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## SUPPLEMENT

# 1

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## INTRODUCTION TO POWER ELECTRONICS

Over the last 40 years, a revolution has occurred in the application of electric motors. The development of solid-state motor drive packages has progressed to the point where practically any power control problem can be solved by using them. With such solid-state drives, it is possible to run dc motors from ac power supplies or ac motors from dc power supplies. It is even possible to change ac power at one frequency to ac power at another frequency.

Furthermore, the costs of solid-state drive systems have decreased dramatically, while their reliability has increased. The versatility and the relatively low cost of solid-state controls and drives have resulted in many new applications for ac motors in which they are doing jobs formerly done by dc machines. DC motors have also gained flexibility from the application of solid-state drives.

This major change has resulted from the development and improvement of a series of high-power solid-state devices. Although the detailed study of such power electronic circuits and components would require a book in itself, some familiarity with them is important to an understanding of modern motor applications.

This chapter is a brief introduction to high-power electronic components and to the circuits in which they are employed. It is placed at this point in the book because the material contained in it is used in the discussions of both ac motor controllers and dc motor controllers.

### **S1.1 POWER ELECTRONIC COMPONENTS**

Several major types of semiconductor devices are used in motor-control circuits. Among the more important are

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1. The diode
2. The two-wire thyristor (or PNP diode)
3. The three-wire thyristor [or silicon controlled rectifier (SCR)]
4. The gate turnoff (GTO) thyristor
5. The DIAC
6. The TRIAC
7. The power transistor (PTR)
8. The insulated-gate bipolar transistor (IGBT)

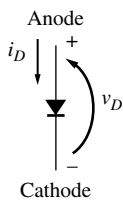
Circuits containing these eight devices are studied in this chapter. Before the circuits are examined, though, it is necessary to understand what each device does.

### The Diode

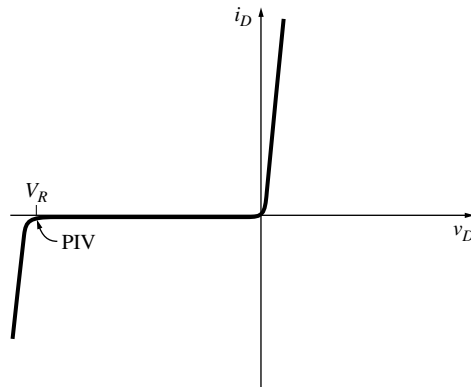
A *diode* is a semiconductor device designed to conduct current in one direction only. The symbol for this device is shown in Figure S1-1. A diode is designed to conduct current from its anode to its cathode, but not in the opposite direction.

The voltage-current characteristic of a diode is shown in Figure S1-2. When a voltage is applied to the diode in the forward direction, a large current flow results. When a voltage is applied to the diode in the reverse direction, the current flow is limited to a very small value (on the order of microamperes or less). If a large enough reverse voltage is applied to the diode, eventually the diode will break down and allow current to flow in the reverse direction. These three regions of diode operation are shown on the characteristic in Figure S1-2.

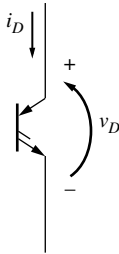
Diodes are rated by the amount of power they can safely dissipate and by the maximum reverse voltage that they can take before breaking down. The power



**FIGURE S1-1**  
The symbol of a diode.



**FIGURE S1-2**  
Voltage-current characteristic of a diode.



**FIGURE S1-3**  
The symbol of a two-wire thyristor or PNPN diode.

dissipated by a diode during forward operation is equal to the forward voltage drop across the diode times the current flowing through it. This power must be limited to protect the diode from overheating. The maximum reverse voltage of a diode is known as its *peak inverse voltage* (PIV). It must be high enough to ensure that the diode does not break down in a circuit and conduct in the reverse direction.

Diodes are also rated by their switching time, that is, by the time it takes to go from the off state to the on state, and vice versa. Because power diodes are large, high-power devices with a lot of stored charge in their junctions, they switch states much more slowly than the diodes found in electronic circuits. Essentially all power diodes can switch states fast enough to be used as rectifiers in 50- or 60-Hz circuits. However, some applications such as pulse-width modulation (PWM) can require power diodes to switch states at rates higher than 10,000 Hz. For these very fast switching applications, special diodes called *fast-recovery high-speed diodes* are employed.

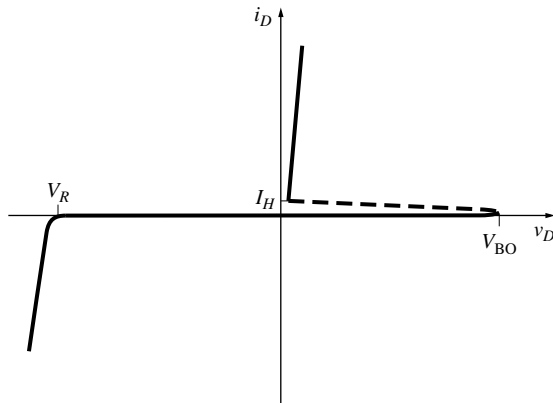
### The Two-Wire Thyristor or PNPN Diode

*Thyristor* is the generic name given to a family of semiconductor devices which are made up of four semiconductor layers. One member of this family is the two-wire thyristor, also known as the *PNPN diode* or *trigger diode*. This device's name in the Institute of Electrical and Electronics Engineers (IEEE) standard for graphic symbols is *reverse-blocking diode-type thyristor*. Its symbol is shown in Figure S1-3.

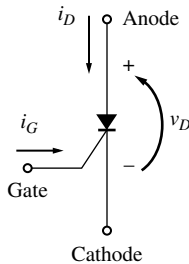
The PNPN diode is a rectifier or diode with an unusual voltage-current characteristic in the forward-biased region. Its voltage-current characteristic is shown in Figure S1-4. The characteristic curve consists of three regions:

1. The reverse-blocking region
2. The forward-blocking region
3. The conducting region

In the reverse-blocking region, the PNPN diode behaves as an ordinary diode and blocks all current flow until the reverse breakdown voltage is reached. In the conducting region, the PNPN diode again behaves as an ordinary diode, allowing large amounts of current to flow with very little voltage drop. It is the forward-blocking region that distinguishes a PNPN diode from an ordinary diode.



**FIGURE S1-4**  
Voltage-current characteristic of  
a PNP diode.



**FIGURE S1-5**  
The symbol of a three-wire thyristor or SCR.

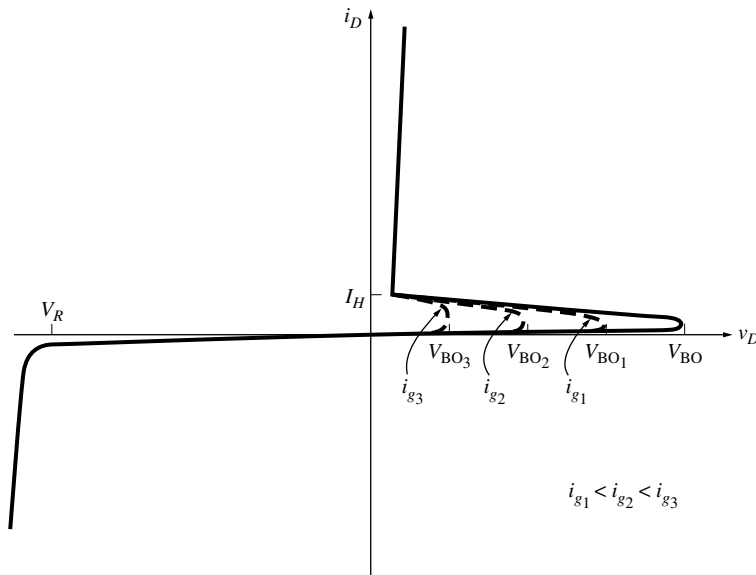
When a PNP diode is forward-biased, no current flows until the forward voltage drop exceeds a certain value called the *breakover voltage*  $V_{BO}$ . When the forward voltage across the PNP diode exceeds  $V_{BO}$ , the PNP diode turns on and *remains on* until the current flowing through it falls below a certain minimum value (typically a few milliamperes). If the current is reduced to a value below this minimum value (called the *holding current*  $I_H$ ), the PNP diode turns off and will not conduct until the forward voltage drop again exceeds  $V_{BO}$ .

In summary, a PNP diode

1. Turns on when the applied voltage  $v_D$  exceeds  $V_{BO}$
2. Turns off when the current  $i_D$  drops below  $I_H$
3. Blocks all current flow in the reverse direction until the maximum reverse voltage is exceeded

### The Three-Wire Thyristor or SCR

The most important member of the thyristor family is the three-wire thyristor, also known as the *silicon controlled rectifier* or SCR. This device was developed and given the name SCR by the General Electric Company in 1958. The name *thyristor* was adopted later by the International Electrotechnical Commission (IEC). The symbol for a three-wire thyristor or SCR is shown in Figure S1-5.



**FIGURE S1-6**  
Voltage-current characteristics of an SCR.

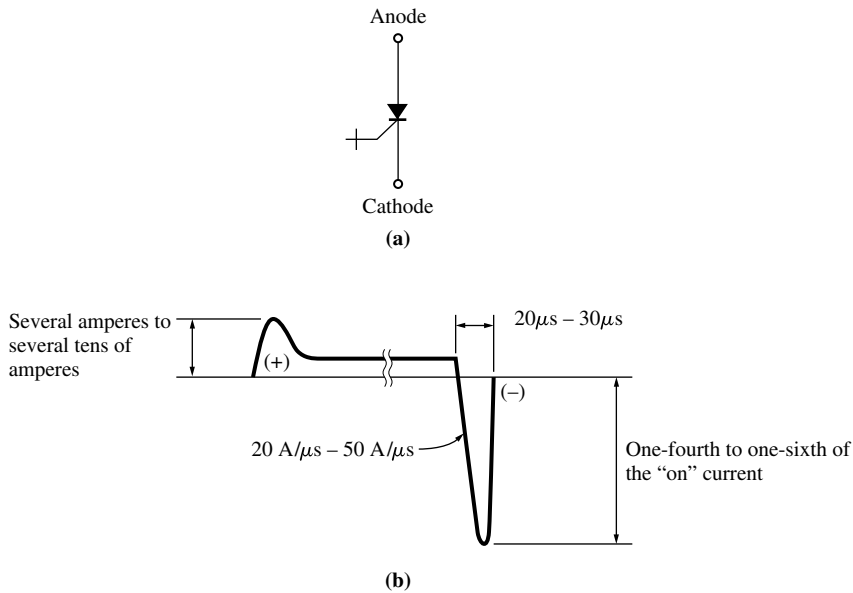
As the name suggests, the SCR is a *controlled* rectifier or diode. Its voltage-current characteristic with the gate lead open is the same as that of a PNP diode.

What makes an SCR especially useful in motor-control applications is that the *breakover or turn-on voltage of the device can be adjusted* by a current flowing into its gate lead. The larger the gate current, the lower  $V_{BO}$  becomes (see Figure S1-6). If an SCR is chosen so that its breakover voltage with no gate signal is larger than the highest voltage in the circuit, then it can *only* be turned on by the application of a gate current. Once it is on, the device stays on until its current falls below  $I_H$ . Therefore, once an SCR is triggered, its gate current may be removed without affecting the on state of the device. In the on state, the forward voltage drop across the SCR is about 1.2 to 1.5 times larger than the voltage drop across an ordinary forward-biased diode.

Three-wire thyristors or SCRs are by far the most common devices used in power-control circuits. They are widely used for switching or rectification applications and are currently available in ratings ranging from a few amperes up to a maximum of about 3000 A.

In summary, an SCR

1. Turns on when the voltage  $v_D$  applied to it exceeds  $V_{BO}$
2. Has a breakover voltage  $V_{BO}$  whose level is controlled by the amount of gate current  $i_G$  present in the SCR

**FIGURE S1-7**

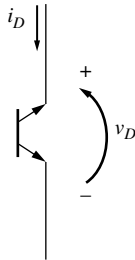
(a) The symbol of a gate turn-off thyristor (GTO). (b) The gate current waveform required to turn a GTO thyristor on and off.

3. Turns off when the current  $i_D$  flowing through it drops below  $I_H$
4. Blocks all current flow in the reverse direction until the maximum reverse voltage is exceeded

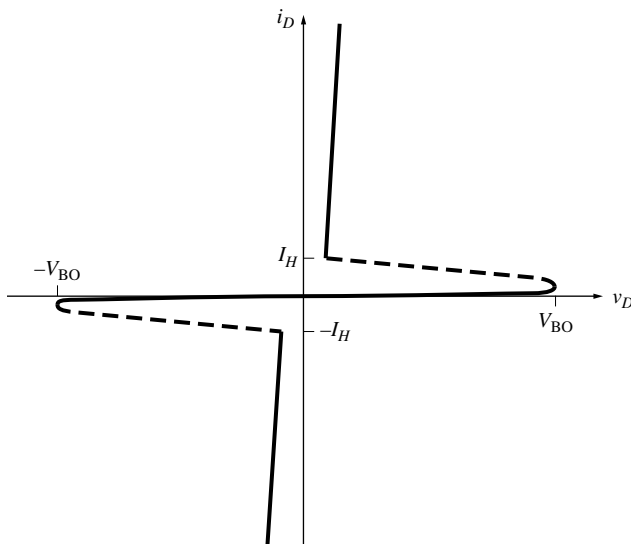
### The Gate Turnoff Thyristor

Among the recent improvements to the thyristor is the gate turnoff (GTO) thyristor. A GTO thyristor is an SCR that can be turned off by a large enough negative pulse at its gate lead even if the current  $i_D$  exceeds  $I_H$ . Although GTO thyristors have been around since the 1960s, they only became practical for motor-control applications in the late 1970s. Such devices are becoming more and more common in motor-control packages, since they eliminate the need for external components to turn off SCRs in dc circuits (see Section S1.5). The symbol for a GTO thyristor is shown in Figure S1-7a.

Figure S1-7b shows a typical gate current waveform for a high-power GTO thyristor. A GTO thyristor typically requires a larger gate current for turn-on than an ordinary SCR. For large high-power devices, gate currents on the order of 10 A or more are necessary. To turn off the device, a large negative current pulse of 20- to 30- $\mu\text{s}$  duration is required. The magnitude of the negative current pulse must be one-fourth to one-sixth that of the current flowing through the device.



**FIGURE S1-8**  
The symbol of a DIAC.



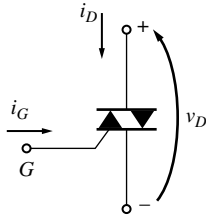
**FIGURE S1-9**  
Voltage-current characteristic of a DIAC.

### The DIAC

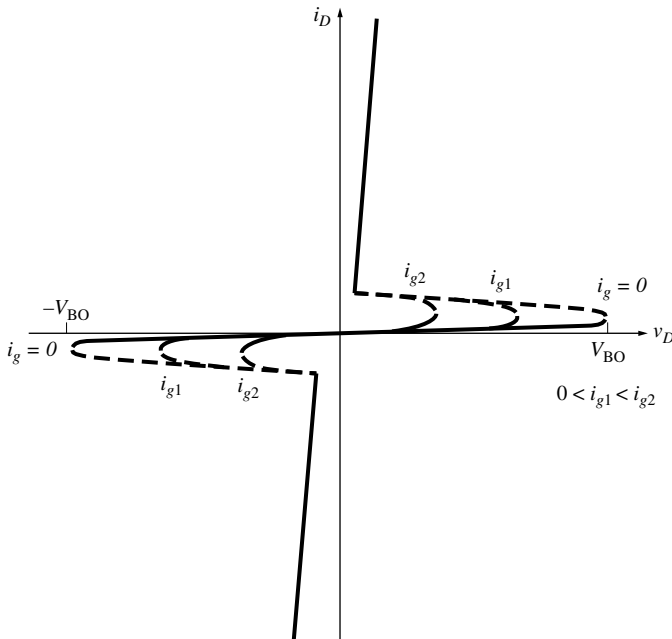
A DIAC is a device containing five semiconductor layers (PNPNP) that behaves like two PNP diodes connected back to back. It can conduct in either direction once the breakover voltage is exceeded. The symbol for a DIAC is shown in Figure S1-8, and its current-voltage characteristic is shown in Figure S1-9. It turns on when the applied voltage *in either direction* exceeds  $V_{BO}$ . Once it is turned on, a DIAC remains on until its current falls below  $I_H$ .

### The TRIAC

A TRIAC is a device that behaves like two SCRs connected back to back with a common gate lead. It can conduct in either direction once its breakover voltage



**FIGURE S1-10**  
The symbol of a TRIAC.

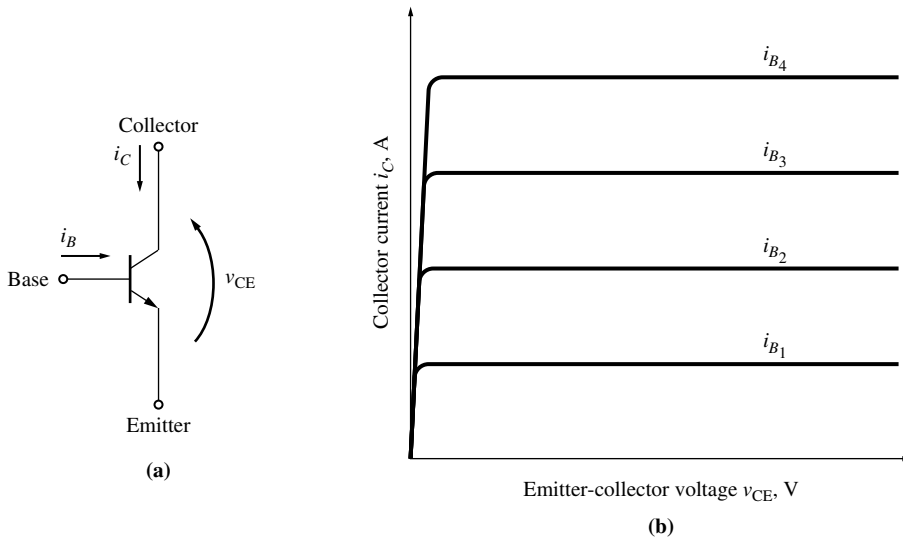


**FIGURE S1-11**  
Voltage-current characteristic of a TRIAC.

is exceeded. The symbol for a TRIAC is shown in Figure S1-10, and its current-voltage characteristic is shown in Figure S1-11. The breakover voltage in a TRIAC decreases with increasing gate current in just the same manner as it does in an SCR, except that a TRIAC responds to either positive or negative pulses at its gate. Once it is turned on, a TRIAC remains on until its current falls below  $I_H$ .

Because a single TRIAC can conduct in both directions, it can replace a more complex pair of back-to-back SCRs in many ac control circuits. However, TRIACs generally switch more slowly than SCRs, and are available only at lower power ratings. As a result, their use is largely restricted to low- to medium-power applications in 50- or 60-Hz circuits, such as simple lighting circuits.





**FIGURE S1-12**  
 (a) The symbol of a power transistor. (b) The voltage-current characteristic of a power transistor.

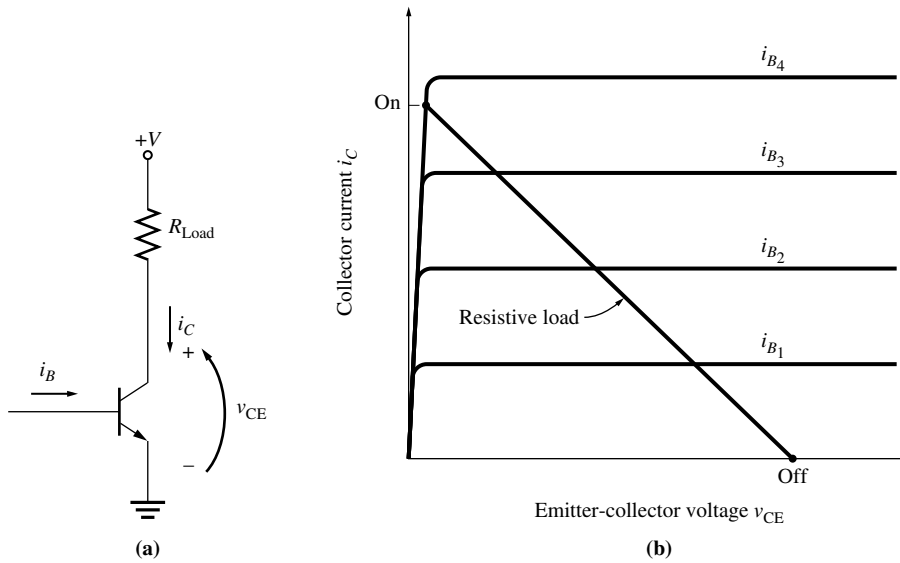
### The Power Transistor

The symbol for a transistor is shown in Figure S1-12a, and the collector-to-emitter voltage versus collector current characteristic for the device is shown in Figure S1-12b. As can be seen from the characteristic in Figure S1-12b, the transistor is a device whose collector current  $i_C$  is directly proportional to its base current  $i_B$  over a very wide range of collector-to-emitter voltages ( $v_{CE}$ ).

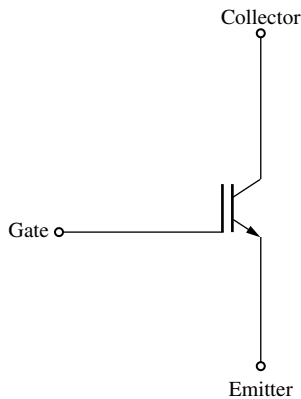
Power transistors (PTRs) are commonly used in machinery-control applications to switch a current on or off. A transistor with a resistive load is shown in Figure S1-13a, and its  $i_C$ - $v_{CE}$  characteristic is shown in Figure S1-13b with the load line of the resistive load. Transistors are normally used in machinery-control applications as switches; as such they should be either completely on or completely off. As shown in Figure S1-13b, a base current of  $i_{B4}$  would completely turn on this transistor, and a base current of zero would completely turn off the transistor.

If the base current of this transistor were equal to  $i_{B3}$ , then the transistor would be neither fully on nor fully off. This is a very undesirable condition, since a large collector current will flow across a large collector-to-emitter voltage  $v_{CE}$ , dissipating a lot of power in the transistor. To ensure that the transistor conducts without wasting a lot of power, it is necessary to have a base current high enough to completely saturate it.

Power transistors are most often used in inverter circuits. Their major drawback in switching applications is that large power transistors are relatively slow in changing from the on to the off state and vice versa, since a relatively large base current has to be applied or removed when they are turned on or off.



**FIGURE S1-13**  
 (a) A transistor with a resistive load. (b) The voltage-current characteristic of this transistor and load.



**FIGURE S1-14**  
 The symbol of an IGBT.

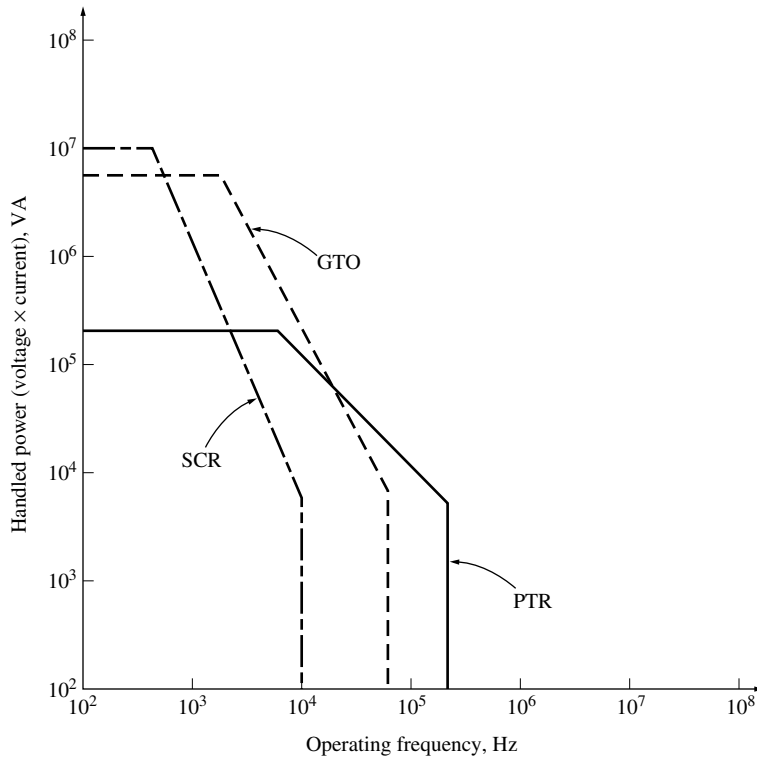
### The Insulated-Gate Bipolar Transistor

The *insulated-gate bipolar transistor* (IGBT) is a relatively recent development. It is similar to the power transistor, except that it is controlled by the voltage applied to a gate rather than the current flowing into the base as in the power transistor. The impedance of the control gate is very high in an IGBT, so the amount of current flowing in the gate is extremely small. The device is essentially equivalent to the combination of a metal-oxide-semiconductor field-effect transistor (MOSFET) and a power transistor. The symbol of an IGBT is shown in Figure S1-14.

Since the IGBT is controlled by a gate voltage with very little current flow, it can switch much more rapidly than a conventional power transistor can. IGBTs are therefore being used in high-power high-frequency applications.

### Power and Speed Comparison of Power Electronic Components

Figure S1–15 shows a comparison of the relative speeds and power-handling capabilities of SCRs, GTO thyristors, and power transistors. Clearly SCRs are capable of higher-power operation than any of the other devices. GTO thyristors can operate at almost as high a power and much faster than SCRs. Finally, power transistors can handle less power than either type of thyristor, but they can switch more than 10 times faster.



**FIGURE S1–15**

A comparison of the relative speeds and power-handling capabilities of SCRs, GTO thyristors, and power transistors.

## S1.2 BASIC RECTIFIER CIRCUITS

A rectifier circuit is a circuit that converts ac power to dc power. There are many different rectifier circuits which produce varying degrees of smoothing in their dc output. The four most common rectifier circuits are

1. The half-wave rectifier
2. The full-wave bridge rectifier
3. The three-phase half-wave rectifier
4. The three-phase full-wave rectifier

A good measure of the smoothness of the dc voltage out of a rectifier circuit is the *ripple factor* of the dc output. The *percentage of ripple* in a dc power supply is defined as the ratio of the rms value of the ac components in the supply's voltage to the dc value of the voltage

$$r = \frac{V_{ac,rms}}{V_{DC}} \times 100\% \quad (S1-1)$$

where  $V_{ac,rms}$  is the rms value of the ac components of the output voltage and  $V_{DC}$  is the dc component of voltage in the output. The smaller the ripple factor in a power supply, the smoother the resulting dc waveform.

The dc component of the output voltage  $V_{DC}$  is quite easy to calculate, since it is just the *average* of the output voltage of the rectifier:

$$V_{DC} = \frac{1}{T} \int v_0(t) dt \quad (S1-2)$$

The rms value of the ac part of the output voltage is harder to calculate, though, since the dc component of the voltage must be subtracted first. However, the ripple factor  $r$  can be calculated from a different but equivalent formula which does not require the rms value of the ac component of the voltage. This formula for ripple is

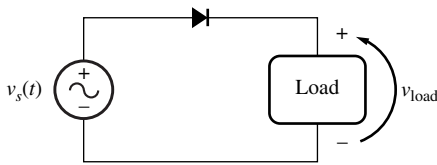
$$r = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1} \times 100\% \quad (S1-3)$$

where  $V_{rms}$  is the rms value of the total output voltage from the rectifier and  $V_{DC}$  is the dc or average output voltage from the rectifier.

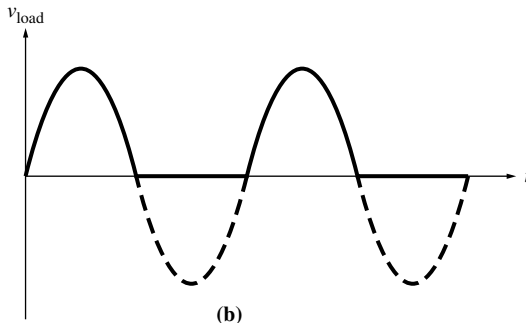
In the following discussion of rectifier circuits, the input ac frequency is assumed to be 60 Hz.

### The Half-Wave Rectifier

A half-wave rectifier is shown in Figure S1-16a, and its output is shown in Figure S1-16b. The diode conducts on the positive half-cycle and blocks current flow on



(a)



(b)

**FIGURE S1-16**  
 (a) A half-wave rectifier circuit.  
 (b) The output voltage of the rectifier circuit.

the negative half-cycle. A simple half-wave rectifier of this sort is an extremely poor approximation to a constant dc waveform—it contains ac frequency components at 60 Hz and all its harmonics. A half-wave rectifier such as the one shown has a ripple factor  $r = 121$  percent, which means it has more ac voltage components in its output than dc voltage components. Clearly, the half-wave rectifier is a very poor way to produce a dc voltage from an ac source.

**Example S1-1.** Calculate the ripple factor for the half-wave rectifier shown in Figure S1-16, both analytically and using MATLAB.

**Solution**

In Figure S1-16, the ac source voltage is  $v_s(t) = V_M \sin \omega t$  volts. The output voltage of the rectifier is

$$v_{\text{load}}(t) = \begin{cases} V_M \sin \omega t & 0 < \omega t < \pi \\ 0 & \pi \leq \omega t \leq 2\pi \end{cases}$$

Both the average voltage and the rms voltage must be calculated in order to calculate the ripple factor analytically. The average voltage out of the rectifier is

$$\begin{aligned} V_{\text{DC}} = V_{\text{avg}} &= \frac{1}{T} \int_0^T v_{\text{load}}(t) dt \\ &= \frac{\omega}{2\pi} \int_0^{\pi/\omega} V_M \sin \omega t dt \\ &= \frac{\omega}{2\pi} \left( -\frac{V_M}{\omega} \cos \omega t \right) \Big|_0^{\pi/\omega} \end{aligned}$$

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$$\begin{aligned}
 &= -\frac{V_M}{2\pi}[(-1) - (1)] \\
 &= \frac{V_M}{\pi}
 \end{aligned}$$

The rms value of the total voltage out of the rectifier is

$$\begin{aligned}
 V_{\text{rms}} &= \sqrt{\frac{1}{T} \int_0^T v_{\text{load}}^2(t) dt} \\
 &= \sqrt{\frac{\omega}{2\pi} \int_0^{\pi/\omega} V_M^2 \sin^2 \omega t dt} \\
 &= V_M \sqrt{\frac{\omega}{2\pi} \int_0^{\pi/\omega} \frac{1 - \cos 2\omega t}{2} dt} \\
 &= V_M \sqrt{\frac{\omega}{2\pi} \int_0^{\pi/\omega} \frac{1}{2} dt - \frac{\omega}{2\pi} \int_0^{\pi/\omega} \frac{1}{2} \cos 2\omega t dt} \\
 &= V_M \sqrt{\left( \frac{\omega}{4\pi} t - \frac{1}{8\pi} \sin 2\omega t \right) \Big|_0^{\pi/\omega}} \\
 &= V_M \sqrt{\left( \frac{1}{4} - \frac{1}{8\pi} \sin 2\pi \right) - \left( 0 - \frac{1}{8\pi} \sin 0 \right)} \\
 &= \frac{V_M}{2}
 \end{aligned}$$

Therefore, the ripple factor of this rectifier circuit is

$$r = \sqrt{\left( \frac{V_M/2}{V_M/\pi} \right)^2 - 1} \times 100\%$$

$$r = 121\%$$

The ripple factor can be calculated with MATLAB by implementing the average and rms voltage calculations in a MATLAB function, and then calculating the ripple from Equation (S1-3). The first part of the function shown below calculates the average of an input waveform, while the second part of the function calculates the rms value of the input waveform. Finally, the ripple factor is calculated directly from Equation (S1-3).

```

function r = ripple(waveform)
% Function to calculate the ripple on an input waveform.

% Calculate the average value of the waveform
nvals = size(waveform,2);
temp = 0;
for ii = 1:nvals
    temp = temp + waveform(ii);
end
average = temp/nvals;

% Calculate rms value of waveform
temp = 0;

```

```

for ii = 1:nvals
    temp = temp + waveform(ii)^2;
end
rms = sqrt(temp/nvals);

% Calculate ripple factor
r = sqrt((rms / average)^2 - 1) * 100;

```

Function `ripple` can be tested by writing an m-file to create a half-wave rectified waveform and supply that waveform to the function. The appropriate M-file is shown below:

```

% M-file: test_halfwave.m
% M-file to calculate the ripple on the output of a half-wave
% wave rectifier.

% First, generate the output of a half-wave rectifier
waveform = zeros(1,128);
for ii = 1:128
    waveform(ii) = halfwave(ii*pi/64);
end

% Now calculate the ripple factor
r = ripple(waveform);

% Print out the result
string = ['The ripple is ' num2str(r) '%.'];
disp(string);

```

The output of the half-wave rectifier is simulated by function `halfwave`.

```

function volts = halfwave(wt)
% Function to simulate the output of a half-wave rectifier.
% wt = Phase in radians (=omega x time)

% Convert input to the range 0 <= wt < 2*pi
while wt >= 2*pi
    wt = wt - 2*pi;
end
while wt < 0
    wt = wt + 2*pi;
end

% Simulate the output of the half-wave rectifier
if wt >= 0 & wt <= pi
    volts = sin(wt);
else
    volts = 0;
end
end

```

When `test_halfwave` is executed, the results are:

```

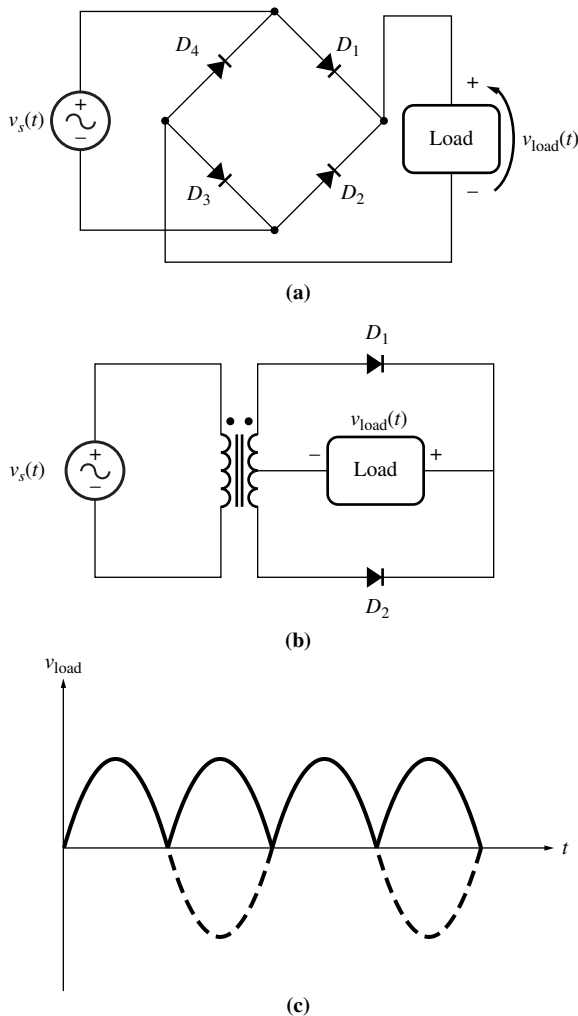
> test_halfwave
The ripple is 121.1772%.

```

This answer agrees with the analytic solution calculated above.

### The Full-Wave Rectifier

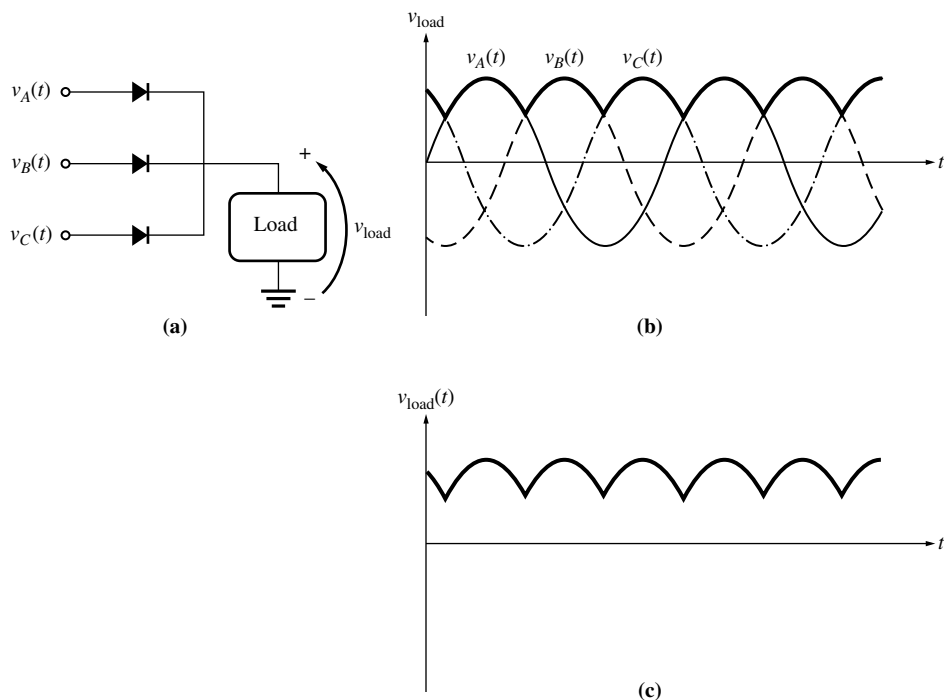
A full-wave bridge rectifier circuit is shown in Figure S1-17a, and its output voltage is shown in Figure S1-17c. In this circuit, diodes  $D_1$  and  $D_3$  conduct on the positive half-cycle of the ac input, and diodes  $D_2$  and  $D_4$  conduct on the negative half-cycle. The output voltage from this circuit is smoother than the output voltage from the half-wave rectifier, but it still contains ac frequency components at 120 Hz and its harmonics. The ripple factor of a full-wave rectifier of this sort is  $r = 48.2$  percent—it is clearly much better than that of a half-wave circuit.



**FIGURE S1-17**

(a) A full-wave bridge rectifier circuit. (b) The output voltage of the rectifier circuit. (c) An alternative full-wave rectifier circuit using two diodes and a center-tapped transformer.



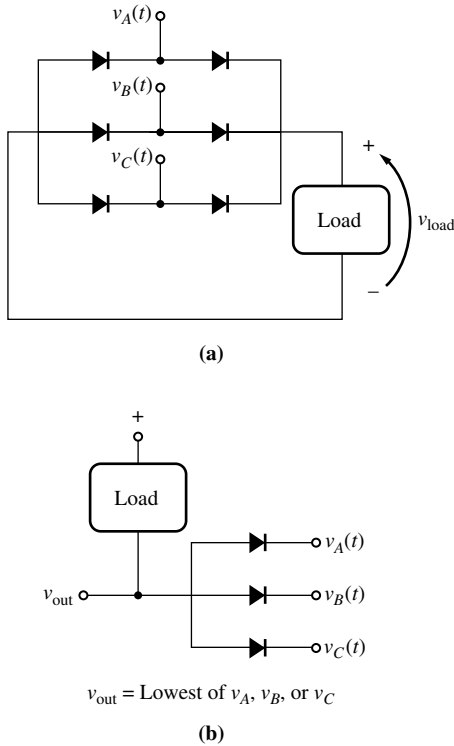
**FIGURE S1-18**

(a) A three-phase half-wave rectifier circuit. (b) The three-phase input voltages to the rectifier circuit. (c) The output voltage of the rectifier circuit.

Another possible full-wave rectifier circuit is shown in Figure S1-17b. In this circuit, diode  $D_1$  conducts on the positive half-cycle of the ac input with the current returning through the center tap of the transformer, and diode  $D_2$  conducts on the negative half-cycle of the ac input with the current returning through the center tap of the transformer. The output waveform is identical to the one shown in Figure S1-17c.

### The Three-Phase Half-Wave Rectifier

A three-phase half-wave rectifier is shown in Figure S1-18a. The effect of having three diodes with their cathodes connected to a common point is that *at any instant the diode with the largest voltage applied to it will conduct, and the other two diodes will be reverse-biased*. The three phase voltages applied to the rectifier circuit are shown in Figure S1-18b, and the resulting output voltage is shown in Figure S1-18c. Notice that the voltage at the output of the rectifier at any time is just the highest of the three input voltages at that moment.

**FIGURE S1-19**

(a) A three-phase full-wave rectifier circuit.  
 (b) This circuit places the *lowest* of its three input voltages at its output.

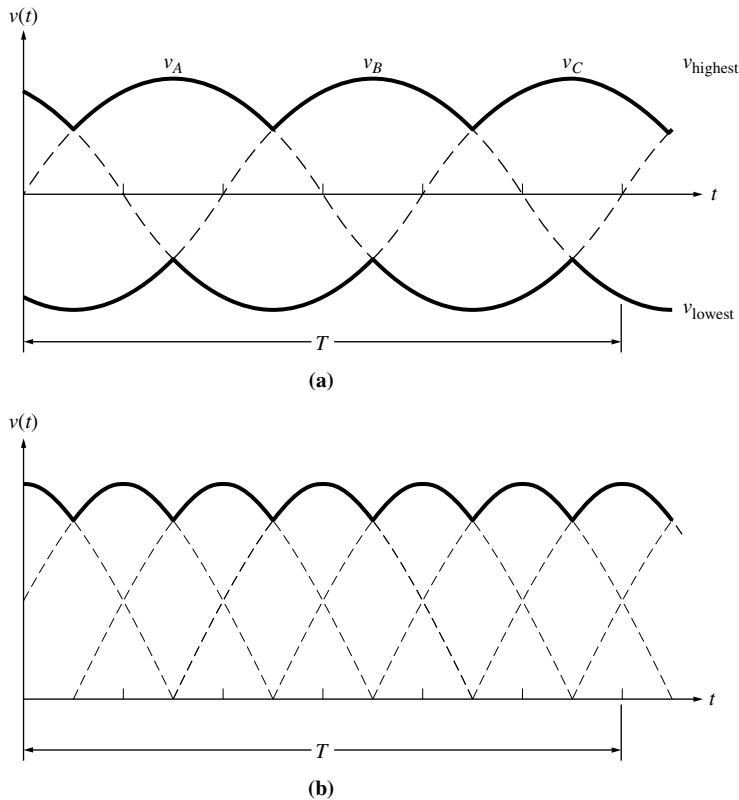
This output voltage is even smoother than that of a full-wave bridge rectifier circuit. It contains ac voltage components at 180 Hz and its harmonics. The ripple factor for a rectifier of this sort is 18.3 percent.

### The Three-Phase Full-Wave Rectifier

A three-phase full-wave rectifier is shown in Figure S1-19a. Basically, a circuit of this sort can be divided into two component parts. One part of the circuit looks just like the three-phase half-wave rectifier in Figure S1-18, and it serves to connect the highest of the three phase voltages at any given instant to the load.

The other part of the circuit consists of three diodes oriented with their anodes connected to the load and their cathodes connected to the supply voltages (Figure S1-19b). This arrangement connects the *lowest* of the three supply voltages to the load at any given time.

Therefore, the three-phase full-wave rectifier at all times connects the *highest* of the three voltages to one end of the load and always connects the *lowest* of the three voltages to the other end of the load. The result of such a connection is shown in Figure S1-20.

**FIGURE S1-20**

(a) The highest and lowest voltages in the three-phase full-wave rectifier. (b) The resulting output voltage.

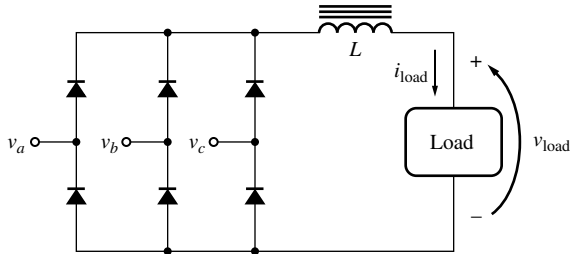
The output of a three-phase full-wave rectifier is even smoother than the output of a three-phase half-wave rectifier. The lowest ac frequency component present in it is 360 Hz, and the ripple factor is only 4.2 percent.

### Filtering Rectifier Output

The output of any of these rectifier circuits may be further smoothed by the use of low-pass filters to remove more of the ac frequency components from the output. Two types of elements are commonly used to smooth the rectifier's output:

1. Capacitors connected across the lines to smooth ac voltage changes
2. Inductors connected in series with the line to smooth ac current changes

A common filter in rectifier circuits used with machines is a single series inductor, or *choke*. A three-phase full-wave rectifier with a choke filter is shown in Figure S1-21.



**FIGURE S1-21**

A three-phase full-wave bridge circuit with an inductive filter for reducing output ripple.

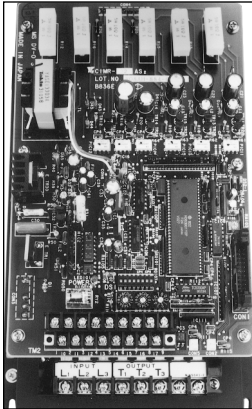
### S1.3 PULSE CIRCUITS

The SCRs, GTO thyristors, and TRIACs described in Section S1.1 are turned on by the application of a pulse of current to their gating circuits. To build power controllers, it is necessary to provide some method of producing and applying pulses to the gates of these devices at the proper time to turn them on. (In addition, it is necessary to provide some method of producing and applying negative pulses to the gates of GTO thyristors at the proper time to turn them off.)

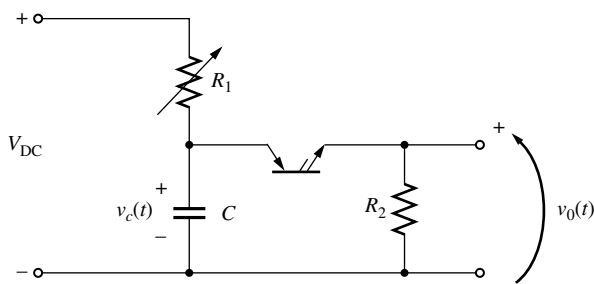
Many techniques are available to produce voltage and current pulses. They may be divided into two broad categories: analog and digital. Analog pulse generation circuits have been used since the earliest days of solid-state machinery controls. They typically rely on devices such as PNP diodes that have voltage-current characteristics with discrete nonconducting and conducting regions. The transition from the nonconducting to the conducting region of the device (or vice versa) is used to generate a voltage and current pulse. Some simple analog pulse generation circuits are described in this section. These circuits are collectively known as *relaxation oscillators*.

Digital pulse generation circuits are becoming very common in modern solid-state motor drives. They typically contain a microcomputer that executes a program stored in *read-only memory* (ROM). The computer program may consider many different inputs in deciding the proper time to generate firing pulses. For example, it may consider the desired speed of the motor, the actual speed of the motor, the rate at which it is accelerating or decelerating, and any specified voltage or current limits in determining the time to generate the firing pulses. The inputs that it considers and the relative weighting applied to those inputs can usually be changed by setting switches on the microcomputer's circuit board, making solid-state motor drives with digital pulse generation circuits very flexible. A typical digital pulse generation circuit board from a pulse-width-modulated induction motor drive is shown in Figure S1-22. Examples of solid-state ac and dc motor drives containing such digital firing circuits are described in Chapters 7 and 9, respectively.

The production of pulses for triggering SCRs, GTOs, and TRIACs is one of the most complex aspects of solid-state power control. The simple analog circuits

**FIGURE S1–22**

A typical digital pulse generation circuit board from a pulse-width-modulated (PWM) induction motor drive. (Courtesy of MagneTek Drives and Systems.)

**FIGURE S1–23**

A relaxation oscillator (or pulse generator) using a PNPN diode.

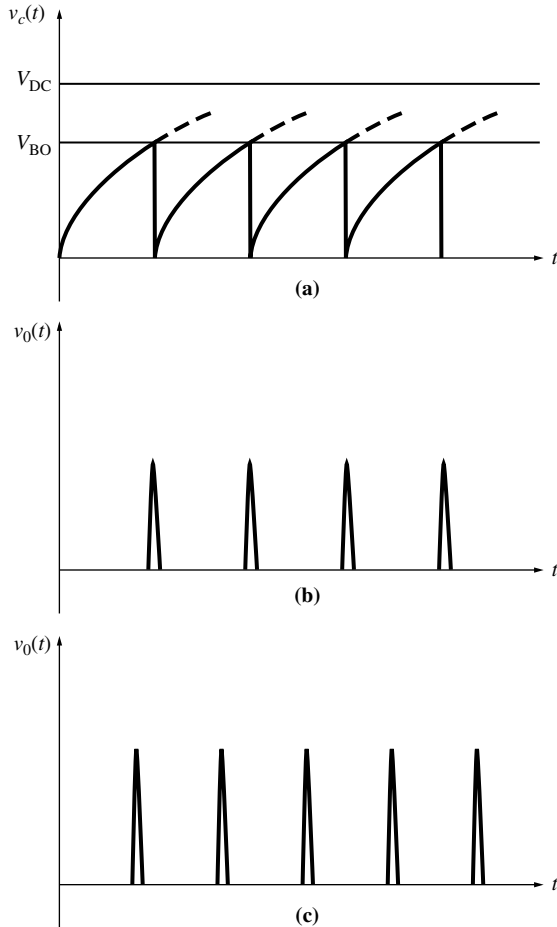
shown here are examples of only the most primitive types of pulse-producing circuits—more advanced ones are beyond the scope of this book.

### A Relaxation Oscillator Using a PNPN Diode

Figure S1–23 shows a relaxation oscillator or pulse-generating circuit built with a PNPN diode. In order for this circuit to work, the following conditions must be true:

1. The power supply voltage  $V_{DC}$  must exceed  $V_{BO}$  for the PNPN diode.
2.  $V_{DC}/R_1$  must be less than  $I_H$  for the PNPN diode.
3.  $R_1$  must be much larger than  $R_2$ .

When the switch in the circuit is first closed, capacitor  $C$  will charge through resistor  $R_1$  with time constant  $\tau = R_1C$ . As the voltage on the capacitor builds up, it will eventually exceed  $V_{BO}$  and the PNPN diode will turn on. Once

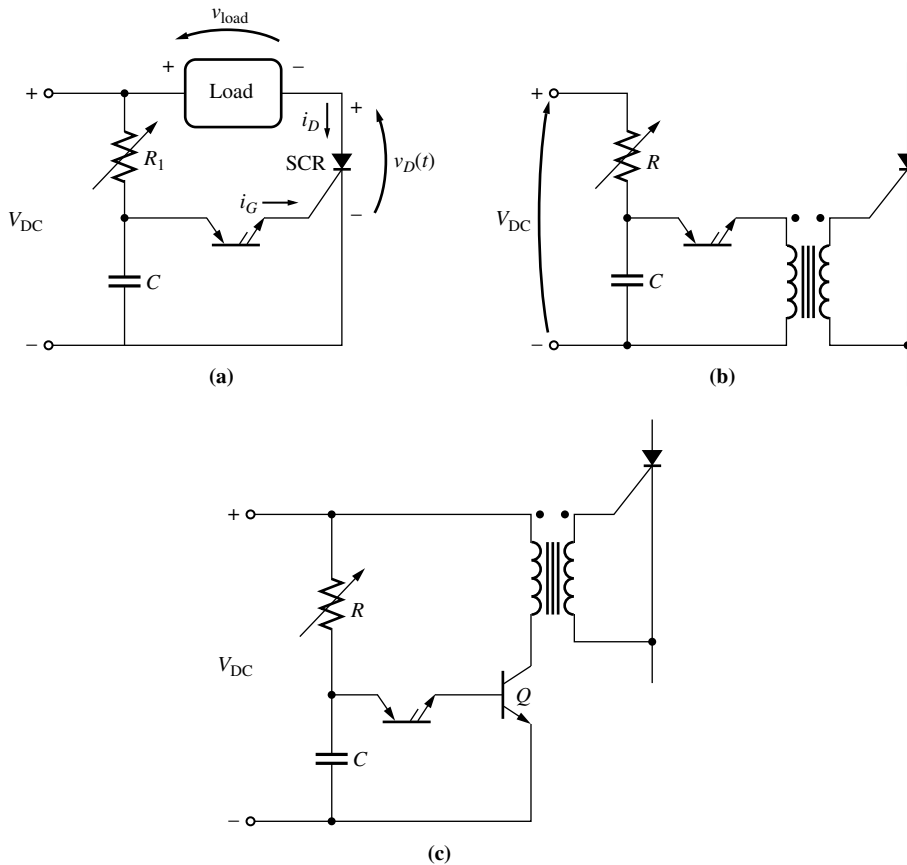
**FIGURE S1-24**

(a) The voltage across the capacitor in the relaxation oscillator. (b) The output voltage of the relaxation oscillator. (c) The output voltage of the oscillator after  $R_1$  is decreased.

the PNPN diode turns on, the capacitor will discharge through it. The discharge will be very rapid because  $R_2$  is very small compared to  $R_1$ . Once the capacitor is discharged, the PNPN diode will turn off, since the steady-state current coming through  $R_1$  is less than the current  $I_H$  of the PNPN diode.

The voltage across the capacitor and the resulting output voltage and current are shown in Figure S1-24a and b, respectively.

The timing of these pulses can be changed by varying  $R_1$ . Suppose that resistor  $R_1$  is decreased. Then the capacitor will charge more quickly, and the PNPN diode will be triggered sooner. The pulses will thus occur closer together (see Figure S1-24c).

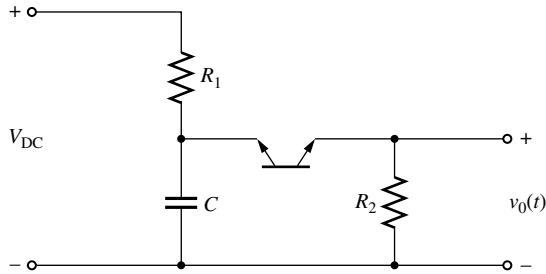

**FIGURE S1-25**

(a) Using a pulse generator to directly trigger an SCR. (b) Coupling a pulse generator to an SCR through a transformer. (c) Connecting a pulse generator to an SCR through a transistor amplifier to increase the strength of the pulse.

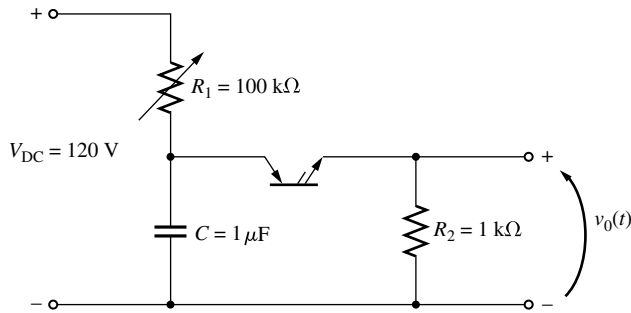
This circuit can be used to trigger an SCR directly by removing  $R_2$  and connecting the SCR gate lead in its place (see Figure S1-25a). Alternatively, the pulse circuit can be coupled to the SCR through a transformer, as shown in Figure S1-25b. If more gate current is needed to drive the SCR or TRIAC, then the pulse can be amplified by an extra transistor stage, as shown in Figure S1-25c.

The same basic circuit can also be built by using a DIAC in place of the PNPN diode (see Figure S1-26). It will function in exactly the same fashion as previously described.

In general, the quantitative analysis of pulse generation circuits is very complex and beyond the scope of this book. However, one simple example using a relaxation oscillator follows. It may be skipped with no loss of continuity, if desired.



**FIGURE S1-26**  
A relaxation oscillator using a DIAC instead of a PNPN diode.



**FIGURE S1-27**  
The relaxation oscillator of Example S1-2.

**Example S1-2.** Figure S1-27 shows a simple relaxation oscillator using a PNPN diode. In this circuit,

$$V_{DC} = 120 \text{ V} \quad R_1 = 100 \text{ k}\Omega$$

$$C = 1 \text{ }\mu\text{F} \quad R_2 = 1 \text{ k}\Omega$$

$$V_{BO} = 75 \text{ V} \quad I_H = 10 \text{ mA}$$

- (a) Determine the firing frequency of this circuit.
- (b) Determine the firing frequency of this circuit if  $R_1$  is increased to 150 k $\Omega$ .

**Solution**

- (a) When the PNPN diode is turned off, capacitor  $C$  charges through resistor  $R_1$  with a time constant  $\tau = R_1C$ , and when the PNPN diode turns on, capacitor  $C$  discharges through resistor  $R_2$  with time constant  $\tau = R_2C$ . (Actually, the discharge rate is controlled by the parallel combination of  $R_1$  and  $R_2$ , but since  $R_1 \gg R_2$ , the parallel combination is essentially the same as  $R_2$  itself.) From elementary circuit theory, the equation for the voltage on the capacitor as a function of time during the charging portion of the cycle is

$$v_C(t) = A + B e^{-t/R_1C}$$



where  $A$  and  $B$  are constants depending on the initial conditions in the circuit. Since  $v_C(0) = 0$  V and  $v_C(\infty) = V_{DC}$ , it is possible to solve for  $A$  and  $B$ :

$$\begin{aligned} A &= v_C(\infty) = V_{DC} \\ A + B &= v_C(0) = 0 \Rightarrow B = -V_{DC} \end{aligned}$$

Therefore,

$$\boxed{v_C(t) = V_{DC} - V_{DC} e^{-t/R_1 C}} \quad (\text{S1-4})$$

The time at which the capacitor will reach the breakover voltage is found by solving for time  $t$  in Equation (S1-4):

$$t_1 = -R_1 C \ln \frac{V_{DC} - V_{BO}}{V_{DC}} \quad (\text{S1-5})$$

In this case,

$$\begin{aligned} t_1 &= -(100 \text{ k}\Omega)(1 \text{ }\mu\text{F}) \ln \frac{120 \text{ V} - 75 \text{ V}}{120 \text{ V}} \\ &= 98 \text{ ms} \end{aligned}$$

Similarly, the equation for the voltage on the capacitor as a function of time during the discharge portion of the cycle turns out to be

$$v_C(t) = V_{BO} e^{-t/R_2 C} \quad (\text{S1-6})$$

so the current flow through the PNP diode becomes

$$i(t) = \frac{V_{BO}}{R_2} e^{-t/R_2 C} \quad (\text{S1-7})$$

If we ignore the continued trickle of current through  $R_1$ , the time at which  $i(t)$  reaches  $I_H$  and the PNP diode turns off is

$$\begin{aligned} t_2 &= -R_2 C \ln \frac{I_H R_2}{V_{BO}} \\ &= -(1 \text{ k}\Omega)(1 \text{ }\mu\text{F}) \ln \frac{(10 \text{ mA})(1 \text{ k}\Omega)}{75 \text{ V}} = 2 \text{ ms} \end{aligned} \quad (\text{S1-8})$$

Therefore, the total period of the relaxation oscillator is

$$T = t_1 + t_2 = 98 \text{ ms} + 2 \text{ ms} = 100 \text{ ms}$$

and the frequency of the relaxation oscillator is

$$f = \frac{1}{T} = 10 \text{ Hz}$$

(b) If  $R_1$  is increased to 150 k $\Omega$ , the capacitor charging time becomes

$$\begin{aligned} t_1 &= -R_1 C \ln \frac{V_{DC} - V_{BO}}{V_{DC}} \\ &= -(150 \text{ k}\Omega)(1 \text{ }\mu\text{F}) \ln \frac{120 \text{ V} - 75 \text{ V}}{120 \text{ V}} \\ &= 147 \text{ ms} \end{aligned}$$

The capacitor discharging time remains unchanged at

$$t_2 = -R_2 C \ln \frac{I_H R_2}{V_{BO}} = 2 \text{ ms}$$

Therefore, the total period of the relaxation oscillator is

$$T = t_1 + t_2 = 147 \text{ ms} + 2 \text{ ms} = 149 \text{ ms}$$

and the frequency of the relaxation oscillator is

$$f = \frac{1}{0.149 \text{ s}} = 6.71 \text{ Hz}$$

### Pulse Synchronization

In ac applications, it is important that the triggering pulse be applied to the controlling SCRs at the same point in each ac cycle. The way this is normally done is to synchronize the pulse circuit to the ac power line supplying power to the SCRs. This can easily be accomplished by making the power supply to the triggering circuit the same as the power supply to the SCRs.

If the triggering circuit is supplied from a half-cycle of the ac power line, the RC circuit will always begin to charge at exactly the beginning of the cycle, so the pulse will always occur at a fixed time with respect to the beginning of the cycle.

Pulse synchronization in three-phase circuits and inverters is much more complex and is beyond the scope of this book.

## S1.4 VOLTAGE VARIATION BY AC PHASE CONTROL

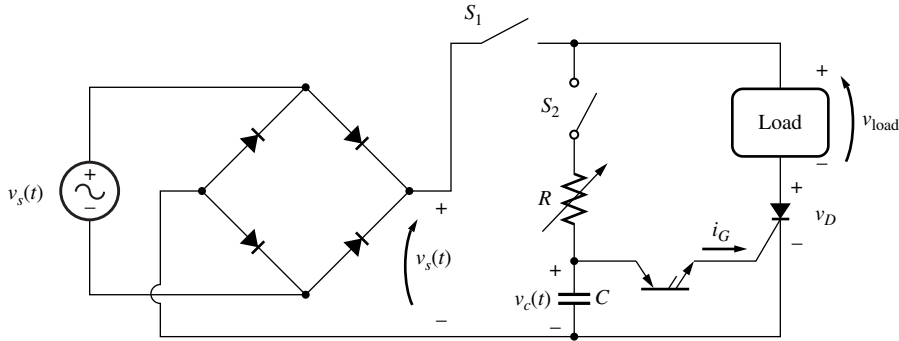
The level of voltage applied to a motor is one of the most common variables in motor-control applications. The SCR and the TRIAC provide a convenient technique for controlling the average voltage applied to a load by changing the phase angle at which the source voltage is applied to it.

### AC Phase Control for a DC Load Driven from an AC Source

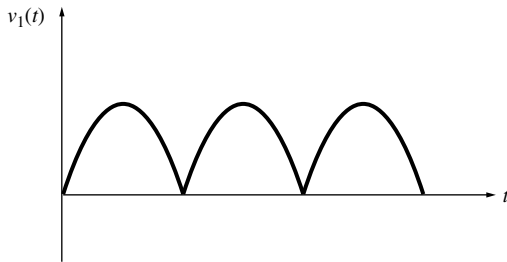
Figure S1–28 illustrates the concept of phase angle power control. The figure shows a voltage-phase-control circuit with a resistive dc load supplied by an ac source. The SCR in the circuit has a breakover voltage for  $i_G = 0 \text{ A}$  that is greater than the highest voltage in the circuit, while the PNP diode has a very low breakover voltage, perhaps 10 V or so. The full-wave bridge circuit ensures that the voltage applied to the SCR and the load will always be dc.

If the switch  $S_1$  in the picture is open, then the voltage  $V_1$  at the terminals of the rectifier will just be a full-wave rectified version of the input voltage (see Figure S1–29).

If switch  $S_1$  is shut but switch  $S_2$  is left open, then the SCR will always be off. This is true because the voltage out of the rectifier will never exceed  $V_{BO}$  for



**FIGURE S1-28**  
A circuit controlling the voltage to a dc load by phase angle control.



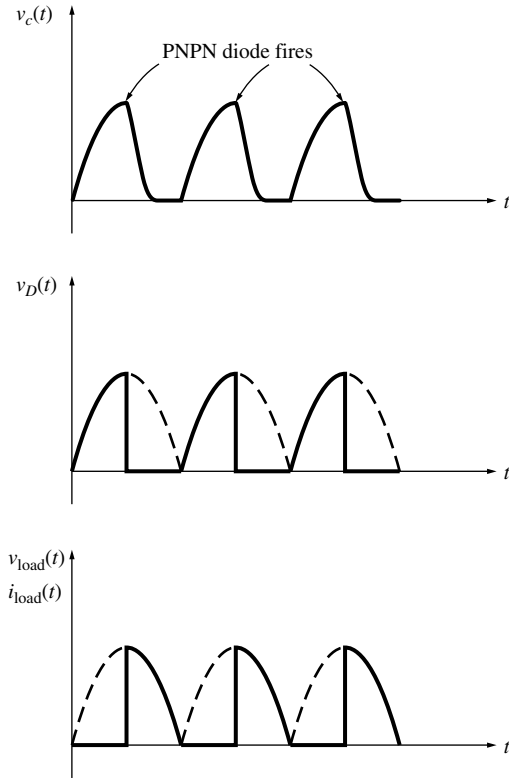
**FIGURE S1-29**  
The voltage at the output of the bridge circuit with switch  $S_1$  open.

the SCR. Since the SCR is always an open circuit, the current through it and the load, and hence the voltage on the load, will still be zero.

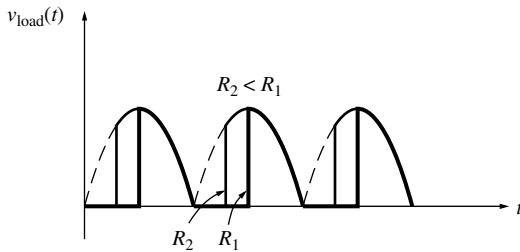
Now suppose that switch  $S_2$  is closed. Then, at the beginning of the first half-cycle after the switch is closed, a voltage builds up across the  $RC$  network, and the capacitor begins to charge. During the time the capacitor is charging, the SCR is off, since the voltage applied to it has not exceeded  $V_{BO}$ . As time passes, the capacitor charges up to the breakover voltage of the PNPN diode, and the PNPN diode conducts. The current flow from the capacitor and the PNPN diode flows through the gate of the SCR, lowering  $V_{BO}$  for the SCR and turning it on. When the SCR turns on, current flows through it and the load. This current flow continues for the rest of the half-cycle, even after the capacitor has discharged, since the SCR turns off only when its current falls below the holding current (since  $I_H$  is a few milliamperes, this does not occur until the extreme end of the half-cycle).

At the beginning of the next half-cycle, the SCR is again off. The  $RC$  circuit again charges up over a finite period and triggers the PNPN diode. The PNPN diode once more sends a current to the gate of the SCR, turning it on. Once on, the SCR remains on for the rest of the cycle again. The voltage and current waveforms for this circuit are shown in Figure S1-30.

Now for the critical question: How can the power supplied to this load be changed? Suppose the value of  $R$  is decreased. Then at the beginning of each



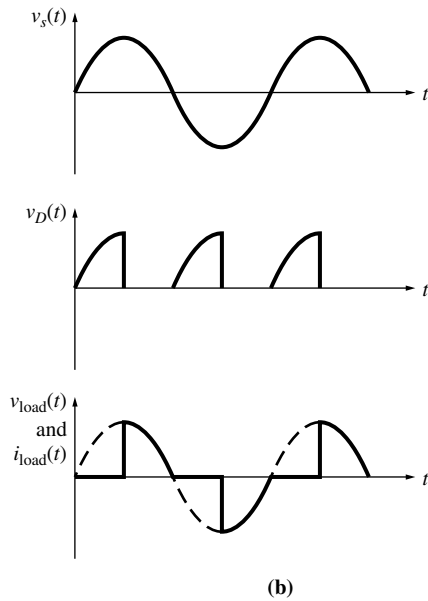
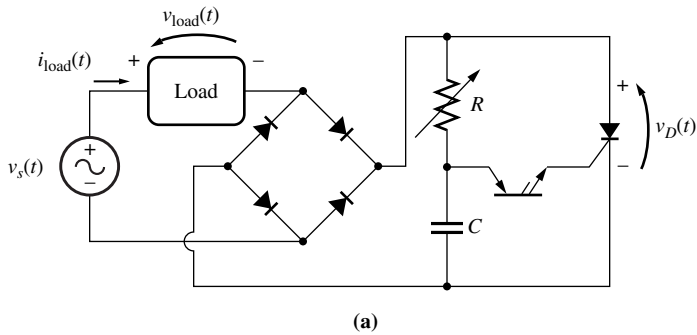
**FIGURE S1-30**  
 The voltages across the capacitor, SCR, and load, and the current through the load, when switches  $S_1$  and  $S_2$  are closed.



**FIGURE S1-31**  
 The effect of decreasing  $R$  on the output voltage applied to the load in the circuit of Figure S1-28.

half-cycle, the capacitor will charge more quickly, and the SCR will fire sooner. Since the SCR will be on for longer in the half-cycle, *more power will be supplied to the load* (see Figure S1-31). The resistor  $R$  in this circuit controls the power flow to the load in the circuit.

The power supplied to the load is a function of the time that the SCR fires; the earlier that it fires, the more power will be supplied. The firing time of the SCR is customarily expressed as a *firing angle*, where the firing angle is the angle of the applied sinusoidal voltage at the time of firing. The relationship between the firing angle and the supplied power will be derived in Example S1-3.



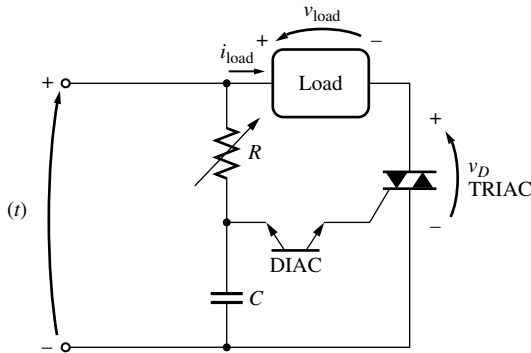
**FIGURE S1-32**

(a) A circuit controlling the voltage to an ac load by phase angle control. (b) Voltages on the source, the load, and the SCR in this controller.

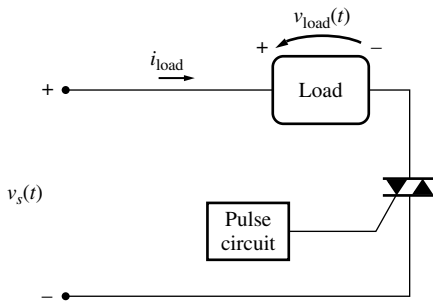
### AC Phase Angle Control for an AC Load

It is possible to modify the circuit in Figure S1-28 to control an ac load simply by moving the load from the dc side of the circuit to a point before the rectifiers. The resulting circuit is shown in Figure S1-32a, and its voltage and circuit waveforms are shown in Figure S1-32b.

However, there is a much easier way to make an ac power controller. If the same basic circuit is used with a DIAC in place of the PNPN diode and a TRIAC in place of the SCR, then the diode bridge circuit can be completely taken out of the circuit. Because both the DIAC and the TRIAC are two-way devices, they



**FIGURE S1-33**  
An ac phase angle controller using a DIAC and TRIAC.



**FIGURE S1-34**  
An ac phase angle controller using a TRIAC triggered by a digital pulse circuit.

operate equally well on either half-cycle of the ac source. An ac phase power controller with a DIAC and a TRIAC is shown in Figure S1-33.

**Example S1-3.** Figure S1-34 shows an ac phase angle controller supplying power to a resistive load. The circuit uses a TRIAC triggered by a digital pulse circuit that can provide firing pulses at any point in each half-cycle of the applied voltage  $v_s(t)$ . Assume that the supply voltage is 120 V rms at 60 Hz.

- Determine the rms voltage applied to the load as a function of the firing angle of the pulse circuit, and plot the relationship between firing angle and the supplied voltage.
- What firing angle would be required to supply a voltage of 75 V rms to the load?

**Solution**

- This problem is ideally suited to solution using MATLAB because it involves a repetitive calculation of the rms voltage applied to the load at many different firing angles. We will solve the problem by calculating the waveform produced by firing the TRIAC at each angle from  $1^\circ$  to  $179^\circ$ , and calculating the rms voltage of the resulting waveform. (Note that only the positive half cycle is considered, since the negative half cycle is symmetrical.)

The first step in the solution process is to produce a MATLAB function that mimics the load voltage for any given  $\omega t$  and firing angle. Function

`ac_phase_controller` does this. It accepts two input arguments, a normalized time  $\omega t$  in radians and a firing angle in degrees. If the time  $\omega t$  is earlier than the firing angle, the load voltage at that time will be 0 V. If the time  $\omega t$  is after the firing angle, the load voltage will be the same as the source voltage for that time.

```
function volts = ac_phase_controller(wt,deg)
% Function to simulate the output of the positive half
% cycle of an ac phase angle controller with a peak
% voltage of 120 * SQRT(2) = 170 V.
%   wt = Phase in radians (=omega x time)
%   deg = Firing angle in degrees

% Degrees to radians conversion factor
deg2rad = pi / 180;

% Simulate the output of the phase angle controller.
if wt > deg * deg2rad;
    volts = 170 * sin(wt);
else
    volts = 0;
end
```

The next step is to write an m-file that creates the load waveform for each possible firing angle, and calculates and plots the resulting rms voltage. The m-file shown below uses function `ac_phase_controller` to calculate the load voltage waveform for each firing angle, and then calculates the rms voltage of that waveform.

```
% M-file: volts_vs_phase_angle.m
% M-file to calculate the rms voltage applied to a load as
% a function of the phase angle firing circuit, and to
% plot the resulting relationship.

% Loop over all firing angles (1 to 179 degrees)
deg = zeros(1,179);
rms = zeros(1,179);
for ii = 1:179

    % Save firing angle
    deg(ii) = ii;

    % First, generate the waveform to analyze.
    waveform = zeros(1,180);
    for jj = 1:180
        waveform(jj) = ac_phase_controller(jj*pi/180,ii);
    end

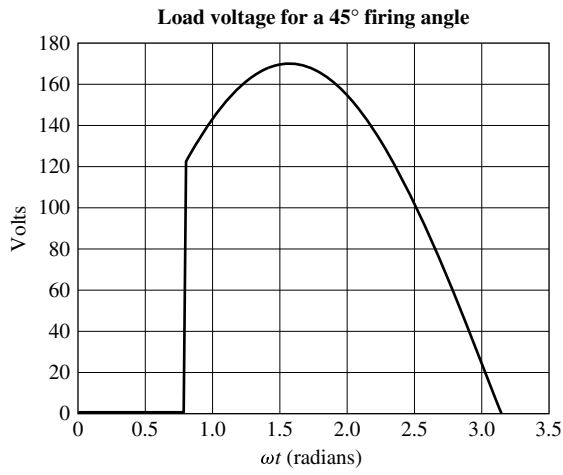
    % Now calculate the rms voltage of the waveform
    temp = sum(waveform.^2);
    rms(ii) = sqrt(temp/180);

end
```

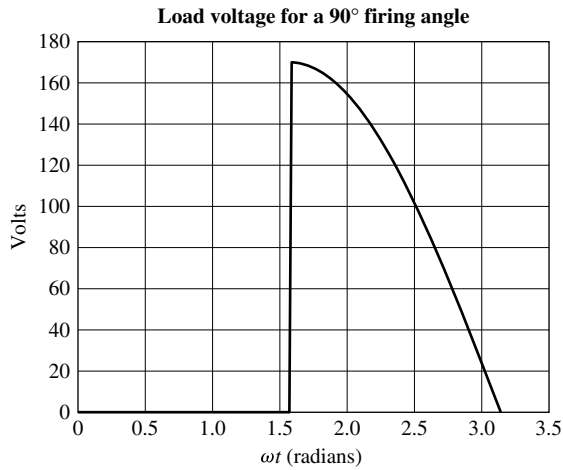
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```
% Plot rms voltage of the load as a function of firing angle
plot(deg,rms);
title('Load Voltage vs. Firing Angle');
xlabel('Firing angle (deg)');
ylabel('RMS voltage (V)');
grid on;
```

Two examples of the waveform generated by this function are shown in Figure S1-35.



(a)

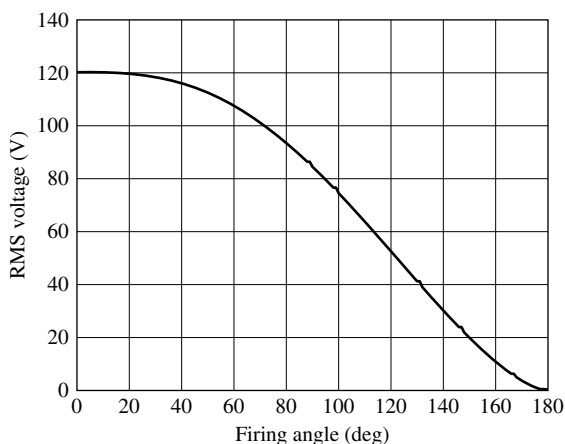


(b)

FIGURE S1-35

Waveform produced by `volts_vs_phase_angle` for a firing angle of (a) 45°; (b) 90°.



**FIGURE S1-36**

Plot of rms load voltage versus TRIAC firing angle.

When this m-file is executed, the plot shown in Figure S1-36 results. Note that the earlier the firing angle, the greater the rms voltage supplied to the load. However, the relationship between firing angle and the resulting voltage is not linear, so it is not easy to predict the required firing angle to achieve a given load voltage.

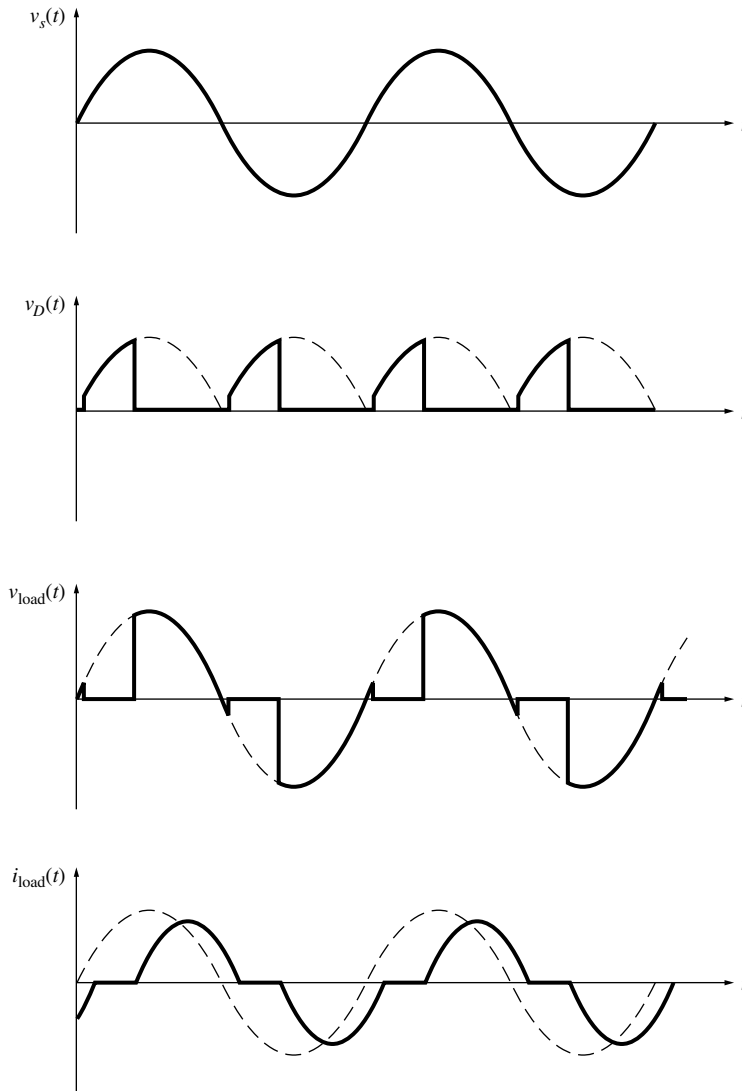
- (b) The firing angle required to supply 75 V to the load can be found from Figure S1-36. It is about 99°.

### The Effect of Inductive Loads on Phase Angle Control

If the load attached to a phase angle controller is inductive (as real machines are), then new complications are introduced to the operation of the controller. By the nature of inductance, *the current in an inductive load cannot change instantaneously*. This means that the current to the load will not rise immediately on firing the SCR (or TRIAC) and that the current will not stop flowing at exactly the end of the half-cycle. At the end of the half-cycle, the inductive voltage on the load will keep the device turned on for some time into the next half-cycle, until the current flowing through the load and the SCR finally falls below  $I_H$ . Figure S1-37 shows the effect of this delay in the voltage and current waveforms for the circuit in Figure S1-32.

A large inductance in the load can cause two potentially serious problems with a phase controller:

1. The inductance can cause the current buildup to be so slow when the SCR is switched on that it does not exceed the holding current before the gate current disappears. If this happens, the SCR will not remain on, because its current is less than  $I_H$ .



**FIGURE S1-37**

The effect of an inductive load on the current and voltage waveforms of the circuit shown in Figure S1-32.

2. If the current continues long enough before decaying to  $I_H$  after the end of a given cycle, the applied voltage could build up high enough in the next cycle to keep the current going, and the SCR will never switch off.

The normal solution to the first problem is to use a special circuit to provide a longer gating current pulse to the SCR. This longer pulse allows plenty of time

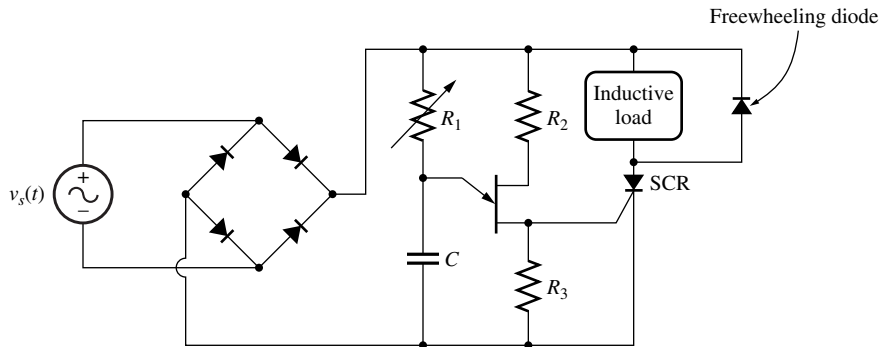


FIGURE S1-38

A phase angle controller illustrating the use of a free-wheeling diode with an inductive load.

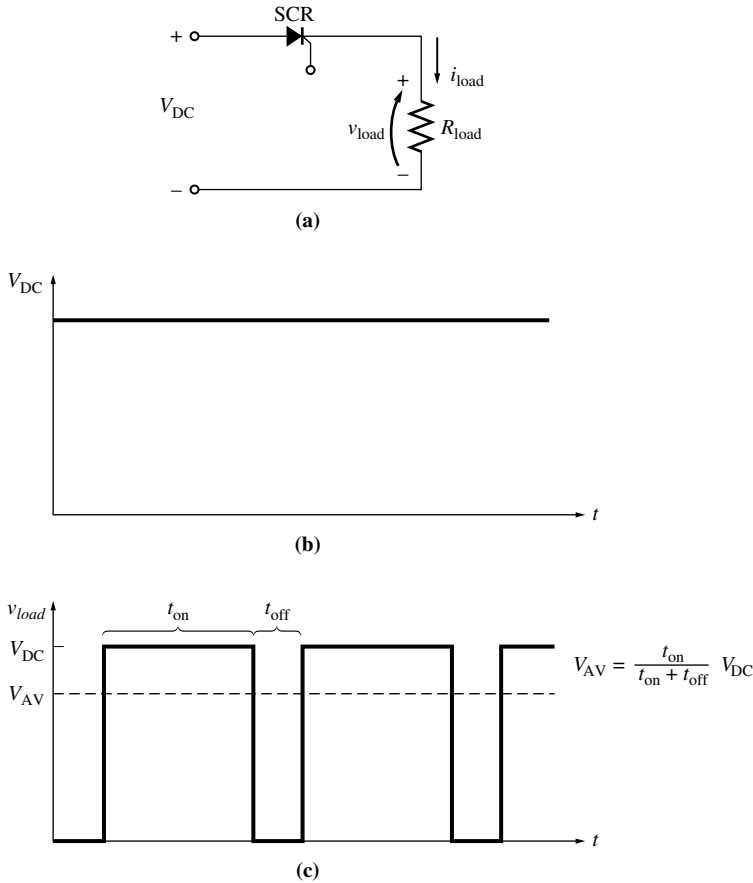
for the current through the SCR to rise above  $I_H$ , permitting the device to remain on for the rest of the half-cycle.

A solution to the second problem is to add a *free-wheeling diode*. A free-wheeling diode is a diode placed across a load and oriented so that it does not conduct during normal current flow. Such a diode is shown in Figure S1-38. At the end of a half-cycle, the current in the inductive load will attempt to keep flowing in the same direction as it was going. A voltage will be built up on the load with the polarity required to keep the current flowing. This voltage will forward-bias the free-wheeling diode, and it will supply a path for the discharge current from the load. In that manner, the SCR can turn off without requiring the current of the inductor to instantly drop to zero.

## S1.5 DC-TO-DC POWER CONTROL— CHOPPERS

Sometimes it is desirable to vary the voltage available from a dc source before applying it to a load. The circuits which vary the voltage of a dc source are called *dc-to-dc converters* or *choppers*. In a chopper circuit, the input voltage is a constant dc voltage source, and the output voltage is varied by varying the *fraction of the time* that the dc source is connected to its load. Figure S1-39 shows the basic principle of a chopper circuit. When the SCR is triggered, it turns on and power is supplied to the load. When it turns off, the dc source is disconnected from the load.

In the circuit shown in Figure S1-39, the load is a resistor, and the voltage on the load is either  $V_{DC}$  or 0. Similarly, the current in the load is either  $V_{DC}/R$  or 0. It is possible to smooth out the load voltage and current by adding a series inductor to filter out some of the ac components in the waveform. Figure S1-40 shows a chopper circuit with an inductive filter. The current through the inductor increases exponentially when the SCR is on and decreases exponentially when the SCR is off. If the inductor is large, the time constant of the current changes ( $\tau = L/R$ ) will

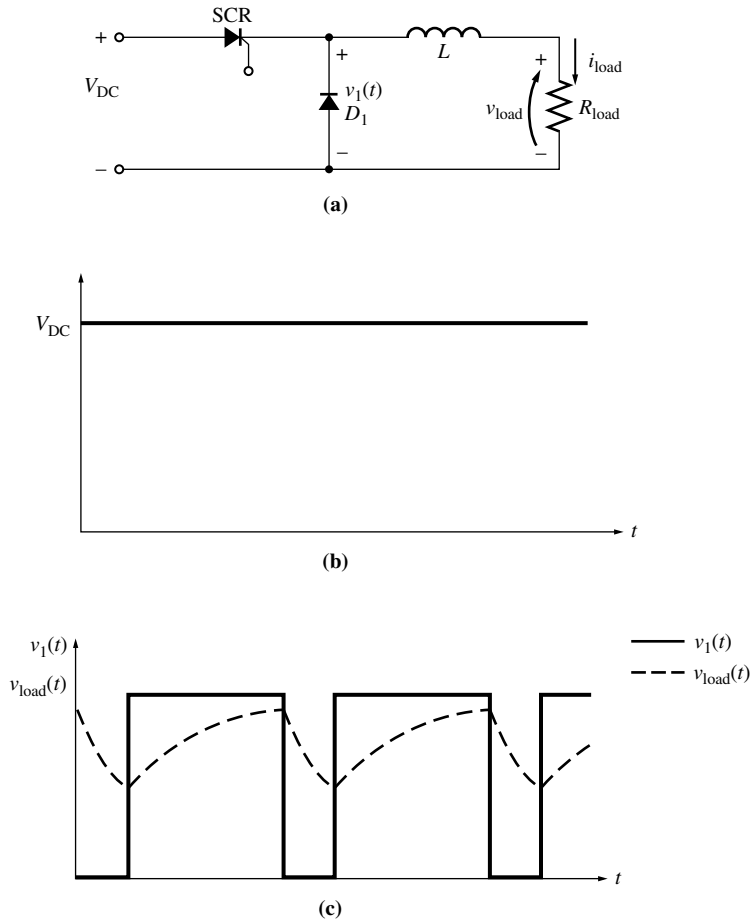


**FIGURE S1-39**  
 (a) The basic principle of a chopper circuit. (b) The input voltage to the circuit. (c) The resulting voltage on the load.

be long relative to the on/off cycle of the SCR and the load voltage and current will be almost constant at some average value.

In the case of ac phase controllers, the SCRs automatically turn off at the end of each half-cycle when their currents go to zero. For dc circuits, there is no point at which the current naturally falls below  $I_H$ , so once an SCR is turned on, it never turns off. To turn the SCR off again at the end of a pulse, it is necessary to apply a reverse voltage to it for a short time. This reverse voltage stops the current flow and turns off the SCR. Once it is off, it will not turn on again until another pulse enters the gate of the SCR. The process of forcing an SCR to turn off at a desired time is known as *forced commutation*.

GTO thyristors are ideally suited for use in chopper circuits, since they are self-commutating. In contrast to SCRs, GTOs can be turned off by a negative current pulse applied to their gates. Therefore, the extra circuitry needed in an SCR



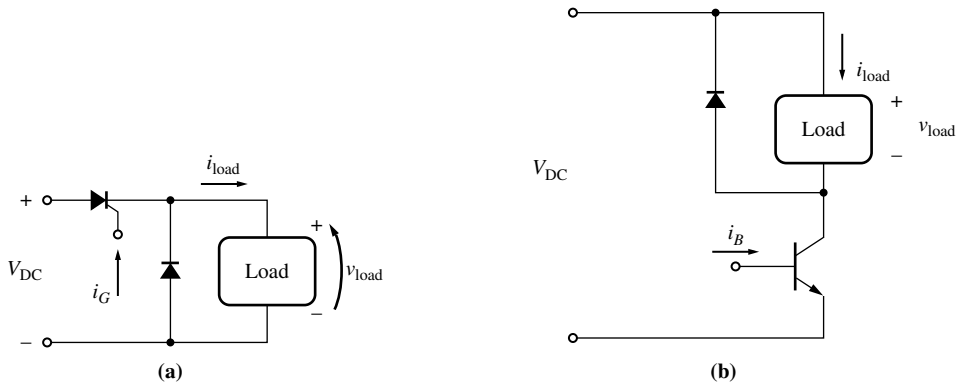
**FIGURE S1-40**  
A chopper circuit with an inductive filter to smooth out the load voltage and current.

chopper circuit to turn off the SCR can be eliminated from a GTO thyristor chopper circuit (Figure S1-41a). Power transistors are also self-commutating and are used in chopper circuits that fall within their power limits (Figure S1-41b).

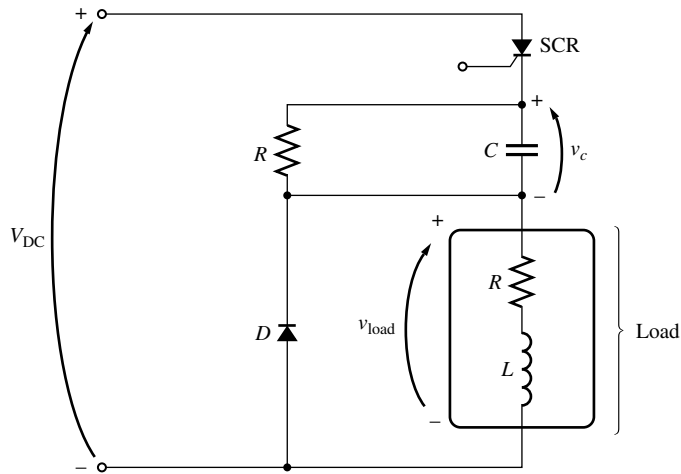
Chopper circuits are used with dc power systems to vary the speed of dc motors. Their greatest advantage for dc speed control compared to conventional methods is that they are more efficient than the systems (such as the Ward-Leonard system described in Chapter 5) that they replace.

### Forced Commutation in Chopper Circuits

When SCRs are used in choppers, a forced-commutation circuit must be included to turn off the SCRs at the desired time. Most such forced-commutation circuits



**FIGURE S1-41**  
 (a) A chopper circuit made with a GTO thyristor. (b) A chopper circuit made with a transistor.



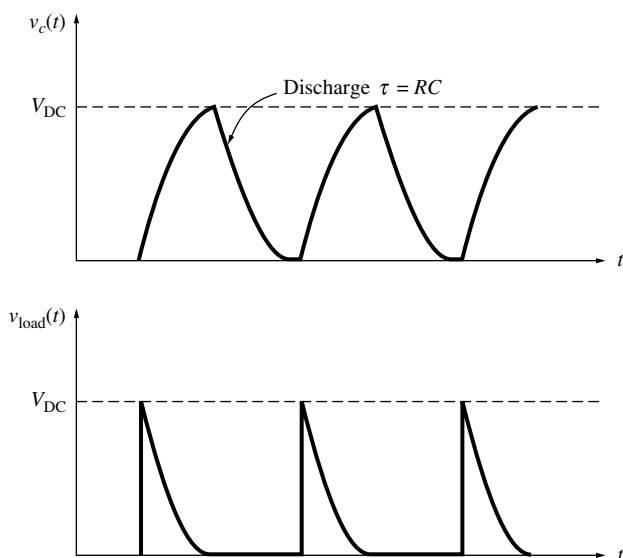
**FIGURE S1-42**  
 A series-capacitor forced-commutation chopper circuit.

depend for their turnoff voltage on a charged capacitor. Two basic versions of capacitor commutation are examined in this brief overview:

1. Series-capacitor commutation circuits
2. Parallel-capacitor commutation circuits

**Series-Capacitor Commutation Circuits**

Figure S1-42 shows a simple dc chopper circuit with series-capacitor commutation. It consists of an SCR, a capacitor, and a load, all in series with each other.

**FIGURE S1-43**

The capacitor and load voltages in the series chopper circuit.

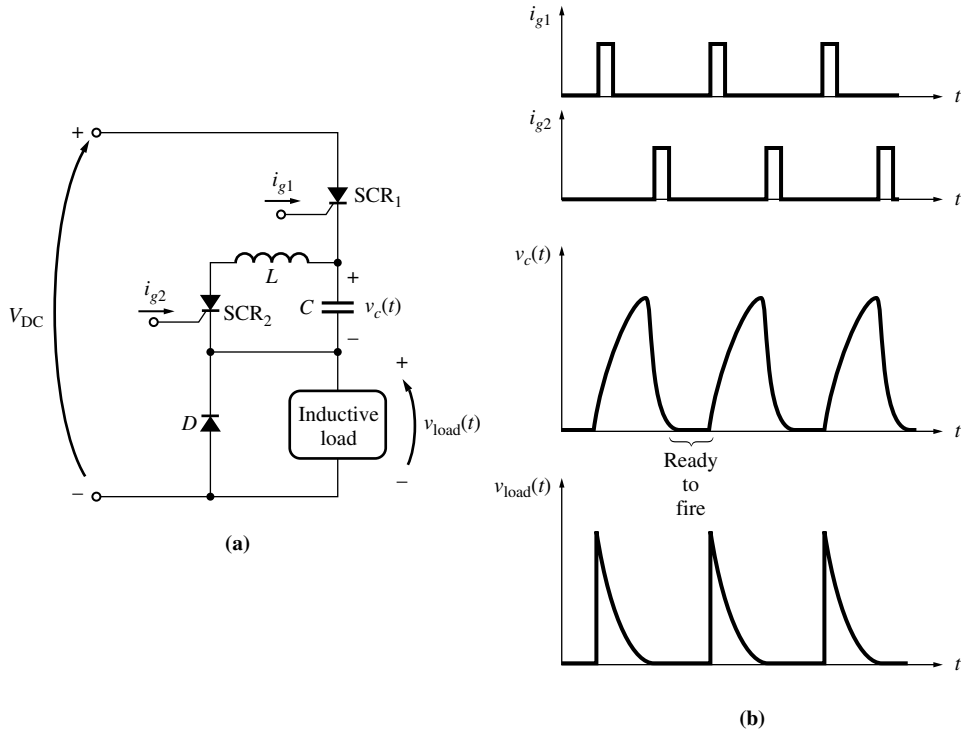
The capacitor has a shunt discharging resistor across it, and the load has a free-wheeling diode across it.

The SCR is initially turned on by a pulse applied to its gate. When the SCR turns on, a voltage is applied to the load and a current starts flowing through it. But this current flows through the series capacitor on the way to the load, and the capacitor gradually charges up. When the capacitor's voltage nearly reaches  $V_{DC}$ , the current through the SCR drops below  $I_H$  and the SCR turns off.

Once the capacitor has turned off the SCR, it gradually discharges through resistor  $R$ . When it is totally discharged, the SCR is ready to be fired by another pulse at its gate. The voltage and current waveforms for this circuit are shown in Figure S1-43.

Unfortunately, this type of circuit is limited in terms of duty cycle, since the SCR cannot be fired again until the capacitor has discharged. The discharge time depends on the time constant  $\tau = RC$ , and  $C$  must be made large in order to let a lot of current flow to the load before it turns off the SCR. But  $R$  must be large, since the current leaking through the resistor has to be less than the holding current of the SCR. These two facts taken together mean that *the SCR cannot be re-fired quickly after it turns off*. It has a long recovery time.

An improved series-capacitor commutation circuit with a shortened recovery time is shown in Figure S1-44. This circuit is similar to the previous one except that the resistor has been replaced by an inductor and SCR in series. When SCR is fired, current will flow to the load and the capacitor will charge up, cutting off SCR<sub>1</sub>. Once it is cut off, SCR<sub>2</sub> can be fired, discharging the capacitor much more



**FIGURE S1-44**  
 (a) A series-capacitor forced-commutation chopper circuit with improved capacitor recovery time.  
 (b) The resulting capacitor and load voltage waveforms. Note that the capacitor discharges much more rapidly, so SCR<sub>1</sub> could be refired sooner than before.

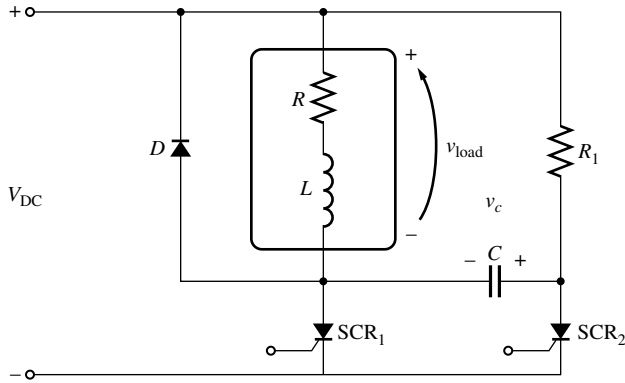
quickly than the resistor would. The inductor in series with SCR<sub>2</sub> protects SCR<sub>2</sub> from instantaneous current surges that exceed its ratings. Once the capacitor discharges, SCR<sub>2</sub> turns off and SCR<sub>1</sub> is ready to fire again.

**Parallel-Capacitor Commutation Circuits**

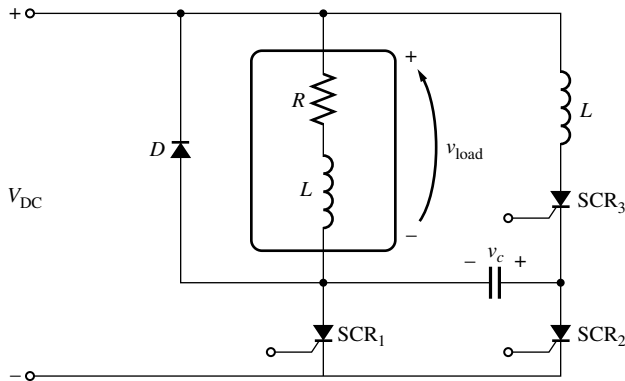
The other common way to achieve forced commutation is via the parallel-capacitor commutation scheme. A simple example of the parallel-capacitor scheme is shown in Figure S1-45. In this scheme, SCR<sub>1</sub> is the main SCR, supplying power to the load, and SCR<sub>2</sub> controls the operation of the commutating capacitor. To apply power to the load, SCR<sub>1</sub> is fired. When this occurs, a current flows through the SCR to the load, supplying power to it. Also, capacitor C charges up through resistor R to a voltage equal to the supply voltage V<sub>DC</sub>.

When the time comes to turn off the power to the load, SCR<sub>2</sub> is fired. When SCR<sub>2</sub> is fired, the voltage across it drops to zero. Since the voltage across a





**FIGURE S1-45**  
A parallel-capacitor forced-commutation chopper circuit.



**FIGURE S1-46**  
A parallel-capacitor forced-commutation chopper circuit with improved capacitor charging time.  $SCR_3$  permits the load power to be turned off more quickly than it could be with the basic parallel-capacitor circuit.

capacitor cannot change instantaneously, the voltage on the left side of the capacitor must instantly drop to  $-V_{DC}$  volts. This turns off  $SCR_1$ , and the capacitor charges through the load and  $SCR_2$  to a voltage of  $V_{DC}$  volts positive on its left side. Once capacitor  $C$  is charged,  $SCR_2$  turns off, and the cycle is ready to begin again.

Again, resistor  $R_1$  must be large in order for the current through it to be less than the holding current of  $SCR_2$ . But a large resistor  $R_1$  means that the capacitor will charge only slowly after  $SCR_1$  fires. This limits how soon  $SCR_1$  can be turned off after it fires, setting a lower limit on the on time of the chopped waveform.

A circuit with a reduced capacitor charging time is shown in Figure S1-46. In this circuit  $SCR_3$  is triggered at the same time as  $SCR_1$  is, and the capacitor can

charge much more rapidly. This allows the current to be turned off much more rapidly if it is desired to do so.

In any circuit of this sort, the free-wheeling diode is *extremely* important. When SCR<sub>1</sub> is forced off, the current through the inductive load *must* have another path available to it, or it could possibly damage the SCR.

## S1.6 INVERTERS

Perhaps the most rapidly growing area in modern power electronics is static frequency conversion, the conversion of ac power at one frequency to ac power at another frequency by means of solid-state electronics. Traditionally there have been two approaches to static ac frequency conversion: the *cycloconverter* and the *rectifier-inverter*. The cycloconverter is a device for directly converting ac power at one frequency to ac power at another frequency, while the rectifier-inverter first converts ac power to dc power and then converts the dc power to ac power again at a different frequency. This section deals with the operation of rectifier-inverter circuits, and Section S1.7 deals with the cycloconverter.

A rectifier-inverter is divided into two parts:

1. A *rectifier* to produce dc power
2. An *inverter* to produce ac power from the dc power.

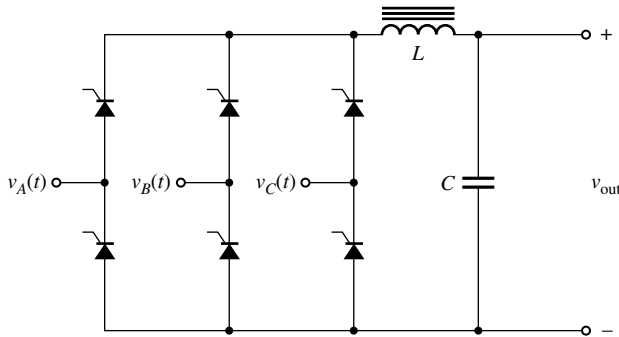
Each part is treated separately.

### The Rectifier

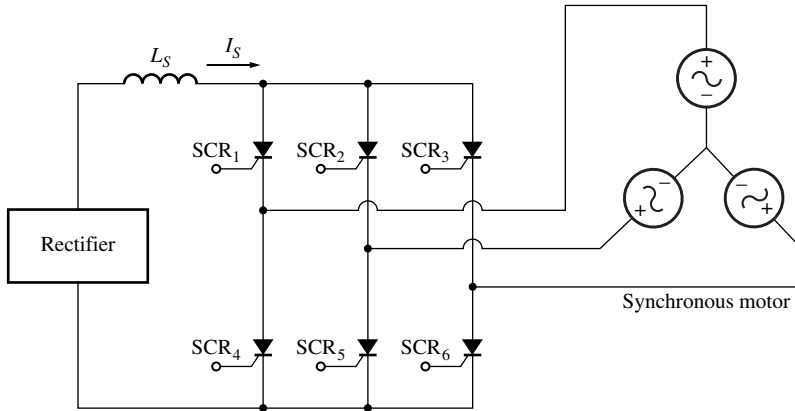
The basic rectifier circuits for converting ac power to dc power are described in Section S1.2. These circuits have one problem from a motor-control point of view—their output voltage is fixed for a given input voltage. This problem can be overcome by replacing the diodes in these circuits with SCRs.

Figure S1–47 shows a three-phase full-wave rectifier circuit with the diodes in the circuits replaced by SCRs. The average dc output voltage from this circuit depends on when the SCRs are triggered during their positive half-cycles. If they are triggered at the beginning of the half-cycle, this circuit will be the same as that of a three-phase full-wave rectifier with diodes. If the SCRs are never triggered, the output voltage will be 0 V. For any other firing angle between 0° and 180° on the waveform, the dc output voltage will be somewhere between the maximum value and 0 V.

When SCRs are used instead of diodes in the rectifier circuit to get control of the dc voltage output, this output voltage will have more harmonic content than a simple rectifier would, and some form of filter on its output is important. Figure S1–47 shows an inductor and capacitor filter placed at the output of the rectifier to help smooth the dc output.



**FIGURE S1-47**  
A three-phase rectifier circuit using SCRs to provide control of the dc output voltage level.



**FIGURE S1-48**  
An external commutation inverter.

**External Commutation Inverters**

Inverters are classified into two basic types by the commutation technique used: external commutation and self-commutation. *External commutation inverters* are inverters in which the energy required to turn off the SCRs is provided by an external motor or power supply. An example of an external commutation inverter is shown in Figure S1-48. The inverter is connected to a three-phase synchronous motor, which provides the countervoltage necessary to turn off one SCR when its companion is fired.

The SCRs in this circuit are triggered in the following order: SCR<sub>1</sub>, SCR<sub>6</sub>, SCR<sub>2</sub>, SCR<sub>4</sub>, SCR<sub>3</sub>, SCR<sub>5</sub>. When SCR<sub>1</sub> fires, the internal generated voltage in the synchronous motor provides the voltage necessary to turn off SCR<sub>3</sub>. Note that if the load were not connected to the inverter, the SCRs would never be turned off and after ½ cycle a short circuit would develop through SCR<sub>1</sub> and SCR<sub>4</sub>.

This inverter is also called a *load-commutated inverter*.

### Self-Commutation Inverters

If it is not possible to guarantee that a load will always provide the proper countervoltage for commutation, then a self-commutation inverter must be used. A self-commutation inverter is an inverter in which the active SCRs are turned off by energy stored in a capacitor when another SCR is switched on. It is also possible to design self-commutation inverters using GTOs or power transistors, in which case commutation capacitors are not required.

There are three major types of self-commutation inverters: current source inverters (CSIs), voltage source inverters (VSIs), and pulse-width modulation (PWM) inverters. Current source inverters and voltage source inverters are simpler than PWM inverters and have been used for a longer time. PWM inverters require more complex control circuitry and faster switching components than CSIs and VSIs. CSIs and VSIs are discussed first. Current source inverters and voltage source inverters are compared in Figure S1–49.

In the current source inverter, a rectifier is connected to an inverter through a large series inductor  $L_S$ . The inductance of  $L_S$  is sufficiently large that the direct current is constrained to be almost constant. The SCR current output waveform will be roughly a square wave, since the current flow  $I_S$  is constrained to be nearly constant. The line-to-line voltage will be approximately triangular. It is easy to limit overcurrent conditions in this design, but the output voltage can swing widely in response to changes in load.

In the voltage source inverter, a rectifier is connected to an inverter through a series inductor  $L_S$  and a parallel capacitor  $C$ . The capacitance of  $C$  is sufficiently large that the voltage is constrained to be almost constant. The SCR line-to-line voltage output waveform will be roughly a square wave, since the voltage  $V_C$  is constrained to be nearly constant. The output current flow will be approximately triangular. Voltage variations are small in this circuit, but currents can vary wildly with variations in load, and overcurrent protection is difficult to implement.

The frequency of both current and voltage source inverters can be easily changed by changing the firing pulses on the gates of the SCRs, so both inverters can be used to drive ac motors at variable speeds (see Chapter 9).

### A Single-Phase Current Source Inverter

A single-phase current source inverter circuit with capacitor commutation is shown in Figure S1–50. It contains two SCRs, a capacitor, and an output transformer. To understand the operation of this circuit, assume initially that both SCRs are off. If SCR<sub>1</sub> is now turned on by a gate current, voltage  $V_{DC}$  will be applied to the upper half of the transformer in the circuit. This voltage induces a voltage  $V_{DC}$  in the lower half of the transformer as well, causing a voltage of  $2V_{DC}$  to be built up across the capacitor. The voltages and currents in the circuit at this time are shown in Figure S1–50b.

Now SCR<sub>2</sub> is turned on. When SCR<sub>2</sub> is turned on, the voltage at the *cathode* of the SCR will be  $V_{DC}$ . Since the voltage across a capacitor cannot change

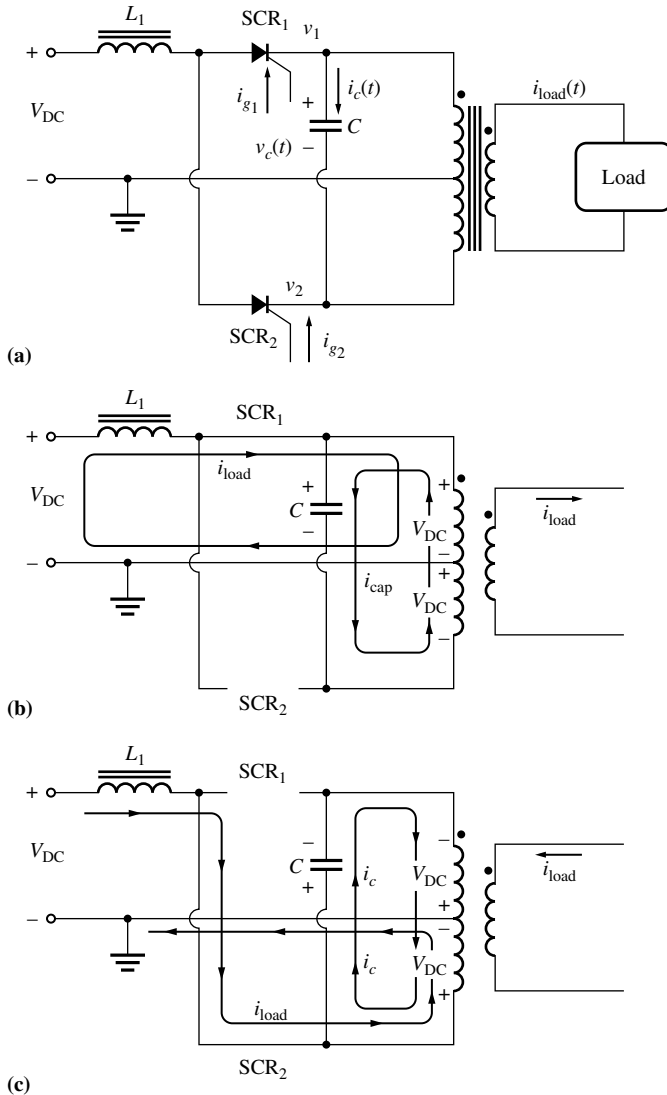
	Current source inverter	Voltage source inverter
Main circuit configuration		
Type of source	Current source $-I_S$ almost constant	Voltage source $-V_S$ almost constant
Output impedance	High	Low
Output waveform		
Characteristics	<ol style="list-style-type: none"> <li>1. Easy to control overcurrent conditions with this design</li> <li>2. Output voltage varies widely with changes in load</li> </ol>	<ol style="list-style-type: none"> <li>1. Difficult to limit current because of capacitor</li> <li>2. Output voltage variations small because of capacitor</li> </ol>

**FIGURE S1-49**

Comparison of current source inverters and voltage source inverters.

instantaneously, this forces the voltage at the top of the capacitor to instantly become  $3V_{DC}$ , turning off  $SCR_1$ . At this point, the voltage on the bottom half of the transformer is built up positive at the bottom to negative at the top of the winding, and its magnitude is  $V_{DC}$ . The voltage in the bottom half induces a voltage  $V_{DC}$  in the upper half of the transformer, charging the capacitor  $C$  up to a voltage of  $2V_{DC}$ , oriented positive at the bottom with respect to the top of the capacitor. The condition of the circuit at this time is shown in Figure S1-50c.

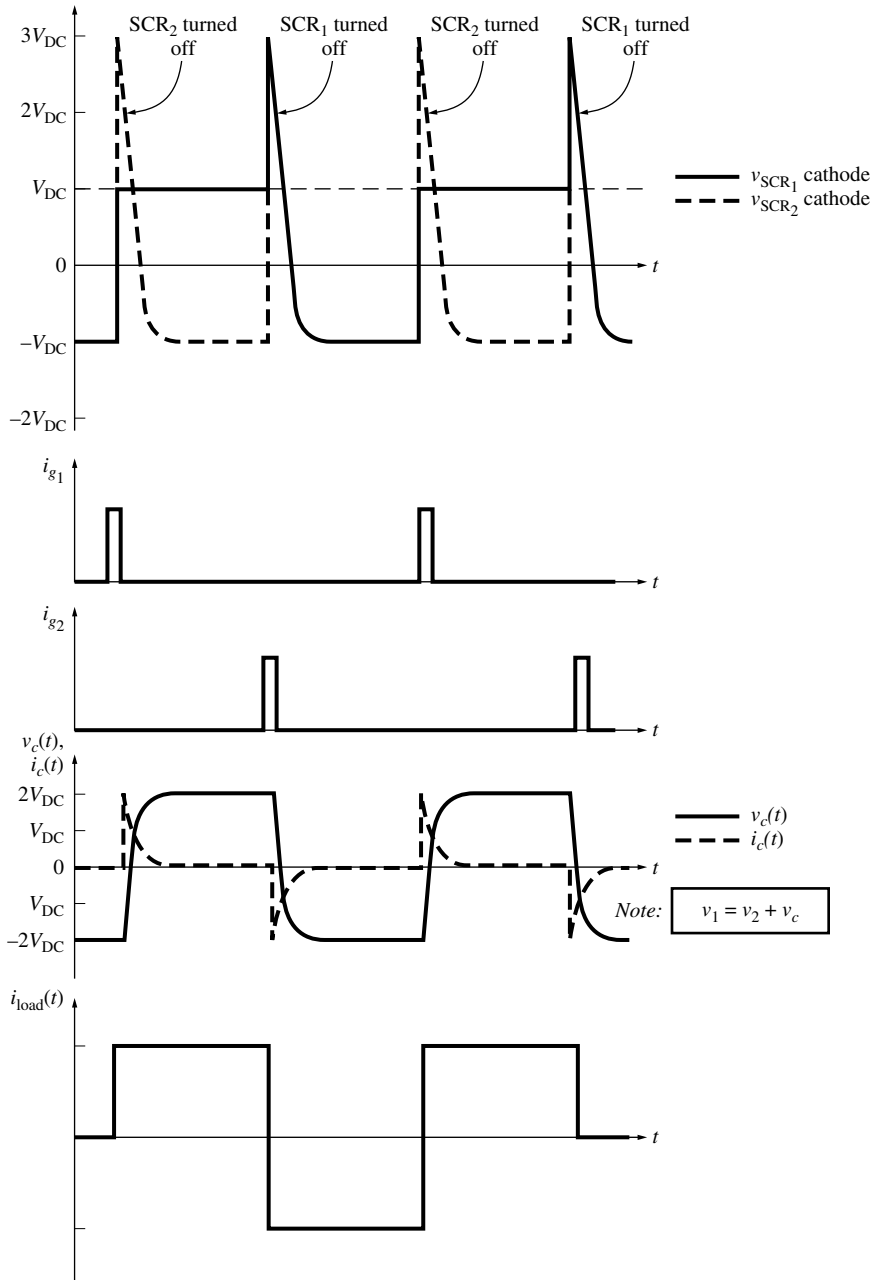
When  $SCR_1$  is fired again, the capacitor voltage cuts off  $SCR_2$ , and this process repeats indefinitely. The resulting voltage and current waveforms are shown in Figure S1-51.



**FIGURE S1-50**  
 (a) A simple single-phase inverter circuit. (b) The voltages and currents in the circuit when  $SCR_1$  is triggered. (c) The voltages and currents in the circuit when  $SCR_2$  is triggered.

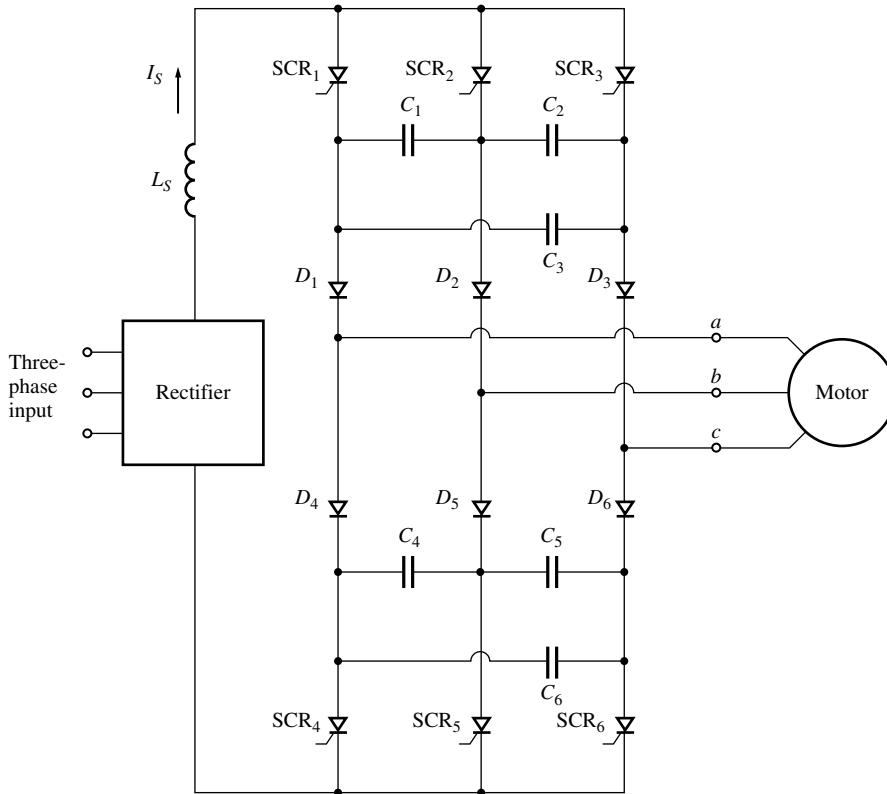
### A Three-Phase Current Source Inverter

Figure S1-52 shows a three-phase current source inverter. In this circuit, the six SCRs fire in the order  $SCR_1$ ,  $SCR_6$ ,  $SCR_2$ ,  $SCR_4$ ,  $SCR_3$ ,  $SCR_5$ . Capacitors  $C_1$  through  $C_6$  provide the commutation required by the SCRs.



**FIGURE S1-51**

Plots of the voltages and current in the inverter circuit:  $V_1$  is the voltage at the cathode of SCR<sub>1</sub>, and  $V_2$  is the voltage at the cathode of SCR<sub>2</sub>. Since the voltage at their anodes is  $V_{DC}$ , any time  $V_1$  or  $V_2$  exceeds  $V_{DC}$ , that SCR is turned off.  $i_{load}$  is the current supplied to the inverter's load.



**FIGURE S1-52**  
A three-phase current source inverter.

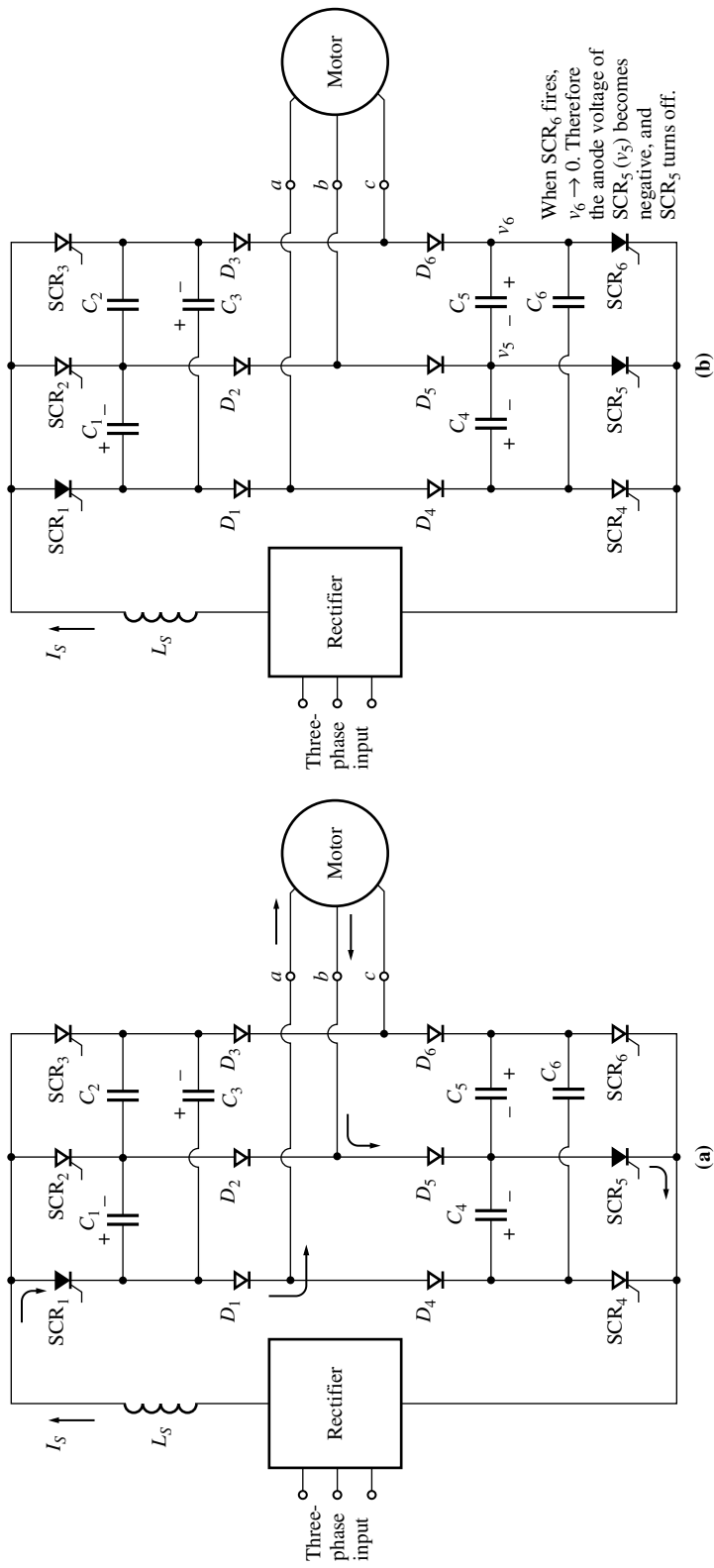
To understand the operation of this circuit, examine Figure S1-53. Assume that initially SCR<sub>1</sub> and SCR<sub>5</sub> are conducting, as shown in Figure S1-53a. Then a voltage will build up across capacitors C<sub>1</sub>, C<sub>3</sub>, C<sub>4</sub>, and C<sub>5</sub> as shown on the diagram. Now assume that SCR<sub>6</sub> is gated on. When SCR<sub>6</sub> is turned on, the voltage at point 6 drops to zero (see Figure S1-53b). Since the voltage across capacitor C<sub>5</sub> cannot change instantaneously, the anode of SCR<sub>5</sub> is biased negative, and SCR<sub>5</sub> is turned off. Once SCR<sub>6</sub> is on, all the capacitors charge up as shown in Figure S1-53c, and the circuit is ready to turn off SCR<sub>6</sub> whenever SCR<sub>4</sub> is turned on. This same commutation process applies to the upper SCR bank as well.

The output phase and line current from this circuit are shown in Figure S1-53d.

### A Three-Phase Voltage Source Inverter

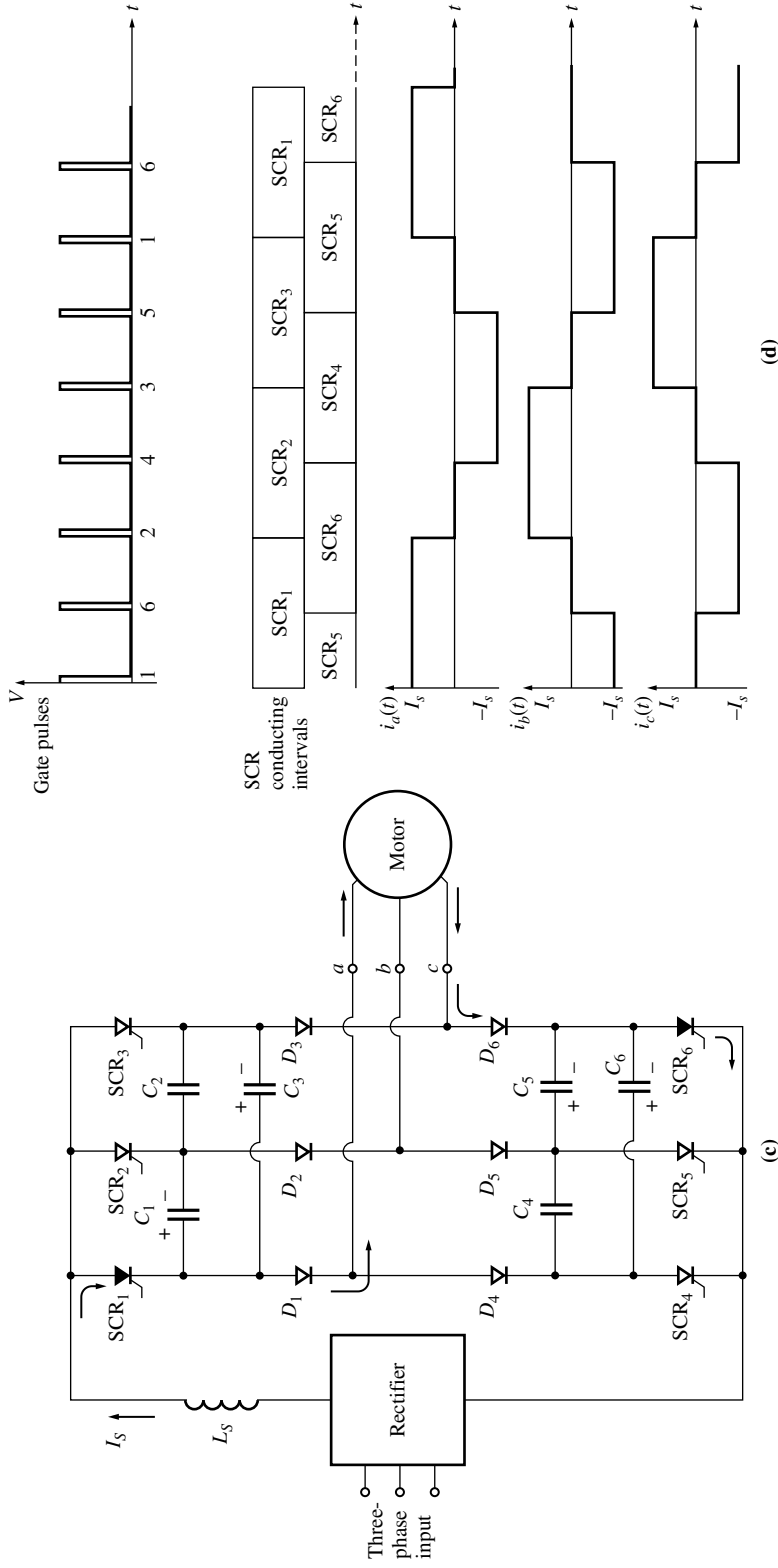
Figure S1-54 shows a three-phase voltage source inverter using power transistors as the active elements. Since power transistors are self-commutating, no special commutation components are included in this circuit.





**FIGURE S1-53**

The operation of the three-phase CSI. (a) Initially, SCR<sub>1</sub> and SCR<sub>5</sub> are conducting. Note how the commutating capacitors have charged up. (b) The situation when SCR<sub>6</sub> fires. The voltage at the anode of SCR<sub>6</sub> falls almost instantaneously to zero. Since the voltage across capacitor C<sub>5</sub> cannot change instantaneously, the voltage at the anode of SCR<sub>5</sub> will become negative, and SCR<sub>5</sub> will turn off.



**FIGURE S1-53 (concluded)**  
 (c) Now SCR<sub>1</sub> and SCR<sub>6</sub> are conducting, and the commutating capacitors charge up as shown. (d) The gating pulses, SCR conducting intervals, and the output current from this inverter.

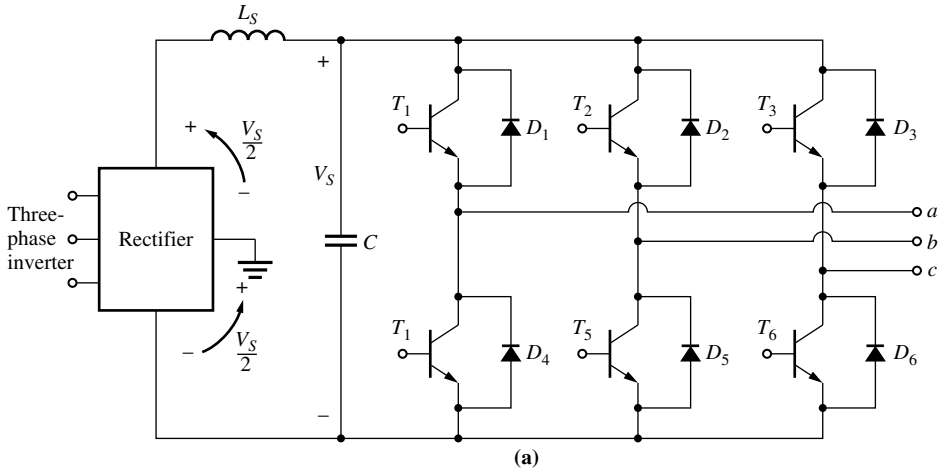


FIGURE S1-54

(a) A three-phase voltage source inverter using power transistors.

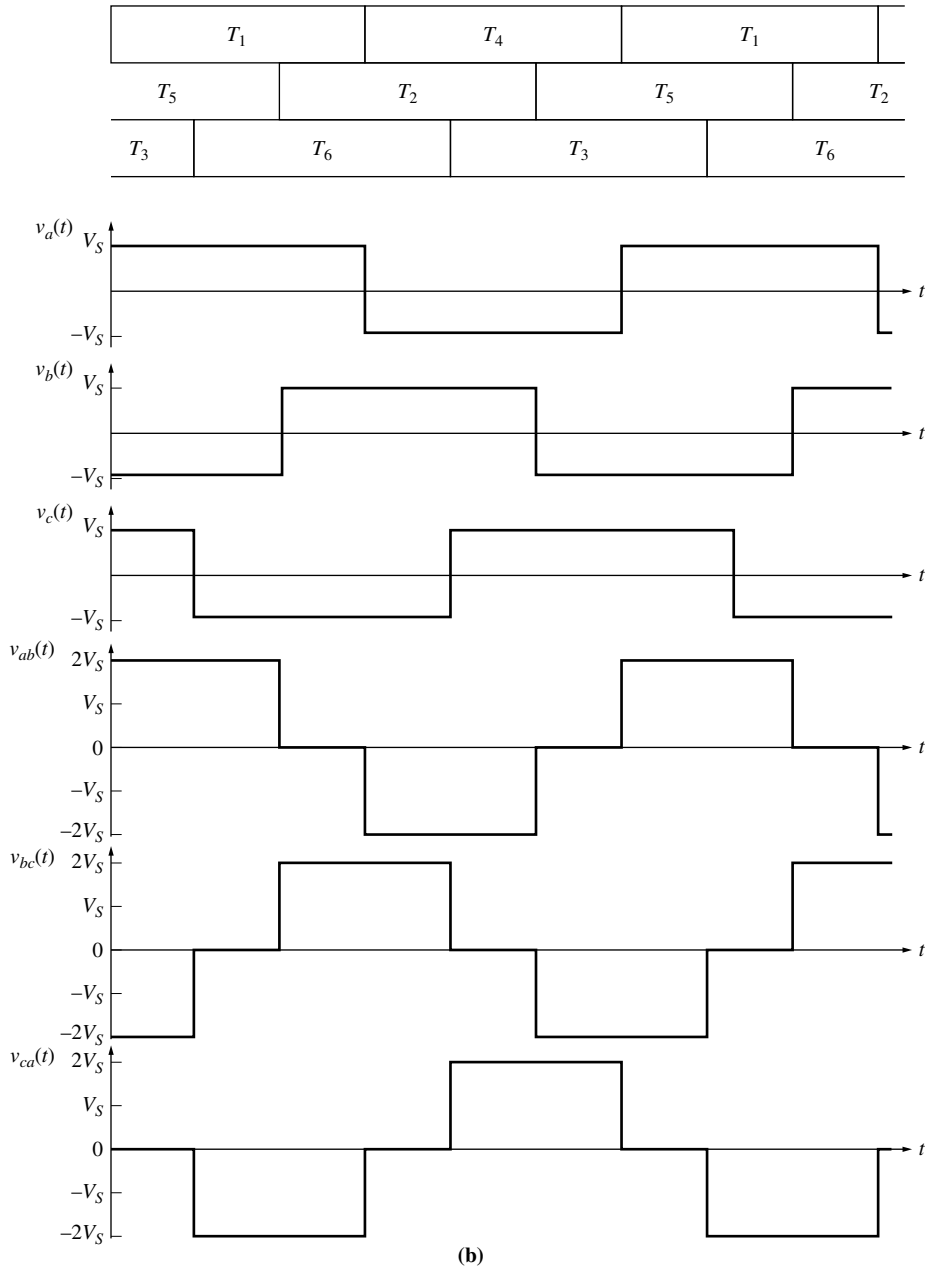
In this circuit, the transistors are made to conduct in the order  $T_1, T_6, T_2, T_4, T_3, T_5$ . The output phase and line voltage from this circuit are shown in Figure S1-54b.

### Pulse-Width Modulation Inverters

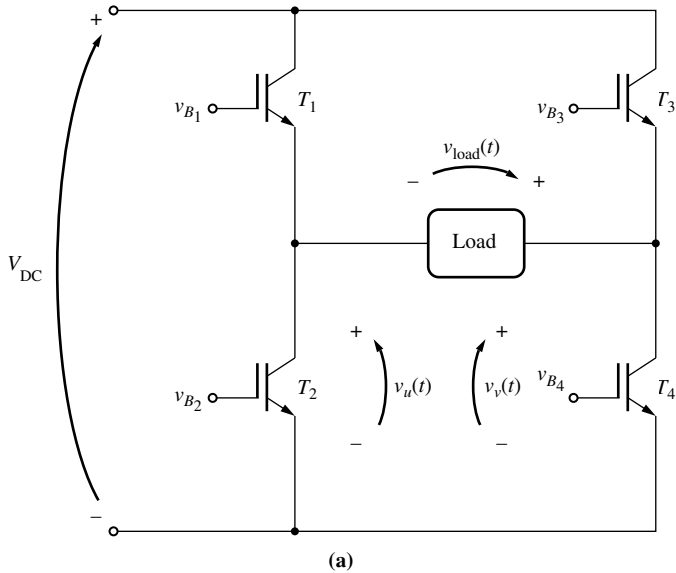
*Pulse-width modulation* is the process of modifying the width of the pulses in a pulse train in direct proportion to a small control signal; the greater the control voltage, the wider the resulting pulses become. By using a sinusoid of the desired frequency as the control voltage for a PWM circuit, it is possible to produce a high-power waveform whose *average* voltage varies sinusoidally in a manner suitable for driving ac motors.

The basic concepts of pulse-width modulation are illustrated in Figure S1-55. Figure S1-55a shows a single-phase PWM inverter circuit using IGBTs. The states of IGBT<sub>1</sub> through IGBT<sub>4</sub> in this circuit are controlled by the two comparators shown in Figure S1-55b.

A *comparator* is a device that compares the input voltage  $v_{in}(t)$  to a reference signal and turns transistors on or off depending on the results of the test. Comparator *A* compares  $v_{in}(t)$  to the reference voltage  $v_x(t)$  and controls IGBTs  $T_1$  and  $T_2$  based on the results of the comparison. Comparator *B* compares  $v_{in}(t)$  to the reference voltage  $v_y(t)$  and controls IGBTs  $T_3$  and  $T_4$  based on the results of the comparison. If  $v_{in}(t)$  is greater than  $v_x(t)$  at any given time  $t$ , then comparator *A* will turn on  $T_1$  and turn off  $T_2$ . Otherwise, it will turn off  $T_1$  and turn on  $T_2$ . Similarly, if  $v_{in}(t)$  is greater than  $v_y(t)$  at any given time  $t$ , then comparator *B* will turn



**FIGURE S1-54 (concluded)**  
 (b) The output phase and line voltages from the inverter.

**FIGURE S1-55**

The basic concepts of pulse-width modulation. (a) A single-phase PWM circuit using IGBTs.

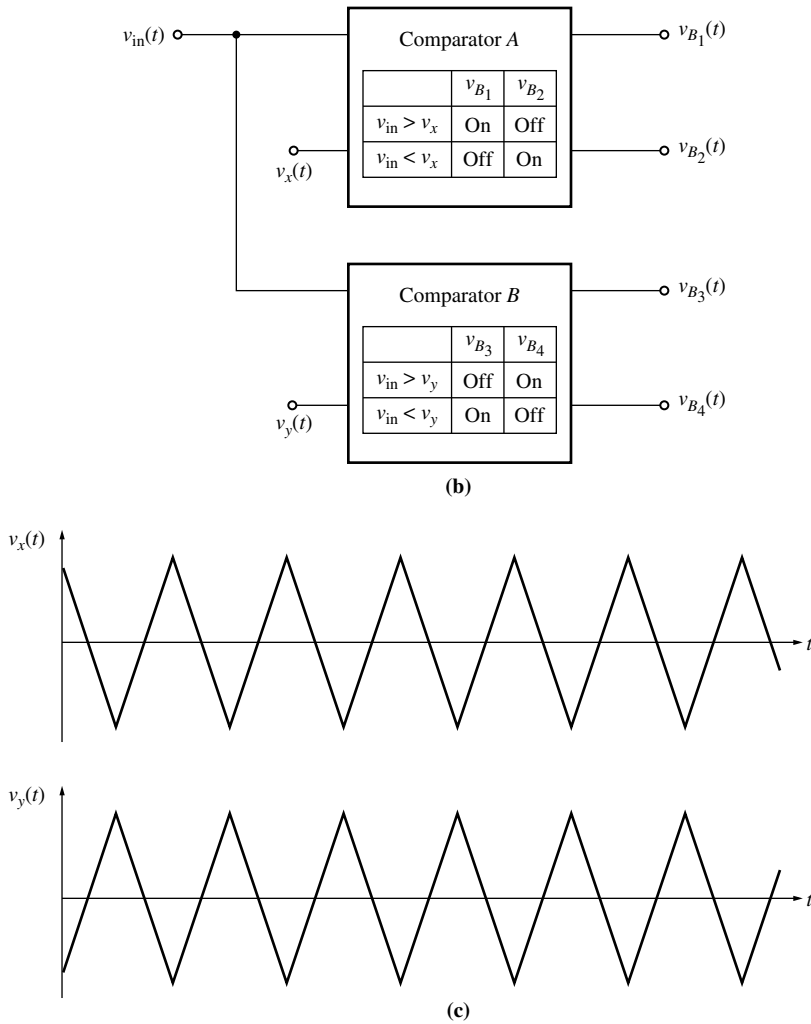
off  $T_3$  and turn on  $T_4$ . Otherwise, it will turn on  $T_3$  and turn off  $T_4$ . The reference voltages  $v_x(t)$  and  $v_y(t)$  are shown in Figure S1-55c.

To understand the overall operation of this PWM inverter circuit, see what happens when different control voltages are applied to it. First, assume that the control voltage is 0 V. Then voltages  $v_u(t)$  and  $v_v(t)$  are identical, and the load voltage out of the circuit  $v_{load}(t)$  is zero (see Figure S1-56).

Next, assume that a constant positive control voltage equal to one-half of the peak reference voltage is applied to the circuit. The resulting output voltage is a train of pulses with a 50 percent duty cycle, as shown in Figure S1-57.

Finally, assume that a sinusoidal control voltage is applied to the circuit as shown in Figure S1-58. The width of the resulting pulse train varies sinusoidally with the control voltage. The result is a high-power output waveform whose average voltage over any small region is directly proportional to the average voltage of the control signal in that region. The *fundamental frequency* of the output waveform is the same as the frequency of the input control voltage. Of course, there are harmonic components in the output voltage, but they are not usually a concern in motor-control applications. The harmonic components may cause additional heating in the motor being driven by the inverter, but the extra heating can be compensated for either by buying a specially designed motor or by *derating* an ordinary motor (running it at less than its full rated power).

A complete three-phase PWM inverter would consist of three of the single-phase inverters described above with control voltages consisting of sinusoids

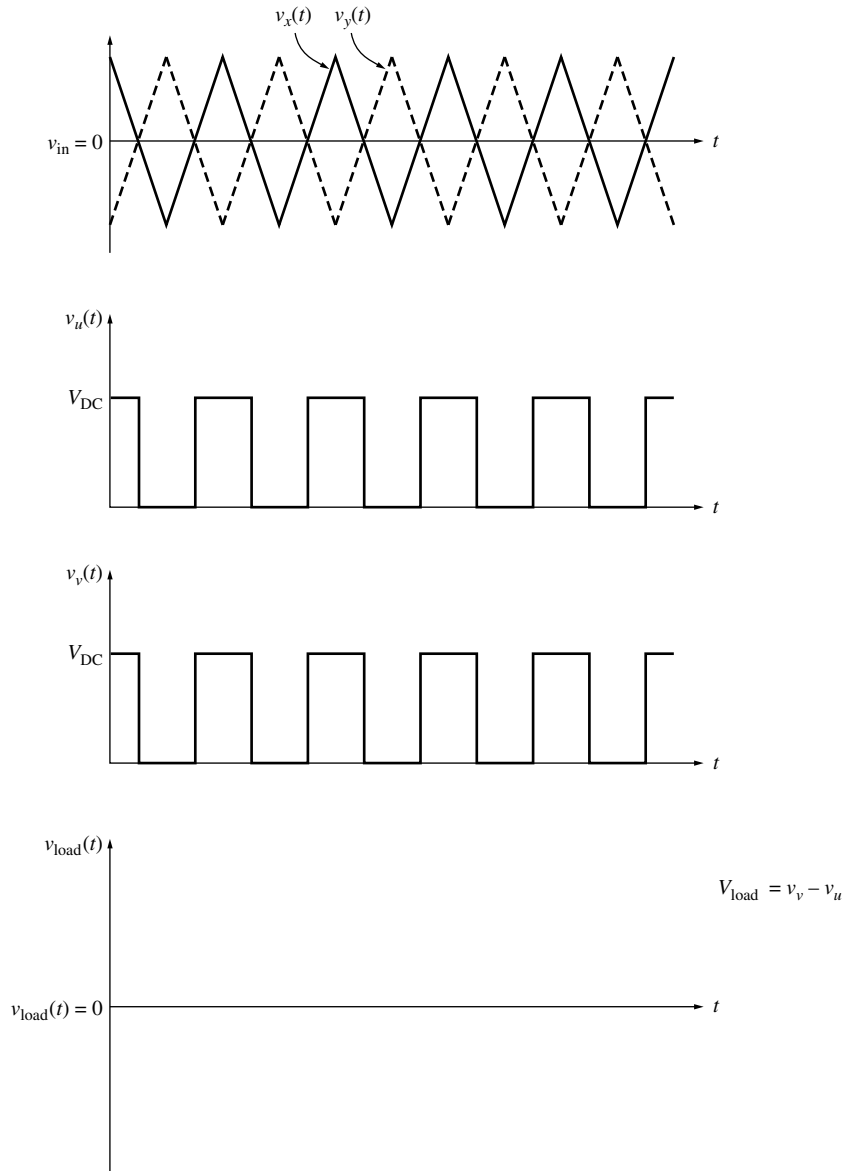


**FIGURE S1-55 (concluded)**

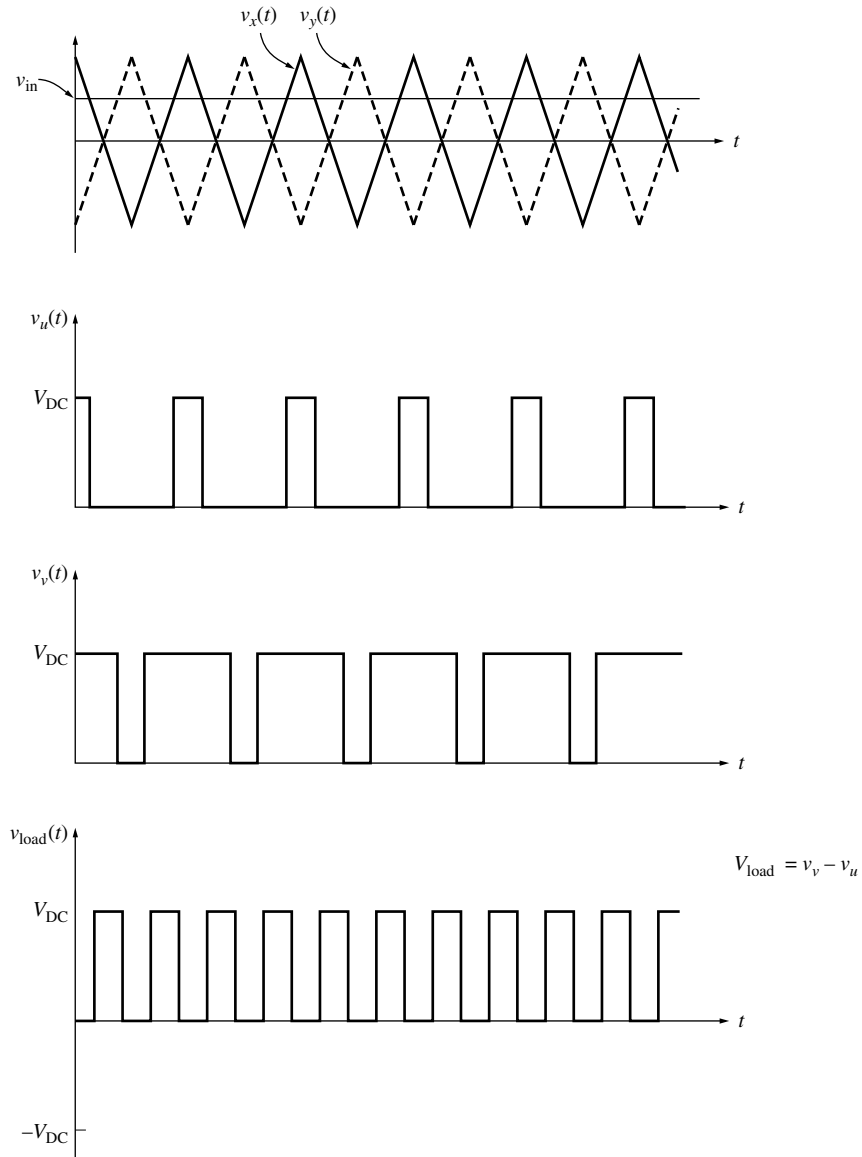
(b) The comparators used to control the on and off states of the transistors. (c) The reference voltages used in the comparators.

shifted by  $120^\circ$  between phases. Frequency control in a PWM inverter of this sort is accomplished by changing the frequency of the input control voltage.

A PWM inverter switches states many times during a single cycle of the resulting output voltage. At the time of this writing, reference voltages with frequencies as high as 12 kHz are used in PWM inverter designs, so the components in a PWM inverter must change states up to 24,000 times per second. This rapid switching means that PWM inverters require faster components than CSIs or VSIs. PWM inverters need high-power high-frequency components such as GTO thyristors,

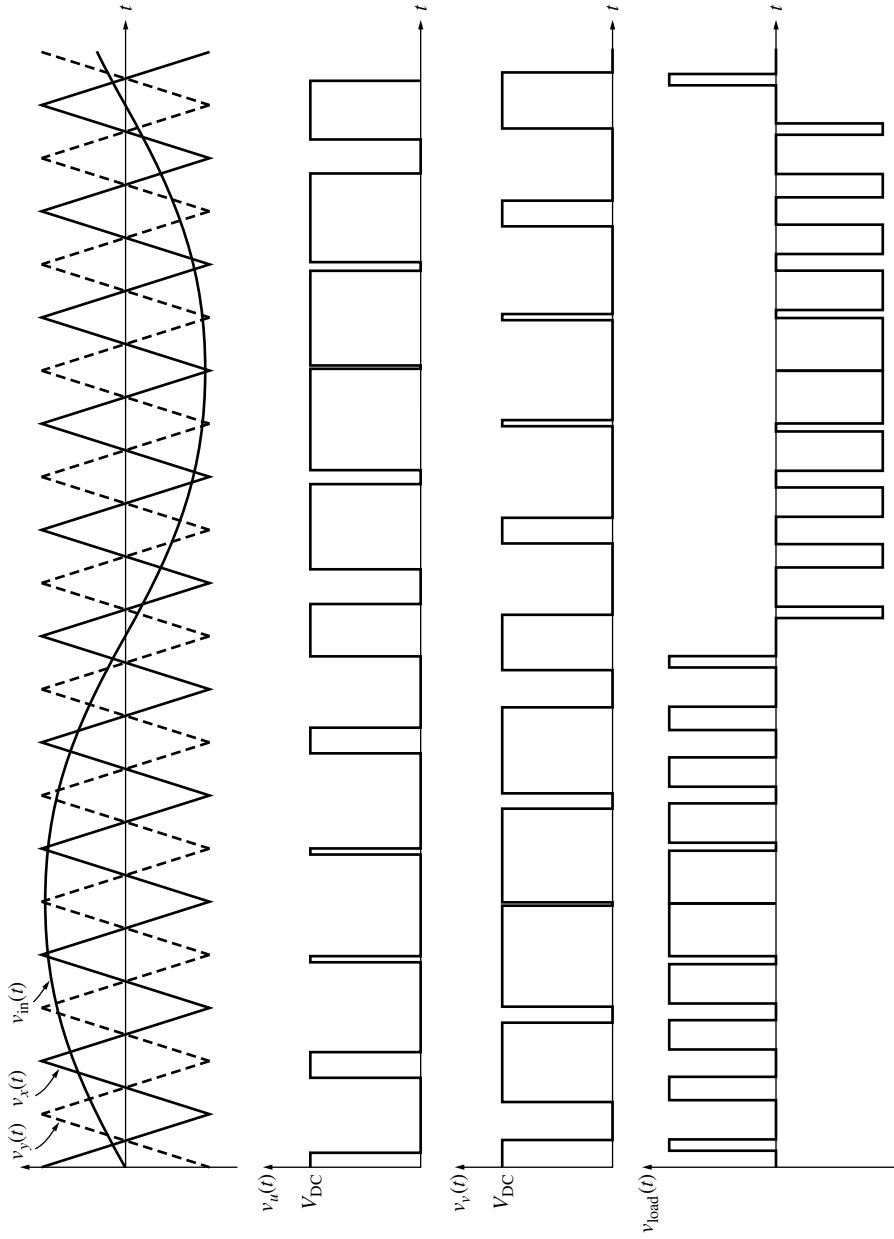


**FIGURE S1-56**  
 The output of the PWM circuit with an input voltage of 0 V. Note that  $v_u(t) = v_v(t)$ , so  $v_{load}(t) = 0$ .



**FIGURE S1-57**  
 The output of the PWM circuit with an input voltage equal to one-half of the peak comparator voltage.





**FIGURE S1-58**  
The output of the PWM circuit with a sinusoidal control voltage applied to its input.

IGBTs, and/or power transistors for proper operation. (At the time of this writing, IGBTs have the advantage for high-speed, high-power switching, so they are the preferred component for building PWM inverters.) The control voltage fed to the comparator circuits is usually implemented digitally by means of a microcomputer mounted on a circuit board within the PWM motor controller. The control voltage (and therefore the output pulse width) can be controlled by the microcomputer in a manner much more sophisticated than that described here. It is possible for the microcomputer to vary the control voltage to achieve different frequencies and voltage levels in any desired manner. For example, the microcomputer could implement various acceleration and deceleration ramps, current limits, and voltage-versus-frequency curves by simply changing options in software.

A real PWM-based induction motor drive circuit is described in Section 6.10.

## S1.7 CYCLOCONVERTERS

The cycloconverter is a device for directly converting ac power at one frequency to ac power at another frequency. Compared to rectifier-inverter schemes, cycloconverters have many more SCRs and much more complex gating circuitry. Despite these disadvantages, cycloconverters can be less expensive than rectifier-inverters at higher power ratings.

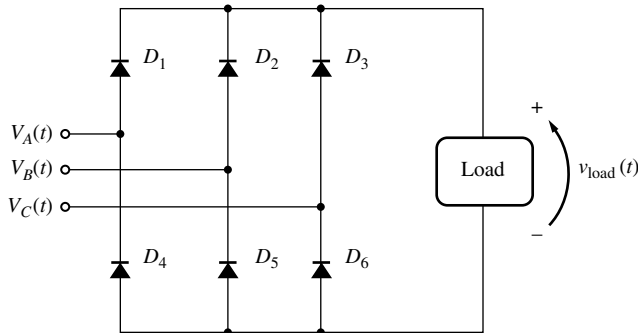
Cycloconverters are now available in constant-frequency and variable-frequency versions. A constant-frequency cycloconverter is used to supply power at one frequency from a source at another frequency (e.g., to supply 50-Hz loads from a 60-Hz source). Variable-frequency cycloconverters are used to provide a variable output voltage and frequency from a constant-voltage and constant-frequency source. They are often used as ac induction motor drives.

Although the details of a cycloconverter can become very complex, the basic idea behind the device is simple. The input to a cycloconverter is a three-phase source which consists of three voltages equal in magnitude and phase-shifted from each other by  $120^\circ$ . The desired output voltage is some specified waveform, usually a sinusoid at a different frequency. *The cycloconverter generates its desired output waveform by selecting the combination of the three input phases which most closely approximates the desired output voltage at each instant of time.*

There are two major categories of cycloconverters, *noncirculating current cycloconverters* and *circulating current cycloconverters*. These types are distinguished by whether or not a current circulates internally within the cycloconverter; they have different characteristics. The two types of cycloconverters are described following an introduction to basic cycloconverter concepts.

### Basic Concepts

A good way to begin the study of cycloconverters is to take a closer look at the three-phase full-wave bridge rectifier circuit described in Section S1.2. This circuit is shown in Figure S1-59 attached to a resistive load. In that figure, the diodes are divided into two halves, a positive half and a negative half. In the positive half, the



$$V_A(t) = V_M \sin \omega t \text{ V}$$

$$V_B(t) = V_M \sin (\omega t - 120^\circ) \text{ V}$$

$$V_C(t) = V_M \sin (\omega t - 240^\circ) \text{ V}$$

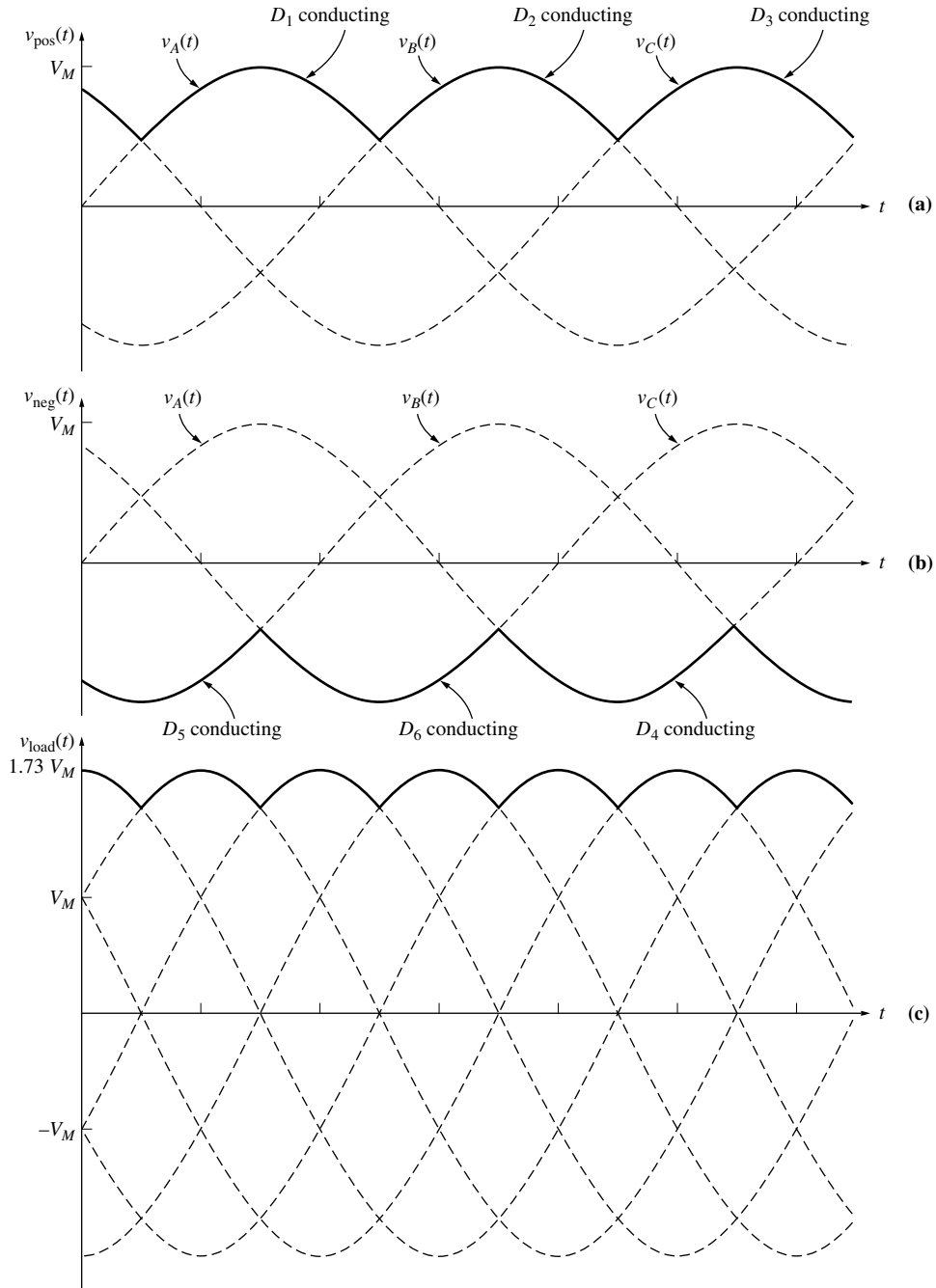
**FIGURE S1-59**

A three-phase full-wave diode bridge circuit connected to a resistive load.

diode with the highest voltage applied to it at any given time will conduct, and it will reverse-bias the other two diodes in the section. In the negative half, the diode with the lowest voltage applied to it at any given time will conduct, and it will reverse-bias the other two diodes in the section. The resulting output voltage is shown in Figure S1-60.

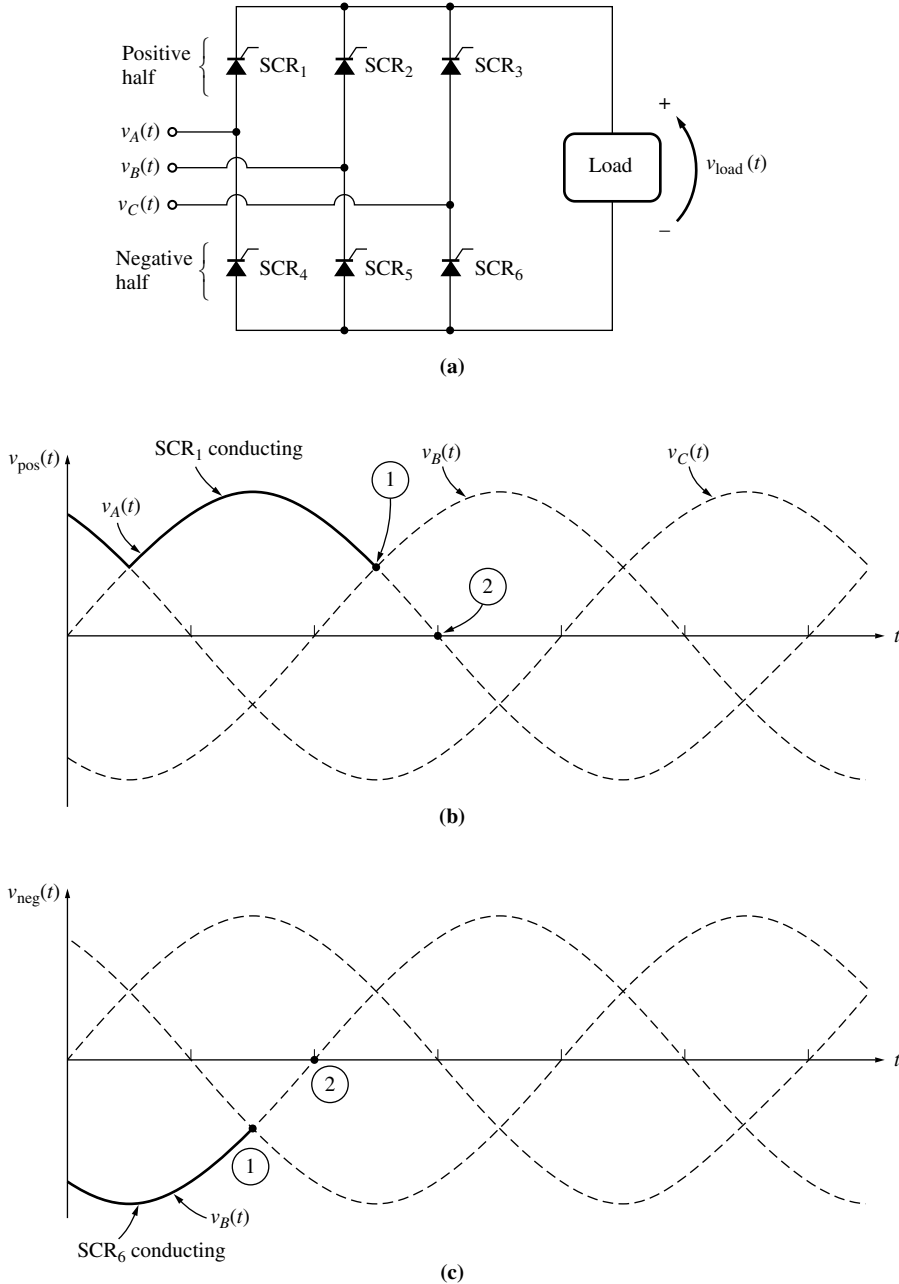
Now suppose that the six diodes in the bridge circuit are replaced by six SCRs as shown in Figure S1-61. Assume that initially SCR<sub>1</sub> is conducting as shown in Figure S1-61b. This SCR will continue to conduct until the current through it falls below  $I_H$ . If no other SCR in the positive half is triggered, then SCR<sub>1</sub> will be turned off when voltage  $v_A$  goes to zero and reverses polarity at point 2. However, if SCR<sub>2</sub> is triggered at any time after point 1, then SCR<sub>1</sub> will be instantly reverse-biased and turned off. The process in which SCR<sub>2</sub> forces SCR<sub>1</sub> to turn off is called *forced commutation*; it can be seen that forced commutation is possible only for the phase angles between points 1 and 2. The SCRs in the negative half behave in a similar manner, as shown in Figure S1-61c. Note that if each of the SCRs is fired as soon as commutation is possible, then the output of this bridge circuit will be the same as the output of the full-wave diode bridge rectifier shown in Figure S1-59.

Now suppose that it is desired to produce a linearly decreasing output voltage with this circuit, as shown in Figure S1-62. To produce such an output, the conducting SCR in the positive half of the bridge circuit must be turned off whenever its voltage falls too far below the desired value. This is done by triggering another SCR voltage above the desired value. Similarly, the conducting SCR in the negative half of the bridge circuit must be turned off whenever its voltage rises too far above the desired value. By triggering the SCRs in the positive and negative halves at the right time, it is possible to produce an output voltage which decreases in a manner roughly corresponding to the desired waveform. It is obvious from examining Figure S1-62 that many harmonic components are present in the resulting output voltage.



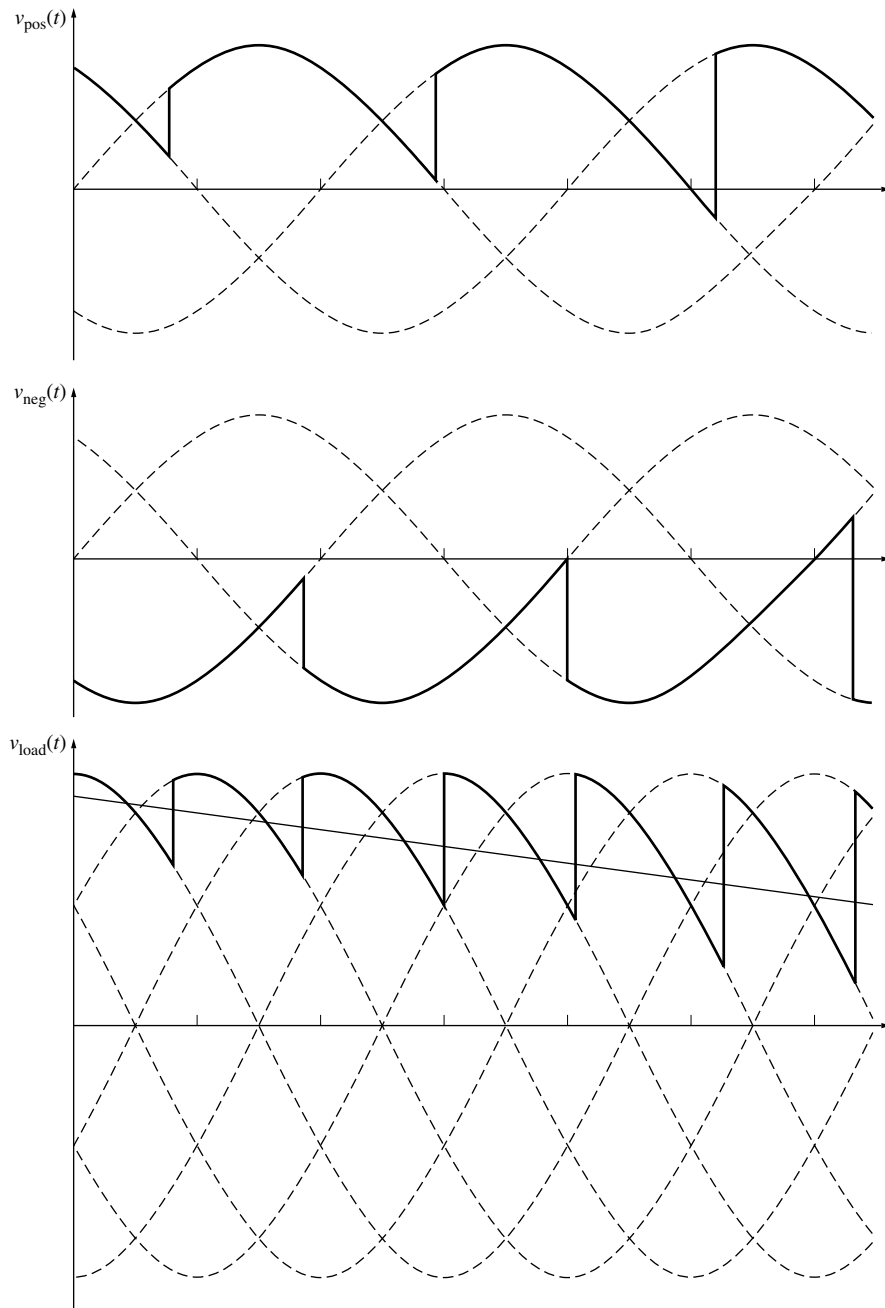
**FIGURE S1-60**

(a) The output voltage from the positive-half diodes. (b) The output voltage from the negative-half diodes. (c) The total voltage applied to the load.

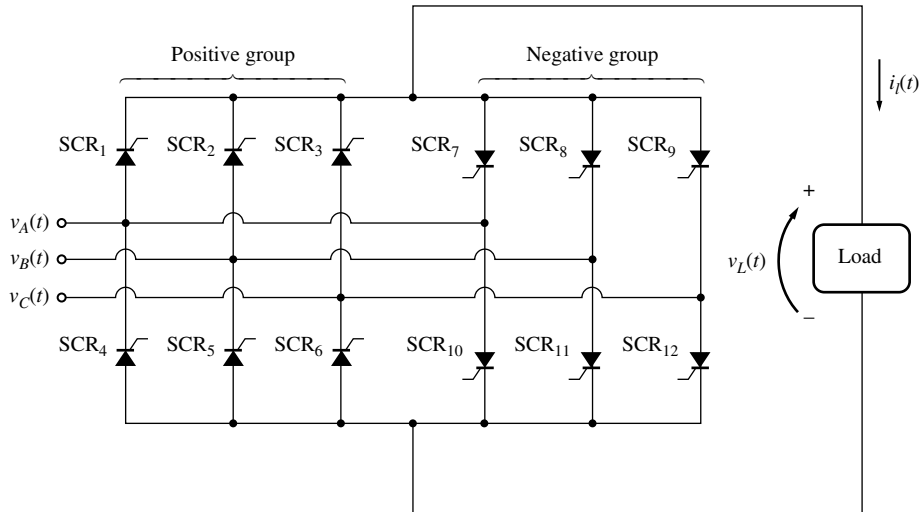


**FIGURE S1-61**

(a) A three-phase full-wave SCR bridge circuit connected to a resistive load. (b) The operation of the positive half of the SCRs. Assume that initially SCR<sub>1</sub> is conducting. If SCR<sub>2</sub> is triggered at any time after point 1, then SCR<sub>1</sub> will be reverse-biased and shut off. (c) The operation of the negative half of the SCRs. Assume that initially SCR<sub>6</sub> is conducting. If SCR<sub>4</sub> is triggered at any time after point 1, then SCR<sub>6</sub> will be reverse-biased and shut off.



**FIGURE S1-62**  
Approximating a linearly decreasing voltage with the three-phase full-wave SCR bridge circuit.



**FIGURE S1-63**  
One phase of a noncirculating current cycloconverter circuit.

If two of these SCR bridge circuits are connected in parallel with opposite polarities, the result is a noncirculating current cycloconverter.

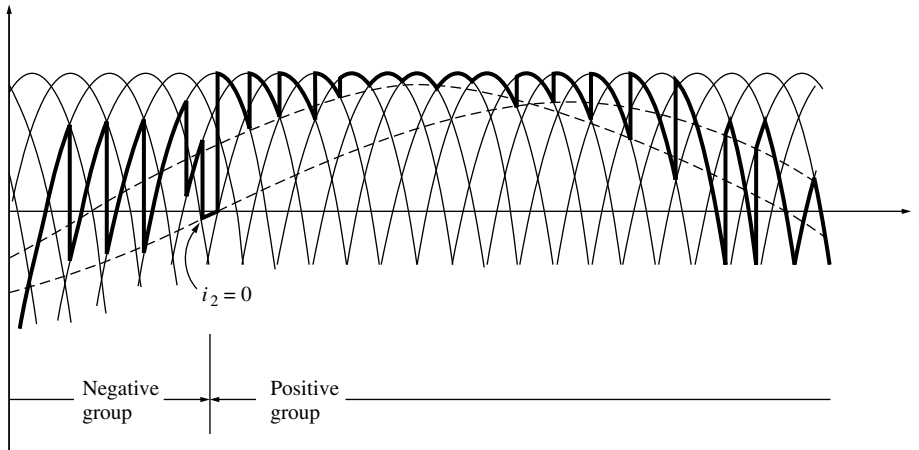
### Noncirculating Current Cycloconverters

One phase of a typical noncirculating current cycloconverter is shown in Figure S1-63. A full three-phase cycloconverter consists of three identical units of this type. Each unit consists of two three-phase full-wave SCR bridge circuits, one conducting current in the positive direction (the *positive group*) and one conducting current in the negative direction (the *negative group*). The SCRs in these circuits are triggered so as to approximate a sinusoidal output voltage, with the SCRs in the positive group being triggered when the current flow is in the positive direction and the SCRs in the negative group being triggered when the current flow is in the negative direction. The resulting output voltage is shown in Figure S1-64.

As can be seen from Figure S1-64, noncirculating current cycloconverters produce an output voltage with a fairly large harmonic component. These high harmonics limit the output frequency of the cycloconverter to a value less than about one-third of the input frequency.

In addition, note that current flow must switch from the positive group to the negative group or vice versa as the load current reverses direction. The cycloconverter pulse-control circuits must detect this current transition with a current polarity detector and switch from triggering one group of SCRs to triggering the other group. There is generally a brief period during the transition in which neither the positive nor the negative group is conducting. This current pause causes additional glitches in the output waveform.

The high harmonic content, low maximum frequency, and current glitches associated with noncirculating current cycloconverters combine to limit their use.

**FIGURE S1-64**

The output voltage and current from a noncirculating current cycloconverter connected to an inductive load. Note the switch from the operation of the negative group to the operation of the positive group at the time the current changes direction.

In any practical noncirculating current cycloconverter, a filter (usually a series inductor or a transformer) is placed between the output of the cycloconverter and the load, to suppress some of the output harmonics.

### Circulating Current Cycloconverters

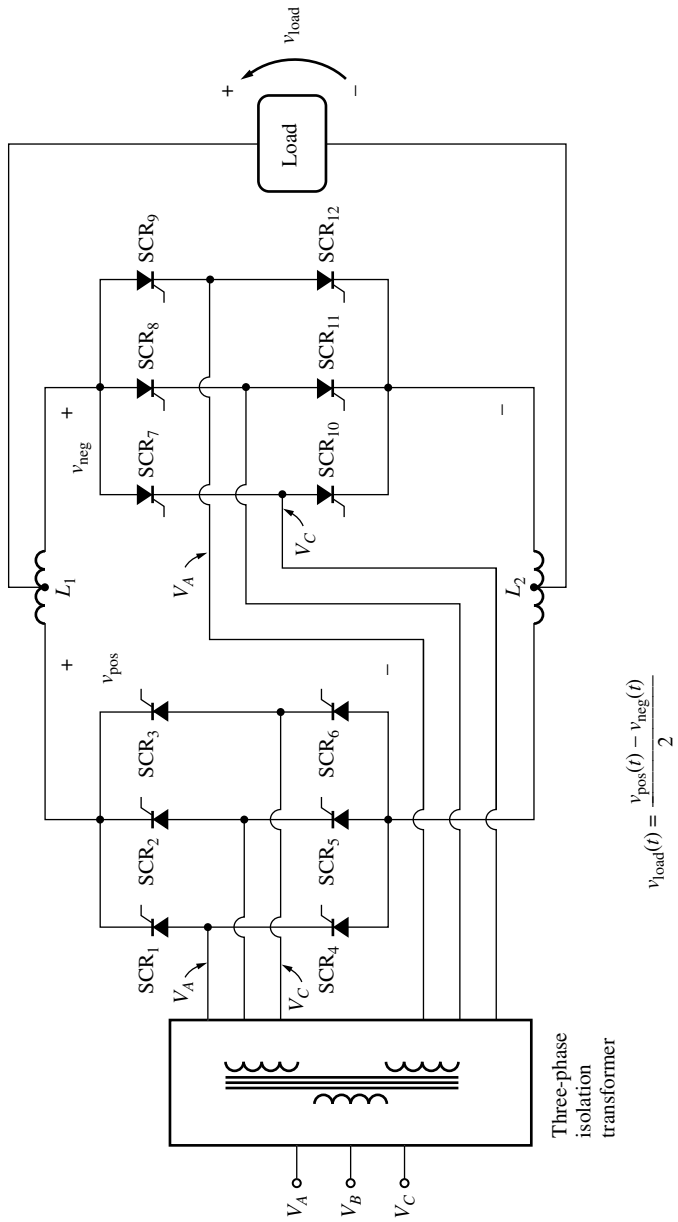
One phase of a typical circulating current cycloconverter is shown in Figure S1-65. It differs from the noncirculating current cycloconverter in that the positive and negative groups are connected through two large inductors, and the load is supplied from center taps on the two inductors. Unlike the noncirculating current cycloconverter, *both the positive and the negative groups are conducting at the same time*, and a circulating current flows around the loop formed by the two groups and the series inductors. The series inductors must be quite large in a circuit of this sort to limit the circulating current to a safe value.

The output voltage from the circulating current cycloconverter has a smaller harmonic content than the output voltage from the noncirculating current cycloconverter, and its maximum frequency can be much higher. It has a low power factor due to the large series inductors, so a capacitor is often used for power-factor compensation.

The reason that the circulating current cycloconverter has a lower harmonic content is shown in Figure S1-66. Figure S1-66a shows the output voltage of the positive group, and Figure S1-66b shows the output voltage of the negative group. The output voltage  $v_{\text{load}}(t)$  across the center taps of the inductors is

$$v_{\text{load}}(t) = \frac{v_{\text{pos}}(t) - v_{\text{neg}}(t)}{2} \quad (\text{S1-9})$$



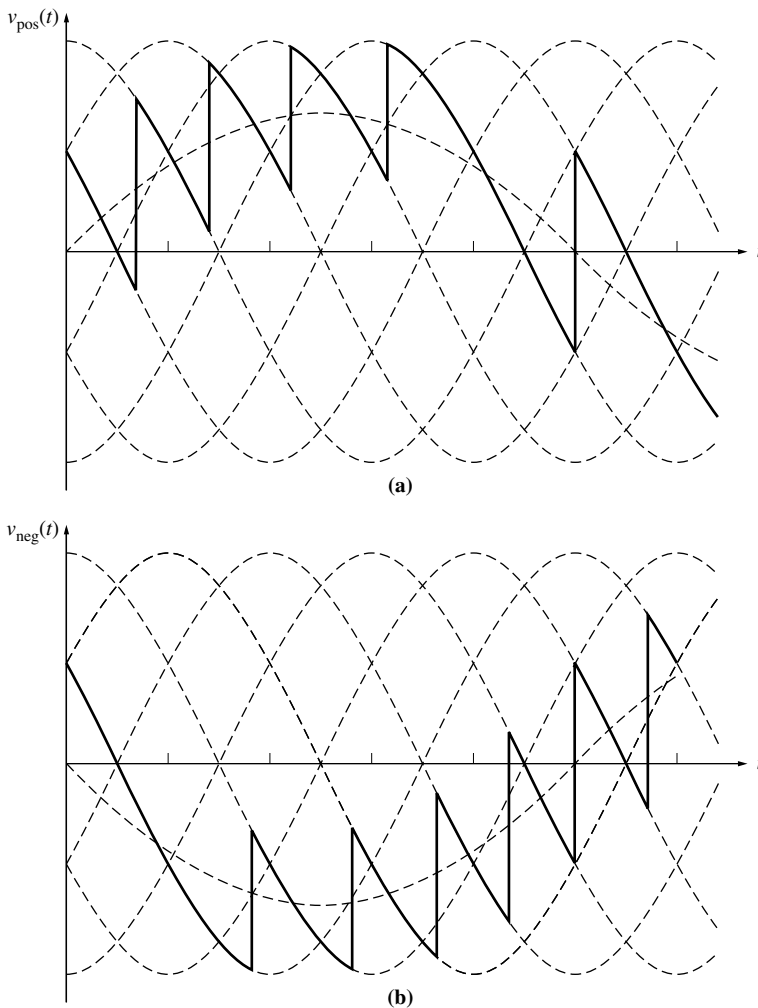


$$v_{load}(t) = \frac{v_{pos}(t) - v_{neg}(t)}{2}$$

**FIGURE S1-65**  
One phase of a six-pulse type of circulating current cycloconverter.

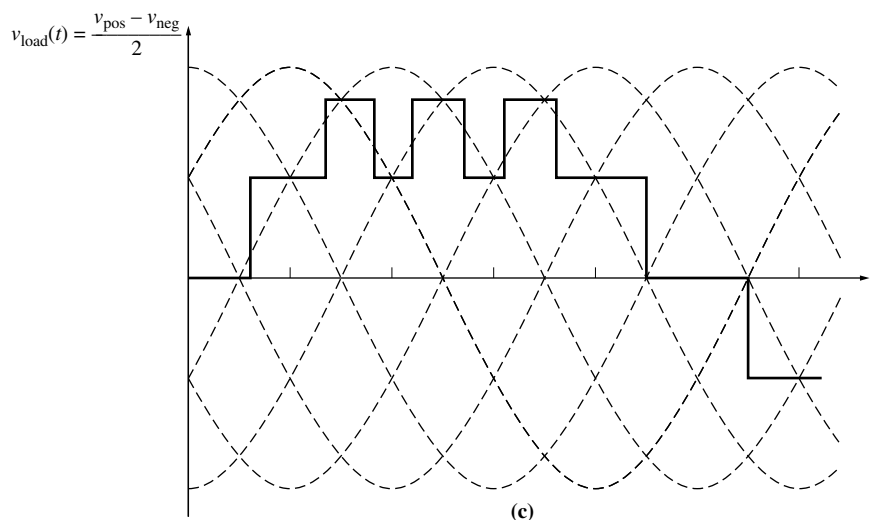
Many of the high-frequency harmonic components which appear when the positive and negative groups are examined separately are common to both groups. As such, they cancel during the subtraction and do not appear at the terminals of the cycloconverter.

Some recirculating current cycloconverters are more complex than the one shown in Figure S1-65. With more sophisticated designs, it is possible to make cycloconverters whose maximum output frequency can be even higher than their input frequency. These more complex devices are beyond the scope of this book.



**FIGURE S1-66**

Voltages in the six-pulse circulating current cycloconverter. (a) The voltage out of the positive group; (b) the voltage out of the negative group.



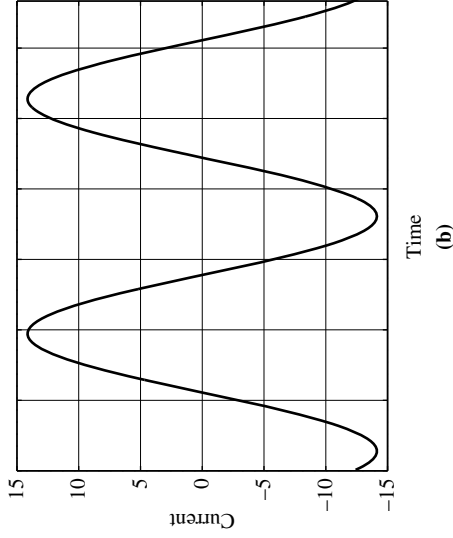
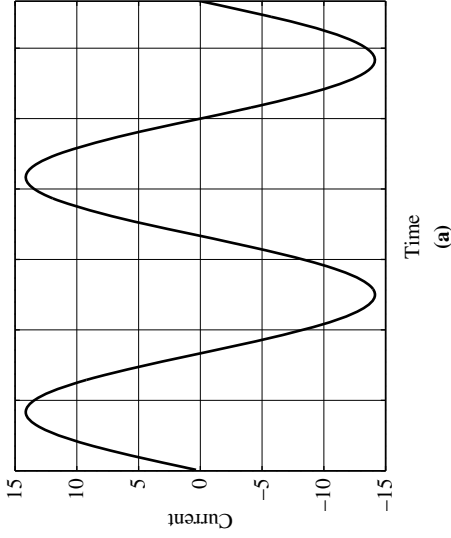
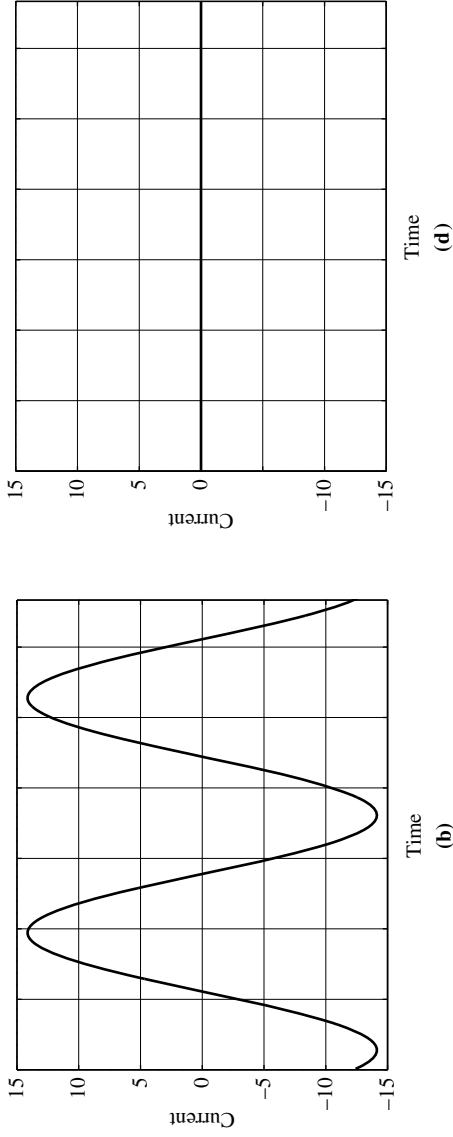
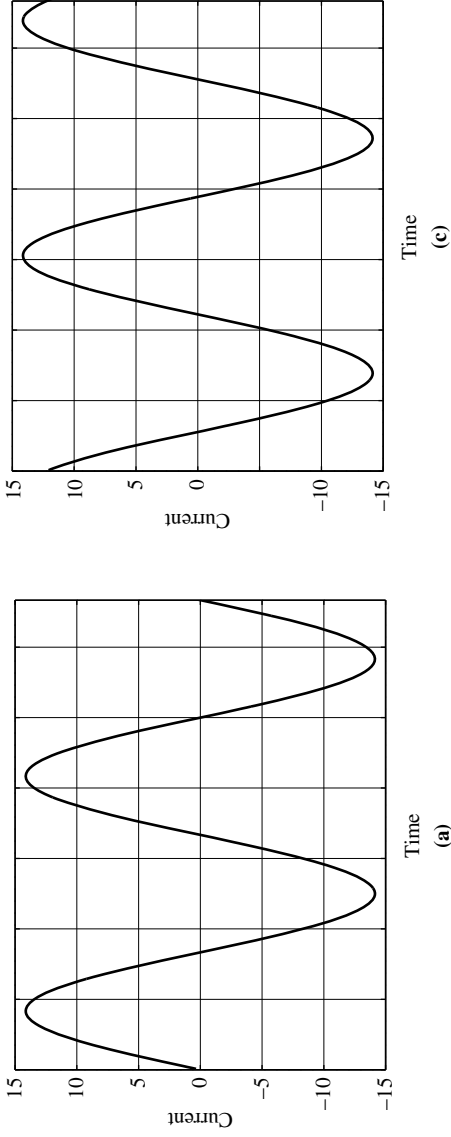
**FIGURE S1-66 (concluded)**  
(c) the resulting load voltage.

## S1.8 HARMONIC PROBLEMS

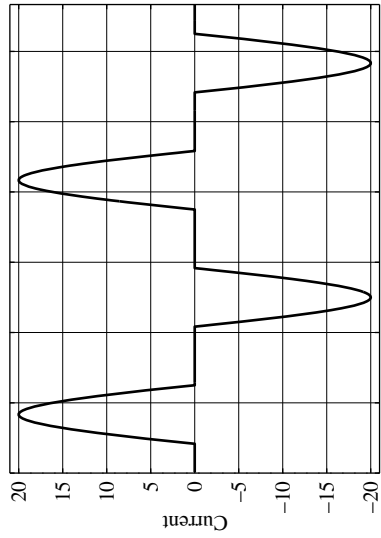
Power electronic components and circuits are so flexible and useful that equipment controlled by them now makes up 50 to 60 percent of the total load on most power systems in the developed world. As a result, the behavior of these power electronic circuits strongly influences the overall operation of the power systems that they are connected to.

The principal problem associated with power electronics is the harmonic components of voltage and current induced in the power system by the switching transients in power electronic controllers. These harmonics increase the total current flows in the lines (especially in the neutral of a three-phase power system). The extra currents cause increased losses and increased heating in power system components, requiring larger components to supply the same total load. In addition, the high neutral currents can trip protective relays, shutting down portions of a power system.

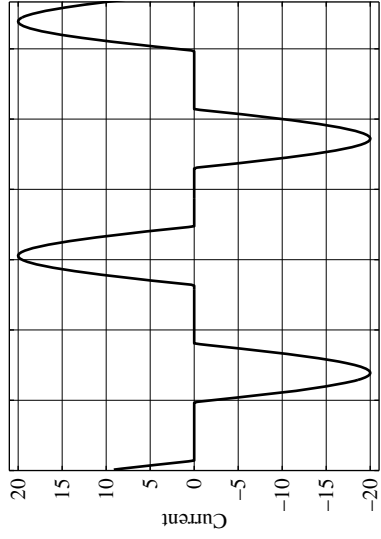
As an example of this problem, consider a balanced three-phase motor with a wye connection that draws 10 A at full load. When this motor is connected to a power system, the currents flowing in each phase will be equal in magnitude and  $120^\circ$  out of phase with each other, and the return current in the neutral will be 0 (see Figure S1-67). Now consider the same motor supplied with the same total power through a rectifier-inverter that produces pulses of current. The currents in the power line now are shown in Figure S1-68. Note that the rms current of each line is still 10 A, but the neutral also has an rms current of 15 A! The current in the neutral consists entirely of harmonic components.



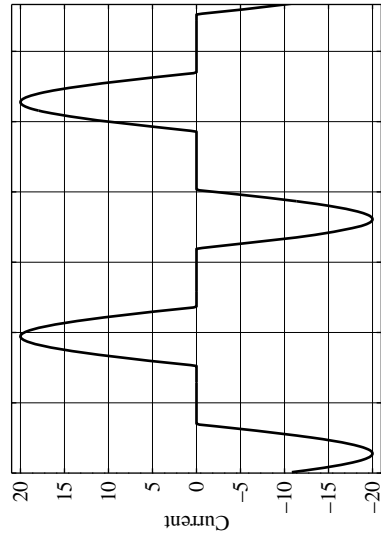
**FIGURE SI-67** Current flow for a balanced three-phase, wye-connected motor: (a) phase *a*; (b) phase *b*; (c) phase *c*; (d) neutral. The rms current flow in phases *a*, *b*, and *c* is 10 A, and the current flow in the neutral is 0.



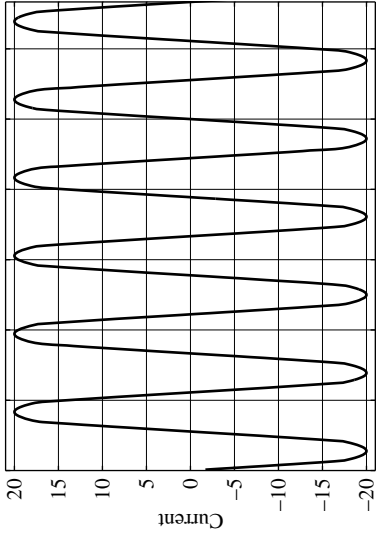
(a)



(b)



(c)



(d)

**FIGURE SI-68**  
 Current flow for a balanced three-phase, wye-connected motor connected to the power line through a power electronic controller that produces current pulses: (a) phase  $a$ ; (b) phase  $b$ ; (c) phase  $c$ ; (d) neutral. The rms current flow in phases  $a$ ,  $b$ , and  $c$  is 10 A, while the rms current flow in the neutral is 15 A.

The spectra of the currents in the three phases and in the neutral are shown in Figure S1–69. For the motor connected directly to the line, only the fundamental frequency is present in the phases, and nothing at all is present in the neutral. For the motor connected through the power controller, the current in the phases includes both the fundamental frequency and all of the odd harmonics. The current in the neutral consists principally of the third, ninth, and fifteenth harmonics.

Since power electronic circuits are such a large fraction of the total load on a modern power system, their high harmonic content causes significant problems for the power system as a whole. New standards\* have been created to limit the amount of harmonics produced by power electronic circuits, and new controllers are designed to minimize the harmonics that they produce.

## S1.9 SUMMARY

Power electronic components and circuits have produced a major revolution in the area of motor controls during the last 40 years or so. Power electronics provide a convenient way to convert ac power to dc power, to change the average voltage level of a dc power system, to convert dc power to ac power, and to change the frequency of an ac power system.

The conversion of ac to dc power is accomplished by rectifier circuits, and the resulting dc output voltage level can be controlled by changing the firing times of the devices (SCRs, TRIACs, GTO thyristors, etc.) in the rectifier circuit.

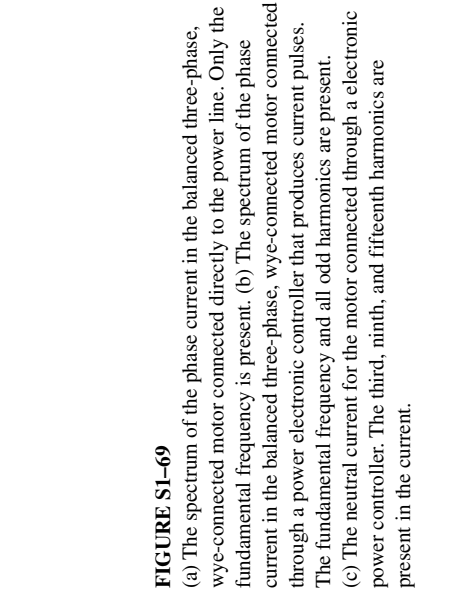
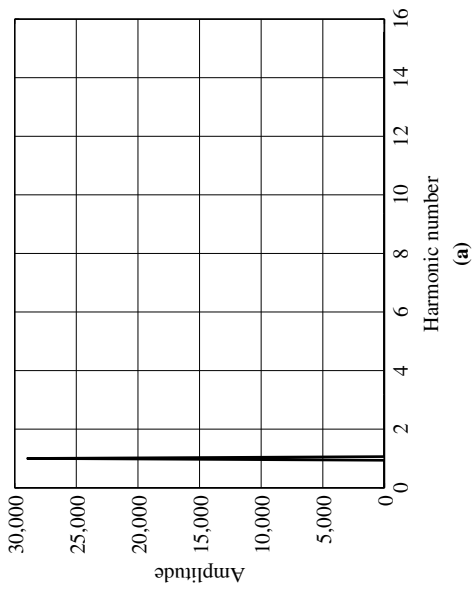
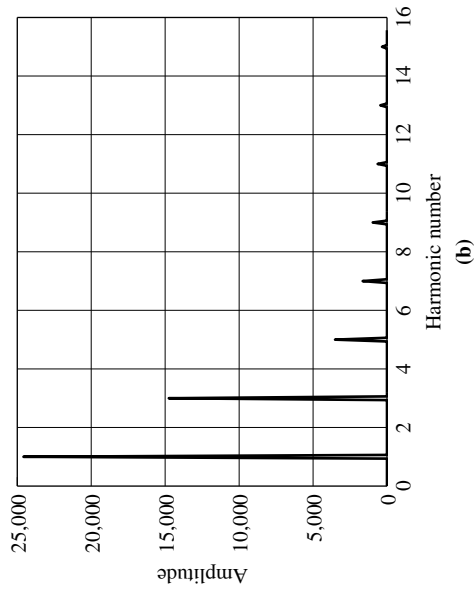
Adjustment of the average dc voltage level on a load is accomplished by chopper circuits, which control the fraction of time for which a fixed dc voltage is applied to a load.

Static frequency conversion is accomplished by either rectifier-inverters or cycloconverters. Inverters are of two basic types: externally commutated and self-commutated. Externally commutated inverters rely on the attached load for commutation voltages; self-commutated inverters either use capacitors to produce the required commutation voltages or use self-commutating devices such as GTO thyristors. Self-commutated inverters include current source inverters, voltage source inverters, and pulse-width modulation inverters.

Cycloconverters are used to directly convert ac power at one frequency to ac power at another frequency. There are two basic types of cycloconverters: non-circulating current and circulating current. Noncirculating current cycloconverters have large harmonic components and are restricted to relatively low frequencies. In addition, they can suffer from glitches during current direction changes. Circulating current cycloconverters have lower harmonic components and are capable of operating at higher frequencies. They require large series inductors to limit the circulating current to a safe value, and so they are bulkier than noncirculating current cycloconverters of the same rating.

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\*See IEC 1000-3-2. EMC: Part 3, Section 2, "Limits for harmonic current emission (equipment input current  $\leq 16$  A per phase)," and ANSI/IEEE Standard 519-1992, "IEEE recommended practices and requirements for harmonic control in power systems."



**FIGURE S1-69**

(a) The spectrum of the phase current in the balanced three-phase, wye-connected motor connected directly to the power line. Only the fundamental frequency is present. (b) The spectrum of the phase current in the balanced three-phase, wye-connected motor connected through a power electronic controller that produces current pulses. The fundamental frequency and all odd harmonics are present. (c) The neutral current for the motor connected through an electronic power controller. The third, ninth, and fifteenth harmonics are present in the current.

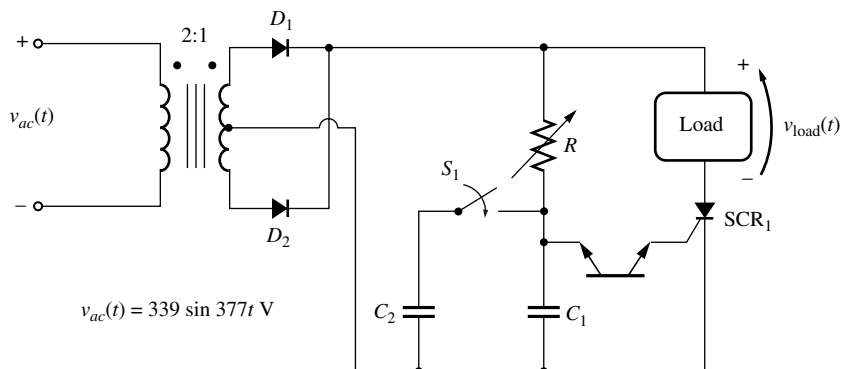
**QUESTIONS**

- S1-1.** Explain the operation and sketch the output characteristic of a diode.
- S1-2.** Explain the operation and sketch the output characteristic of a PNP diode.
- S1-3.** How does an SCR differ from a PNP diode? When does an SCR conduct?
- S1-4.** What is a GTO thyristor? How does it differ from an ordinary three-wire thyristor (SCR)?
- S1-5.** What is an IGBT? What are its advantages compared to other power electronic devices?
- S1-6.** What is a DIAC? A TRIAC?
- S1-7.** Does a single-phase full-wave rectifier produce a better or worse dc output than a three-phase half-wave rectifier? Why?
- S1-8.** Why are pulse-generating circuits needed in motor controllers?
- S1-9.** What are the advantages of digital pulse-generating circuits compared to analog pulse-generating circuits?
- S1-10.** What is the effect of changing resistor  $R$  in Figure S1-32? Explain why this effect occurs.
- S1-11.** What is forced commutation? Why is it necessary in dc-to-dc power-control circuits?
- S1-12.** What device(s) could be used to build dc-to-dc power-control circuits without forced commutation?
- S1-13.** What is the purpose of a free-wheeling diode in a control circuit with an inductive load?
- S1-14.** What is the effect of an inductive load on the operation of a phase angle controller?
- S1-15.** Can the on time of a chopper with series-capacitor commutation be made arbitrarily long? Why or why not?
- S1-16.** Can the on time of a chopper with parallel-capacitor commutation be made arbitrarily long? Why or why not?
- S1-17.** What is a rectifier-inverter? What is it used for?
- S1-18.** What is a current-source inverter?
- S1-19.** What is a voltage-source inverter? Contrast the characteristics of a VSI with those of a CSI.
- S1-20.** What is pulse-width modulation? How do PWM inverters compare to CSI and VSI inverters?
- S1-21.** Are power transistors more likely to be used in PWM inverters or in CSI inverters? Why?

**PROBLEMS**

- S1-1.** Calculate the ripple factor of a three-phase half-wave rectifier circuit, both analytically and using MATLAB.
- S1-2.** Calculate the ripple factor of a three-phase full-wave rectifier circuit, both analytically and using MATLAB.
- S1-3.** Explain the operation of the circuit shown in Figure PS1-1. What would happen in this circuit if switch  $S_1$  were closed?




**FIGURE PS1-1**

The circuit of Problems S1-3 through S1-6.

**S1-4.** What would the rms voltage on the load in the circuit in Figure PS1-1 be if the firing angle of the SCR were (a)  $0^\circ$ , (b)  $30^\circ$ , (c)  $90^\circ$ ?

**\*S1-5.** For the circuit in Figure PS1-1, assume that  $V_{BO}$  for the DIAC is 30 V,  $C_1$  is  $1 \mu\text{F}$ ,  $R$  is adjustable in the range 1 to 20 k $\Omega$ , and switch  $S_1$  is open. What is the firing angle of the circuit when  $R$  is 10 k $\Omega$ ? What is the rms voltage on the load under these conditions? (*Caution:* This problem is hard to solve analytically because the voltage charging the capacitor varies as a function of time.)

**S1-6.** One problem with the circuit shown in Figure PS1-1 is that it is very sensitive to variations in the input voltage  $v_{ac}(t)$ . For example, suppose the peak value of the input voltage were to decrease. Then the time that it takes capacitor  $C_1$  to charge up to the breakover voltage of the DIAC will increase, and the SCR will be triggered later in each half-cycle. Therefore, the rms voltage supplied to the load will be reduced *both* by the lower peak voltage *and* by the later firing. This same effect happens in the opposite direction if  $v_{ac}(t)$  increases. How could this circuit be modified to reduce its sensitivity to variations in input voltage?

**S1-7.** Explain the operation of the circuit shown in Figure PS1-2, and sketch the output voltage from the circuit.

**S1-8.** Figure PS1-3 shows a relaxation oscillator with the following parameters:

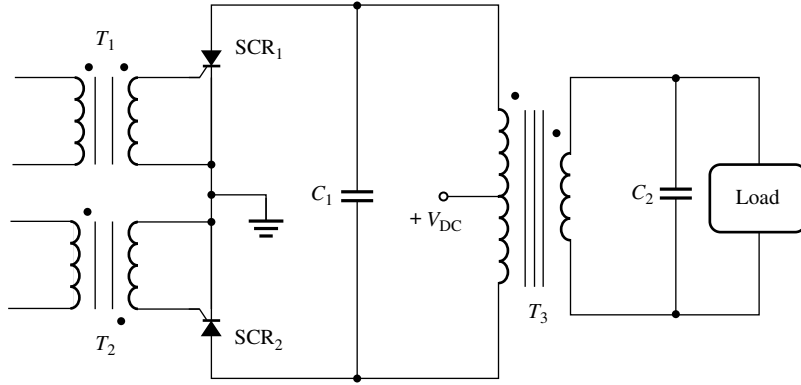
$$\begin{array}{ll} R_1 = \text{variable} & R_2 = 1500 \Omega \\ C = 1 \mu\text{F} & V_{DC} = 100 \text{ V} \\ V_{BO} = 30 \text{ V} & I_H = 0.5 \text{ mA} \end{array}$$

(a) Sketch the voltages  $v_C(t)$ ,  $v_D(t)$ , and  $v_0(t)$  for this circuit.

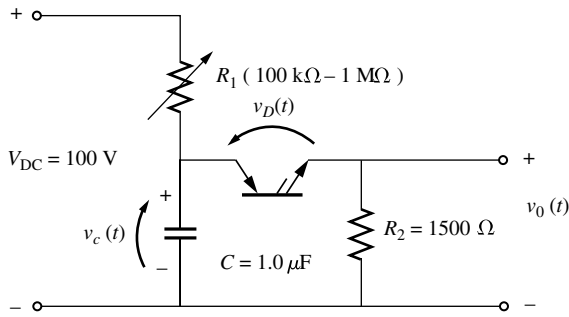
(b) If  $R_1$  is currently set to 500 k $\Omega$ , calculate the period of this relaxation oscillator.

**S1-9.** In the circuit in Figure PS1-4,  $T_1$  is an autotransformer with the tap exactly in the center of its winding. Explain the operation of this circuit. Assuming that the load is inductive, sketch the voltage and current applied to the load. What is the purpose of  $\text{SCR}_2$ ? What is the purpose of  $D_2$ ? (This chopper circuit arrangement is known as a *Jones circuit*.)

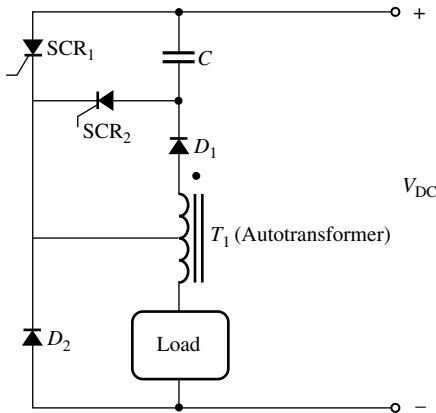
\*The asterisk in front of a problem number indicates that it is a more difficult problem.



**FIGURE PS1-2**  
The inverter circuit of Problem S1-7.



**FIGURE PS1-3**  
The relaxation oscillator circuit of Problem S1-8.

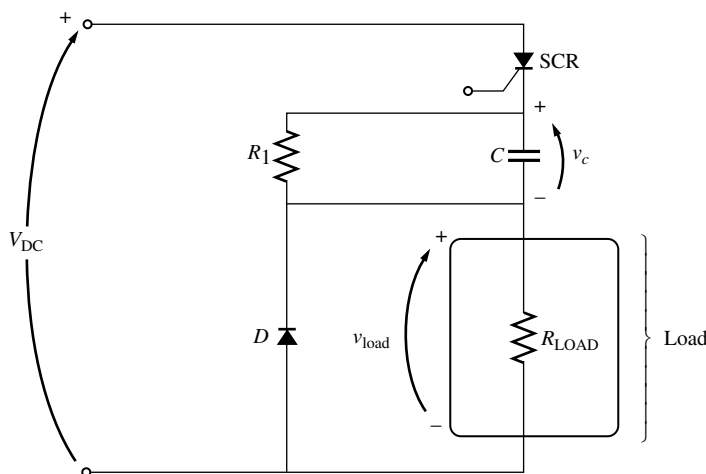


**FIGURE PS1-4**  
The chopper circuit of Problem S1-9.

**S1-10.** A series-capacitor forced commutation chopper circuit supplying a purely resistive load is shown in Figure PS1-5.

$$\begin{array}{ll} V_{DC} = 120 \text{ V} & R_1 = 20 \text{ k}\Omega \\ I_H = 8 \text{ mA} & R_{load} = 250 \Omega \\ V_{BO} = 200 \text{ V} & C = 150 \mu\text{F} \end{array}$$

- When  $SCR_1$  is turned on, how long will it remain on? What causes it to turn off?
- When  $SCR_1$  turns off, how long will it be until the SCR can be turned on again? (Assume that 3 time constants must pass before the capacitor is discharged.)
- What problem or problems do these calculations reveal about this simple series-capacitor forced-commutation chopper circuit?
- How can the problem(s) described in part *c* be eliminated?



**FIGURE PS1-5**

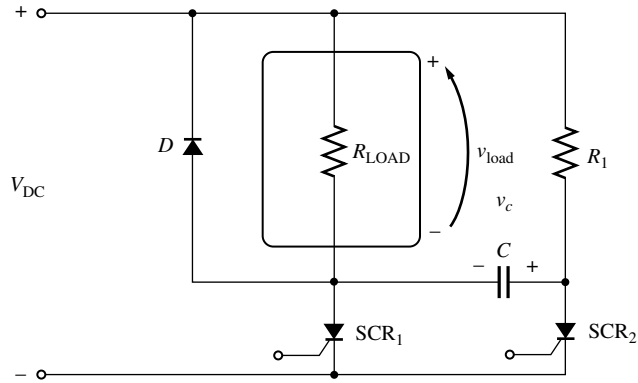
The simple series-capacitor forced-commutation circuit of Problem S1-10.

**S1-11.** A parallel-capacitor forced-commutation chopper circuit supplying a purely resistive load is shown in Figure PS1-6.

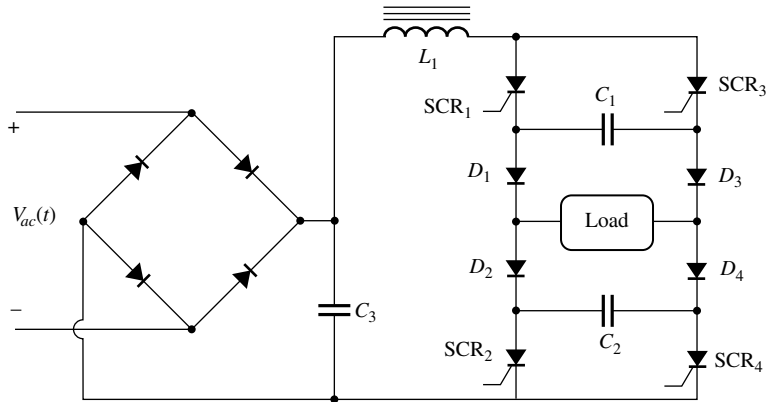
$$\begin{array}{ll} V_{DC} = 120 \text{ V} & R_1 = 20 \text{ k}\Omega \\ I_H = 5 \text{ mA} & R_{load} = 250 \Omega \\ V_{BO} = 250 \text{ V} & C = 15 \mu\text{F} \end{array}$$

- When  $SCR_1$  is turned on, how long will it remain on? What causes it to turn off?
- What is the earliest time that  $SCR_1$  can be turned off after it is turned on? (Assume that 3 time constants must pass before the capacitor is charged.)
- When  $SCR_1$  turns off, how long will it be until the SCR can be turned on again?
- What problem or problems do these calculations reveal about this simple parallel-capacitor forced-commutation chopper circuit?
- How can the problem(s) described in part *d* be eliminated?

**S1-12.** Figure PS1-7 shows a single-phase rectifier-inverter circuit. Explain how this circuit functions. What are the purposes of  $C_1$  and  $C_2$ ? What controls the output frequency of the inverter?



**FIGURE PS1-6**  
The simple parallel-capacitor forced commutation circuit of Problem S1-11.



**FIGURE PS1-7**  
The single-phase rectifier-inverter circuit of Problem S1-12.

**\*S1-13.** A simple full-wave ac phase angle voltage controller is shown in Figure PS1-8.

The component values in this circuit are

$R = 20$  to  $300 \text{ k}\Omega$ , currently set to  $80 \text{ k}\Omega$

$C = 0.15 \text{ }\mu\text{F}$

$V_{BO} = 40 \text{ V}$  (for PNP diode  $D_1$ )

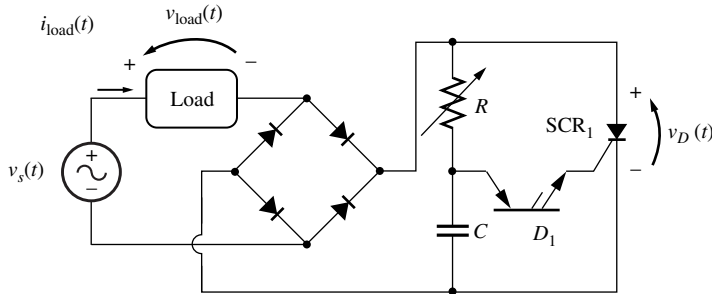
$V_{BO} = 250 \text{ V}$  (for  $\text{SCR}_1$ )

$v_S(t) = V_M \sin \omega t \text{ V}$  where  $V_M = 169.7 \text{ V}$  and  $\omega = 377 \text{ rad/s}$

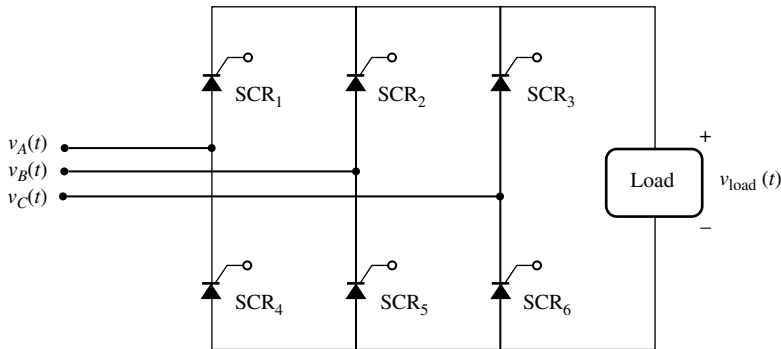
(a) At what phase angle do the PNP diode and the SCR turn on?

(b) What is the rms voltage supplied to the load under these circumstances?

**\*S1-14.** Figure PS1-9 shows a three-phase full-wave rectifier circuit supplying power to a dc load. The circuit uses SCRs instead of diodes as the rectifying elements.



**FIGURE PS1-8**  
The full-wave phase angle voltage controller of Problem S1-13.



**FIGURE PS1-9**  
The three-phase full-wave rectifier circuit of Problem S1-14.

- (a) What will the rms load voltage and ripple be if each SCR is triggered as soon as it becomes forward-biased? At what phase angle should the SCRs be triggered in order to operate this way? Sketch or plot the output voltage for this case.
- (b) What will the rms load voltage and ripple be if each SCR is triggered at a phase angle of  $90^\circ$  (that is, halfway through the half-cycle in which it is forward biased)? Sketch or plot the output voltage for this case.

**\*S1-15.** Write a MATLAB program that imitates the operation of the pulse-width modulation circuit shown in Figure S1-55, and answer the following questions.

- (a) Assume that the comparison voltages  $v_x(t)$  and  $v_y(t)$  have peak amplitudes of 10 V and a frequency of 500 Hz. Plot the output voltage when the input voltage is  $v_{in}(t) = 10 \sin 2\pi ft$  V, and  $f = 60$  Hz.
- (b) What does the spectrum of the output voltage look like? What could be done to reduce the harmonic content of the output voltage?
- (c) Now assume that the frequency of the comparison voltages is increased to 1000 Hz. Plot the output voltage when the input voltage is  $v_{in}(t) = 10 \sin 2\pi ft$  V and  $f = 60$  Hz.
- (d) What does the spectrum of the output voltage in c look like?
- (e) What is the advantage of using a higher comparison frequency and more rapid switching in a PWM modulator?

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