Preface

OBJECTIVES

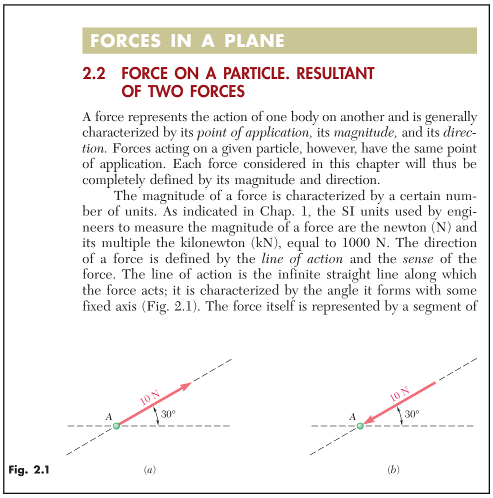
The main objective of a first course in mechanics should be to develop in the engineering student the ability to analyze any problem in a simple and logical manner and to apply to its solution a few, well-understood, basic principles. It is hoped that this text, designed for the first course in statics offered in the sophomore year, and the volume that follows, *Vector Mechanics for Engineers: Dynamics,* will help the instructor achieve this goal.†

GENERAL APPROACH

Vector analysis is introduced early in the text and is used in the presentation and discussion of the fundamental principles of mechanics. Vector methods are also used to solve many problems, particularly three-dimensional problems where these techniques result in a simpler and more concise solution. The emphasis in this text, however, remains on the correct understanding of the principles of mechanics and on their application to the solution of engineering problems, and vector analysis is presented chiefly as a convenient tool.‡

Practical Applications Are Introduced Early. One of the characteristics of the approach used in these volumes is that mechanics of *particles* is clearly separated from the mechanics of *rigid bodies.* This approach makes it possible to consider simple practical applications at an early stage and to postpone the introduction of the more difficult concepts. For example:

- In *Statics,* statics of particles is treated first (Chap. 2); after the rules of addition and subtraction of vectors are introduced, the principle of equilibrium of a particle is immediately applied to practical situations involving only concurrent forces. The statics of rigid bodies is considered in Chaps. 3 and 4. In Chap. 3, the vector and scalar products of two vectors are introduced and used to define the moment of a force about a point and about an axis. The presentation of these new concepts is followed by a thorough and rigorous discussion of equivalent systems of forces leading, in Chap. 4, to many practical applications involving the equilibrium of rigid bodies under general force systems.
- In *Dynamics,* the same division is observed. The basic concepts of force, mass, and acceleration, of work and energy, and


FORCES IN A PLANE

2.2 FORCE ON A PARTICLE. RESULTANT OF TWO FORCES

A force represents the action of one body on another and is generally characterized by its *point of application,* its *magnitude,* and its *direction.* Forces acting on a given particle, however, have the same point of application. Each force considered in this chapter will thus be completely defined by its magnitude and direction.

The magnitude of a force is characterized by a certain number of units. As indicated in Chap. 1, the SI units used by engineers to measure the magnitude of a force are the newton (N) and its multiple the kilonewton (kN), equal to 1000 N. The direction of a force is defined by the *line of action* and the *sense* of the force. The line of action is the infinite straight line along which the force acts; it is characterized by the angle it forms with some fixed axis (Fig. 2.1). The force itself is represented by a segment of

Fig. 2.1

†This text is available in a single volume, *Vector Mechanics for Engineers: Statics and Dynamics,* tenth edition.

‡In a parallel text, *Mechanics for Engineers: Statics,* fifth edition, the use of vector algebra is limited to the addition and subtraction of vectors.

4.1 INTRODUCTION

We saw in the preceding chapter that the external forces acting on a rigid body can be reduced to a force-couple system at some arbitrary point O. When the force and the couple are both equal to zero, the external forces form a system equivalent to zero, and the rigid body is said to be in *equilibrium.*

The necessary and sufficient conditions for the equilibrium of a rigid body, therefore, can be obtained by setting $\mathbf{R}$ and $\mathbf{M}_O^R$ equal to zero in the relations (3.52) of Sec. 3.17:

$$\Sigma \mathbf{F} = 0 \qquad \Sigma \mathbf{M}_O = \Sigma(\mathbf{r} \times \mathbf{F}) = 0 \tag{4.1}$$

Resolving each force and each moment into its rectangular components, we can express the necessary and sufficient conditions for the equilibrium of a rigid body with the following six scalar equations:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{4.2}$$
$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0 \tag{4.3}$$

The equations obtained can be used to determine unknown forces applied to the rigid body or unknown reactions exerted on it by its supports. We note that Eqs. (4.2) express the fact that the components of the external forces in the x, y, and z directions are balanced; Eqs. (4.3) express the fact that the moments of the external forces about the x, y, and z axes are balanced. Therefore, for a rigid body in equilibrium, the system of the external forces will impart no translational or rotational motion to the body considered.

In order to write the equations of equilibrium for a rigid body, it is essential to first identify all of the forces acting on that body and then to draw the corresponding *free-body diagram.* In this chapter we first consider the equilibrium of *two-dimensional structures* subjected to forces contained in their planes and learn how to draw their free-body diagrams. In addition to the forces *applied* to a structure, the *reactions* exerted on the structure by its supports will be considered. A specific reaction will be associated with each type of support. You will learn how to determine whether the structure is properly supported, so that you can know in advance whether the equations of equilibrium can be solved for the unknown forces and reactions.

Later in the chapter, the equilibrium of three-dimensional structures will be considered, and the same kind of analysis will be given to these structures and their supports.

4.2 FREE-BODY DIAGRAM

4.2 Free-Body Diagram 161

In solving a problem concerning the equilibrium of a rigid body, it is essential to consider *all* of the forces acting on the body; it is equally important to exclude any force which is not directly applied to the body. Omitting a force or adding an extraneous one would destroy the conditions of equilibrium. Therefore, the first step in the solution of the problem should be to draw a *free-body diagram* of the rigid body under consideration. Free-body diagrams have already been used on many occasions in Chap. 2. However, in view of their importance to the solution of equilibrium problems, we summarize here the various steps which must be followed in drawing a free-body diagram.

1. A clear decision should be made regarding the choice of the free body to be used. This body is then detached from the ground and is separated from all other bodies. The contour of the body thus isolated is sketched.
2. All external forces should be indicated on the free-body diagram. These forces represent the actions exerted *on* the free body *by* the ground and *by* the bodies which have been detached; they should be applied at the various points where the free body was supported by the ground or was connected to the other bodies. The *weight* of the free body should also be included among the external forces, since it represents the attraction exerted by the earth on the various particles forming the free body. As will be seen in Chap. 5, the weight should be applied at the center of gravity of the body. When the free body is made of several parts, the forces the various parts exert on each other should *not* be included among the external forces. These forces are internal forces as far as the free body is concerned.
3. The magnitudes and directions of the *known external forces* should be clearly marked on the free-body diagram. When indicating the directions of these forces, it must be remembered that the forces shown on the free-body diagram must be those which are exerted *on,* and not *by,* the free body. Known external forces generally include the *weight* of the free body and *forces applied* for a given purpose.
4. *Unknown external forces* usually consist of the *reactions,* through which the ground and other bodies oppose a possible motion of the free body. The reactions constrain the free body to remain in the same position, and, for that reason, are sometimes called *constraining forces.* Reactions are exerted at the points where the free body is *supported by* or *connected to* other bodies and should be clearly indicated. Reactions are discussed in detail in Secs. 4.3 and 4.8.
5. The free-body diagram should also include dimensions, since these may be needed in the computation of moments of forces. Any other detail, however, should be omitted.

Photo 4.1 A free-body diagram of the tractor shown would include all of the external forces acting on the tractor: the weight of the tractor, the weight of the load in the bucket, and the forces exerted by the ground on the tires.

Photo 4.2 In Chap. 6, we will discuss how to determine the internal forces in structures made of several connected pieces, such as the forces in the members that support the bucket of the tractor of Photo 4.1.

of impulse and momentum are introduced and first applied to problems involving only particles. Thus, students can familiarize themselves with the three basic methods used in dynamics and learn their respective advantages before facing the difficulties associated with the motion of rigid bodies.

New Concepts Are Introduced in Simple Terms. Since this text is designed for the first course in statics, new concepts are presented in simple terms and every step is explained in detail. On the other hand, by discussing the broader aspects of the problems considered, and by stressing methods of general applicability, a definite maturity of approach is achieved. For example, the concepts of partial constraints and statical indeterminacy are introduced early and are used throughout.

Fundamental Principles Are Placed in the Context of Simple Applications. The fact that mechanics is essentially a *deductive* science based on a few fundamental principles is stressed. Derivations have been presented in their logical sequence and with all the rigor warranted at this level. However, the learning process being largely *inductive,* simple applications are considered first. For example:

- The statics of particles precedes the statics of rigid bodies, and problems involving internal forces are postponed until Chap. 6.
- In Chap. 4, equilibrium problems involving only coplanar forces are considered first and solved by ordinary algebra, while problems involving three-dimensional forces and requiring the full use of vector algebra are discussed in the second part of the chapter.

Free-Body Diagrams Are Used Both to Solve Equilibrium Problems and to Express the Equivalence of Force Systems. Free-body diagrams are introduced early, and their importance is emphasized throughout the text. They are used not only to solve equilibrium problems but also to express the equivalence of two systems of forces or, more generally, of two systems of vectors. The advantage of this approach becomes apparent in the study of the dynamics of rigid bodies, where it is used to solve three-dimensional as well as two-dimensional problems. By placing the emphasis on "free-body-diagram equations" rather than on the standard algebraic equations of motion, a more intuitive and more complete understanding of the fundamental principles of dynamics can be achieved. This approach, which was first introduced in 1962 in the first edition of *Vector Mechanics for Engineers,* has now gained wide acceptance among mechanics teachers in this country. It is, therefore, used in preference to the method of dynamic equilibrium and to the equations of motion in the solution of all sample problems in this book.

A Four-Color Presentation Uses Color to Distinguish Vectors. Color has been used, not only to enhance the quality of the illustrations, but also to help students distinguish among the various types of vectors they will encounter. While there was no intention to "color code" this text, the same color is used in any given chapter to represent vectors of the same type. Throughout *Statics,* for example, red is used exclusively to represent forces and couples, while position vectors are shown in blue and dimensions in black. This makes it easier for the students to identify the forces acting on a given particle or rigid body and to follow the discussion of sample problems and other examples given in the text.

Optional Sections Offer Advanced or Specialty Topics. A large number of optional sections have been included. These sections are indicated by asterisks and thus are easily distinguished from those which form the core of the basic statics course. They may be omitted without prejudice to the understanding of the rest of the text.

Among the topics covered in these additional sections are the reduction of a system of forces to a wrench, applications to hydrostatics, shear and bending-moment diagrams for beams, equilibrium of cables, products of inertia and Mohr's circle, the determination of the principal axes and the mass moments of inertia of a body of arbitrary shape, and the method of virtual work. The sections on beams are especially useful when the course in statics is immediately followed by a course in mechanics of materials, while the sections on the inertia properties of three-dimensional bodies are primarily intended for students who will later study in dynamics the three-dimensional motion of rigid bodies.

The material presented in the text and most of the problems require no previous mathematical knowledge beyond algebra, trigonometry, and elementary calculus; all the elements of vector algebra necessary to the understanding of the text are carefully presented in Chaps. 2 and 3. In general, a greater emphasis is placed on the correct understanding of the basic mathematical concepts involved than on the nimble manipulation of mathematical formulas. In this connection, it should be mentioned that the determination of the centroids of composite areas precedes the calculation of centroids by integration, thus making it possible to establish the concept of the moment of an area firmly before introducing the use of integration.