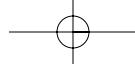
Chapter 13   Nonparametric Statistics

z test value in the sign test: $z = \dfrac{(X + 0.5) - (n/2)}{\sqrt{n}/2}$

where n = sample size (greater than or equal to 26)

X = smaller number of $+$ or $-$ signs

Wilcoxon rank sum test: $z = \dfrac{R - \mu_R}{\sigma_R}$

where

$\mu_R = \dfrac{n_1(n_1 + n_2 + 1)}{2}$

$\sigma_R = \sqrt{\dfrac{n_1 n_2(n_1 + n_2 + 1)}{12}}$

R = sum of the ranks for the smaller sample size (n_1)

n_1 = smaller of the sample sizes

n_2 = larger of the sample sizes

$n_1 \geq 10$ and $n_2 \geq 10$

Wilcoxon signed-rank test: $z = \dfrac{w_s - \dfrac{n(n + 1)}{4}}{\sqrt{\dfrac{n(n + 1)(2n + 1)}{24}}}$

where

n = number of pairs where the difference is not 0

w_s = smaller sum in absolute value of the signed ranks

Kruskal-Wallis test:

$$H = \frac{12}{N(N + 1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \cdots + \frac{R_k^2}{n_k} \right) - 3(N + 1)$$

where

R_1 = sum of the ranks of sample 1

n_1 = size of sample 1

R_2 = sum of the ranks of sample 2

n_2 = size of sample 2

$\vdots$

R_k = sum of the ranks of sample k

n_k = size of sample k

$N = n_1 + n_2 + \cdots + n_k$

k = number of samples

Spearman rank correlation coefficient:

$$r_S = 1 - \frac{6 \Sigma d^2}{n(n^2 - 1)}$$

where

d = difference in the ranks

n = number of data pairs

Procedure Table

Solving Hypothesis-Testing Problems (Traditional Method)

STEP 1   State the hypotheses, and identify the claim.

STEP 2   Find the critical value(s) from the appropriate table in Appendix C.

STEP 3   Compute the test value.

STEP 4   Make the decision to reject or not reject the null hypothesis.

STEP 5   Summarize the results.

Procedure Table

Solving Hypothesis-Testing Problems (*P*-value Method)

STEP 1   State the hypotheses and identify the claim.

STEP 2   Compute the test value.

STEP 3   Find the *P*-value.

STEP 4   Make the decision.

STEP 5   Summarize the results.
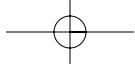
Table E	The Standard Normal Distribution									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Note: Use 0.4999 for *z* values above 3.09.

Source: Frederick Mosteller and Robert E. K. Rourke, *Sturdy Statistics,* Table A–1 (Reading, Mass.: Addison-Wesley, 1973). Reprinted with permission of the copyright owners.


Area given in table

0   z
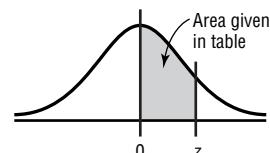
Table F	The *t* Distribution						
	Confidence intervals	**50%**	**80%**	**90%**	**95%**	**98%**	**99%**
	One tail, α	**0.25**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
d.f.	**Two tails, α**	**0.50**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**
1		1.000	3.078	6.314	12.706	31.821	63.657
2		.816	1.886	2.920	4.303	6.965	9.925
3		.765	1.638	2.353	3.182	4.541	5.841
4		.741	1.533	2.132	2.776	3.747	4.604
5		.727	1.476	2.015	2.571	3.365	4.032
6		.718	1.440	1.943	2.447	3.143	3.707
7		.711	1.415	1.895	2.365	2.998	3.499
8		.706	1.397	1.860	2.306	2.896	3.355
9		.703	1.383	1.833	2.262	2.821	3.250
10		.700	1.372	1.812	2.228	2.764	3.169
11		.697	1.363	1.796	2.201	2.718	3.106
12		.695	1.356	1.782	2.179	2.681	3.055
13		.694	1.350	1.771	2.160	2.650	3.012
14		.692	1.345	1.761	2.145	2.624	2.977
15		.691	1.341	1.753	2.131	2.602	2.947
16		.690	1.337	1.746	2.120	2.583	2.921
17		.689	1.333	1.740	2.110	2.567	2.898
18		.688	1.330	1.734	2.101	2.552	2.878
19		.688	1.328	1.729	2.093	2.539	2.861
20		.687	1.325	1.725	2.086	2.528	2.845
21		.686	1.323	1.721	2.080	2.518	2.831
22		.686	1.321	1.717	2.074	2.508	2.819
23		.685	1.319	1.714	2.069	2.500	2.807
24		.685	1.318	1.711	2.064	2.492	2.797
25		.684	1.316	1.708	2.060	2.485	2.787
26		.684	1.315	1.706	2.056	2.479	2.779
27		.684	1.314	1.703	2.052	2.473	2.771
28		.683	1.313	1.701	2.048	2.467	2.763
(z) ∞		.674	1.282^a	1.645^b	1.960	2.326^c	2.576^d

[a]This value has been rounded to 1.28 in the textbook.
[b]This value has been rounded to 1.65 in the textbook.
[c]This value has been rounded to 2.33 in the textbook.
[d]This value has been rounded to 2.58 in the textbook.

Source: Adapted from W. H. Beyer, *Handbook of Tables for Probability and Statistics,* 2nd ed., CRC Press, Boca Raton, Florida, 1986. Reprinted with permission.

One tail

Area α

t

Two tails

Area $\frac{\alpha}{2}$

Area $\frac{\alpha}{2}$

$-t$ $+t$
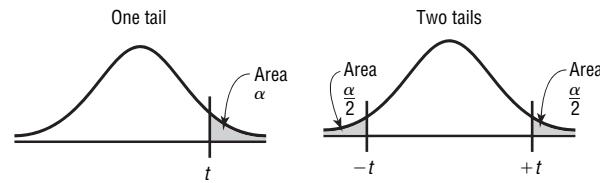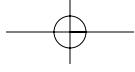
Table G The Chi-Square Distribution

| Degrees of freedom	α									
	0.995	*0.99*	*0.975*	*0.95*	*0.90*	*0.10*	*0.05*	*0.025*	*0.01*	*0.005*
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.262	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169
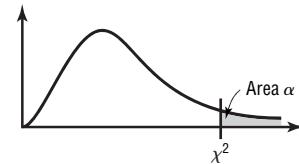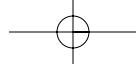
Table I   Critical Values for PPMC

Reject H_0: $\rho = 0$ if the absolute value of r is greater than the value given in the table. The values are for a two-tailed test; d.f. $= n - 2$.

d.f	$\alpha = 0.05$	$\alpha = 0.01$
1	0.999	0.999
2	0.950	0.999
3	0.878	0.959
4	0.811	0.917
5	0.754	0.875
6	0.707	0.834
7	0.666	0.798
8	0.632	0.765
9	0.602	0.735
10	0.576	0.708
11	0.553	0.684
12	0.532	0.661
13	0.514	0.641
14	0.497	0.623
15	0.482	0.606
16	0.468	0.590
17	0.456	0.575
18	0.444	0.561
19	0.433	0.549
20	0.423	0.537
25	0.381	0.487
30	0.349	0.449
35	0.325	0.418
40	0.304	0.393
45	0.288	0.372
50	0.273	0.354
60	0.250	0.325
70	0.232	0.302
80	0.217	0.283
90	0.205	0.267
100	0.195	0.254

Source: From *Biometrika Tables for Statisticians,* Vol. 1 (1962), p. 138. Reprinted with permission.