11
Kinematics of Particles

The motion of the paraglider can be described in terms of its *position*, *velocity*, and *acceleration*. When landing, the pilot of the paraglider needs to consider the wind velocity and the *relative motion* of the glider with respect to the wind. The study of motion is known as *kinematics* and is the subject of this chapter.

Objectives

- **Describe** the basic kinematic relationships between position, velocity, acceleration, and time.
- **Solve** problems using these basic kinematic relationships and calculus or graphical methods.
- **Define** position, velocity, and acceleration in terms of Cartesian, tangential and normal, and radial and transverse coordinates.
- **Analyze** the relative motion of multiple particles by using a translating coordinate system.
- **Determine** the motion of a particle that depends on the motion of another particle.
- **Determine** which coordinate system is most appropriate for solving a curvilinear kinematics problem.
- **Calculate** the position, velocity, and acceleration of a particle undergoing curvilinear motion using Cartesian, tangential and normal, and radial and transverse coordinates.

Introduction

Chapters 1 to 10 were devoted to **statics**, i.e., to the analysis of bodies at rest. We now begin the study of **dynamics**, which is the part of mechanics that deals with the analysis of bodies in motion.

Although the study of statics goes back to the time of the Greek philosophers, the first significant contribution to dynamics was made by Galileo (1564–1642). Galileo's experiments on uniformly accelerated bodies led Newton (1642–1727) to formulate his fundamental laws of motion.

Dynamics includes two broad areas of study:

1. **Kinematics**, which is the study of the geometry of motion. The principles of kinematics relate the displacement, velocity, acceleration, and time of a body's motion, without reference to the cause of the motion.
2. **Kinetics**, which is the study of the relation between the forces acting on a body, the mass of the body, and the motion of the body. We use kinetics to predict the motion caused by given forces or to determine the forces required to produce a given motion.

Chapters 11 through 14 describe the **dynamics of particles**; in Chap. 11, we consider the **kinematics of particles**. The use of the word *particles* does not mean that our study is restricted to small objects; rather, it indicates that in these first chapters we study the motion of bodies—possibly as large as cars, rockets, or airplanes—without regard to their size or shape. By saying that we analyze the bodies as particles, we mean that we consider only their motion as an entire unit; we neglect any rotation about their own centers of mass. In some cases, however, such a rotation is not negligible, and we cannot treat the bodies as particles. Such motions are analyzed in later chapters dealing with the **dynamics of rigid bodies**.

In the first part of Chap. 11, we describe the rectilinear motion of a particle; that is, we determine the position, velocity, and acceleration of a particle at every instant as it moves along a straight line. We first use general methods of analysis to study the motion of a particle; we then consider two important particular cases, namely, the uniform motion and the uniformly accelerated motion of a particle (Sec. 11.2). We then discuss the simultaneous motion of several particles and introduce the concept of the relative motion of one particle with respect to another. The first part of this chapter concludes with a study of graphical methods of analysis and their application to the solution of problems involving the rectilinear motion of particles.

In the second part of this chapter, we analyze the motion of a particle as it moves along a curved path. We define the position, velocity, and acceleration of a particle as vector quantities and introduce the derivative of a vector function to add to our mathematical tools. We consider applications in which we define the motion of a particle by the rectangular components of its velocity and acceleration; at this point, we analyze the motion of a projectile (Sec. 11.4C). Then we examine the motion of a particle relative to a reference frame in translation. Finally, we analyze the curvilinear motion of a particle in terms of components other than rectangular. In Sec. 11.5, we introduce the tangential and normal components of an object's velocity and acceleration and then examine the radial and transverse components.

11.1 RECTILINEAR MOTION OF PARTICLES

A particle moving along a straight line is said to be in **rectilinear motion**. The only variables we need to describe this motion are the time, t, and the distance along the line, x, as a function of time. With these variables, we can define the particle's position, velocity, and acceleration, which completely describe the particle's motion. When we study the motion of a particle moving in a plane (two dimensions) or in space (three dimensions), we will use a more general position vector rather than simply the distance along a line.

11.1A Position, Velocity, and Acceleration

At any given instant t, a particle in rectilinear motion occupies some position on the straight line. To define the particle's position P, we choose a fixed origin O on the straight line and a positive direction along the line. We measure the distance x from O to P and record it with a plus or minus sign, according to whether we reach P from O by moving along the line in the positive or negative direction. The distance x, with the appropriate sign, completely defines the position of the particle; it is called the **position coordinate** of the particle. For example, the position coordinate corresponding to P in Fig. 11.1a is $x = +5$ m; the coordinate corresponding to P' in Fig. 11.1b is $x' = -2$ m.

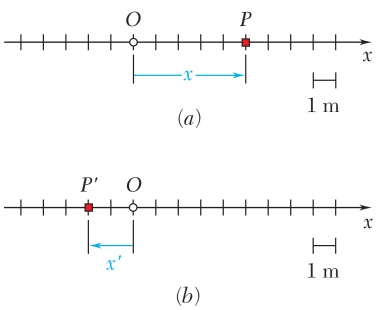


Fig. 11.1 Position is measured from a fixed origin. (a) A positive position coordinate; (b) a negative position coordinate.

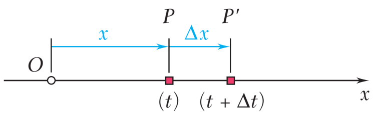


Fig. 11.2 A small displacement Δx from time t to time $t + \Delta t$.

Photo 11.1 The motion of this solar car can be described by its position, velocity, and acceleration.

When we know the position coordinate x of a particle for every value of time t, we say that the motion of the particle is known. We can provide a "timetable" of the motion in the form of an equation in x and t, such as $x = 6t^2 - t^3$, or in the form of a graph of x versus t, as shown in Fig. 11.6. The units most often used to measure the position coordinate x are the meter (m) in the SI system of units† and the foot (ft) in the U.S. customary system of units. Time t is usually measured in seconds (s).

Now consider the position P occupied by the particle at time t and the corresponding coordinate x (Fig. 11.2). Consider also the position P' occupied by the particle at a later time $t + \Delta t$. We can obtain the position coordinate of P' by adding the small displacement Δx to the coordinate x of P. This displacement is positive or negative according to whether P' is to the right or to the left of P. We define the **average velocity** of the particle over the time interval Δt as the quotient of the displacement Δx and the time interval Δt as

$$\text{Average velocity} = \frac{\Delta x}{\Delta t}$$

If we use SI units, Δx is expressed in meters and Δt in seconds; the average velocity is then expressed in meters per second (m/s). If we use U.S. customary units, Δx is expressed in feet and Δt in seconds; the average velocity is then expressed in feet per second (ft/s).

We can determine the **instantaneous velocity** v of a particle at the instant t by allowing the time interval Δt to become infinitesimally small. Thus,

$$\text{Instantaneous velocity} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

The instantaneous velocity is also expressed in m/s or ft/s. Observing that the limit of the quotient is equal, by definition, to the derivative of x with respect to t, we have

Velocity of a particle along a line

$$v = \frac{dx}{dt} \tag{11.1}$$

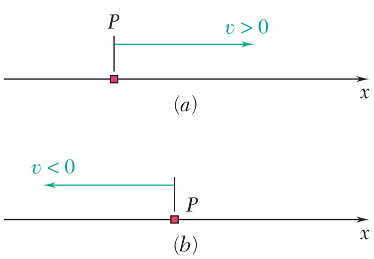


Fig. 11.3 In rectilinear motion, velocity can be only (*a*) positive or (*b*) negative along the line.

We represent the velocity v by an algebraic number that can be positive or negative.‡ A positive value of v indicates that x increases, i.e., that the particle moves in the positive direction (Fig. 11.3*a*). A negative value of v indicates that x decreases, i.e., that the particle moves in the negative direction (Fig. 11.3*b*). The magnitude of v is known as the **speed** of the particle.

Consider the velocity v of the particle at time t and also its velocity $v + \Delta v$ at a later time $t + \Delta t$ (Fig. 11.4). We define the **average acceleration** of the particle over the time interval Δt as the quotient of Δv and Δt as

$$\text{Average acceleration} = \frac{\Delta v}{\Delta t}$$

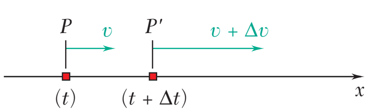


Fig. 11.4 A change in velocity from v to $v + \Delta v$ corresponding to a change in time from t to $t + \Delta t$.

†See Sec. 1.3.

‡As you will see in Sec. 11.4A, velocity is actually a vector quantity. However, since we are considering here the rectilinear motion of a particle where the velocity has a known and fixed direction, we need only specify its sense and magnitude. We can do this conveniently by using a scalar quantity with a plus or minus sign. This is also true of the acceleration of a particle in rectilinear motion.

If we use SI units, Δv is expressed in m/s and Δt in seconds; the average acceleration is then expressed in m/s^2. If we use U.S. customary units, Δv is expressed in ft/s and Δt in seconds; the average acceleration is then expressed in ft/s^2.

We obtain the **instantaneous acceleration** a of the particle at the instant t by again allowing the time interval Δt to approach zero. Thus,

$$\text{Instantaneous acceleration} = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

The instantaneous acceleration is also expressed in m/s^2 or ft/s^2. The limit of the quotient, which is by definition the derivative of v with respect to t, measures the rate of change of the velocity. We have

Acceleration of a particle along a line

$$a = \frac{dv}{dt} \tag{11.2}$$

or substituting for v from Eq. (11.1),

$$a = \frac{d^2x}{dt^2} \tag{11.3}$$

We represent the acceleration a by an algebraic number that can be positive or negative (see the footnote on the preceding page). A positive value of a indicates that the velocity (i.e., the algebraic number v) increases. This may mean that the particle is moving faster in the positive direction (Fig. 11.5*a*) or that it is moving more slowly in the negative direction (Fig. 11.5*b*); in both cases, Δv is positive. A negative value of a indicates that the velocity decreases; either the particle is moving more slowly in the positive direction (Fig. 11.5*c*), or it is moving faster in the negative direction (Fig. 11.5*d*).

Sometimes we use the term *deceleration* to refer to a when the speed of the particle (i.e., the magnitude of v) decreases; the particle is then moving more slowly. For example, the particle of Fig. 11.5 is decelerating in parts *b* and *c*; it is truly accelerating (i.e., moving faster) in parts *a* and *d*.

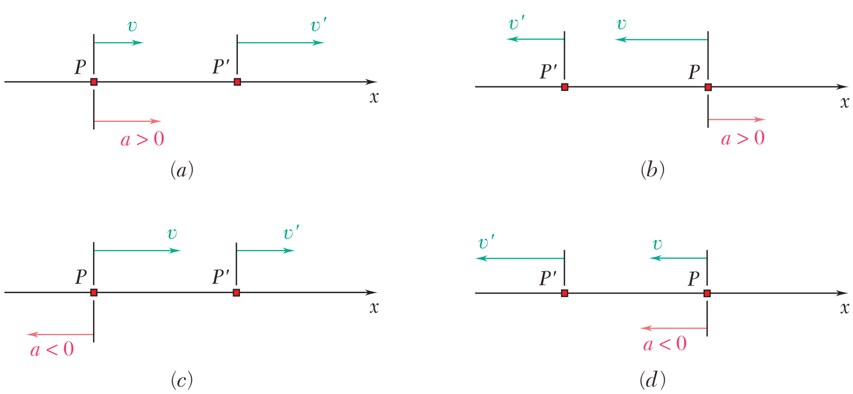


Fig. 11.5 Velocity and acceleration can be in the same or different directions. (*a, d*) When *a* and *v* are in the same direction, the particle speeds up; (*b, c*) when *a* and *v* are in opposite directions, the particle slows down.

We can obtain another expression for the acceleration by eliminating the differential dt in Eqs. (11.1) and (11.2). Solving Eq. (11.1) for dt, we have $dt = dx/v$; substituting into Eq. (11.2) gives us

$$a = v\frac{dv}{dx} \qquad \textbf{(11.4)}$$

Concept Application 11.1

Consider a particle moving in a straight line, and assume that its position is defined by

$$x = 6t^2 - t^3$$

where t is in seconds and x in meters. We can obtain the velocity v at any time t by differentiating x with respect to t as

$$v = \frac{dx}{dt} = 12t - 3t^2$$

We can obtain the acceleration a by differentiating again with respect to t. Hence,

$$a = \frac{dv}{dt} = 12 - 6t$$

In Fig. 11.6, we have plotted the position coordinate, the velocity, and the acceleration. These curves are known as *motion curves*. Keep in mind, however, that the particle does not move along any of these curves; the particle moves in a straight line.

Since the derivative of a function measures the slope of the corresponding curve, the slope of the x–t curve at any given time is equal to the value of v at that time. Similarly, the slope of the v–t curve is equal to the value of a. Since $a = 0$ at $t = 2$ s, the slope of the v–t curve must be zero at $t = 2$ s; the velocity reaches a maximum at this instant. Also, since $v = 0$ at $t = 0$ and at $t = 4$ s, the tangent to the x–t curve must be horizontal for both of these values of t.

A study of the three motion curves of Fig. 11.6 shows that the motion of the particle from $t = 0$ to $t = \infty$ can be divided into four phases:

1. The particle starts from the origin, $x = 0$, with no velocity but with a positive acceleration. Under this acceleration, the particle gains a positive velocity and moves in the positive direction. From $t = 0$ to $t = 2$ s, x, v, and a are all positive.
2. At $t = 2$ s, the acceleration is zero; the velocity has reached its maximum value. From $t = 2$ s to $t = 4$ s, v is positive, but a is negative. The particle still moves in the positive direction but more slowly; the particle is decelerating.
3. At $t = 4$ s, the velocity is zero; the position coordinate x has reached its maximum value (32 m). From then on, both v and a are negative; the particle is accelerating and moves in the negative direction with increasing speed.
4. At $t = 6$ s, the particle passes through the origin; its coordinate x is then zero, while the total distance traveled since the beginning of the motion is 64 m (i.e., twice its maximum value). For values of t larger than 6 s, x, v, and a are all negative. The particle keeps moving in the negative direction—away from O—faster and faster. ■

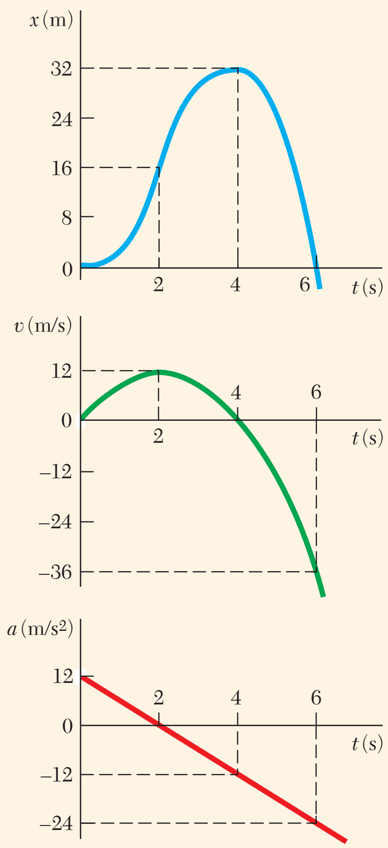


Fig. 11.6 Graphs of position, velocity, and acceleration as functions of time for Concept Application 11.1.

11.1B Determining the Motion of a Particle

We have just seen that the motion of a particle is said to be known if we know its position for every value of the time t. In practice, however, a motion is seldom defined by a relation between x and t. More often, the conditions of the motion are specified by the type of acceleration that the particle possesses. For example, a freely falling body has a constant acceleration that is directed downward and equal to 9.81 m/s^2 or 32.2 ft/s^2, a mass attached to a stretched spring has an acceleration proportional to the instantaneous elongation of the spring measured from its equilibrium position, etc. In general, we can express the acceleration of the particle as a function of one or more of the variables x, v, and t. Thus, in order to determine the position coordinate x in terms of t, we need to perform two successive integrations.

Let us consider three common classes of motion.

1. $a = f(t)$. **The Acceleration Is a Given Function of t.** Solving Eq. (11.2) for dv and substituting $f(t)$ for a, we have

$$dv = a\,dt$$
$$dv = f(t)\,dt$$

Integrating both sides of the equation, we obtain

$$\int dv = \int f(t)\,dt$$

This equation defines v in terms of t. Note, however, that an arbitrary constant is introduced after the integration is performed. This is due to the fact that many motions correspond to the given acceleration $a = f(t)$. In order to define the motion of the particle uniquely, it is necessary to specify the **initial conditions** of the motion, i.e., the value v_0 of the velocity and the value x_0 of the position coordinate at $t = 0$. Rather than use an arbitrary constant that is determined by the initial conditions, it is often more convenient to replace the indefinite integrals with **definite integrals**. Definite integrals have lower limits corresponding to the initial conditions $t = 0$ and $v = v_0$ and upper limits corresponding to $t = t$ and $v = v$. This gives us

$$\int_{v_0}^{v} dv = \int_0^t f(t)\,dt$$

$$v - v_0 = \int_0^t f(t)\,dt$$

which yields v in terms of t.

We can now solve Eq. (11.1) for dx as

$$dx = v\,dt$$

and substitute for v the expression obtained from the first integration. Then we integrate both sides of this equation via the left-hand side with respect to x from $x = x_0$ to $x = x$ and the right-hand side with respect to t from $t = 0$ to $t = t$. In this way, we obtain the position coordinate x in terms of t; the motion is completely determined.

We will study two important cases in greater detail in Sec. 11.2: the case when $a = 0$, corresponding to a *uniform motion*, and the case when $a =$ constant, corresponding to a *uniformly accelerated motion*.

2. $a = f(x)$. **The Acceleration Is a Given Function of** x. Rearranging Eq. (11.4) and substituting $f(x)$ for a, we have

$$v\,dv = a\,dx$$
$$v\,dv = f(x)\,dx$$

Since each side contains only one variable, we can integrate the equation. Denoting again the initial values of the velocity and of the position coordinate by v_0 and x_0, respectively, we obtain

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} f(x)\,dx$$

$$\tfrac{1}{2}v^2 - \tfrac{1}{2}v_0^2 = \int_{x_0}^{x} f(x)\,dx$$

which yields v in terms of x. We now solve Eq. (11.1) for dt, giving

$$dt = \frac{dx}{v}$$

and substitute for v the expression just obtained. We can then integrate both sides to obtain the desired relation between x and t. However, in most cases, this last integration cannot be performed analytically, and we must resort to a numerical method of integration.

3. $a = f(v)$. **The Acceleration Is a Given Function of** v. We can now substitute $f(v)$ for a in either Eqs. (11.2) or (11.4) to obtain either

$$f(v) = \frac{dv}{dt} \qquad f(v) = v\frac{dv}{dx}$$

$$dt = \frac{dv}{f(v)} \qquad dx = \frac{v\,dv}{f(v)}$$

Integration of the first equation yields a relation between v and t; integration of the second equation yields a relation between v and x. Either of these relations can be used in conjunction with Eq. (11.1) to obtain the relation between x and t that characterizes the motion of the particle.

Sample Problem 11.1

The position of a particle moving along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$, where x is expressed in meters and t in seconds. Determine (*a*) the time at which the velocity is zero, (*b*) the position and distance traveled by the particle at that time, (*c*) the acceleration of the particle at that time, (*d*) the distance traveled by the particle from $t = 4$ s to $t = 6$ s.

STRATEGY: You need to use the basic kinematic relationships between position, velocity, and acceleration. Because the position is given as a function of time, you can differentiate it to find equations for the velocity and acceleration. Once you have these equations, you can solve the problem.

MODELING and ANALYSIS: Taking the derivative of position, you obtain

$$x = t^3 - 6t^2 - 15t + 40 \qquad (1)$$

$$v = \frac{dx}{dt} = 3t^2 - 12t - 15 \qquad (2)$$

$$a = \frac{dv}{dt} = 6t - 12 \qquad (3)$$

These equations are graphed in Fig. 1.

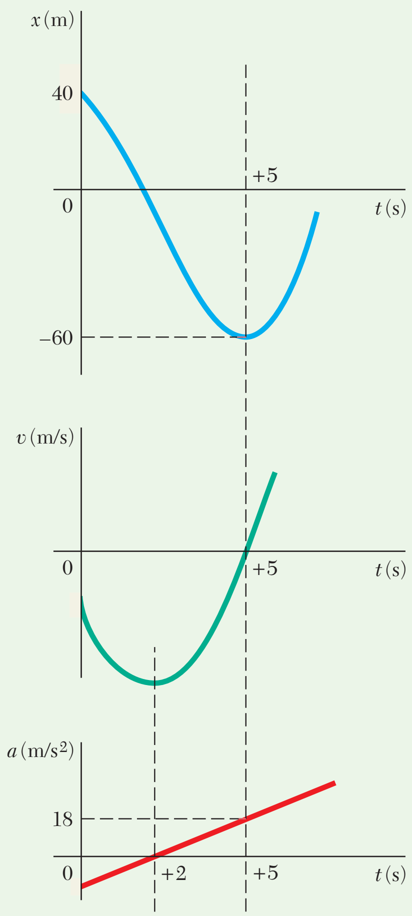


Fig. 1 Motion curves for the particle.

a. Time at Which $v = 0$. Set $v = 0$ in Eq. (2) for

$$3t^2 - 12t - 15 = 0 \qquad t = -1 \text{ s} \quad \text{and} \quad t = +5 \text{ s} \blacktriangleleft$$

Only the root $t = +5$ s corresponds to a time after the motion has begun: for $t < 5$ s, $v < 0$ and the particle moves in the negative direction; for $t > 5$ s, $v > 0$ and the particle moves in the positive direction.

b. Position and Distance Traveled When $v = 0$. Substitute $t = +5$ s into Eq. (1), yielding

$$x_5 = (5)^3 - 6(5)^2 - 15(5) + 40 \qquad x_5 = -60 \text{ m} \blacktriangleleft$$

The initial position at $t = 0$ was $x_0 = +40$ m. Since $v \neq 0$ during the interval $t = 0$ to $t = 5$ s, you have

$$\text{Distance traveled} = x_5 - x_0 = -60 \text{ m} - 40 \text{ m} = -100 \text{ m}$$

$$\text{Distance traveled} = 100 \text{ m in the negative direction} \blacktriangleleft$$

c. Acceleration When $v = 0$. Substitute $t = +5$ s into Eq. (3) for

$$a_5 = 6(5) - 12 \qquad a_5 = +18 \text{ m/s}^2 \blacktriangleleft$$

d. Distance Traveled from $t = 4$ s to $t = 6$ s. The particle moves in the negative direction from $t = 4$ s to $t = 5$ s and in the positive direction from $t = 5$ s to $t = 6$ s; therefore, the distance traveled during each of these time intervals must be computed separately.

From $t = 4$ s to $t = 5$ s: $\quad x_5 = -60$ m

$$x_4 = (4)^3 - 6(4)^2 - 15(4) + 40 = -52 \text{ m}$$

(continued)

$$\text{Distance traveled} = x_5 - x_4 = -60\text{ m} - (-52\text{ m}) = -8\text{ m}$$
$$= 8\text{ m in the negative direction}$$

From $t = 5$ s to $t = 6$ s: $x_5 = -60$ m

$$x_6 = (6)^3 - 6(6)^2 - 15(6) + 40 = -50\text{ m}$$
$$\text{Distance traveled} = x_6 - x_5 = -50\text{ m} - (-60\text{ m}) = +10\text{ m}$$
$$= 10\text{ m in the positive direction}$$

Total distance traveled from $t = 4$ s to $t = 6$ s is 8 m + 10 m $= 18$ m

REFLECT and THINK: The total distance traveled by the particle in the 2-second interval is 18 m, but because one distance is positive and one is negative, the net change in position is only 2 m (in the positive direction). This illustrates the difference between total distance traveled and net change in position. Note that the maximum displacement occurs at $t = 5$ s, when the velocity is zero.

Sample Problem 11.2

You throw a ball vertically upward with a velocity of 10 m/s from a window located 20 m above the ground. Knowing that the acceleration of the ball is constant and equal to 9.81 m/s^2 downward, determine (*a*) the velocity v and elevation y of the ball above the ground at any time t, (*b*) the highest elevation reached by the ball and the corresponding value of t, (*c*) the time when the ball hits the ground and the corresponding velocity. Draw the v–t and y–t curves.

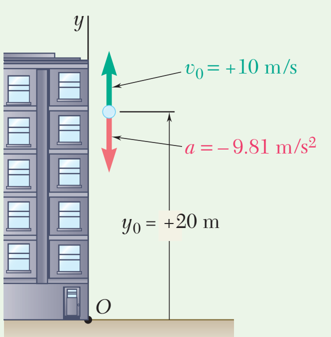


Fig. 1 Acceleration, initial velocity, and initial position of the ball.

STRATEGY: The acceleration is constant, so you can integrate the defining kinematic equation for acceleration once to find the velocity equation and a second time to find the position relationship. Once you have these equations, you can solve the problem.

MODELING and ANALYSIS: Model the ball as a particle with negligible drag.

a. Velocity and Elevation. Choose the y axis measuring the position coordinate (or elevation) with its origin O on the ground and its positive sense upward. The value of the acceleration and the initial values of v and y are as indicated in Fig. 1. Substituting for a in $a = dv/dt$ and noting that, when $t = 0$, $v_0 = +10$ m/s, you have

$$\frac{dv}{dt} = a = -9.81\,\text{m/s}^2$$
$$\int_{v_0=10}^{v} dv = -\int_0^t 9.81\,dt$$
$$[v]_{10}^{v} = -[9.81t]_0^t$$
$$v - 10 = -9.81t$$
$$v = 10 - 9.81t \quad (1)$$

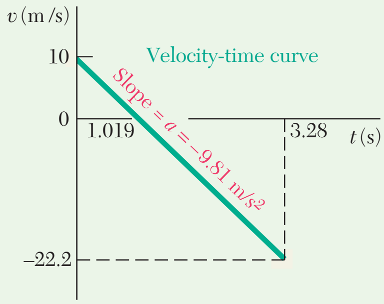


Fig. 2 Velocity of the ball as a function of time.

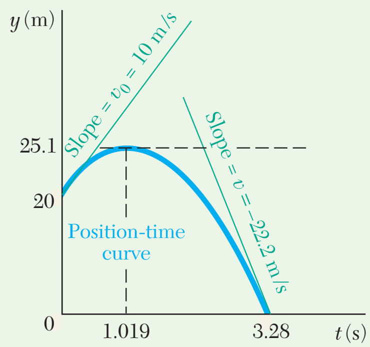


Fig. 3 Height of the ball as a function of time.

Substituting for v in $v = dy/dt$ and noting that when $t = 0$, $y_0 = 20$ m, you have

$$\frac{dy}{dt} = v = 10 - 9.81t$$

$$\int_{y_0=20}^{y} dy = \int_0^t (10 - 9.81t)\,dt$$

$$[y]_{20}^{y} = [10t - 4.905t^2]_0^t$$

$$y - 20 = 10t - 4.905t^2$$

$$y = 20 + 10t - 4.905t^2 \quad \textbf{(2)} \quad ◀$$

Graphs of these equations are shown in Figs. 2 and 3.

b. Highest Elevation. The ball reaches its highest elevation when $v = 0$. Substituting into Eq. (1), you obtain

$$10 - 9.81t = 0 \qquad t = 1.019 \text{ s} \quad ◀$$

Substituting $t = 1.019$ s into Eq. (2), you find

$$y = 20 + 10(1.019) - 4.905(1.019)^2 \qquad y = 25.1 \text{ m} \quad ◀$$

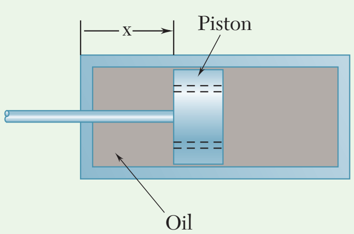
c. Ball Hits the Ground. The ball hits the ground when $y = 0$. Substituting into Eq. (2), you obtain

$$20 + 10t - 4.905t^2 = 0 \qquad t = -1.243 \text{ s} \quad \text{and} \quad t = +3.28 \text{ s} \quad ◀$$

Only the root $t = +3.28$ s corresponds to a time after the motion has begun. Carrying this value of t into Eq. (1), you find

$$v = 10 - 9.81(3.28) = -22.2 \text{ m/s} \qquad v = 22.2 \text{ m/s} \downarrow \quad ◀$$

REFLECT and THINK: When the acceleration is constant, the velocity changes linearly, and the position is a quadratic function of time. You will see in Sec. 11.2 that the motion in this problem is an example of free fall, where the acceleration in the vertical direction is constant and equal to $-g$.



Sample Problem 11.3

Many mountain bike shocks utilize a piston that travels in an oil-filled cylinder to provide shock absorption; this system is shown schematically. When the front tire goes over a bump, the cylinder is given an initial velocity v_0. The piston, which is attached to the fork, then moves with respect to the cylinder, and oil is forced through orifices in the piston. This causes the piston to decelerate at a rate proportional to the velocity at $a = -kv$. At time $t = 0$, the position of the piston is $x = 0$. Express (*a*) the velocity v in terms of t, (*b*) the position x in terms of t, (*c*) the velocity v in terms of x. Draw the corresponding motion curves.

(continued)

STRATEGY: Because the acceleration is given as a function of velocity, you need to use either $a = dv/dt$ or $a = v\,dv/dx$ and then separate variables and integrate. Which one you use depends on what you are asked to find. Since part *a* asks for v in terms of *t*, use $a = dv/dt$. You can integrate this again using $v = dx/dt$ for part *b*. Since part *c* asked for $v(x)$, you should use $a = v\,dv/dx$ and then separate the variables and integrate.

MODELING and ANALYSIS: Rotation of the piston is not relevant, so you can model it as a particle undergoing rectilinear motion.

a. *v* in Terms of *t*. Substitute $-kv$ for a in the fundamental formula defining acceleration, $a = dv/dt$. You obtain

$$-kv = \frac{dv}{dt} \qquad \frac{dv}{v} = -k\,dt \qquad \int_{v_0}^{v} \frac{dv}{v} = -k\int_0^t dt$$

$$\ln\frac{v}{v_0} = -kt \qquad\qquad v = v_0 e^{-kt} \blacktriangleleft$$

b. *x* in Terms of *t*. Substitute the expression just obtained for v into $v = dx/dt$. You get

$$v_0 e^{-kt} = \frac{dx}{dt}$$

$$\int_0^x dx = v_0\int_0^t e^{-kt}\,dt$$

$$x = -\frac{v_0}{k}[e^{-kt}]_0^t = -\frac{v_0}{k}(e^{-kt} - 1)$$

$$x = \frac{v_0}{k}(1 - e^{-kt}) \blacktriangleleft$$

c. *v* in Terms of *x*. Substitute $-kv$ for a in $a = v\,dv/dx$. You have

$$-kv = v\frac{dv}{dx}$$

$$dv = -k\,dx$$

$$\int_{v_0}^{v} dv = -k\int_0^x dx$$

$$v - v_0 = -kx \qquad\qquad v = v_0 - kx \blacktriangleleft$$

The motion curves are shown in Fig. 1.

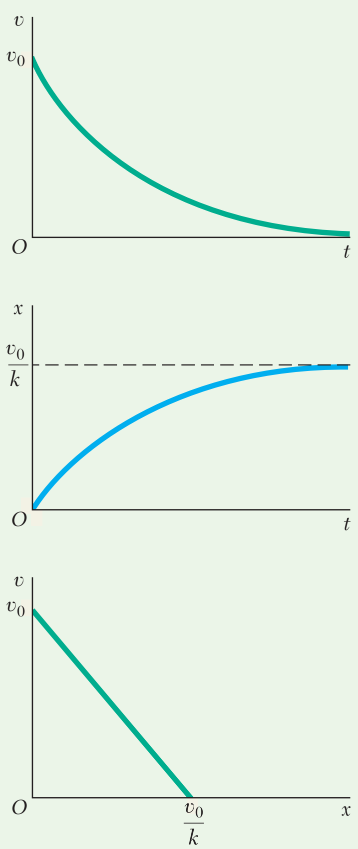


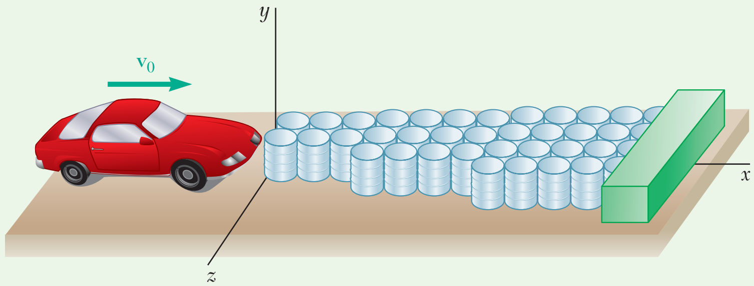
Fig. 1 Motion curves for the piston

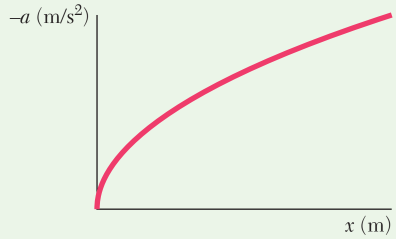
REFLECT and THINK: You could have solved part *c* by eliminating *t* from the answers obtained for parts *a* and *b*. You could use this alternative method as a check. From part *a*, you obtain $e^{-kt} = v/v_0$; substituting into the answer of part *b*, you have

$$x = \frac{v_0}{k}(1 - e^{-kt}) = \frac{v_0}{k}\left(1 - \frac{v}{v_0}\right) \qquad v = v_0 - kx \qquad \text{(checks)}$$

Sample Problem 11.4

An uncontrolled automobile traveling at 72 km/h strikes a highway crash barrier square on. After initially hitting the barrier, the automobile decelerates at a rate proportional to the distance x the automobile has moved into the barrier; specifically, $a = -30\sqrt{x}$, where a and x are expressed in m/s^2 and m, respectively. Determine the distance the automobile will move into the barrier before it comes to rest.





STRATEGY: Since you are given the deceleration as a function of displacement, you should start with the basic kinematic relationship $a = v\,dv/dx$.

MODELING and ANALYSIS: Model the car as a particle. First find the initial speed in ft/s,

$$v_0 = \left(72\,\frac{\text{km}}{\text{hr}}\right)\left(\frac{1\text{ hr}}{3600\text{ s}}\right)\left(\frac{1000\text{ m}}{\text{km}}\right) = 20\,\frac{\text{m}}{\text{s}}$$

Substituting $a = -30\sqrt{x}$ into $a = v\,dv/dx$ gives

$$a = -30\sqrt{x} = \frac{v\,dv}{dx}$$

Separating variables and integrating gives

$$v\,dv = -30\sqrt{x}\,dx \longrightarrow \int_{v_0}^{0} v\,dv = -\int_0^x 30\sqrt{x}\,dx$$

$$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = -20x^{3/2} \longrightarrow x = \left(\frac{1}{40}(v_0^2 - v^2)\right)^{2/3} \qquad (1)$$

Substituting $v = 0$, $v_0 = 20$ m/s gives

$d = 4.64$ m ◀

REFLECT and THINK: A distance of 4.64 m seems reasonable for a barrier of this type. If you substitute d into the equation for a, you find a maximum deceleration of about 7 g's. Note that this problem would have been much harder to solve if you had been asked to find the time for the automobile to stop. In this case, you would need to determine $v(t)$ from Eq. (1). This gives $v = \sqrt{v_0^2 - 40x^{3/2}}$. Using the basic kinematic relationship $v = dx/dt$, you can easily show that

$$\int_0^t dt = \int_0^x \frac{dx}{\sqrt{v_0^2 - 40x^{3/2}}}$$

Unfortunately, there is no closed-form solution to this integral, so you would need to solve it numerically.

SOLVING PROBLEMS ON YOUR OWN

In the problems for this section, you will be asked to determine the **position, velocity,** and/or **acceleration** of a particle in **rectilinear motion**. As you read each problem, it is important to identify both the independent variable (typically t or x) and what is required (for example, the need to express v as a function of x). You may find it helpful to start each problem by writing down both the given information and a simple statement of what is to be determined.

1. Determining $v(t)$ and $a(t)$ for a given $x(t)$. As explained in Sec. 11.1A, the first and second derivatives of x with respect to t are equal to the velocity and the acceleration, respectively, of the particle [Eqs. (11.1) and (11.2)]. If the velocity and acceleration have opposite signs, the particle can come to rest and then move in the opposite direction [Sample Prob. 11.1]. Thus, when computing the total distance traveled by a particle, you should first determine if the particle comes to rest during the specified interval of time. Constructing a diagram similar to that of Sample Prob. 11.1, which shows the position and the velocity of the particle at each critical instant ($v = v_{max}$, $v = 0$, etc.), will help you to visualize the motion.

2. Determining $v(t)$ and $x(t)$ for a given $a(t)$. We discussed the solution of problems of this type in the first part of Sec. 11.1B. We used the initial conditions, $t = 0$ and $v = v_0$, for the lower limits of the integrals in t and v, but any other known state (for example, $t = t_1$ and $v = v_1$) could be used instead. Also, if the given function $a(t)$ contains an unknown constant (for example, the constant k if $a = kt$), you will first have to determine that constant by substituting a set of known values of t and a in the equation defining $a(t)$.

3. Determining $v(x)$ and $x(t)$ for a given $a(x)$. This is the second case considered in Sec. 11.1B. We again note that the lower limits of integration can be any known state (for example, $x = x_1$ and $v = v_1$). In addition, since $v = v_{max}$ when $a = 0$, you can determine the positions where the maximum or minimum values of the velocity occur by setting $a(x) = 0$ and solving for x.

4. Determining $v(x)$, $v(t)$, and $x(t)$ for a given $a(v)$. This is the last case treated in Sec. 11.1B; the appropriate solution techniques for problems of this type are illustrated in Sample Probs. 11.3 and 11.4. All of the general comments for the preceding cases once again apply. Note that Sample Prob. 11.3 provides a summary of how and when to use the equations $v = dx/dt$, $a = dv/dt$, and $a = v\, dv/dx$.

We can summarize these relationships in Table 11.1.

Table 11.1

If....	Kinematic relationship	Integrate
$a = a(t)$	$\frac{dv}{dt} = a(t)$	$\int_{v_0}^{v} dv = \int_0^t a(t)dt$
$a = a(x)$	$v\frac{dv}{dx} = a(x)$	$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)dx$
$a = a(v)$	$\frac{dv}{dt} = a(v)$	$\int_{v_0}^{v} \frac{dv}{a(v)} = \int_0^t dt$
	$v\frac{dv}{dx} = a(v)$	$\int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v\,dv}{a(v)}$

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Problems†

CONCEPT QUESTIONS

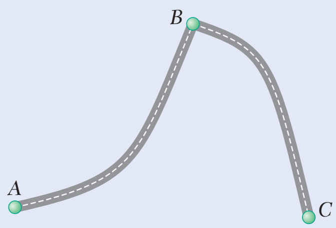

Fig. P11.CQ1

11.CQ1 A bus travels the 100 km between A and B at 50 km/h and then another 100 km between B and C at 70 km/h. The average speed of the bus for the entire 200 km trip is:
a. More than 60 km/h.
b. Equal to 60 km/h.
c. Less than 60 km/h.

11.CQ2 Two cars A and B race each other down a straight road. The position of each car as a function of time is shown. Which of the following statements are true (more than one answer can be correct)?
a. At time t_2 both cars have traveled the same distance.
b. At time t_1 both cars have the same speed.
c. Both cars have the same speed at some time $t < t_1$.
d. Both cars have the same acceleration at some time $t < t_1$.
e. Both cars have the same acceleration at some time $t_1 < t < t_2$.

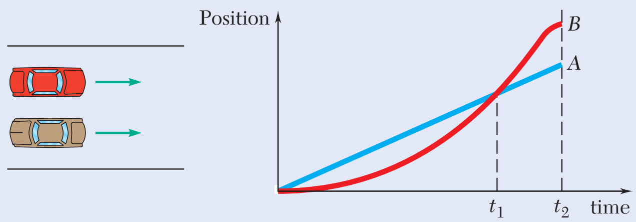

Fig. P11.CQ2

END-OF-SECTION PROBLEMS

11.1 A snowboarder starts from rest at the top of a double black diamond hill. As she rides down the slope, GPS coordinates are used to determine her displacement as a function of time: $x = 0.5t^3 + t^2 + 2t$, where x and t are expressed in meters and seconds, respectively. Determine the position, velocity, and acceleration of the boarder when $t = 5$ seconds.

11.2 The motion of a particle is defined by the relation $x = 2t^3 - 9t^2 + 12t + 10$, where x and t are expressed in meters and seconds, respectively. Determine the time, the position, and the acceleration of the particle when $v = 0$.

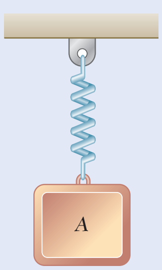

Fig. P11.3

11.3 The vertical motion of mass A is defined by the relation $x = 10 \sin 2t + 15 \cos 2t + 100$, where x and t are expressed in millimeters and seconds, respectively. Determine (*a*) the position, velocity, and acceleration of A when $t = 1$ s, (*b*) the maximum velocity and acceleration of A.

†Answers to all problems set in straight type (such as **11.1**) are given at the end of the book. Answers to problems with a number set in italic type (such as ***11.6***) are not given.

11.4 A loaded railroad car is rolling at a constant velocity when it couples with a spring and dashpot bumper system. After the coupling, the motion of the car is defined by the relation $x = 60e^{-4.8t} \sin 16t$, where x and t are expressed in millimeters and seconds, respectively. Determine the position, the velocity, and the acceleration of the railroad car when (*a*) $t = 0$, (*b*) $t = 0.3$ s.

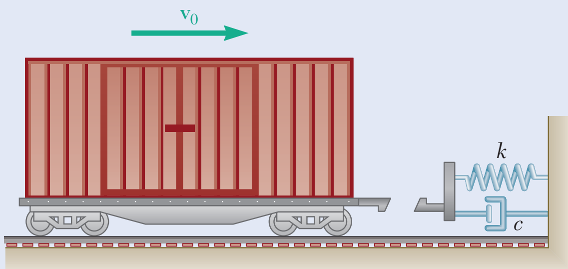

Fig. P11.4

11.5 The motion of a particle is defined by the relation $x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$, where x and t are expressed in meters and seconds, respectively. Determine the time, the position, and the velocity when $a = 0$.

11.6 The motion of a particle is defined by the relation $x = t^3 - 9t^2 + 24t - 8$, where x and t are expressed in meters and seconds, respectively. Determine (*a*) when the velocity is zero, (*b*) the position and the total distance traveled when the acceleration is zero.

11.7 A girl operates a radio-controlled model car in a vacant parking lot. The girl's position is at the origin of the xy coordinate axes, and the surface of the parking lot lies in the x-y plane. She drives the car in a straight line so that the x coordinate is defined by the relation $x(t) = 0.5t^3 - 3t^2 + 3t + 2$, where x and t are expressed in meters and seconds, respectively. Determine (*a*) when the velocity is zero, (*b*) the position and total distance travelled when the acceleration is zero.

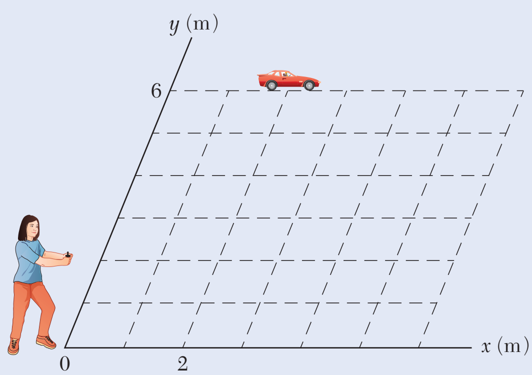

Fig. P11.7

11.8 The motion of a particle is defined by the relation $x = t^2 - (t - 2)^3$, where x and t are expressed in meters and seconds, respectively. Determine (*a*) the two positions at which the velocity is zero (*b*) the total distance traveled by the particle from $t = 0$ to $t = 4$ s.

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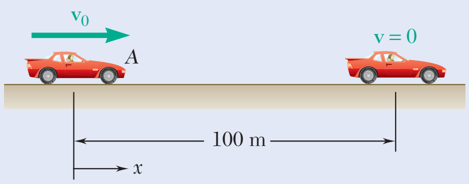


Fig. P11.9

11.9 The brakes of a car are applied, causing it to slow down at a rate of 3 m/s^2. Knowing that the car stops in 100 m, determine (*a*) how fast the car was traveling immediately before the brakes were applied, (*b*) the time required for the car to stop.

11.10 The acceleration of a particle is defined by the relation $a = 3e^{-0.2t}$, where a and t are expressed in m/s^2 and seconds, respectively. Knowing that $x = 0$ and $v = 0$ at $t = 0$, determine the velocity and position of the particle when $t = 0.5$ s.

11.11 The acceleration of a particle is directly proportional to the square of the time t. When $t = 0$, the particle is at $x = 24$ m. Knowing that at $t = 6$ s, $x = 96$ m and $v = 18$ m/s, express x and v in terms of t.

11.12 The acceleration of a particle is defined by the relation $a = kt^2$. (*a*) Knowing that $v = -8$ m/s when $t = 0$ and that $v = +8$ m/s when $t = 2$ s, determine the constant k. (*b*) Write the equations of motion, knowing also that $x = 0$ when $t = 2$ s.

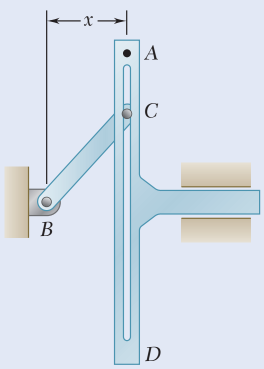


Fig. *P11.13* and *P11.14*

11.13 A Scotch yoke is a mechanism that transforms the circular motion of a crank into the reciprocating motion of a shaft (or vice versa). It has been used in a number of different internal combustion engines and in control valves. In the Scotch yoke shown, the acceleration of point A is defined by the relation $a = -1.8 \sin kt$, where a and t are expressed in m/s^2 and seconds, respectively, and $k = 3$ rad/s. Knowing that $x = 0$ and $v = 0.6$ m/s when $t = 0$, determine the velocity and position of point A when $t = 0.5$ s.

11.14 For the Scotch yoke mechanism shown, the acceleration of point A is defined by the relation $a = -1.08 \sin kt - 1.44 \cos kt$, where a and t are expressed in m/s^2 and seconds, respectively, and $k = 3$ rad/s. Knowing that $x = 0.16$ m and $v = 0.36$ m/s when $t = 0$, determine the velocity and position of point A when $t = 0.5$ s.

11.15 A piece of electronic equipment that is surrounded by packing material is dropped so that it hits the ground with a speed of 4 m/s. After contact the equipment experiences an acceleration of $a = -kx$, where k is a constant and x is the compression of the packing material. If the packing material experiences a maximum compression of 20 mm, determine the maximum acceleration of the equipment.

Fig. P11.15

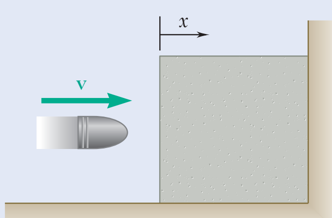


Fig. P11.16

11.16 A projectile enters a resisting medium at $x = 0$ with an initial velocity $v_0 = 270$ m/s and travels 100 mm. before coming to rest. Assuming that the velocity of the projectile is defined by the relation $v = v_0 - kx$, where v is expressed in m/s and x is in meters, determine (*a*) the initial acceleration of the projectile, (*b*) the time required for the projectile to penetrate 97.5 mm into the resisting medium.

11.17 The acceleration of a particle is defined by the relation $a = -k/x$. It has been experimentally determined that $v = 5$ m/s when $x = 0.2$ m and that $v = 3$ m/s when $x = 0.4$ m. Determine (*a*) the velocity of the particle when $x = 0.5$ m, (*b*) the position of the particle at which its velocity is zero.

11.18 A brass (nonmagnetic) block *A* and a steel magnet *B* are in equilibrium in a brass tube under the magnetic repelling force of another steel magnet *C* located at a distance $x = 0.004$ m from *B*. The force is inversely proportional to the square of the distance between *B* and *C*. If block *A* is suddenly removed, the acceleration of block *B* is $a = -9.81 + k/x^2$, where a and x are expressed in m/s^2 and meters, respectively, and $k = 4 \times 10^{-4}$ m^3/s^2. Determine the maximum velocity and acceleration of *B*.

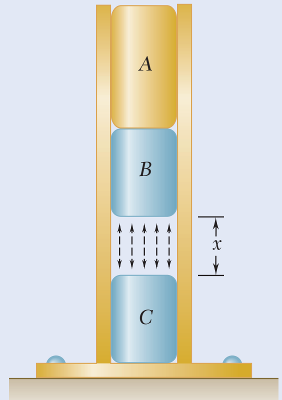


Fig. P11.18

11.19 Based on experimental observations, the acceleration of a particle is defined by the relation $a = -(0.1 + \sin x/b)$, where a and x are expressed in m/s^2 and meters, respectively. Knowing that $b = 0.8$ m and that $v = 1$ m/s when $x = 0$, determine (*a*) the velocity of the particle when $x = -1$ m, (*b*) the position where the velocity is maximum, (*c*) the maximum velocity.

11.20 A spring *AB* is attached to a support at *A* and to a collar. The unstretched length of the spring is l. Knowing that the collar is released from rest at $x = x_0$ and has an acceleration defined by the relation $a = -100(x - lx/\sqrt{l^2 + x^2})$, determine the velocity of the collar as it passes through point *C*.

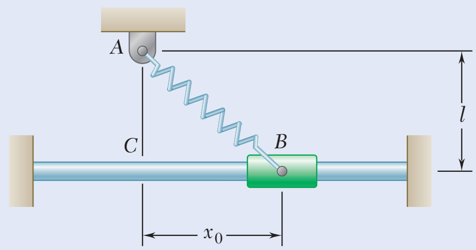


Fig. *P11.20*

11.21 The acceleration of a particle is defined by the relation $a = k(1 - e^{-x})$, where k is a constant. Knowing that the velocity of the particle is $v = +9$ m/s when $x = -3$ m and that the particle comes to rest at the origin, determine (*a*) the value of k, (*b*) the velocity of the particle when $x = -2$ m.

11.22 Starting from $x = 0$ with no initial velocity, a particle is given an acceleration $a = 0.1\sqrt{v^2 + 49}$, where a and v are expressed in m/s^2 and m/s, respectively. Determine (*a*) the position of the particle when $v = 24$ m/s, (*b*) the speed and acceleration of the particle when $x = 40$ m.

11.23 A bowling ball is dropped from a boat so that it strikes the surface of a lake with a speed of 8 m/s. Assuming the ball experiences a downward acceleration of $a = 3 - 0.1v^2$ (where a and v are expressed in m/s^2 and m/s, respectively) when in the water, determine the velocity of the ball when it strikes the bottom of the lake.

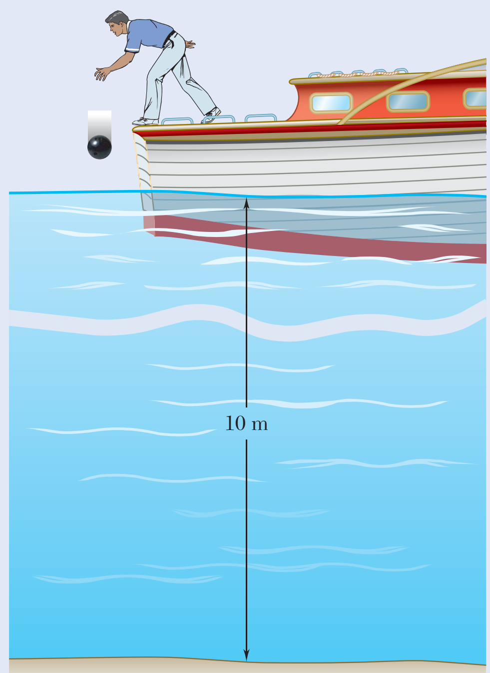


Fig. P11.23

11.24 The acceleration of a particle is defined by the relation $a = -k\sqrt{v}$, where k is a constant. Knowing that $x = 0$ and $v = 81$ m/s at $t = 0$ and that $v = 36$ m/s when $x = 18$ m, determine (*a*) the velocity of the particle when $x = 20$ m, (*b*) the time required for the particle to come to rest.

11.25 The acceleration of a particle is defined by the relation $a = -kv^{2.5}$, where k is a constant. The particle starts at $x = 0$ with a velocity of 16 mm/s, and when $x = 6$ mm, the velocity is observed to be 4 mm/s. Determine (*a*) the velocity of the particle when $x = 5$ mm, (*b*) the time at which the velocity of the particle is 9 mm/s.

Fig. P11.26

11.26 A human-powered vehicle (HPV) team wants to model the acceleration during the 260-m sprint race (the first 60 m is called a flying start) using $a = A - Cv^2$, where a is acceleration in m/s^2 and v is the velocity in m/s. From wind tunnel testing, they found that $C = 0.0012\ \text{m}^{-1}$. Knowing that the cyclist is going 100 km/h at the 260-meter mark, what is the value of A?

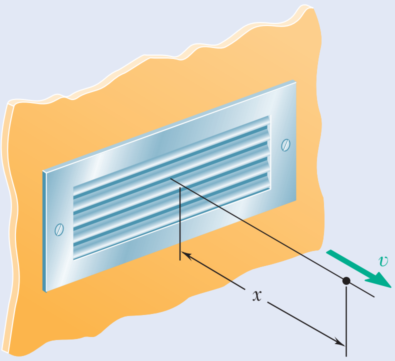


Fig. P11.27

11.27 Experimental data indicate that in a region downstream of a given louvered supply vent the velocity of the emitted air is defined by $v = 0.18v_0/x$, where v and x are expressed in m/s and meters, respectively, and v_0 is the initial discharge velocity of the air. For $v_0 = 3.6$ m/s, determine (*a*) the acceleration of the air at $x = 2$ m, (*b*) the time required for the air to flow from $x = 1$ to $x = 3$ m.

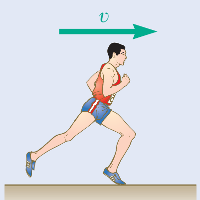


Fig. P11.28

11.28 Based on observations, the speed of a jogger can be approximated by the relation $v = 12(1 - 0.06x)^{0.3}$, where v and x are expressed in km/h and km, respectively. Knowing that $x = 0$ at $t = 0$, determine (*a*) the distance the jogger has run when $t = 1$ h, (*b*) the jogger's acceleration in m/s^2 at $t = 0$, (*c*) the time required for the jogger to run 9 Km.

11.29 The acceleration due to gravity at an altitude y above the surface of the earth can be expressed as

$$a = \frac{-9.81}{\left[1 + \left(\dfrac{y}{6.37 \times 10^6}\right)\right]^2}$$

where a and y are expressed in m/s^2 and metre, respectively. Using this expression, compute the height reached by a projectile fired vertically upward from the surface of the earth if its initial velocity is (*a*) 540 m/s, (*b*) 900 m/s, (*c*) 11,180 m/s.

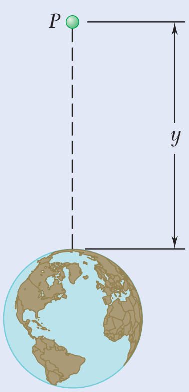


Fig. P11.29

11.30 The acceleration due to gravity of a particle falling toward the earth is $a = -gR^2/r^2$, where r is the distance from the *center* of the earth to the particle, R is the radius of the earth, and g is the acceleration due to gravity at the surface of the earth. If $R = 6370$ km, calculate the *escape velocity,* that is, the minimum velocity with which a particle must be projected vertically upward from the surface of the earth if it is not to return to the earth. (*Hint:* $v = 0$ for $r = \infty$.)

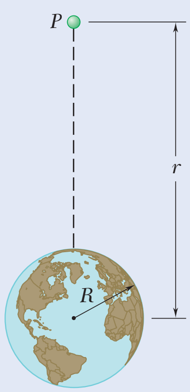


Fig. P11.30

11.31 The velocity of a particle is $v = v_0[1 - \sin(\pi t/T)]$. Knowing that the particle starts from the origin with an initial velocity v_0, determine (*a*) its position and its acceleration at $t = 3T$, (*b*) its average velocity during the interval $t = 0$ to $t = T$.

11.32 An eccentric circular cam, which serves a similar function as the Scotch yoke mechanism in Problem 11.13, is used in conjunction with a flat face follower to control motion in pumps and in steam engine valves. Knowing that the eccentricity is denoted by e, the maximum range of the displacement of the follower is d_{max} and the maximum velocity of the follower is v_{max}, determine the displacement, velocity, and acceleration of the follower.

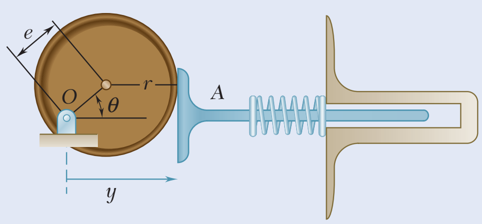


Fig. P11.32

11.2 SPECIAL CASES AND RELATIVE MOTION

In this section, we derive the equations that describe uniform rectilinear motion and uniformly accelerated rectilinear motion. We also introduce the concept of relative motion, which is of fundamental importance whenever we consider the motion of more than one particle at the same time.

11.2A Uniform Rectilinear Motion

Uniform rectilinear motion is a type of straight-line motion that is frequently encountered in practical applications. In this motion, the acceleration a of the particle is zero for every value of t. The velocity v is therefore constant, and Eq. (11.1) becomes

$$\frac{dx}{dt} = v = \text{constant}$$

We can obtain the position coordinate x by integrating this equation. Denoting the initial value of x by x_0, we have

Distance in uniform rectilinear motion

$$\int_{x_0}^{x} dx = v\int_{0}^{t} dt$$

$$x - x_0 = vt$$

$$x = x_0 + vt \qquad \textbf{(11.5)}$$

This equation can be used *only if the velocity of the particle is known to be constant*. For example, this would be true for an airplane in steady flight or a car cruising along a highway at a constant speed.

11.2B Uniformly Accelerated Rectilinear Motion

Uniformly accelerated rectilinear motion is another common type of motion. In this case, the acceleration a of the particle is constant, and Eq. (11.2) becomes

$$\frac{dv}{dt} = a = \text{constant}$$

We obtain the velocity v of the particle by integrating this equation as

$$\int_{v_0}^{v} dv = a\int_{0}^{t} dt$$

$$v - v_0 = at$$

$$v = v_0 + at \qquad \textbf{(11.6)}$$

where v_0 is the initial velocity. Substituting for v in Eq, (11.1), we have

$$\frac{dx}{dt} = v_0 + at$$

Denoting by x_0 the initial value of x and integrating, we have

$$\int_{x_0}^{x} dx = \int_0^t (v_0 + at)dt$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad (11.7)$$

We can also use Eq. (11.4) and write

$$v\frac{dv}{dx} = a = \text{constant}$$

$$v\,dv = a\,dx$$

Integrating both sides, we obtain

$$\int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx$$

$$\tfrac{1}{2}(v^2 - v_0^2) = a(x - x_0)$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad (11.8)$$

The three equations we have derived provide useful relations among position, velocity, and time in the case of constant acceleration, once you have provided appropriate values for a, v_0, and x_0. You first need to define the origin O of the x axis and choose a positive direction along the axis; this direction determines the signs of a, v_0, and x_0. Equation (11.6) relates v and t and should be used when the value of v corresponding to a given value of t is desired, or inversely. Equation (11.7) relates x and t; Eq. (11.8) relates v and x. An important application of uniformly accelerated motion is the motion of a body in **free fall**. The acceleration of a body in free fall (usually denoted by g) is equal to 9.81 m/s^2 or 32.2 ft/s^2 (we ignore air resistance in this case).

It is important to keep in mind that the three equations can be used *only when the acceleration of the particle is known to be constant*. If the acceleration of the particle is variable, you need to determine its motion from the fundamental Eqs. (11.1) through (11.4) according to the methods outlined in Sec. 11.1B.

11.2C Motion of Several Particles

When several particles move independently along the same line, you can write independent equations of motion for each particle. Whenever possible, you should record time from the same initial instant for all particles and measure displacements from the same origin and in the same direction. In other words, use a single clock and a single measuring tape.

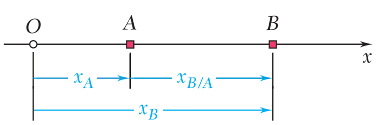


Fig. 11.7 Two particles A and B in motion along the same straight line.

Relative Motion of Two Particles. Consider two particles A and B moving along the same straight line (Fig. 11.7). If we measure the position coordinates x_A and x_B from the same origin, the difference $x_B - x_A$ defines the **relative position coordinate of** B **with respect to** A, which is denoted by $x_{B/A}$. We have

Relative position of two particles

$$x_{B/A} = x_B - x_A \quad \text{or} \quad x_B = x_A + x_{B/A} \qquad (11.9)$$

Regardless of the positions of A and B with respect to the origin, a positive sign for $x_{B/A}$ means that B is to the right of A, and a negative sign means that B is to the left of A.

The rate of change of $x_{B/A}$ is known as the **relative velocity of** B **with respect to** A and is denoted by $v_{B/A}$. Differentiating Eq. (11.9), we obtain

Relative velocity of two particles

$$v_{B/A} = v_B - v_A \quad \text{or} \quad v_B = v_A + v_{B/A} \qquad \textbf{(11.10)}$$

A positive sign for $v_{B/A}$ means that B is *observed from* A to move in the positive direction; a negative sign means that it is observed to move in the negative direction.

The rate of change of $v_{B/A}$ is known as the **relative acceleration of** B **with respect to** A and is denoted by $a_{B/A}$. Differentiating Eq. (11.10), we obtain[†]

Relative acceleration of two particles

$$a_{B/A} = a_B - a_A \quad \text{or} \quad a_B = a_A + a_{B/A} \qquad \textbf{(11.11)}$$

Dependent Motion of Particles. Sometimes, the position of a particle depends upon the position of another particle or of several other particles. These motions are called **dependent**. For example, the position of block B in Fig. 11.8 depends upon the position of block A. Since the rope $ACDEFG$ is of constant length, and since the lengths of the portions of rope CD and EF wrapped around the pulleys remain constant, it follows that the sum of the lengths of the segments AC, DE, and FG is constant. Observing that the length of the segment AC differs from x_A only by a constant and that, similarly, the lengths of the segments DE and FG differ from x_B only by a constant, we have

$$x_A + 2x_B = \text{constant}$$

Since only one of the two coordinates x_A and x_B can be chosen arbitrarily, we say that the system shown in Fig. 11.8 has **one degree of freedom**. From the relation between the position coordinates x_A and x_B, it follows that if x_A is given an increment Δx_A—that is, if block A is lowered by an amount Δx_A—the coordinate x_B receives an increment $\Delta x_B = -\frac{1}{2}\Delta x_A$. In other words, block B rises by half the same amount. You can check this directly from Fig. 11.8.

In the case of the three blocks of Fig. 11.9, we can again observe that the length of the rope that passes over the pulleys is constant. Thus, the following relation must be satisfied by the position coordinates of the three blocks:

$$2x_A + 2x_B + x_C = \text{constant}$$

Since two of the coordinates can be chosen arbitrarily, we say that the system shown in Fig. 11.9 has **two degrees of freedom**.

When the relation existing between the position coordinates of several particles is *linear,* a similar relation holds between the velocities and between the accelerations of the particles. In the case of the blocks of Fig. 11.9, for instance, we can differentiate the position equation twice and obtain

$$2\frac{dx_A}{dt} + 2\frac{dx_B}{dt} + \frac{dx_C}{dt} = 0 \quad \text{or} \quad 2v_A + 2v_B + v_C = 0$$

$$2\frac{dv_A}{dt} + 2\frac{dv_B}{dt} + \frac{dv_C}{dt} = 0 \quad \text{or} \quad 2a_A + 2a_B + a_C = 0$$

[†]Note that the product of the subscripts A and B/A used in the right-hand sides of Eqs. (11.9), (11.10), and (11.11) is equal to the subscript B that appears in the left-hand sides. This may help you remember the correct order of subscripts in various situations.

Photo 11.2 Multiple cables and pulleys are used by this shipyard crane.

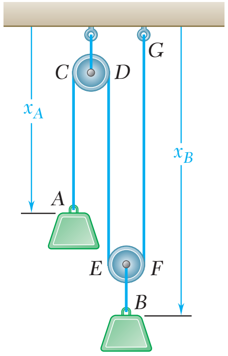


Fig. 11.8 A system of blocks and pulleys with one degree of freedom.

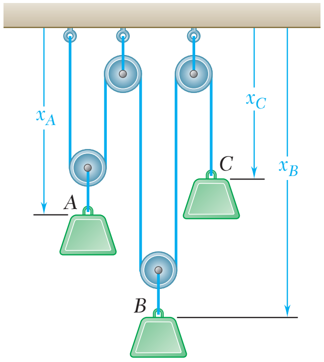


Fig. 11.9 A system of blocks and pulleys with two degrees of freedom.

Sample Problem 11.5

In an elevator shaft, a ball is thrown vertically upward with an initial velocity of 18 m/s from a height of 12 m above ground. At the same instant, an open-platform elevator passes the 5-m level, moving upward with a constant velocity of 2 m/s. Determine (*a*) when and where the ball hits the elevator (*b*) the relative velocity of the ball with respect to the elevator when the ball hits the elevator.

STRATEGY: The ball has a constant acceleration, so its motion is *uniformly accelerated*. The elevator has a constant velocity, so its motion is *uniform*. You can write equations to describe each motion and then set the position coordinates equal to each other to find when the particles meet. The relative velocity is determined from the calculated motion of each particle.

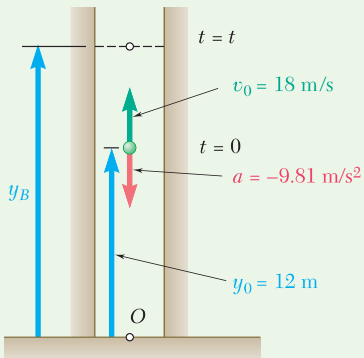


Fig. 1 Acceleration, initial velocity, and initial position of the ball.

MODELING and ANALYSIS:

Motion of Ball. Place the origin O of the y axis at ground level and choose its positive direction upward (Fig. 1). Then the initial position of the ball is $y_0 = +12$ m, its initial velocity is $v_0 = +18$ m/s, and its acceleration is $a = -9.81$ m/s^2. Substituting these values in the equations for uniformly accelerated motion, you get

$$v_B = v_0 + at \qquad v_B = 18 - 9.81t \qquad \textbf{(1)}$$

$$y_B = y_0 + v_0 t + \frac{1}{2}at^2 \qquad y_B = 12 + 18t - 4.905t^2 \qquad \textbf{(2)}$$

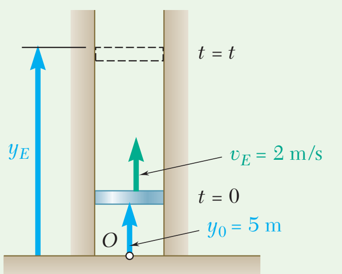


Fig. 2 Initial velocity and initial position of the elevator.

Motion of Elevator. Again place the origin O at ground level and choose the positive direction upward (Fig. 2). Noting that $y_0 = +5$ m, you have

$$v_E = +2 \text{ m/s} \qquad \textbf{(3)}$$

$$y_E = y_0 + v_E t \qquad y_E = 5 + 2t \qquad \textbf{(4)}$$

Ball Hits Elevator. First note that you used the same time t and the same origin O in writing the equations of motion for both the ball and the elevator. From Fig. 3, when the ball hits the elevator,

$$y_E = y_B \qquad \textbf{(5)}$$

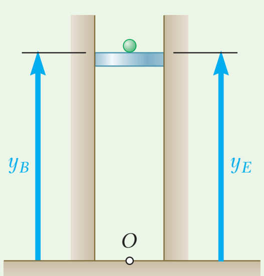


Fig. 3 Position of ball and elevator at time *t*.

Substituting for y_E and y_B from Eqs. (2) and (4) into Eq. (5), you have

$$5 + 2t = 12 + 18t - 4.905t^2$$

$$t = -0.39 \text{ s} \quad \text{and} \quad t = 3.65 \text{ s} \quad ◀$$

Only the root $t = 3.65$ s corresponds to a time after the motion has begun. Substituting this value into Eq. (4), you obtain

$$y_E = 5 + 2(3.65) = 12.30 \text{ m}$$

$$\text{Elevation from ground} = 12.30 \text{ m} \quad ◀$$

Relative Velocity. The relative velocity of the ball with respect to the elevator is

$$v_{B/E} = v_B - v_E = (18 - 9.81t) - 2 = 16 - 9.81t$$

When the ball hits the elevator at time $t = 3.65$ s, you have

$$v_{B/E} = 16 - 9.81(3.65) \qquad v_{B/E} = -19.81 \text{ m/s} \quad \blacktriangleleft$$

The negative sign means that if you are riding on the elevator, it will appear as if the ball is moving downward.

REFLECT and THINK: The key insight is that, when two particles collide, their position coordinates must be equal. Also, although you can use the basic kinematic relationships in this problem, you may find it easier to use the equations relating a, v, x, and t when the acceleration is constant or zero.

Sample Problem 11.6

Car A is travelling at a constant 135 km/h when she passes a parked police officer B, who gives chase when the car passes her. The officer accelerates at a constant rate until she reaches the speed of 150 km/h. Thereafter, her speed remains constant. The police officer catches the car 4.5 km from her starting point. Determine the initial acceleration of the police officer.

STRATEGY: One car is traveling at a constant speed and the other has a constant acceleration, so you can start with the algebraic relationships found in Sec. 11.2 rather than separating and integrating the basic kinematic relationships.

MODELING and ANALYSIS: A clearly labeled picture will help you understand the problem better (Fig. 1). The position, x, is defined from the point the car passes the officer.

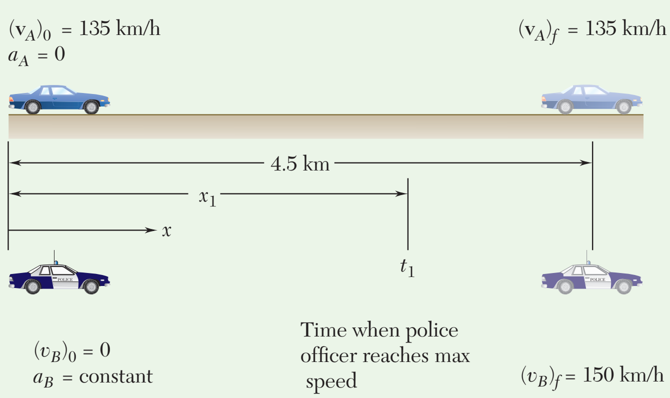


Fig. 1 Velocities and accelerations of the cars at various times.

(continued)

Unit Conversions. First you should convert everything to units of feet and seconds. Use the subscript A for the car and B for the officer

$$v_A = \left(135\ \frac{\text{km}}{\text{hr}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right)\left(\frac{1000\ \text{m}}{\text{km}}\right) = 37.5\ \frac{\text{m}}{\text{s}}$$

$$v_B = \left(150\ \frac{\text{km}}{\text{hr}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right)\left(\frac{1000\ \text{m}}{\text{km}}\right) = \frac{125}{3}\ \frac{\text{m}}{\text{s}}$$

Motion of the Speeding Car *A*. Since the car has a constant speed,

$$x_A = v_A t = 37.5t \qquad (1)$$

Motion of the Officer *B*. The officer has a constant acceleration until she reaches a final speed of 105 mph. This time is labeled t_1 in Fig. 1. Therefore, from time $0 < t < t_1$, the officer has a velocity of

$$v_B = a_B t \quad \text{for } 0 < t < t_1$$

or at time $t = t_1$, it is

$$\frac{125}{3} = a_B t_1 \qquad (2)$$

The distance the officer travels is going to be the distance from 0 to t_1 and then from t_1 to t_f. Hence,

$$x_B = \frac{1}{2}a_B t_1^2 + v_B(t - t_1) \quad \text{for } t > t_1 \qquad (3)$$

The officer catches the speeder when $x_A = x_B = 4.5$ km $= 4{,}500$ m. From Eq. (1), you can solve for the time $t_f = (4500\ \text{m})/(37.5\ \text{m/s}) = 120$ s. Therefore, you have two equations: Eq. (2) and

$$4500 = \frac{1}{2}a_B t_1^2 + \frac{125}{3}(120 - t_1) \qquad (4)$$

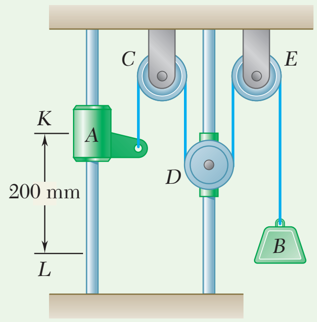
Substituting Eq. (2) into Eq. (4) allows you to solve for t_1:

$$t_1 = 24.0\ \text{s}$$

Substituting this into Eq. (2) gives

$$a_B = 1.736\ \text{m/s}^2 \blacktriangleleft$$

REFLECT and THINK: It is important to use the same origin for the position of both vehicles. The time to accelerate from 0 to 150 km/h seems reasonable, although it is perhaps longer than you would expect. A high-performance sports car can go from 0 to 90 km/h in less than 5 seconds. It is very likely that the officer could have accelerated to 150 km/h in less time if she had wanted to, but perhaps she had to consider the safety of other motorists.



Sample Problem 11.7

Collar A and block B are connected by a cable passing over three pulleys C, D, and E as shown. Pulleys C and E are fixed, while D is attached to a collar which is pulled downward with a constant velocity of 75 mm/s. At $t = 0$, collar A starts moving downward from position K with a constant acceleration and no initial velocity. Knowing that the velocity of collar A is 300 mm/s as it passes through point L, determine the change in elevation, the velocity, and the acceleration of block B when collar A passes through L.

STRATEGY: You have multiple objects connected by cables, so this is a problem in *dependent motion*. Use the given data to write a single equation relating the changes in position coordinates of collar A, pulley D, and block B. Based on the given information, you will also need to use the algebraic relationships we found for uniformly accelerated motion.

MODELING and ANALYSIS:

Motion of Collar *A*. Place the origin O at the upper horizontal surface and choose the positive direction downward. Then when $t = 0$, collar A is at position K and $(v_A)_0 = 0$ (Fig. 1). Since $v_A = 300$ mm/s and $x_A - (x_A)_0 = 200$ mm when the collar passes through L, you have

$$v_A^2 = (v_A)_0^2 + 2a_A[x_A - (x_A)_0] \qquad (300)^2 = 0 + 2a_A(200)$$
$$a_A = 225 \text{ mm/s}^2$$

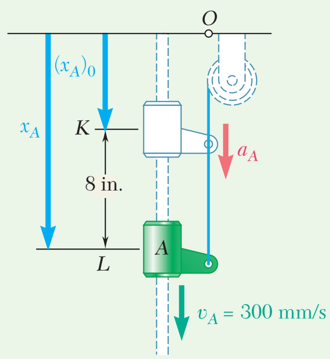


Fig. 1 Position, velocity, and acceleration of collar *A*.

To find the time at which collar A reaches point L, use the equation for velocity as a function of time with uniform acceleration. Thus,

$$v_A = (v_A)_0 + a_A t \qquad 300 = 0 + 225t \qquad t = 1.333 \text{ s}$$

Motion of Pulley *D*. Since the positive direction is downward, you have (Fig. 2)

$$a_D = 0 \qquad v_D = 75 \text{ mm/s} \qquad x_D = (x_D)_0 + v_D t = (x_D)_0 + 75t$$

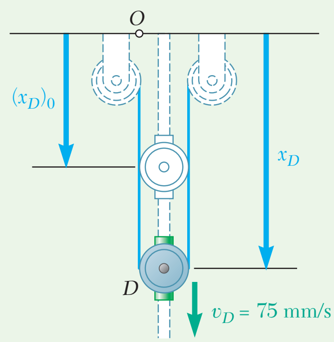


Fig. 2 Position and velocity of pulley *D*.

When collar A reaches L at $t = 1.333$ s, the position of pulley D is

$$x_D = (x_D)_0 + 75(1.333) = (x_D)_0 + 100$$

Thus, $$x_D - (x_D)_0 = 100 \text{ mm}$$

Motion of Block *B*. Note that the total length of cable $ACDEB$ differs from the quantity $(x_A + 2x_D + x_B)$ only by a constant. Since the cable length is constant during the motion, this quantity must also remain constant. Thus, considering the times $t = 0$ and $t = 1.333$ s, you can write

$$x_A + 2x_D + x_B = (x_A)_0 + 2(x_D)_0 + (x_B)_0 \qquad \textbf{(1)}$$

$$[x_A - (x_A)_0] + 2[x_D - (x_D)_0] + [x_B - (x_B)_0] = 0 \qquad \textbf{(2)}$$

But you know that $x_A - (x_A)_0 = 200$ mm and $x_D - (x_D)_0 = 100$ mm Substituting these values in Eq. (2), you find

$$200 + 2(100) + [x_B - (x_B)_0] = 0 \qquad x_B - (x_B)_0 = -400 \text{ mm}$$

Thus, Change in elevation of B = 400 mm ↑ ◀

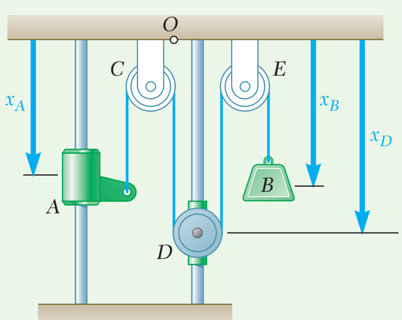


Fig. 3 Position of *A*, *B*, and *D*.

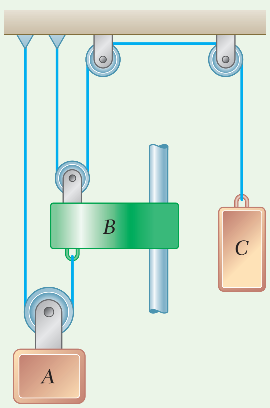
(continued)

Differentiating Eq. (1) twice, you obtain equations relating the velocities and the accelerations of *A*, *B*, and *D*. Substituting for the velocities and accelerations of *A* and *D* at $t = 1.333$ s, you have

$$v_A + 2v_D + v_B = 0: \quad 300 + 2(75) + v_B = 0$$
$$v_B = -450 \text{ mm/s} \qquad v_B = 450 \text{ mm/s} \uparrow \blacktriangleleft$$

$$a_A + 2a_D + a_B = 0: \quad 225 + 2(0) + a_B = 0$$
$$a_B = -225 \text{ mm/s}^2 \qquad a_B = 225 \text{ mm/s}^2 \uparrow \blacktriangleleft$$

REFLECT and THINK: In this case, the relationship we needed was not between position coordinates, but between changes in position coordinates at two different times. The key step is to clearly define your position vectors. This is a two-degree-of-freedom system, because two coordinates are required to completely describe it.



Sample Problem 11.8

Block *C* starts from rest and moves down with a constant acceleration. Knowing that after block *A* has moved 450 mm its velocity is 180 mm/s, determine (*a*) the acceleration of *A* and *C*, (*b*) the change in velocity and the change in position of block *B* after 2.5 seconds.

STRATEGY: Since you have blocks connected by cables, this is a dependent-motion problem. You should define coordinates for each mass and write constraint equations for both cables.

MODELING and ANALYSIS: Define position vectors as shown in Fig. 1, where positive is defined to be down.

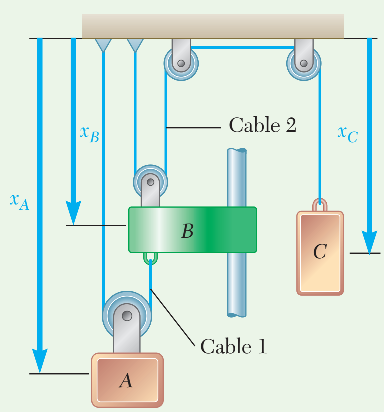


Fig. 1 Position of *A*, *B*, and *C*.

Constraint Equations. Assuming the cables are inextensible, you can write the lengths in terms of the defined coordinates and then differentiate.

Cable 1: $x_A + (x_A - x_B) = \text{constant}$

Differentiating this, you find

$$2v_A = v_B \quad \text{and} \quad 2a_A = a_B \tag{1}$$

Cable 2: $2x_B + x_C = \text{constant}$

Differentiating this, you find

$$v_C = -2v_B \quad \text{and} \quad a_C = -2a_B \tag{2}$$

Substituting Eq. (1) into Eq. (2) gives

$$v_C = -4v_A \quad \text{and} \quad a_C = -4a_A \tag{3}$$

Motion of *A*. You can use the constant-acceleration equations for block *A*:, as

$$v_A^2 - v_{A_0}^2 = 2a_A[x_A - (x_A)_0] \quad \text{or} \quad a_A = \frac{v_A^2 - (v_A)_0^2}{2[x_A - (x_A)_0]} \tag{4}$$

a. Acceleration of *A* and *C*. You know v_C and a_C are down, so from Eq. (3), you also know v_A and a_A are up. Substituting the given values into Eq. (4), you find

$$a_A = \frac{(180\text{ mm/s})^2 - 0}{2(-450\text{ mm})} = -36\text{ mm/s}^2 \qquad \mathbf{a}_A = 36\text{ mm/s}^2 \uparrow \blacktriangleleft$$

Substituting this value into $a_C = -4a_A$, you obtain

$$\mathbf{a}_C = 144\text{ mm/s}^2 \downarrow \blacktriangleleft$$

b. Velocity and change in position of *B* after 2.5 s. Substituting a_A in $a_B = 2a_A$ gives

$$a_B = 2(-36\text{ mm/s}^2) = -72\text{ mm/s}^2$$

You can use the equations of constant acceleration to find

$$\Delta v_B = a_B t = (-72\text{ mm/s}^2)(2.5\text{ s}) = -180\text{ mm/s} \qquad \Delta v_B = 180\text{ mm/s}\uparrow \blacktriangleleft$$

$$\Delta x_B = \tfrac{1}{2}a_B t = \tfrac{1}{2}(-72\text{ mm/s}^2)(2.5\text{ s})^2 = -225\text{ mm} \qquad \Delta x_B = 225\text{ mm}\uparrow \blacktriangleleft$$

REFLECT and THINK: One of the keys to solving this problem is recognizing that since there are two cables, you need to write two constraint equations. The directions of the answers also make sense. If block *C* is accelerating downward, you would expect *A* and *B* to accelerate upward.

SOLVING PROBLEMS ON YOUR OWN

In this section, we derived the equations that describe **uniform rectilinear motion** (constant velocity) and **uniformly accelerated rectilinear motion** (constant acceleration). We also introduced the concept of **relative motion**. We can apply the equations for relative motion [Eqs. (11.9) through (11.11)] to the independent or dependent motions of any two particles moving along the same straight line.

A. Independent motion of one or more particles. Organize the solution of problems of this type as follows.

1. Begin your solution by listing the given information, sketching the system, and selecting the origin and the positive direction of the coordinate axis [Sample Prob. 11.5]. It is always advantageous to have a visual representation of problems of this type.

2. Write the equations that describe the motions of the various particles as well as those that describe how these motions are related [Eq. (5) of Sample Prob. 11.5].

3. Define the initial conditions, i.e., specify the state of the system corresponding to $t = 0$. This is especially important if the motions of the particles begin at different times. In such cases, either of two approaches can be used.

a. Let $t = 0$ be the time when the last particle begins to move. You must then determine the initial position x_0 and the initial velocity v_0 of each of the other particles.

b. Let $t = 0$ be the time when the first particle begins to move. You must then, in each of the equations describing the motion of another particle, replace t with $t - t_0$, where t_0 is the time at which that specific particle begins to move. It is important to recognize that the equations obtained in this way are valid only for $t \geq t_0$.

B. Dependent motion of two or more particles. In problems of this type, the particles of the system are connected to each other, typically by ropes or cables. The method of solution of these problems is similar to that of the preceding group of problems, except that it is now necessary to describe the *physical connections* between the particles. In the following problems, the connection is provided by one or more cables. For each cable, you will have to write equations similar to the last three equations of Sec. 11.2C. We suggest that you use the following procedure.

1. Draw a sketch of the system and select a coordinate system, indicating clearly a positive sense for each of the coordinate axes. For example, in Sample Probs. 11.7 and 11.8, we measured lengths downward from the upper horizontal support. It thus follows that those displacements, velocities, and accelerations that have positive values are directed downward.

2. Write the equation describing the constraint imposed by each cable on the motion of the particles involved. Differentiating this equation twice, you obtain the corresponding relations among velocities and accelerations.

3. If several directions of motion are involved, you must select a coordinate axis and a positive sense for each of these directions. You should also try to locate the origins of your coordinate axes so that the equations of constraints are as simple as possible. For example, in Sample Prob. 11.7, it is easier to define the various coordinates by measuring them downward from the upper support than by measuring them upward from the bottom support.

Finally, keep in mind that the method of analysis described in this section and the corresponding equations can be used only for particles moving with *uniform* or *uniformly accelerated rectilinear motion*.

Problems

11.33 An airplane begins its take-off run at A with zero velocity and a constant acceleration a. Knowing that it becomes airborne 30 s later at B and that the distance AB is 900 m, determine (a) the acceleration a (b) the take-off velocity v_B.

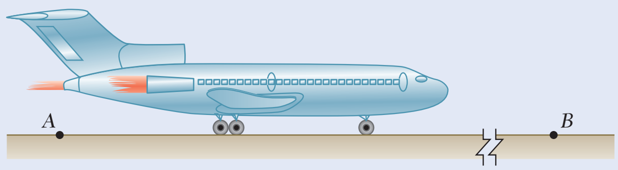


Fig. P11.33

11.34 A motorist is traveling at 54 km/h when she observes that a traffic light 240 m ahead of her turns red. The traffic light is timed to stay red for 24 s. If the motorist wishes to pass the light without stopping just as it turns green again, determine (a) the required uniform deceleration of the car, (b) the speed of the car as it passes the light.

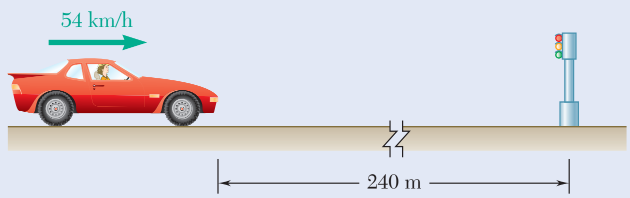


Fig. P11.34

11.35 Steep safety ramps are built beside mountain highways to enable vehicles with defective brakes to stop safely. A truck enters a 225-m ramp at a high speed v_0 and travels 160 m in 6 s at constant deceleration before its speed is reduced to $v_0/2$. Assuming the same constant deceleration, determine (a) the additional time required for the truck to stop (b) the additional distance traveled by the truck.

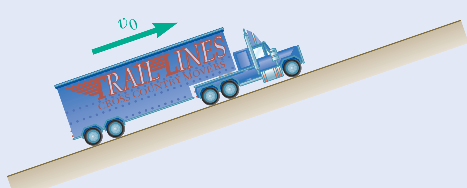


Fig. P11.35

11.36 A group of students launches a model rocket in the vertical direction. Based on tracking data, they determine that the altitude of the rocket was 27 m at the end of the powered portion of the flight and that the rocket landed 16 s later. Knowing that the descent parachute failed to deploy so that the rocket fell freely to the ground after reaching its maximum altitude and assuming that $g = 9.81\ \text{m/s}^2$, determine (a) the speed v_1 of the rocket at the end of powered flight, (b) the maximum altitude reached by the rocket.

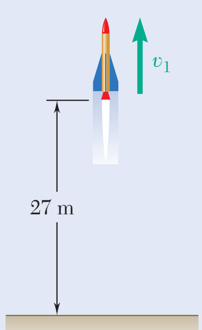


Fig. P11.36

11.37 A small package is released from rest at A and moves along the skate wheel conveyor $ABCD$. The package has a uniform acceleration of 4.8 m/s^2 as it moves down sections AB and CD, and its velocity is constant between B and C. If the velocity of the package at D is 7.2 m/s, determine (*a*) the distance d between C and D, (*b*) the time required for the package to reach D.

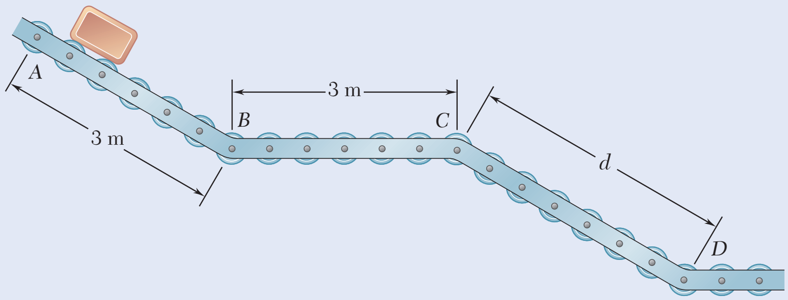


Fig. P11.37

11.38 A sprinter in a 100-m race accelerates uniformly for the first 35 m and then runs with constant velocity. If the sprinter's time for the first 35 m is 5.4 s, determine (*a*) his acceleration, (*b*) his final velocity, (*c*) his time for the race.

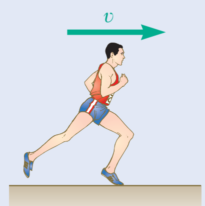


Fig. P11.38

11.39 Automobile A starts from O and accelerates at the constant rate of 0.75 m/s^2. A short time later it is passed by bus B which is traveling in the opposite direction at a constant speed of 6 m/s. Knowing that bus B passes point O 20 s after automobile A started from there, determine when and where the vehicles passed each other.

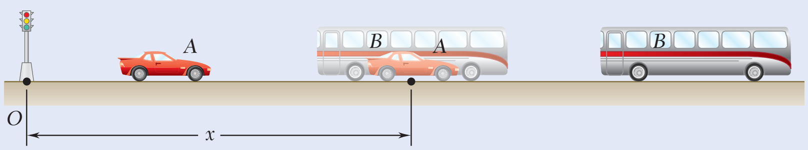


Fig. P11.39

11.40 In a boat race, boat A is leading boat B by 50 m and both boats are traveling at a constant speed of 180 km/h. At $t = 0$, the boats accelerate at constant rates. Knowing that when B passes A, $t = 8$ s and $v_A = 225$ km/h, determine (*a*) the acceleration of A, (*b*) the acceleration of B.

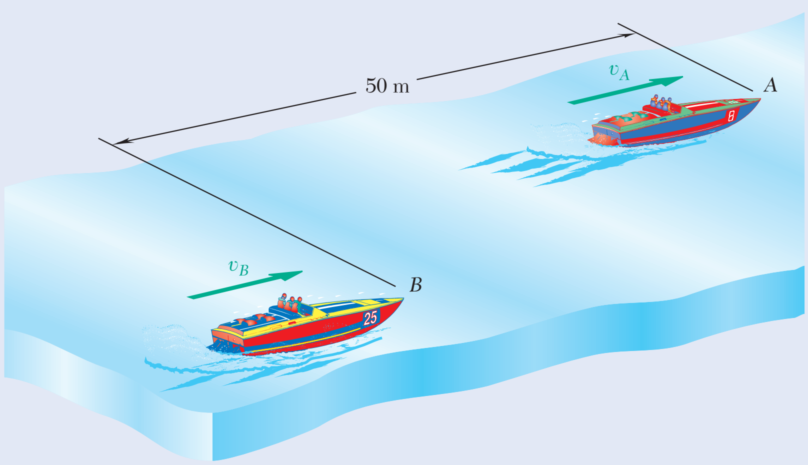


Fig. P11.40

11.41 As relay runner A enters the 20-m-long exchange zone with a speed of 12.9 m/s, he begins to slow down. He hands the baton to runner B 1.82 s later as they leave the exchange zone with the same velocity. Determine (*a*) the uniform acceleration of each of the runners, (*b*) when runner B should begin to run.

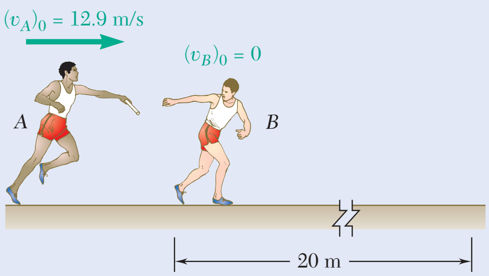


Fig. P11.41

11.42 Automobiles A and B are traveling in adjacent highway lanes and at $t = 0$ have the positions and speeds shown. Knowing that automobile A has a constant acceleration of 0.54 m/s^2 and that B has a constant deceleration of 0.36 m/s^2, determine (*a*) when and where A will overtake B, (*b*) the speed of each automobile at that time.

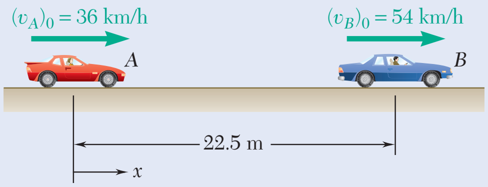


Fig. P11.42

11.43 Two automobiles A and B are approaching each other in adjacent highway lanes. At $t = 0$, A and B are 1 km apart, their speeds are $v_A = 108$ km/h and $v_B = 63$ km/h, and they are at points P and Q, respectively. Knowing that A passes point Q 40 s after B was there and that B passes point P 42 s after A was there, determine (*a*) the uniform accelerations of A and B, (*b*) when the vehicles pass each other, (*c*) the speed of B at that time.

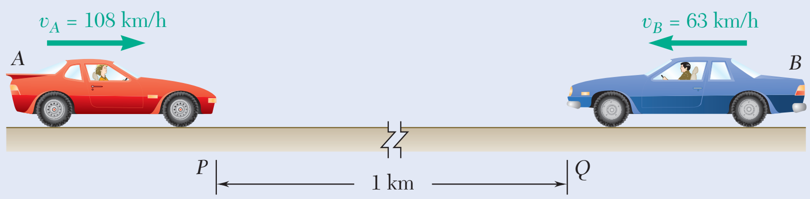


Fig. P11.43

11.44 An elevator is moving upward at a constant speed of 4 m/s. A man standing 10 m above the top of the elevator throws a ball upward with a speed of 3 m/s. Determine (*a*) when the ball will hit the elevator, (*b*) where the ball will hit the elevator with respect to the location of the man.

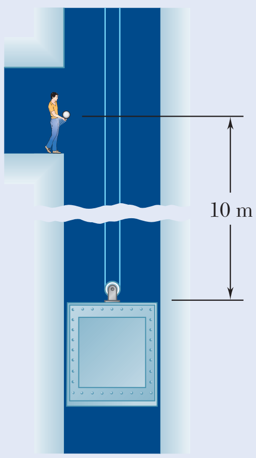


Fig. P11.44

11.45 Two rockets are launched at a fireworks display. Rocket A is launched with an initial velocity $v_0 = 100$ m/s and rocket B is launched t_1 seconds later with the same initial velocity. The two rockets are timed to explode simultaneously at a height of 300 m as A is falling and B is rising. Assuming a constant acceleration $g = 9.81$ m/s^2, determine (*a*) the time t_1, (*b*) the velocity of B relative to A at the time of the explosion.

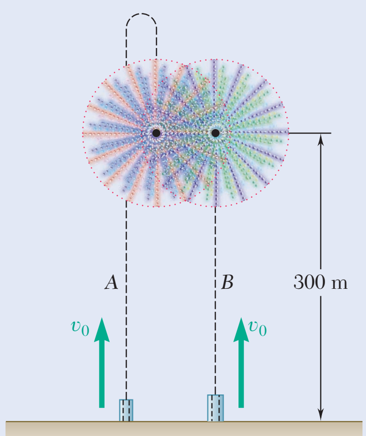


Fig. ***P11.45***

11.46 Car A is parked along the northbound lane of a highway, and car B is traveling in the southbound lane at a constant speed of 90 km/h. At $t = 0$, A starts and accelerates at a constant rate a_A, while at $t = 5$ s, B begins to slow down with a constant deceleration of magnitude $a_A/6$. Knowing that when the cars pass each other $x = 90$ m and $v_A = v_B$, determine (*a*) the acceleration a_A, (*b*) when the vehicles pass each other, (*c*) the distance d between the vehicles at $t = 0$.

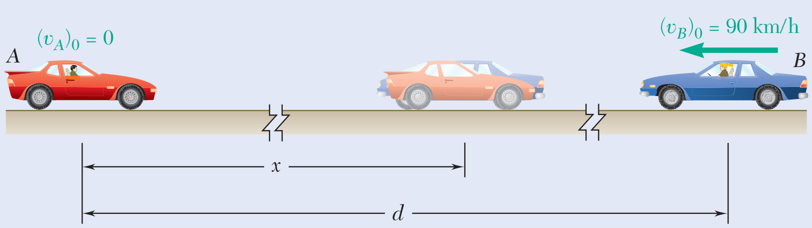


Fig. ***P11.46***

11.47 The elevator E shown in the figure moves downward with a constant velocity of 4 m/s. Determine (*a*) the velocity of the cable C, (*b*) the velocity of the counterweight W, (*c*) the relative velocity of the cable C with respect to the elevator, (*d*) the relative velocity of the counterweight W with respect to the elevator.

11.48 The elevator E shown starts from rest and moves upward with a constant acceleration. If the counterweight W moves through 10 m in 5 s, determine (*a*) the acceleration of the elevator and the cable C, (*b*) the velocity of the elevator after 5 s.

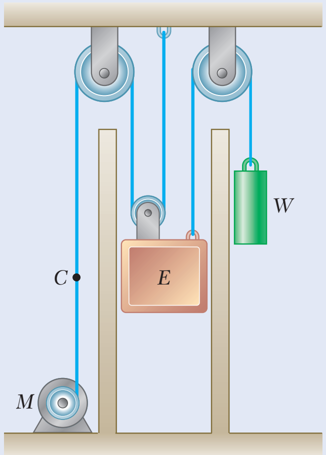


Fig. P11.47 and P11.48

11.49 An athlete pulls handle A to the left with a constant velocity of 0.5 m/s. Determine (*a*) the velocity of the weight B, (*b*) the relative velocity of weight B with respect to the handle A.

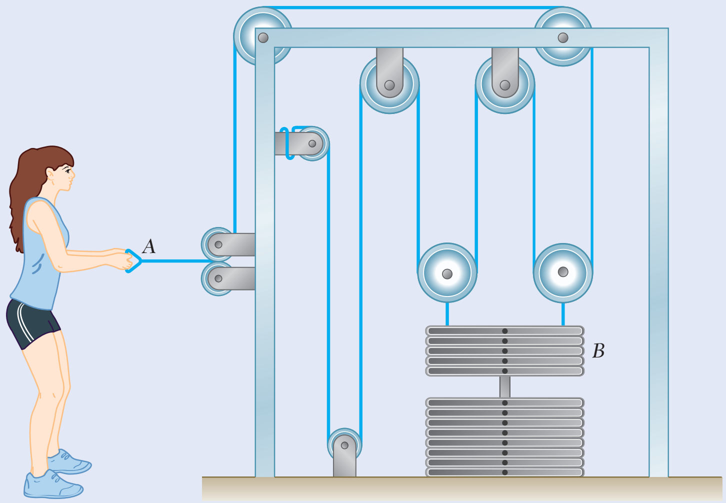


Fig. P11.49

11.50 An athlete pulls handle A to the left with a constant acceleration. Knowing that after the weight B has been lifted 100 mm its velocity is 0.6 m/s, determine (*a*) the accelerations of handle A and weight B, (*b*) the velocity and change in position of handle A after 0.5 sec.

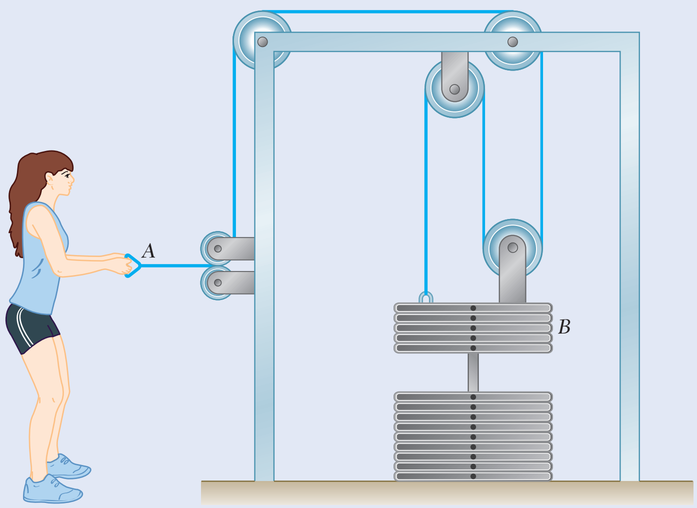


Fig. P11.50

11.51 Slider block B moves to the right with a constant velocity of 300 mm/s. Determine (a) the velocity of slider block A, (b) the velocity of portion C of the cable, (c) the velocity of portion D of the cable, (d) the relative velocity of portion C of the cable with respect to slider block A.

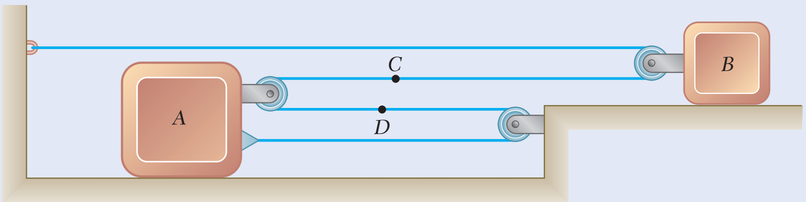


Fig. P11.51 and P11.52

11.52 At the instant shown, slider block B is moving with a constant acceleration, and its speed is 150 mm/s. Knowing that after slider block A has moved 240 mm to the right its velocity is 60 mm/s, determine (a) the accelerations of A and B, (b) the acceleration of portion D of the cable, (c) the velocity and the change in position of slider block B after 4 s.

11.53 Slider block A moves to the left with a constant velocity of 6 m/s. Determine (a) the velocity of block B, (b) the velocity of portion D of the cable, (c) the relative velocity of portion C of the cable with respect to portion D.

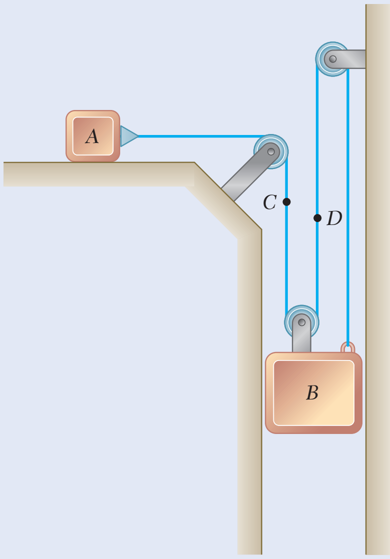


Fig. *P11.53*

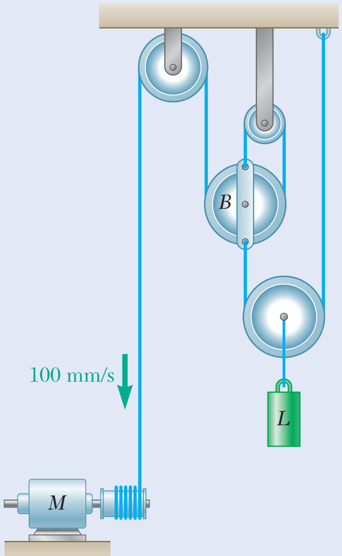


Fig. *P11.54*

11.54 The motor M reels in the cable at a constant rate of 100 mm/s. Determine (a) the velocity of load L, (b) the velocity of pulley B with respect to load L.

11.55 Collar A starts from rest and moves upward with a constant acceleration. Knowing that after 8 s the relative velocity of collar B with respect to collar A is 0.6 m/s, determine (*a*) the accelerations of A and B, (*b*) the velocity and the change in position of B after 6 s.

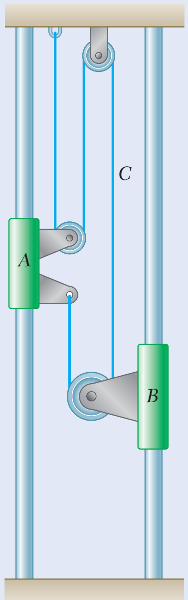


Fig. P11.55

11.56 Block A starts from rest at $t = 0$ and moves downward with a constant acceleration of 150 mm/s^2. Knowing that block B moves up with a constant velocity of 75 mm/s, determine (*a*) the time when the velocity of block C is zero, (*b*) the corresponding position of block C.

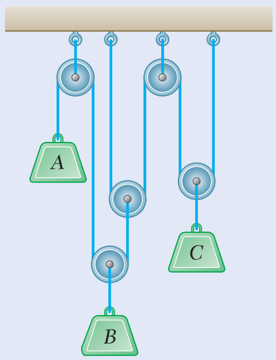


Fig. P11.56

11.57 Block B starts from rest, block A moves with a constant acceleration, and slider block C moves to the right with a constant acceleration of 75 mm/s^2. Knowing that at $t = 2$ s the velocities of B and C are 480 mm/s downward and 280 mm/s to the right, respectively, determine (*a*) the accelerations of A and B, (*b*) the initial velocities of A and C, (*c*) the change in position of slider block C after 3 s.

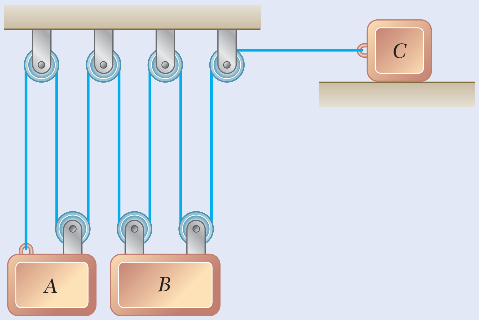


Fig. P11.57 and P11.58

11.58 Block B moves downward with a constant velocity of 20 mm/s. At $t = 0$, block A is moving upward with a constant acceleration, and its velocity is 30 mm/s. Knowing that at $t = 3$ s slider block C has moved 57 mm to the right, determine (*a*) the velocity of slider block C at $t = 0$, (*b*) the accelerations of A and C, (*c*) the change in position of block A after 5 s.

11.59 The system shown starts from rest, and each component moves with a constant acceleration. If the relative acceleration of block C with respect to collar B is 60 mm/s^2 upward and the relative acceleration of block D with respect to block A is 110 mm/s^2 downward, determine (*a*) the velocity of block C after 3 s, (*b*) the change in position of block D after 5 s.

****11.60*** The system shown starts from rest, and the length of the upper cord is adjusted so that A, B, and C are initially at the same level. Each component moves with a constant acceleration, and after 2 s the relative change in position of block C with respect to block A is 280 mm upward. Knowing that when the relative velocity of collar B with respect to block A is 80 mm/s downward, the displacements of A and B are 160 mm downward and 320 mm downward, respectively, determine (*a*) the accelerations of A and B if $a_B > 10$ mm/s^2, (*b*) the change in position of block D when the velocity of block C is 600 mm/s upward.

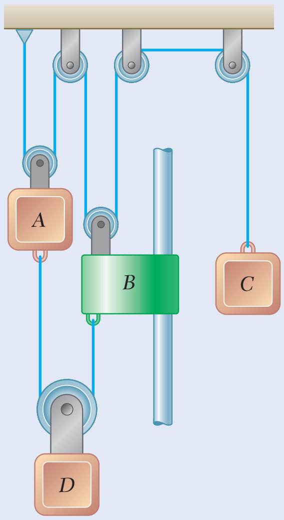


Fig. *P11.59* and *P11.60*

*11.3 GRAPHICAL SOLUTIONS

In analyzing problems in rectilinear motion, it is often useful to draw graphs of position, velocity, or acceleration versus time. Sometimes these graphs can provide insight into the situation by indicating when quantities increase, decrease, or stay the same. In other cases, the graphs can provide numerical solutions when analytical methods are not available. In many experimental situations, data are collected as a function of time, and the methods of this section are very useful for the analysis.

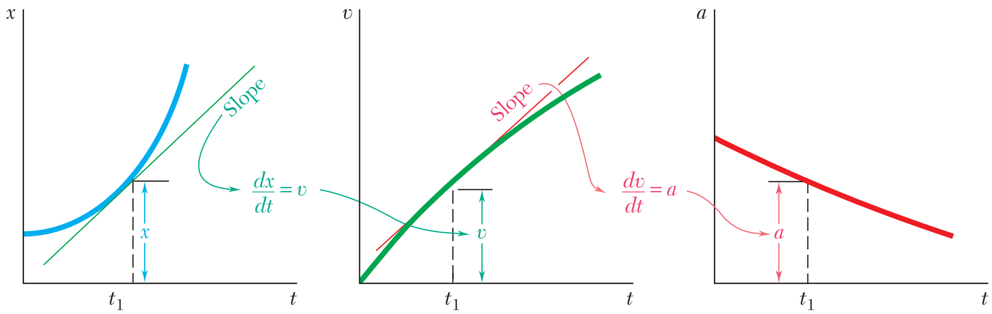


Fig. 11.10 The slope of an x–t curve at time t_1 equals the velocity v at that time; the slope of the v–t curve at time t_1 equals the acceleration a at that time.

We observed in Sec. 11.1 that the fundamental formulas

$$v = \frac{dx}{dt} \quad \text{and} \quad a = \frac{dv}{dt}$$

have a geometrical significance. The first formula says that the velocity at any instant is equal to the slope of the x–t curve at that instant (Fig. 11.10). The second formula states that the acceleration is equal to the slope of the v–t curve. We can use these two properties to determine graphically the v–t and a–t curves of a motion when the x–t curve is known.

Integrating the two fundamental formulas from a time t_1 to a time t_2, we have

$$x_2 - x_1 = \int_{t_1}^{t_2} v\,dt \quad \text{and} \quad v_2 - v_1 = \int_{t_1}^{t_2} a\,dt \tag{11.12}$$

The first formula says that the area measured under the $v-t$ curve from t_1 to t_2 is equal to the change in x during that time interval (Fig. 11.11). Similarly, the second formula states that the area measured under the a–t curve from t_1 to t_2 is equal to the change in v during that time interval. We can use these two properties to determine graphically the x–t curve of a motion when its v–t curve or its a–t curve is known (see Sample Prob. 11.9).

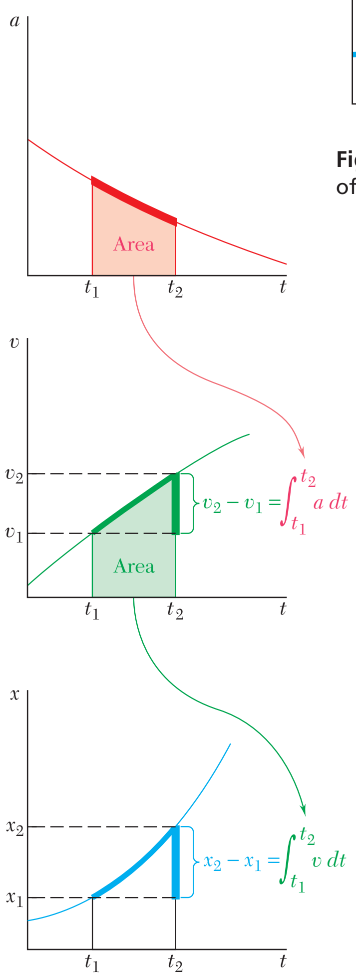
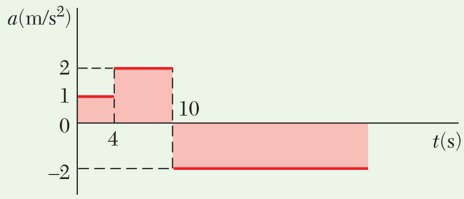


Fig. 11.11 The area under an a–t curve equals the change in velocity during that time interval; the area under the v–t curve equals the change in position during that time interval.

Graphical solutions are particularly useful when the motion considered is defined from experimental data and when x, v, and a are not analytical functions of t. They also can be used to advantage when the motion consists of distinct parts and when its analysis requires writing a different equation for each of its parts. When using a graphical solution, however, be careful to note that (1) the area under the v–t curve measures the *change in x*—not x itself—and similarly, that the area under the a–t curve measures the change in v; (2) an area above the t axis corresponds to an *increase* in x or v, whereas an area located below the t axis measures a *decrease* in x or v.

In drawing motion curves, it is useful to remember that, if the velocity is constant, it is represented by a horizontal straight line; the position coordinate x is then a linear function of t and is represented by an oblique straight line. If the acceleration is constant and different from zero, it is represented by a horizontal straight line; v is then a linear function of t and is represented by an oblique straight line, and x is a second-degree polynomial in t and is represented by a parabola. If the acceleration is a linear function of t, the velocity and the position coordinate are equal, respectively, to second-degree and third-degree polynomials; a is then represented by an oblique straight line, v by a parabola, and x by a cubic. In general, if the acceleration is a polynomial of degree n in t, the velocity is a polynomial of degree $n + 1$, and the position coordinate is a polynomial of degree $n + 2$. These polynomials are represented by motion curves of a corresponding degree.

Sample Problem 11.9



A particle moves in a straight line with the acceleration shown in the figure. Knowing that it starts from the origin with $v_0 = -6$ m/s, (*a*) plot the $v - t$ and $x - t$ curves for $0 < t < 20$ s, (*b*) determine its velocity, its position, and the total distance traveled when $t = 12$ s.

STRATEGY: You are given the graph of a versus t. You can calculate areas under the curve to determine the v–t curve and calculate areas under the v–t curve to determine the x–t curve.

MODELING and ANALYSIS: The particle is moving under rectilinear acceleration.

a Acceleration-Time Curve.

Initial conditions: $t = 0$, $v_0 = -6$ m/s, $x_0 = 0$

Change in v = area under $a-t$ curve:

$$v_0 = -6 \text{ m/s}$$

$$0 < t < 4\text{s}: \quad v_4 - v_0 = (1 \text{ m/s}^2)(4\text{s}) = +4 \text{ m/s} \qquad v_4 = -2 \text{ m/s}$$

$$4\text{s} < t < 10\text{s}: \quad v_{10} - v_4 = (2 \text{ m/s}^2)(6\text{s}) = +12 \text{ m/s} \qquad v_{10} = +10 \text{ m/s}$$

$$10\text{s} < t < 12\text{s}: \quad v_{12} - v_{10} = (-2 \text{ m/s}^2)(2\text{s}) = -4 \text{ m/s} \qquad v_{12} = +6 \text{ m/s}$$

$$12\text{s} < t < 20\text{s}: \quad v_{20} - v_{12} = (-2 \text{ m/s}^2)(8\text{s}) = -16 \text{ m/s} \qquad v_{20} = -10 \text{ m/s}$$

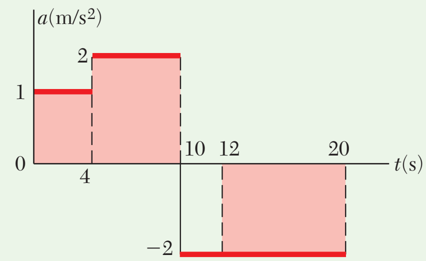


Fig. 1 Acceleration of the particle as a function of time

Change in x = area under $v - t$ curve: $\qquad x_0 = 0$

$$0 < t < 4\text{s}: \quad x_4 - x_0 = \frac{1}{2}(-6-2)(4) = -16 \text{ m} \qquad x_4 = -16 \text{m}$$

$$4\text{s} < t < 5\text{s}: \quad x_5 - x_4 = \frac{1}{2}(-2)(1) = -1 \text{ m} \qquad x_5 = -17 \text{m}$$

$$5\text{s} < t < 10\text{s}: \quad x_{10} - x_5 = \frac{1}{2}(+10)(5) = +25 \text{ m} \qquad x_{10} = 8 \text{m}$$

$$10\text{s} < t < 12\text{s}: \quad x_{12} - x_{10} = \frac{1}{2}(+10+6)(2) = +16 \text{ m} \qquad x_{12} = +24 \text{m}$$

$$12\text{s} < t < 15\text{s}: \quad x_{15} - x_{12} = \frac{1}{2}(+6)(3) = +9 \text{ m} \qquad x_{16} = +33 \text{m}$$

$$15\text{s} < t < 20\text{s}: \quad x_{20} - x_{15} = \frac{1}{2}(-10)(5) = -25 \text{ m} \qquad x_{20} = +8 \text{m}$$

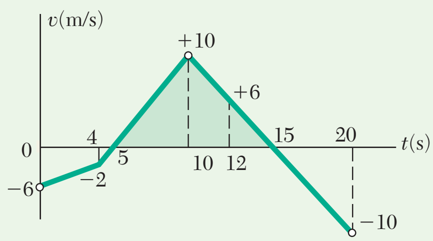


Fig. 2 Velocity of the particle as a function of time

b From above curves, you read

For $t = 12$ s: $\quad v_{12} = +6$ m/s, $x_{12} = +24$ m

Distance traveled $t = 0$ to $t = 12$ s

From $t = 0$ s to $t = 5$ s: Distance traveled = 17 m

From $t = 5$ s to $t = 12$ s: Distance traveled = (17 + 24) = 41 m

Total distance traveled = 58 m ◀

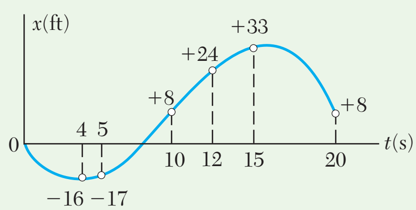


Fig. 3 Position of the particle as a function of time

REFLECT and THINK: This problem also could have been solved using the uniform motion equations for each interval of time that has a different acceleration, but it would have been much more difficult and time consuming. For a real particle, the acceleration does not instantaneously change from one value to another.

SOLVING PROBLEMS ON YOUR OWN

In this section, we reviewed and developed several **graphical techniques** for the solution of problems involving rectilinear motion. These techniques can be used to solve problems directly or to complement analytical methods of solution by providing a visual description, and thus a better understanding, of the motion of a given body. We suggest that you sketch one or more motion curves for several of the problems in this section, even if these problems are not part of your homework assignment.

1. Drawing x–t, v–t, and a–t curves and applying graphical methods. We described the following properties in Sec. 11.3, and they should be kept in mind as you use a graphical method of solution.

a. The slopes of the x–t and v–t curves at a time t_1 are equal to the velocity and the acceleration at time t_1, respectively.

b. The areas under the a–t and v–t curves between the times t_1 and t_2 are equal to the change Δv in the velocity and to the change Δx in the position coordinate, respectively, during that time interval.

c. If you know one of the motion curves, the fundamental properties we have summarized in paragraphs *a* and *b* will enable you to construct the other two curves. However, when using the properties of paragraph *b*, you must know the velocity and the position coordinate at time t_1 in order to determine the velocity and the position coordinate at time t_2. Thus, in Sample Prob. 11.9, knowing that the initial value of the velocity was zero allowed us to find the velocity at $t = 6$ s: $v_6 = v_0 + \Delta v = 0 + 24 \text{ ft/s} = 24 \text{ ft/s}$.

If you have studied the shear and bending-moment diagrams for a beam previously, you should recognize the analogy between the three motion curves and the three diagrams representing, respectively, the distributed load, the shear, and the bending moment in the beam. Thus, any techniques that you have learned regarding the construction of these diagrams can be applied when drawing the motion curves.

2. Using approximate methods. When the a–t and v–t curves are not represented by analytical functions or when they are based on experimental data, it is often necessary to use approximate methods to calculate the areas under these curves. In those cases, the given area is approximated by a series of rectangles of width Δt. The smaller the value of Δt, the smaller is the error introduced by the approximation. You can obtain the velocity and the position coordinate from

$$v = v_0 + \Sigma a_{ave}\,\Delta t \qquad x = x_0 + \Sigma v_{ave}\,\Delta t$$

where a_{ave} and v_{ave} are the heights of an acceleration rectangle and a velocity rectangle, respectively.

Problems

11.61 A particle moves in a straight line with a constant acceleration of -4 m/s^2 for 6 s, zero acceleration for the next 4 s, and a constant acceleration of $+4$ m/s^2 for the next 4 s. Knowing that the particle starts from the origin and that its velocity is -8 m/s during the zero acceleration time interval, (*a*) construct the v–t and x–t curves for $0 \le t \le 14$ s, (*b*) determine the position and the velocity of the particle and the total distance traveled when $t = 14$ s.

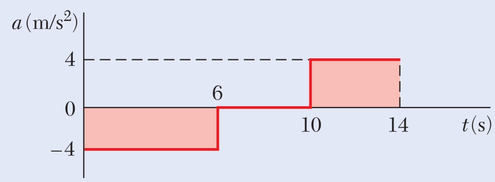


Fig. P11.61 and P11.62

11.62 A particle moves in a straight line with a constant acceleration of -4 m/s^2 for 6 s, zero acceleration for the next 4 s, and a constant acceleration of $+4$ m/s^2 for the next 4 s. Knowing that the particle starts from the origin with $v_0 = 16$ m/s, (*a*) construct the v–t and x–t curves for $0 \le t \le 14$ s, (*b*) determine the amount of time during which the particle is further than 16 m from the origin.

11.63 A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -540$ m at $t = 0$, (*a*) construct the a–t and x–t curves for $0 < t < 50$ s, and determine (*b*) the total distance traveled by the particle when $t = 50$ s, (*c*) the two times at which $x = 0$.

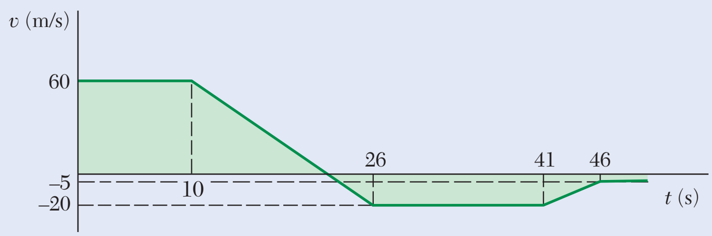


Fig. P11.63 and P11.64

11.64 A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -540$ m at $t = 0$, (*a*) construct the a–t and x–t curves for $0 < t < 50$ s, and determine (*b*) the maximum value of the position coordinate of the particle, (*c*) the values of t for which the particle is at $x = 100$ m.

11.65 A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -48$ m at $t = 0$, draw the a–t and x–t curves for $0 < t < 40$ s and determine (*a*) the maximum value of the position coordinate of the particle, (*b*) the values of t for which the particle is at a distance of 108 m from the origin.

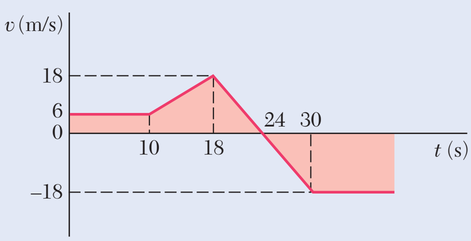


Fig. P11.65

11.66 A parachutist is in free fall at a rate of 200 km/h when he opens his parachute at an altitude of 600 m. Following a rapid and constant deceleration, he then descends at a constant rate of 50 km/h from 586 m to 30 m, where he maneuvers the parachute into the wind to further slow his descent. Knowing that the parachutist lands with a negligible downward velocity, determine (*a*) the time required for the parachutist to land after opening his parachute, (*b*) the initial deceleration.

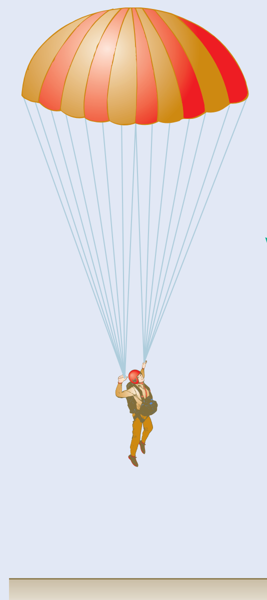


Fig. P11.66

11.67 A commuter train traveling at 60 km/h is 4.5 km from a station. The train then decelerates so that its speed is 30 km/h when it is 0.75 km from the station. Knowing that the train arrives at the station 7.5 min after beginning to decelerate and assuming constant decelerations, determine (*a*) the time required for the train to travel the first 3.75 km, (*b*) the speed of the train as it arrives at the station, (*c*) the final constant deceleration of the train.

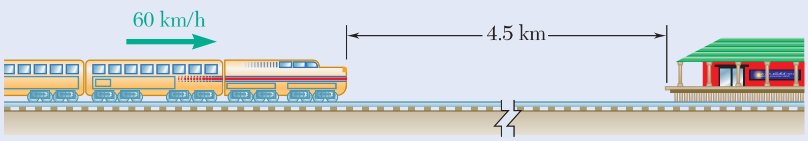


Fig. *P11.67*

11.68 A temperature sensor is attached to slider *AB* which moves back and forth through 1500 mm. The maximum velocities of the slider are 300 mm/s to the right and 750 mm/s to the left. When the slider is moving to the right, it accelerates and decelerates at a constant rate of 150 mm/s^2; when moving to the left, the slider accelerates and decelerates at a constant rate of 500 mm/s^2. Determine the time required for the slider to complete a full cycle, and construct the *v*–*t* and *x*–*t* curves of its motion.

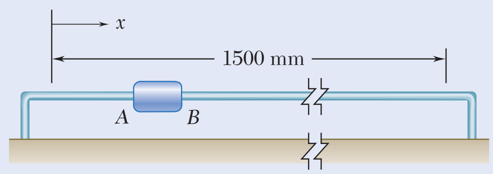


Fig. *P11.68*

11.69 In a water-tank test involving the launching of a small model boat, the model's initial horizontal velocity is 6 m/s and its horizontal acceleration varies linearly from -12 m/s^2 at $t = 0$ to -2 m/s^2 at $t = t_1$ and then remains equal to -2 m/s^2 until $t = 1.4$ s. Knowing that $v = 1.8$ m/s when $t = t_1$, determine (*a*) the value of t_1, (*b*) the velocity and the position of the model at $t = 1.4$ s.

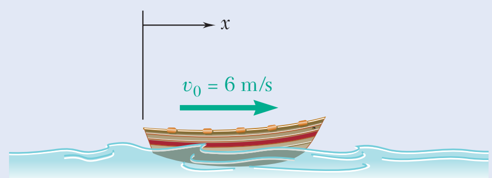


Fig. P11.69

11.70 The acceleration record shown was obtained for a small airplane traveling along a straight course. Knowing that $x = 0$ and $v = 60$ m/s when $t = 0$, determine (*a*) the velocity and position of the plane at $t = 20$ s, (*b*) its average velocity during the interval $6 \text{ s} < t < 14 \text{ s}$.

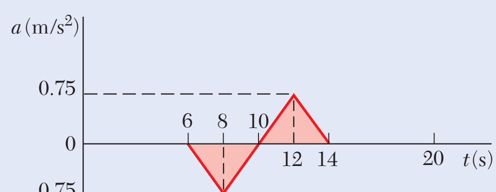


Fig. P11.70

11.71 In a 400-m race, runner *A* reaches her maximum velocity v_A in 4 s with constant acceleration and maintains that velocity until she reaches the halfway point with a split time of 25 s. Runner *B* reaches her maximum velocity v_B in 5 s with constant acceleration and maintains that velocity until she reaches the halfway point with a split time of 25.2 s. Both runners then run the second half of the race with the same constant deceleration of 0.1 m/s^2. Determine (*a*) the race times for both runners, (*b*) the position of the winner relative to the loser when the winner reaches the finish line.

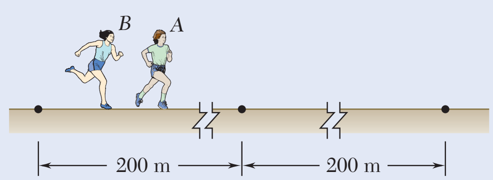


Fig. P11.71

11.72 A car and a truck are both traveling at the constant speed of 50 km/h; the car is 12 m behind the truck. The driver of the car wants to pass the truck, i.e., he wishes to place his car at B, 12 m in front of the truck, and then resume the speed of 50 km/h. The maximum acceleration of the car is 1.5 m/s^2 and the maximum deceleration obtained by applying the brakes is 6 m/s^2. What is the shortest time in which the driver of the car can complete the passing operation if he does not at any time exceed a speed of 75 km/h? Draw the v–t curve.

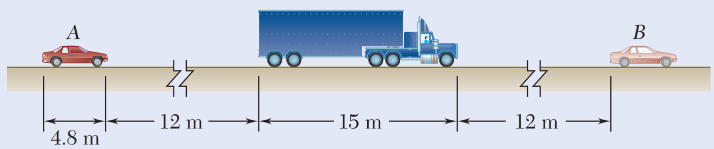


Fig. P11.72

11.73 Solve Prob. 11.72, assuming that the driver of the car does not pay any attention to the speed limit while passing and concentrates on reaching position B and resuming a speed of 50 km/h in the shortest possible time. What is the maximum speed reached? Draw the v–t curve.

11.74 Car A is traveling on a highway at a constant speed $(v_A)_0 = 90$ km/h and is 120 m from the entrance of an access ramp when car B enters the acceleration lane at that point at a speed $(v_B)_0 = 25$ km/h. Car B accelerates uniformly and enters the main traffic lane after traveling 60 m in 5 s. It then continues to accelerate at the same rate until it reaches a speed of 90 km/h, which it then maintains. Determine the final distance between the two cars.

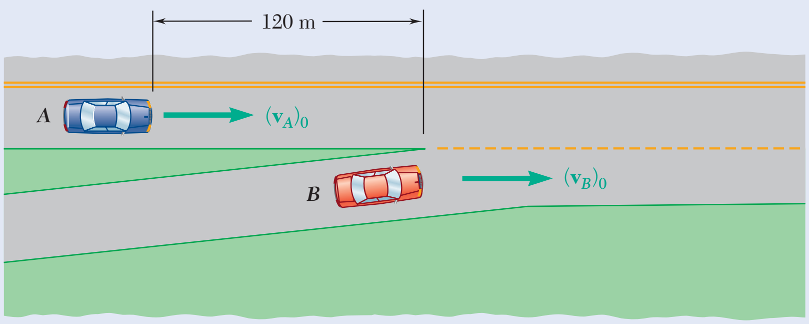


Fig. P11.74

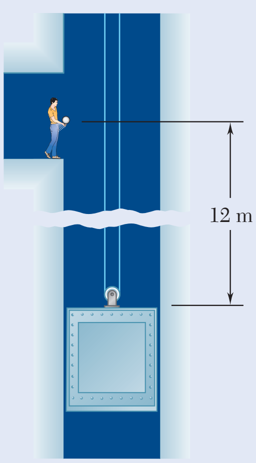


Fig. P11.75

11.75 An elevator starts from rest and moves upward, accelerating at a rate of 1.2 m/s^2 until it reaches a speed of 7.8 m/s, which it then maintains. Two seconds after the elevator begins to move, a man standing 12 m above the initial position of the top of the elevator throws a ball upward with an initial velocity of 20 m/s. Determine when the ball will hit the elevator.

11.76 Car A is traveling at 60 km/h when it enters a 40 km/h speed zone. The driver of car A decelerates at a rate of 5 m/s^2 until reaching a speed of 40 km/h, which she then maintains. When car B, which was initially 20 m behind car A and traveling at a constant speed of 70 km/h, enters the speed zone, its driver decelerates at a rate of 6 m/s^2 until reaching a speed of 35 km/h. Knowing that the driver of car B maintains a speed of 35 km/h, determine (*a*) the closest that car B comes to car A, (*b*) the time at which car A is 25 m in front of car B.

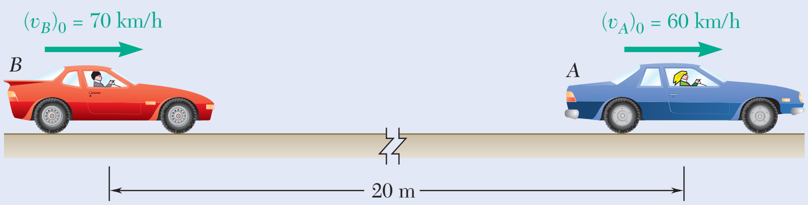


Fig. P11.76

11.77 An accelerometer record for the motion of a given part of a mechanism is approximated by an arc of a parabola for 0.2 s and a straight line for the next 0.2 s as shown in the figure. Knowing that $v = 0$ when $t = 0$ and $x = 0.4$ m when $t = 0.4$ s, (*a*) construct the v–t curve for $0 \leq t \leq 0.4$ s, (*b*) determine the position of the part at $t = 0.3$ s and $t = 0.2$ s.

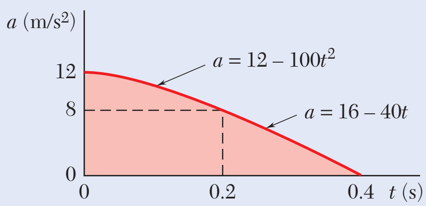


Fig. P11.77

11.78 A car is traveling at a constant speed of 54 km/h when its driver sees a child run into the road. The driver applies her brakes until the child returns to the sidewalk and then accelerates to resume her original speed of 54 km/h; the acceleration record of the car is shown in the figure. Assuming $x = 0$ when $t = 0$, determine (*a*) the time t_1 at which the velocity is again 54 km/h, (*b*) the position of the car at that time, (*c*) the average velocity of the car during the interval $1 \text{ s} \leq t \leq t_1$.

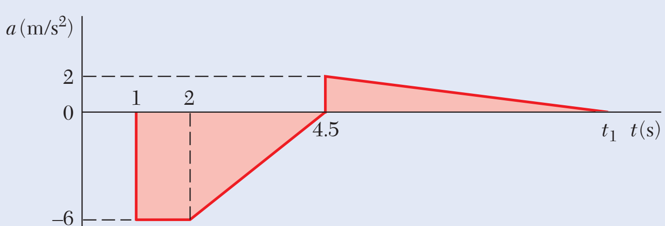


Fig. P11.78

11.79 An airport shuttle train travels between two terminals that are 2.5 km apart. To maintain passenger comfort, the acceleration of the train is limited to ± 1.2 m/s^2, and the jerk, or rate of change of acceleration, is limited to ± 0.24 m/s^2 per second. If the shuttle has a maximum speed of 30 km/h, determine (*a*) the shortest time for the shuttle to travel between the two terminals, (*b*) the corresponding average velocity of the shuttle.

11.80 During a manufacturing process, a conveyor belt starts from rest and travels a total of 400 mm before temporarily coming to rest. Knowing that the jerk, or rate of change of acceleration, is limited to ± 1.5 m/s^2 per second, determine (*a*) the shortest time required for the belt to move 400 mm, (*b*) the maximum and average values of the velocity of the belt during that time.

11.81 Two seconds are required to bring the piston rod of an air cylinder to rest; the acceleration record of the piston rod during the 2 s is as shown. Determine by approximate means (*a*) the initial velocity of the piston rod, (*b*) the distance traveled by the piston rod as it is brought to rest.

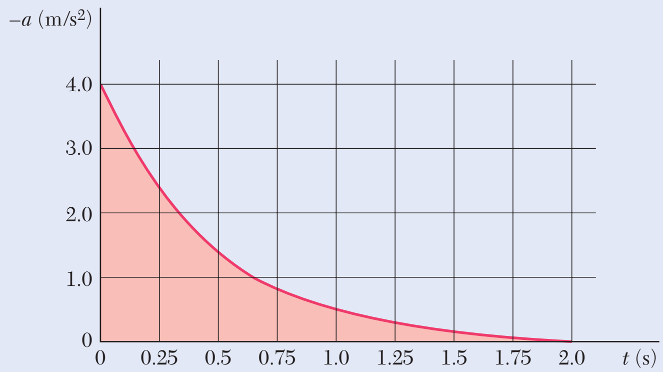


Fig. ***P11.81***

11.82 The acceleration record shown was obtained during the speed trials of a sports car. Knowing that the car starts from rest, determine by approximate means (*a*) the velocity of the car at $t = 8$ s, (*b*) the distance the car has traveled at $t = 20$ s.

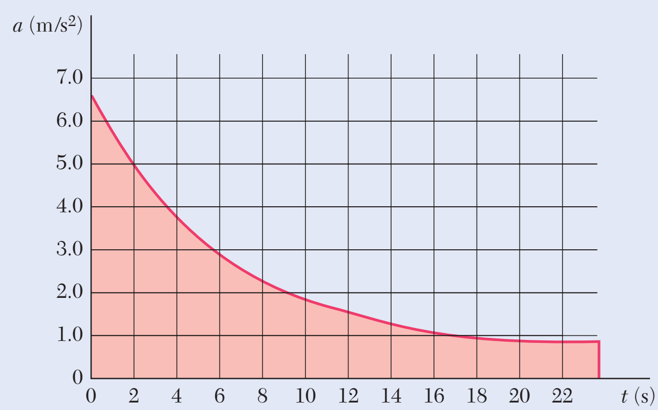


Fig. ***P11.82***

11.83 A training airplane has a velocity of 38 m/s when it lands on an aircraft carrier. As the arresting gear of the carrier brings the airplane to rest, the velocity and the acceleration of the airplane are recorded; the results are shown (solid curve) in the figure. Determine by approximate means (*a*) the time required for the airplane to come to rest, (*b*) the distance traveled in that time.

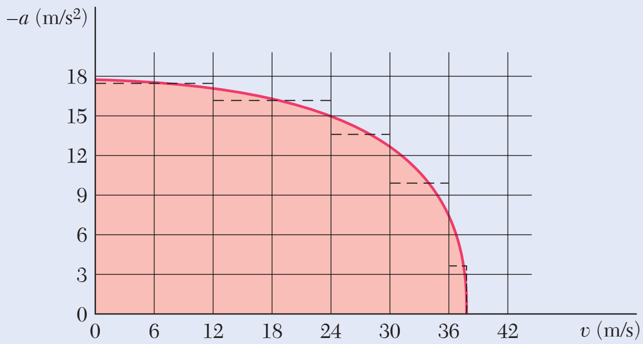


Fig. P11.83

11.84 Shown in the figure is a portion of the experimentally determined v–x curve for a shuttle cart. Determine by approximate means the acceleration of the cart when (*a*) $x = 250$ mm, (*b*) $v = 2000$ mm/s.

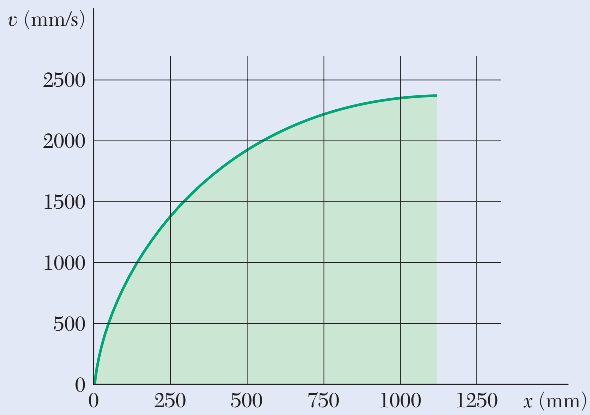


Fig. P11.84

11.85 An elevator starts from rest and rises 40 m to its maximum velocity in T s with the acceleration record shown in the figure. Determine (*a*) the required time T, (*b*) the maximum velocity, (*c*) the velocity and position of the elevator at $t = T/2$.

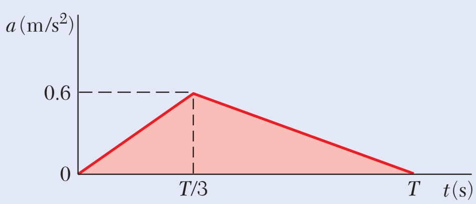


Fig. P11.85

11.86 The acceleration of an object subjected to the pressure wave of a large explosion is defined approximately by the curve shown. The object is initially at rest and is again at rest at time t_1. Using the method of Sec. 11.8, determine (*a*) the time t_1, (*b*) the distance through which the object is moved by the pressure wave.

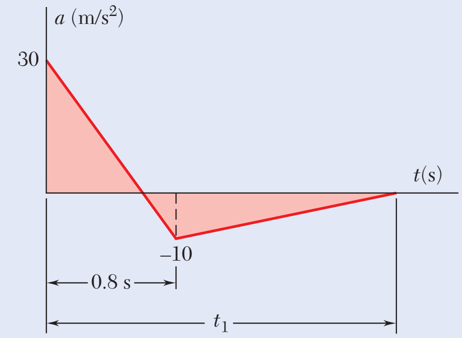


Fig. ***P11.86***

11.87 As shown in the figure, from $t = 0$ to $t = 4$ s, the acceleration of a given particle is represented by a parabola. Knowing that $x = 0$ and $v = 8$ m/s when $t = 0$, (*a*) construct the v–t and x–t curves for $0 < t < 4$ s, (*b*) determine the position of the particle at $t = 3$ s. (*Hint:* Use table inside the front cover.)

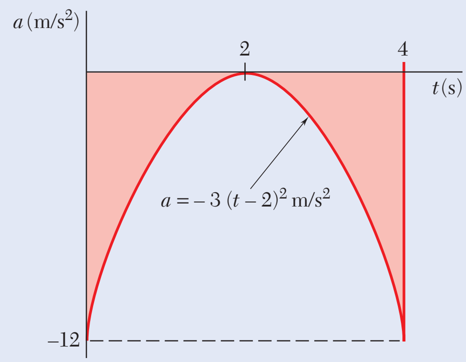


Fig. ***P11.87***

11.88 A particle moves in a straight line with the acceleration shown in the figure. Knowing that the particle starts from the origin with $v_0 = -2$ m/s, (a) construct the v–t and x–t curves for $0 < t < 18$ s, (*b*) determine the position and the velocity of the particle and the total distance traveled when $t = 18$ s.

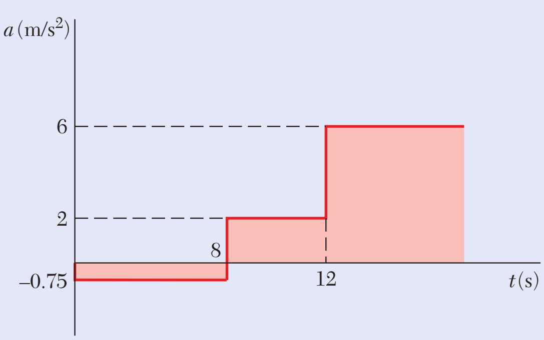


Fig. ***P11.88***

11.4 CURVILINEAR MOTION OF PARTICLES

When a particle moves along a curve other than a straight line, we say that the particle is in **curvilinear motion**. We can use position, velocity, and acceleration to describe the motion, but now we must treat these quantities as vectors because they can have directions in two or three dimensions.

11.4A Position, Velocity, and Acceleration Vectors

To define the position P occupied by a particle in curvilinear motion at a given time t, we select a fixed reference system, such as the x, y, z axes shown in Fig. 11.12*a*, and draw the vector $\mathbf{r}$ joining the origin O and point P. The vector $\mathbf{r}$ is characterized by its magnitude r and its direction with respect to the reference axes, so it completely defines the position of the particle with respect to those axes. We refer to vector $\mathbf{r}$ as the **position vector** of the particle at time t.

Consider now the vector $\mathbf{r}'$ defining the position P' occupied by the same particle at a later time $t + \Delta t$. The vector $\Delta\mathbf{r}$ joining P and P' represents the change in the position vector during the time interval Δt and is called the **displacement vector**. We can check this directly from Fig. 11.12*a*, where we obtain the vector $\mathbf{r}'$ by adding the vectors $\mathbf{r}$ and $\Delta\mathbf{r}$ according to the triangle rule. Note that $\Delta\mathbf{r}$ represents a change in *direction* as well as a change in *magnitude* of the position vector $\mathbf{r}$.

We define the **average velocity** of the particle over the time interval Δt as the quotient of $\Delta\mathbf{r}$ and Δt. Since $\Delta\mathbf{r}$ is a vector and Δt is a scalar, the quotient $\Delta\mathbf{r}/\Delta t$ is a vector attached at P with the same direction as $\Delta\mathbf{r}$ and a magnitude equal to the magnitude of $\Delta\mathbf{r}$ divided by Δt (Fig. 11.12*b*).

We obtain the **instantaneous velocity** of the particle at time t by taking the limit as the time interval Δt approaches zero. The instantaneous velocity is thus represented by the vector

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{r}}{\Delta t} \tag{11.13}$$

As Δt and $\Delta\mathbf{r}$ become shorter, the points P and P' get closer together. Thus, the vector $\mathbf{v}$ obtained in the limit must be tangent to the path of the particle (Fig. 11.12*c*).

Because the position vector $\mathbf{r}$ depends upon the time t, we can refer to it as a **vector function** of the scalar variable t and denote it by $\mathbf{r}(t)$. Extending the concept of the derivative of a scalar function introduced in elementary calculus, we refer to the limit of the quotient $\Delta\mathbf{r}/\Delta t$ as the **derivative** of the vector function $\mathbf{r}(t)$. We have

Velocity vector

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \tag{11.14}$$

The magnitude v of the vector $\mathbf{v}$ is called the **speed** of the particle. We can obtain the speed by substituting the magnitude of this vector,

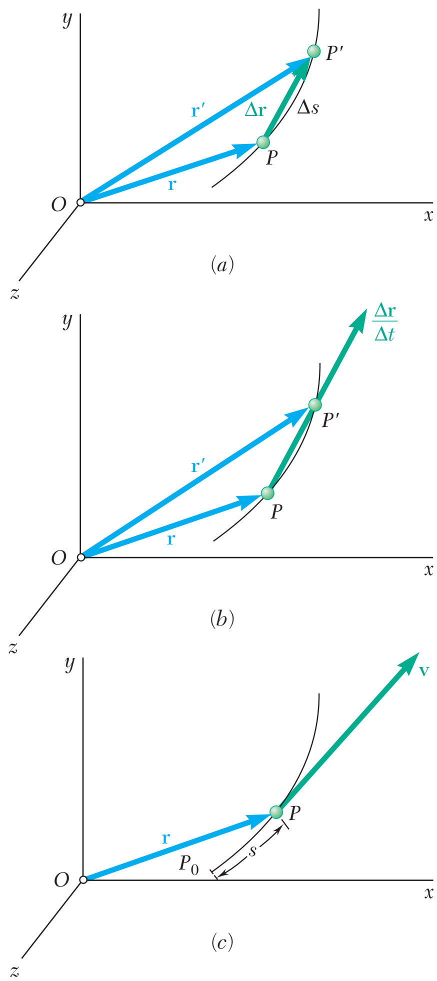
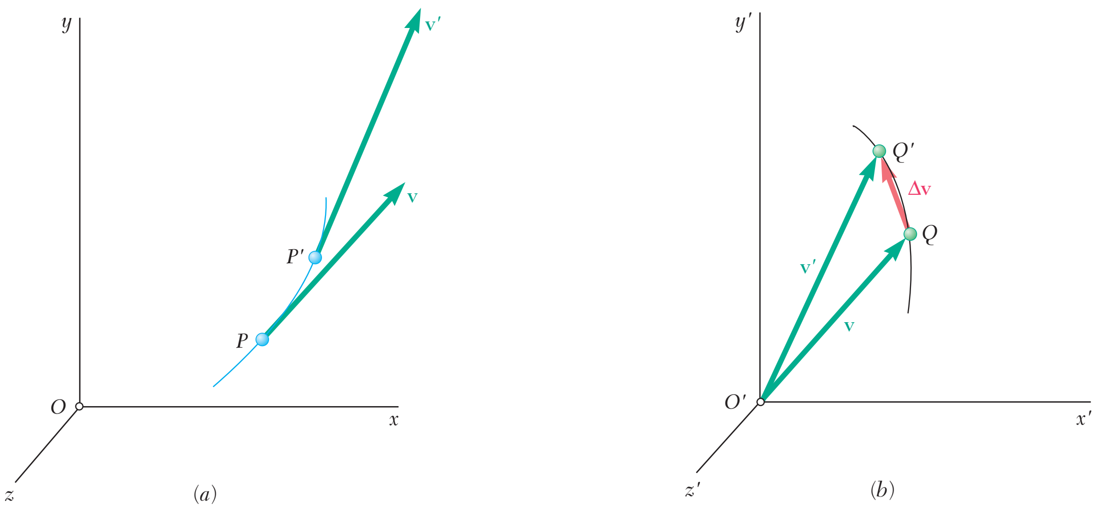
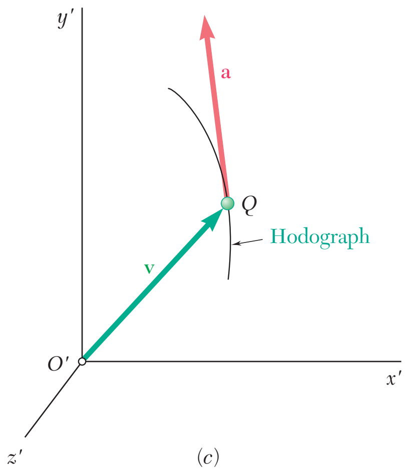


Fig. 11.12 (*a*) Position vectors for a particle moving along a curve from P to P'; (*b*) the average velocity vector is the quotient of the change in position to the elapsed time interval; (*c*) the instantaneous velocity vector is tangent to the particle's path.





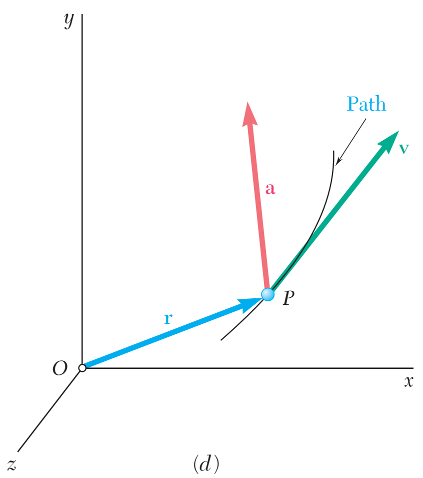


Fig. 11.13 (*a*) Velocities *v* and *v′* of a particle at two different times; (*b*) the vector change in the particle's velocity during the time interval; (*c*) the instantaneous acceleration vector is tangent to the hodograph; (*d*) in general, the acceleration vector is not tangent to the particle's path.

which is represented by the straight-line segment PP', for the vector $\Delta\mathbf{r}$ in formula (11.13). However, the length of segment PP' approaches the length Δs of arc PP' as Δt decreases (Fig. 11.12*a*). Therefore, we can write

$$v = \lim_{\Delta t \to 0} \frac{PP'}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \qquad v = \frac{ds}{dt} \tag{11.15}$$

Thus, we obtain the speed v by finding the length s of the arc described by the particle and differentiating it with respect to t.

Now let's consider the velocity $\mathbf{v}$ of the particle at time t and its velocity $\mathbf{v}'$ at a later time $t + \Delta t$ (Fig. 11.13*a*). Let us draw both vectors $\mathbf{v}$ and $\mathbf{v}'$ from the same origin O' (Fig. 11.13*b*). The vector $\Delta\mathbf{v}$ joining Q and Q' represents the change in the velocity of the particle during the time interval Δt, since we can obtain the vector $\mathbf{v}'$ by adding the vectors $\mathbf{v}$ and $\Delta\mathbf{v}$. Again, note that $\Delta\mathbf{v}$ represents a change in the *direction* of the velocity as well as a change in *speed*. We define the **average acceleration** of the particle over the time interval Δt as the quotient of $\Delta\mathbf{v}$ and Δt. Since $\Delta\mathbf{v}$ is a vector and Δt is a scalar, the quotient $\Delta\mathbf{v}/\Delta t$ is a vector in the same direction as $\Delta\mathbf{v}$.

We obtain the **instantaneous acceleration** of the particle at time t by choosing increasingly smaller values for Δt and $\Delta\mathbf{v}$. The instantaneous acceleration is thus represented by the vector

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}}{\Delta t} \tag{11.16}$$

Noting that the velocity $\mathbf{v}$ is a vector function $\mathbf{v}(t)$ of the time t, we can refer to the limit of the quotient $\Delta\mathbf{v}/\Delta t$ as the derivative of $\mathbf{v}$ with respect to t. We have

Acceleration vector

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} \tag{11.17}$$

Observe that the acceleration **a** is tangent to the curve described by the tip Q of the vector **v** when we draw **v** from a fixed origin O' (Fig. 11.13*c*). However, in general, the acceleration is *not* tangent to the path of the particle (Fig. 11.13*d*). The curve described by the tip of **v** and shown in Fig. 11.13*c* is called the *hodograph* of the motion.

11.4B Derivatives of Vector Functions

We have just seen that we can represent the velocity **v** of a particle in curvilinear motion by the derivative of the vector function $\mathbf{r}(t)$ characterizing the position of the particle. Similarly, we can represent the acceleration **a** of the particle by the derivative of the vector function $\mathbf{v}(t)$. Here we give a formal definition of the derivative of a vector function and establish a few rules governing the differentiation of sums and products of vector functions.

Let $\mathbf{P}(u)$ be a vector function of the scalar variable u. By that, we mean that the scalar u completely defines the magnitude and direction of the vector **P**. If the vector **P** is drawn from a fixed origin O and the scalar u is allowed to vary, the tip of **P** describes a given curve in space. Consider the vectors **P** corresponding, respectively, to the values u and $u + \Delta u$ of the scalar variable (Fig. 11.14*a*). Let $\Delta\mathbf{P}$ be the vector joining the tips of the two given vectors. Then we have

$$\Delta\mathbf{P} = \mathbf{P}(u + \Delta u) - \mathbf{P}(u)$$

Dividing through by Δu and letting Δu approach zero, we define the derivative of the vector function $\mathbf{P}(u)$ as

$$\frac{d\mathbf{P}}{du} = \lim_{\Delta u \to 0} \frac{\Delta\mathbf{P}}{\Delta u} = \lim_{\Delta u \to 0} \frac{\mathbf{P}(u + \Delta u) - \mathbf{P}(u)}{\Delta u} \tag{11.18}$$

As Δu approaches zero, the line of action of $\Delta\mathbf{P}$ becomes tangent to the curve of Fig. 11.14*a*. Thus, the derivative $d\mathbf{P}/du$ of the vector function $\mathbf{P}(u)$ *is tangent to the curve described by the tip of* $\mathbf{P}(u)$ (Fig. 11.14*b*).

The standard rules for the differentiation of the sums and products of scalar functions extend to vector functions. Consider first the **sum of two vector functions** $\mathbf{P}(u)$ and $\mathbf{Q}(u)$ of the same scalar variable u. According to the definition given in Eq. (11.18), the derivative of the vector $\mathbf{P} + \mathbf{Q}$ is

$$\frac{d(\mathbf{P} + \mathbf{Q})}{du} = \lim_{\Delta u \to 0} \frac{\Delta(\mathbf{P} + \mathbf{Q})}{\Delta u} = \lim_{\Delta u \to 0} \left(\frac{\Delta\mathbf{P}}{\Delta u} + \frac{\Delta\mathbf{Q}}{\Delta u} \right)$$

or since the limit of a sum is equal to the sum of the limits of its terms,

$$\frac{d(\mathbf{P} + \mathbf{Q})}{du} = \lim_{\Delta u \to 0} \frac{\Delta\mathbf{P}}{\Delta u} + \lim_{\Delta u \to 0} \frac{\Delta\mathbf{Q}}{\Delta u}$$

$$\frac{d(\mathbf{P} + \mathbf{Q})}{du} = \frac{d\mathbf{P}}{du} + \frac{d\mathbf{Q}}{du} \tag{11.19}$$

That is, the derivative of a sum of vector functions equals the sum of the derivative of each function separately.

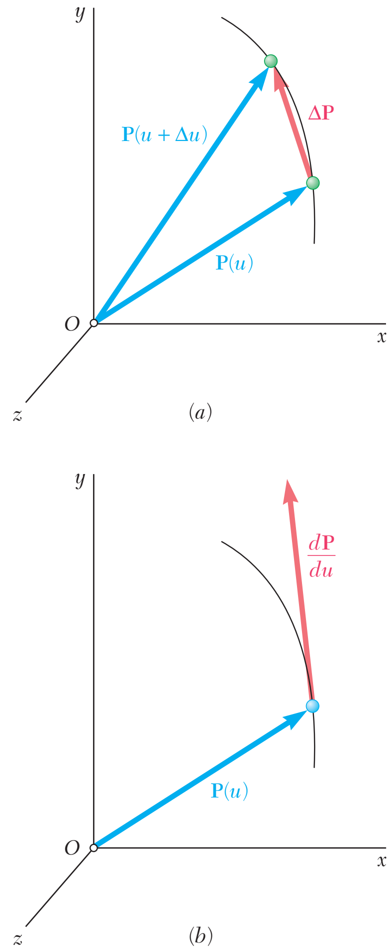


Fig. 11.14 (*a*) The change in vector function for a particle moving along a curvilinear path; (*b*) the derivative of the vector function is tangent to the path described by the tip of the function.

We now consider the **product of a scalar function** $f(u)$ **and a vector function** $\mathbf{P}(u)$ of the same scalar variable u. The derivative of the vector $f\mathbf{P}$ is

$$\frac{d(f\mathbf{P})}{du} = \lim_{\Delta u \to 0} \frac{(f + \Delta f)(\mathbf{P} + \Delta\mathbf{P}) - f\mathbf{P}}{\Delta u} = \lim_{\Delta u \to 0} \left(\frac{\Delta f}{\Delta u}\mathbf{P} + f\frac{\Delta\mathbf{P}}{\Delta u} \right)$$

or recalling the properties of the limits of sums and products,

$$\frac{d(f\mathbf{P})}{du} = \frac{df}{du}\mathbf{P} + f\frac{d\mathbf{P}}{du} \tag{11.20}$$

In a similar way, we can obtain the derivatives of the **scalar product** and the **vector product** of two vector functions $\mathbf{P}(u)$ and $\mathbf{Q}(u)$. Thus,

$$\frac{d(\mathbf{P} \cdot \mathbf{Q})}{du} = \frac{d\mathbf{P}}{du} \cdot \mathbf{Q} + \mathbf{P} \cdot \frac{d\mathbf{Q}}{du} \tag{11.21}$$

$$\frac{d(\mathbf{P} \times \mathbf{Q})}{du} = \frac{d\mathbf{P}}{du} \times \mathbf{Q} + \mathbf{P} \times \frac{d\mathbf{Q}}{du} \tag{11.22}$$ †

We can use the properties just established to determine the **rectangular components of the derivative of a vector function** $\mathbf{P}(u)$. Resolving $\mathbf{P}$ into components along fixed rectangular axes x, y, and z, we have

$$\mathbf{P} = P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k} \tag{11.23}$$

where P_x, P_y, and P_z are the rectangular scalar components of the vector $\mathbf{P}$, and $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the unit vectors corresponding, respectively, to the x, y, and z axes (Sec. 2.12 or Appendix A). From Eq. (11.19), the derivative of $\mathbf{P}$ is equal to the sum of the derivatives of the terms in the right-hand side. Since each of these terms is the product of a scalar and a vector function, we should use Eq. (11.20). However, the unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ have a constant magnitude (equal to 1) and fixed directions. Their derivatives are therefore zero, and we obtain

$$\frac{d\mathbf{P}}{du} = \frac{dP_x}{du}\mathbf{i} + \frac{dP_y}{du}\mathbf{j} + \frac{dP_z}{du}\mathbf{k} \tag{11.24}$$

Note that the coefficients of the unit vectors are, by definition, the scalar components of the vector $d\mathbf{P}/du$. We conclude that we can obtain the rectangular scalar components of the derivative $d\mathbf{P}/du$ of the vector function $\mathbf{P}(u)$ by differentiating the corresponding scalar components of $\mathbf{P}$.

Rate of Change of a Vector. When the vector $\mathbf{P}$ is a function of the time t, its derivative $d\mathbf{P}/dt$ represents the **rate of change** of $\mathbf{P}$ with respect to the frame $Oxyz$. Resolving $\mathbf{P}$ into rectangular components and using Eq. (11.24), we have

$$\frac{d\mathbf{P}}{dt} = \frac{dP_x}{dt}\mathbf{i} + \frac{dP_y}{dt}\mathbf{j} + \frac{dP_z}{dt}\mathbf{k}$$

†Since the vector product is not commutative (see Sec. 3.4), the order of the factors in Eq. (11.22) must be maintained.

Alternatively, using dots to indicate differentiation with respect to t gives

$$\dot{\mathbf{P}} = \dot{P}_x\mathbf{i} + \dot{P}_y\mathbf{j} + \dot{P}_z\mathbf{k} \qquad \textbf{(11.24}'\textbf{)}$$

As you will see in Sec. 15.5, the rate of change of a vector as observed from a *moving frame of reference* is, in general, different from its rate of change as observed from a fixed frame of reference. However, if the moving frame $O'x'y'z'$ is in *translation*, i.e., if its axes remain parallel to the corresponding axes of the fixed frame $Oxyz$ (Fig. 11.15), we can use the same unit vectors **i**, **j**, and **k** in both frames, and at any given instant, the vector **P** has the same components P_x, P_y, and P_z in both frames. It follows from Eq. (11.24′) that the rate of change $\dot{\mathbf{P}}$ is the same with respect to the frames $Oxyz$ and $O'x'y'z'$. Therefore,

The rate of change of a vector is the same with respect to a fixed frame and with respect to a frame in translation.

This property will greatly simplify our work, since we will be concerned mainly with frames in translation.

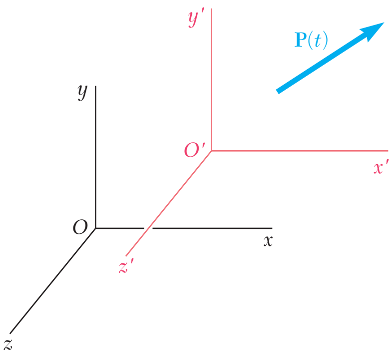


Fig. 11.15 The rate of change of a vector is the same with respect to a fixed frame of reference and with respect to a frame in translation.

11.4C Rectangular Components of Velocity and Acceleration

Suppose the position of a particle P is defined at any instant by its rectangular coordinates x, y, and z. In this case, it is often convenient to resolve the velocity **v** and the acceleration **a** of the particle into rectangular components (Fig. 11.16).

To resolve the position vector **r** of the particle into rectangular components, we write

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \qquad \textbf{(11.25)}$$

Here the coordinates x, y, and z are functions of t. Differentiating twice, we obtain

Velocity and acceleration in rectangular components

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k} \qquad \textbf{(11.26)}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j} + \ddot{z}\mathbf{k} \qquad \textbf{(11.27)}$$

where $\dot{x}$, $\dot{y}$, and $\dot{z}$ and $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$ represent, respectively, the first and second derivatives of x, y, and z with respect to t. It follows from Eqs. (11.26) and (11.27) that the scalar components of the velocity and acceleration are

$$v_x = \dot{x} \qquad v_y = \dot{y} \qquad v_z = \dot{z} \qquad \textbf{(11.28)}$$

$$a_x = \ddot{x} \qquad a_y = \ddot{y} \qquad a_z = \ddot{z} \qquad \textbf{(11.29)}$$

A positive value for v_x indicates that the vector component $\mathbf{v}_x$ is directed to the right, and a negative value indicates that it is directed to the left. The sense of each of the other vector components is determined in a similar way from the sign of the corresponding scalar component. If desired, we can obtain the magnitudes and directions of the velocity and acceleration from their scalar components using the methods of Secs. 2.2A and 2.4A (or Appendix A).

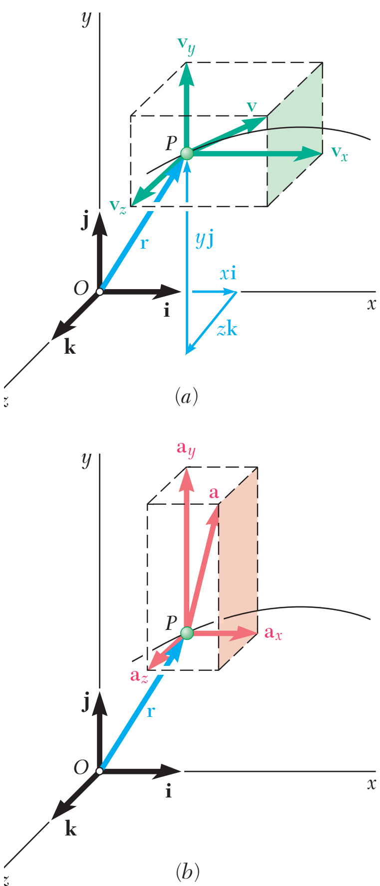
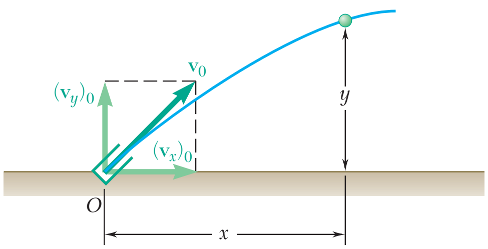
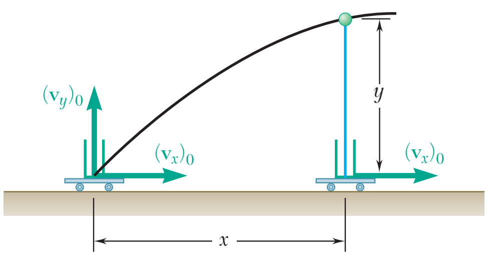


Fig. 11.16 (*a*) Rectangular components of position and velocity for a particle *P*; (*b*) rectangular components of acceleration for particle *P*.

Photo 11.3 The motion of this snowboarder in the air is a parabola, assuming we can neglect air resistance.



(*a*) Motion of a projectile



(*b*) Equivalent rectilinear motions

Fig. 11.17 The motion of a projectile (*a*) consists of uniform horizontal motion and uniformly accelerated vertical motion and (*b*) is equivalent to two independent rectilinear motions.

The use of rectangular components to describe the position, velocity, and acceleration of a particle is particularly effective when the component a_x of the acceleration depends only upon t, x, and/or v_x, and similarly when a_y depends only upon t, y, and/or v_y, and when a_z depends upon t, z, and/or v_z. In this case, we can integrate Equations (11.28) and (11.29) independently. In other words, the motion of the particle in the x direction, its motion in the y direction, and its motion in the z direction can be treated separately.

In the case of the **motion of a projectile**, we can show (see Sec. 12.1D) that the components of the acceleration are

$$a_x = \ddot{x} = 0 \qquad a_y = \ddot{y} = -g \qquad a_z = \ddot{z} = 0$$

if the resistance of the air is neglected. Denoting the coordinates of a gun by x_0, y_0, and z_0 and the components of the initial velocity $\mathbf{v}_0$ of the projectile by $(v_x)_0$, $(v_y)_0$, and $(v_z)_0$, we can integrate twice in t and obtain

$$v_x = \dot{x} = (v_x)_0 \qquad v_y = \dot{y} = (v_y)_0 - gt \qquad v_z = \dot{z} = (v_z)_0$$
$$x = x_0 + (v_x)_0 t \qquad y = y_0 + (v_y)_0 t - \tfrac{1}{2}gt^2 \qquad z = z_0 + (v_z)_0 t$$

If the projectile is fired in the xy plane from the origin O, we have $x_0 = y_0 = z_0 = 0$ and $(v_z)_0 = 0$, so the equations of motion reduce to

$$v_x = (v_x)_0 \qquad v_y = (v_y)_0 - gt \qquad v_z = 0$$
$$x = (v_x)_0 t \qquad y = (v_y)_0 t - \tfrac{1}{2}gt^2 \qquad z = 0$$

These equations show that the projectile remains in the xy plane, that its motion in the horizontal direction is uniform, and that its motion in the vertical direction is uniformly accelerated. Thus, we can replace the motion of a projectile by two independent rectilinear motions, which are easily visualized if we assume that the projectile is fired vertically with an initial velocity $(\mathbf{v}_y)_0$ from a platform moving with a constant horizontal velocity $(\mathbf{v}_x)_0$ (Fig. 11.17). The coordinate x of the projectile is equal at any instant to the distance traveled by the platform, and we can compute its coordinate y as if the projectile were moving along a vertical line. Additionally, because the $(\mathbf{v}_x)_0$ values are the same, the projectile will land on the platform regardless of the value of $(\mathbf{v}_y)_0$.

Note that the equations defining the coordinates x and y of a projectile at any instant are the parametric equations of a parabola. Thus, the trajectory of a projectile is *parabolic*. This result, however, ceases to be valid if we take into account the resistance of the air or the variation with altitude of the acceleration due to gravity.

11.4D Motion Relative to a Frame in Translation

We have just seen how to describe the motion of a particle by using a single frame of reference. In most cases, this frame was attached to the earth and was considered to be fixed. Now we want to analyze situations in which it is convenient to use several frames of reference simultaneously. If one of the frames is attached to the earth, it is called a **fixed frame of reference**, and the other frames are referred to as **moving frames of reference**. You should recognize, however, that the selection of a fixed frame of reference is purely arbitrary. Any frame can be designated as "fixed"; all other frames not rigidly attached to this frame are then described as "moving."

Consider two particles A and B moving in space (Fig. 11.18). The vectors $\mathbf{r}_A$ and $\mathbf{r}_B$ define their positions at any given instant with respect to the fixed frame of reference $Oxyz$. Consider now a system of axes x', y', *and* z' centered at A and parallel to the x, y, and z axes. Suppose that, while the origin of these axes moves, their orientation remains the same; then the frame of reference $Ax'y'z'$ is in *translation* with respect to $Oxyz$. The vector $\mathbf{r}_{B/A}$ joining A and B defines **the position of B relative to the moving frame** $Ax'y'z'$ (or for short, **the position of B relative to** A).

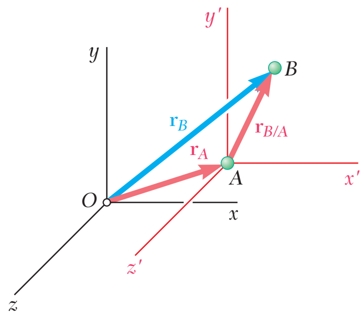


Fig. 11.18 The vector $\mathbf{r}_{B/A}$ defines the position of B with respect to moving frame A.

Figure 11.18 shows that the position vector $\mathbf{r}_B$ of particle B is the sum of the position vector $\mathbf{r}_A$ of particle A and of the position vector $\mathbf{r}_{B/A}$ of B relative to A; that is,

Relative position

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A} \qquad \textbf{(11.30)}$$

Differentiating Eq. (11.30) with respect to t within the fixed frame of reference, and using dots to indicate time derivatives, we have

$$\dot{\mathbf{r}}_B = \dot{\mathbf{r}}_A + \dot{\mathbf{r}}_{B/A} \qquad \textbf{(11.31)}$$

The derivatives $\dot{\mathbf{r}}_A$ and $\dot{\mathbf{r}}_B$ represent, respectively, the velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ of the particles A and B. Since $Ax'y'z'$ is in translation, the derivative $\dot{\mathbf{r}}_{B/A}$ represents the rate of change of $\mathbf{r}_{B/A}$ with respect to the frame $Ax'y'z'$ as well as with respect to the fixed frame (Sec. 11.4B). This derivative, therefore, defines **the velocity $\mathbf{v}_{B/A}$ of B relative to the frame** $Ax'y'z'$ (or for short, **the velocity $\mathbf{v}_{B/A}$ of B relative to** A). We have

Photo 11.4 The pilot of a helicopter landing on a moving carrier must take into account the relative motion of the ship.

Relative velocity

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \qquad \textbf{(11.32)}$$

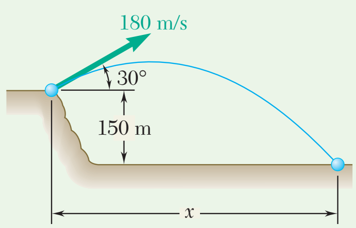
Differentiating Eq. (11.32) with respect to t, and using the derivative $\dot{\mathbf{v}}_{B/A}$ to define **the acceleration $\mathbf{a}_{B/A}$** of B **relative to the frame** $Ax'y'z'$ (or for short, **the acceleration $\mathbf{a}_{B/A}$ of B relative to** A), we obtain

Relative acceleration

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} \qquad \textbf{(11.33)}$$

We refer to the motion of B with respect to the fixed frame $Oxyz$ as the **absolute motion of** B. The equations derived in this section show that **we can obtain the absolute motion of B by combining the motion of A and the relative motion of B with respect to the moving frame attached to** A. Equation (11.32), for example, expresses that the absolute velocity $\mathbf{v}_B$ of particle B can be obtained by vectorially adding the velocity of A and the velocity of B relative to the frame $Ax'y'z'$. Equation (11.33) expresses a similar property in terms of the accelerations. (Note that the product of the subscripts A and B/A used in the right-hand sides of Eqs. (11.30) through (11.33) is equal to the subscript B used in their left-hand sides.) Keep in mind, however, that the frame $Ax'y'z'$ *is* in *translation;* that is, while it moves with A, it maintains the same orientation. As you will see later (Sec. 15.7), you must use different relations in the case of a rotating frame of reference.

Sample Problem 11.10



A projectile is fired from the edge of a 150-m cliff with an initial velocity of 180 m/s at an angle of 30° with the horizontal. Neglecting air resistance, find (*a*) the horizontal distance from the gun to the point where the projectile strikes the ground, (*b*) the greatest elevation above the ground reached by the projectile.

STRATEGY: This is a projectile motion problem, so you can consider the vertical and horizontal motions separately. First determine the equations governing each direction, and then use them to find the distances.

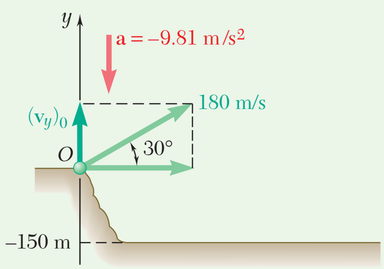


Fig. 1 Acceleration and initial velocity of the projectile in the *y*-direction.

MODELING and ANALYSIS: Model the projectile as a particle and neglect the effects of air resistance. The vertical motion has a constant acceleration. Choosing the positive sense of the y axis upward and placing the origin O at the gun (Fig. 1), you have

$$(v_y)_0 = (180 \text{ m/s}) \sin 30° = +90 \text{ m/s}$$
$$a = -9.81 \text{ m/s}^2$$

Substitute these values into the equations for motion with constant acceleration. Thus,

$$v_y = (v_y)_0 + at \qquad v_y = 90 - 9.81t \qquad \textbf{(1)}$$
$$y = (v_y)_0 t + \tfrac{1}{2}at^2 \qquad y = 90t - 4.90t^2 \qquad \textbf{(2)}$$
$$v_y^2 = (v_y)_0^2 + 2ay \qquad v_y^2 = 8100 - 19.62y \qquad \textbf{(3)}$$

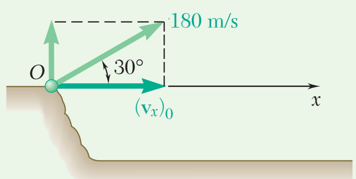


Fig. 2 Initial velocity of the projectile in the *x*-direction.

The horizontal motion has zero acceleration. Choose the positive sense of the x axis to the right (Fig. 2), which gives you

$$(v_x)_0 = (180 \text{ m/s}) \cos 30° = +155.9 \text{ m/s}$$

Substituting into the equation for constant acceleration, you obtain

$$x = (v_x)_0 t \qquad x = 155.9t \qquad \textbf{(4)}$$

a. Horizontal Distance. When the projectile strikes the ground,

$$y = -150 \text{ m}$$

Substituting this value into Eq. (2) for the vertical motion, you have

$$-150 = 90t - 4.90t^2 \qquad t^2 - 18.37t - 30.6 = 0 \qquad t = 19.91 \text{ s}$$

Substituting $t = 19.91$ s into Eq. (4) for the horizontal motion, you obtain

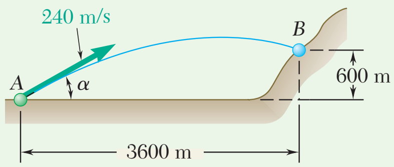
$$x = 155.9(19.91) \qquad x = 3100 \text{ m} \blacktriangleleft$$

b. Greatest Elevation. When the projectile reaches its greatest elevation, $v_y = 0$; substituting this value into Eq. (3) for the vertical motion, you have

$$0 = 8100 - 19.62y \qquad y = 413 \text{ m}$$
$$\text{Greatest elevation above ground} = 150 \text{ m} + 413 \text{ m} = 563 \text{ m} \blacktriangleleft$$

REFLECT and THINK: Because there is no air resistance, you can treat the vertical and horizontal motions separately and can immediately write down the algebraic equations of motion. If you did want to include air resistance, you must know the acceleration as a function of the speed (you will see how to derive this in Chapter 12), and then you need to use the basic kinematic relationships, separate variables, and integrate.

Sample Problem 11.11



A projectile is fired with an initial velocity of 240 m/s at a target B located 600 m above the gun A and at a horizontal distance of 3,600 m. Neglecting air resistance, determine the value of the firing angle α needed to hit the target.

STRATEGY: This is a projectile motion problem, so you can consider the vertical and horizontal motions separately. First determine the equations governing the motion in each direction, and then use them to find the firing angle.

MODELING and ANALYSIS:

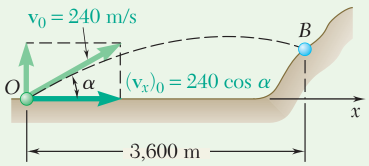


Fig. 1 Initial velocity of the projectile in the *x*-direction.

Horizontal Motion. Place the origin of the coordinate axes at the gun (Fig. 1). Then

$$(v_x)_0 = 240 \cos \alpha$$

Substituting into the equation of uniform horizontal motion, you obtain

$$x = (v_x)_0 t \qquad x = (240 \cos \alpha)t$$

Obtain the time required for the projectile to move through a horizontal distance of 3,600 m by setting x equal to 3,600 m.

$$3600 = (240 \cos \alpha)t$$

$$t = \frac{3{,}600}{240 \cos \alpha} = \frac{15}{\cos \alpha}$$

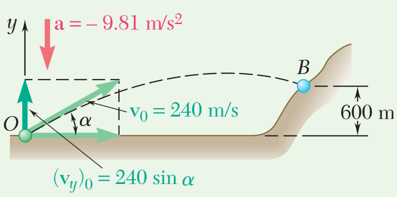


Fig. 2 Acceleration and initial velocity of the projectile in the *y*-direction.

Vertical Motion. Again, place the origin at the gun (Fig. 2).

$$(v_y)_0 = 240 \sin \alpha \qquad a = -9.81 \text{ m/s}^2$$

Substituting into the equation for constant acceleration in the vertical direction, you obtain

$$y = (v_y)_0 t + \tfrac{1}{2}at^2 \qquad y = (240 \sin \alpha)t - 4.905t^2$$

Projectile Hits Target. When $x = 3600$ m, you want $y = 600$ m. Substituting for y and setting t equal to the value found previously, you have

$$600 = 240 \sin \alpha \frac{15}{\cos \alpha} - 4.905\left(\frac{15}{\cos \alpha}\right)^2 \qquad \textbf{(1)}$$

Since $1/\cos^2 \alpha = \sec^2 \alpha = 1 + \tan^2 \alpha$, we have

$$600 = 240(15) \tan \alpha - 4.905(15^2)(1 + \tan^2 \alpha)$$
$$1104 \tan^2 \alpha - 3600 \tan \alpha + 1704 = 0$$

Solving this quadratic equation for $\tan \alpha$, we have

$$\tan \alpha = 0.575 \qquad \text{and} \qquad \tan \alpha = 2.69$$
$$\alpha = 29.9° \qquad \text{and} \qquad \alpha = 69.6° \quad ◀$$

The target will be hit if either of these two firing angles is used (Fig. 3).

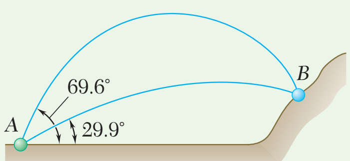
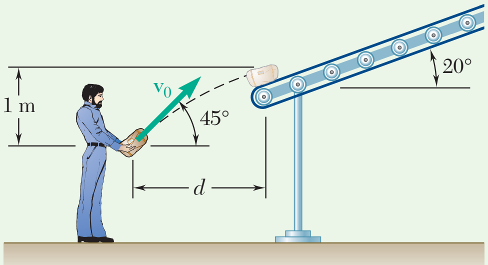


Fig. 3 Firing angles that will hit target *B*.

REFLECT and THINK: It is a well-known characteristic of projectile motion that you can hit the same target by using either of two firing angles. We used trigonometry to write the equation in terms of $\tan \alpha$, but most calculators or computer programs like Maple, Matlab, or Mathematica also can be used to solve (1) for α. You must be careful when using these tools, however, to make sure that you find both angles.

Sample Problem 11.12



A conveyor belt at an angle of 20° with the horizontal is used to transfer small packages to other parts of an industrial plant. A worker tosses a package with an initial velocity $\mathbf{v}_0$ at an angle of 45° so that its velocity is parallel to the belt as it lands 1 m above the release point. Determine (*a*) the magnitude of v_0, (*b*) the horizontal distance d.

STRATEGY: This is a projectile motion problem, so you can consider the vertical and the horizontal motions separately. First determine the equations governing the motion in each direction, then use them to determine the unknown quantities.

MODELING and ANALYSIS:

Horizontal Motion. Placing the axes of your origin at the location where the package leaves the workers hands (Fig. 1), you can write

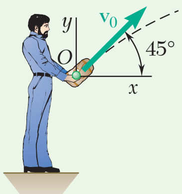


Fig. 1 Initial velocity of the package.

Horizontal: $v_x = v_0 \cos 45°$ and $x = (v_0 \cos 45°)t$

Vertical: $v_y = v_0 \sin 45° - gt$ and $y = (v_0 \sin 45°)t - \frac{1}{2}gt^2$

Landing on the Belt. The problem statement indicates that when the package lands on the belt, its velocity vector will be in the same direction as the belt is moving. If this happens when $t = t_1$, you can write

$$\frac{v_y}{v_x} = \tan 20° = \frac{v_0 \sin 45° - gt_1}{v_0 \cos 45°} = 1 - \frac{gt_1}{v_0 \cos 45°} \qquad \textbf{(1)}$$

This equation has two unknown quantities: t_1 and v_0. Therefore, you need more equations. Substituting $t = t_1$ into the remaining projectile motion equations gives

$$d = (v_0 \cos 45°)t \qquad \textbf{(2)}$$

$$1 \text{ m} = (v_0 \sin 45°)t_1 - \frac{1}{2}gt_1^2 \qquad \textbf{(3)}$$

You now have three equations (1), (2), and (3) and three unknowns t_1, v_0, and d. Using $g = 9.81 \text{ m/s}^2$ and solving these three equations give $t_1 = 0.3083$ s and

$$v_0 = 6.73 \text{ m/s} \quad ◀$$

$$d = 1.466 \text{ m} \quad ◀$$

REFLECT and THINK: All of these projectile problems are similar. You write down the governing equations for motion in the horizontal and vertical directions and then use additional information in the problem statement to solve the problem. In this case, the distance is just less than 1.5 meters, which is a reasonable distance for a worker to toss a package.

Sample Problem 11.13

Airplane B, which is travelling at a constant 560 km/h, is pursuing airplane A, which is travelling northeast at a constant 800 km/hr. At time $t = 0$, airplane A is 640 km east of airplane B. Determine (a) the direction of the course airplane B should follow (measured from the east) to intercept plane A, (b) the rate at which the distance between the airplanes is decreasing, (c) how long it takes for airplane B to catch airplane A.

STRATEGY: To find when B intercepts A, you just need to find out when the two planes are at the same location. The rate at which the distance is decreasing is the magnitude of $v_{B/A}$, so you can use the relative velocity equation.

MODELING and ANALYSIS: Choose x to be east, y to be north, and place the origin of your coordinate system at B (Fig. 1).

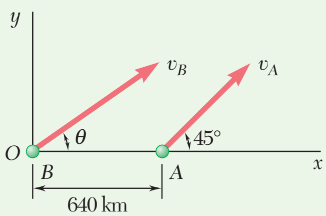


Fig. 1 Initial velocity of airplanes A and B.

Positions of the Planes: You know that each plane has a constant speed, so you can write a position vector for each plane. Thus,

$$\mathbf{r}_A = [(v_A \cos 45°)t + 640 \text{ km}]\mathbf{i} + [(v_A \sin 45°)t]\mathbf{j} \quad (1)$$
$$\mathbf{r}_B = [(v_B \cos \theta)t]\mathbf{i} + [(v_B \sin \theta)t]\mathbf{j} \quad (2)$$

a. Direction of *B*. Plane B will catch up when they are at the same location, that is, $\mathbf{r}_A = \mathbf{r}_B$. You can equate components in the $\mathbf{j}$ direction to find

$$v_A \sin 45° t_1 = v_B \sin \theta\, t_1$$

After you substitute in values,

$$\sin \theta = \frac{(v_A \sin 45°)t_1}{v_B t_1} = \frac{(560 \text{ km/hr})\sin 45°}{800 \text{ km/hr}} = 0.4950$$
$$\theta = \sin^{-1} 0.4950 = 29.67° \qquad \theta = 29.7° \blacktriangleleft$$

b. Rate. The rate at which the distance is decreasing is the magnitude of $\mathbf{v}_{B/A}$, so

$$\begin{aligned}\mathbf{v}_{B/A} &= \mathbf{v}_B - \mathbf{v}_A = (v_B \cos \theta\, \mathbf{i} + v_B \sin \theta\, \mathbf{j}) - (v_A \cos 45°\, \mathbf{i} + v_A \sin 45°\, \mathbf{j})\\ &= [(800 \text{ km/h})\cos 29.668° - (560 \text{ km/h})\cos 45°]\mathbf{i}\\ &\quad + [(800 \text{ km/h})\sin 29.668° - (560 \text{ km/h})\sin 45°]\mathbf{j}\\ &= 299.15 \text{ km/h}\, \mathbf{i} \end{aligned} \qquad |\mathbf{v}_{B/A}| = 299 \text{ km/h} \blacktriangleleft$$

c. Time for *B* to catch up with *A*. To find the time, you equate the $\mathbf{i}$ components of each position vector, giving

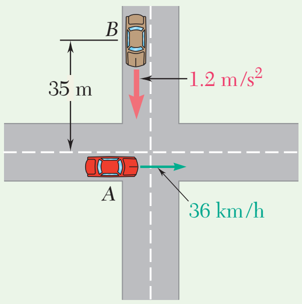
$$(v_A \cos 45°)t_1 + 640 \text{ km} = (v_B \cos \theta)t_1$$

Solve this for t_1. Thus,

$$t_1 = \frac{640 \text{ km}}{v_B \cos \theta - v_A \cos 45°}$$
$$= \frac{640 \text{ km}}{(800 \text{ km/h})\cos 29.67° - (560 \text{ km/h})\cos 45°} = 2.139 \text{ h}$$
$$t_1 = 2.14 \text{ h} \blacktriangleleft$$

REFLECT and THINK: The relative velocity is only in the horizontal (eastern) direction. This makes sense, because the vertical (northern) components have to be equal in order for the two planes to intersect.

Sample Problem 11.14



Automobile A is traveling east at the constant speed of 36 km/h. As automobile A crosses the intersection shown, automobile B starts from rest 35 m north of the intersection and moves south with a constant acceleration of 1.2 m/s^2. Determine the position, velocity, and acceleration of B relative to A 5 s after A crosses the intersection.

STRATEGY: This is a relative motion problem. Determine the motion of each vehicle independently, and then use the definition of relative motion to determine the desired quantities.

MODELING and ANALYSIS:

Motion of Automobile *A*. Choose x and y axes with the origin at the intersection of the two streets and with positive senses directed east and north, respectively. First express the speed in m/s, as

$$v_A = \left(36\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 10\text{ m/s}$$

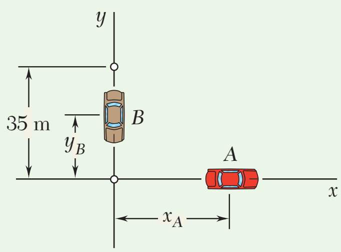


Fig. 1 Initial positions of car A and B.

The motion of A is uniform, so for any time t

$$a_A = 0$$
$$v_A = +10\text{ m/s}$$
$$x_A = (x_A)_0 + v_A t = 0 + 10t$$

For $t = 5$ s, you have (Fig. 1)

$$a_A = 0 \qquad \mathbf{a}_A = 0$$
$$v_A = +10\text{ m/s} \qquad \mathbf{v}_A = 10\text{ m/s} \rightarrow$$
$$x_A = +(10\text{ m/s})(5\text{ s}) = +50\text{ m} \qquad \mathbf{r}_A = 50\text{ m} \rightarrow$$

Motion of Automobile *B*. The motion of B is uniformly accelerated, so

$$a_B = -1.2\text{ m/s}^2$$
$$v_B = (v_B)_0 + at = 0 - 1.2t$$
$$y_B = (y_B)_0 + (v_B)_0 t + \tfrac{1}{2}a_B t^2 = 35 + 0 - \tfrac{1}{2}(1.2)t^2$$

For $t = 5$ s, you have (Fig. 1)

$$a_B = -1.2\text{ m/s}^2 \qquad \mathbf{a}_B = 1.2\text{ m/s}^2 \downarrow$$
$$v_B = -(1.2\text{ m/s}^2)(5\text{ s}) = -6\text{ m/s} \qquad \mathbf{v}_B = 6\text{ m/s} \downarrow$$
$$y_B = 35 - \tfrac{1}{2}(1.2\text{ m/s}^2)(5\text{ s})^2 = +20\text{ m} \qquad \mathbf{r}_B = 20\text{ m} \uparrow$$

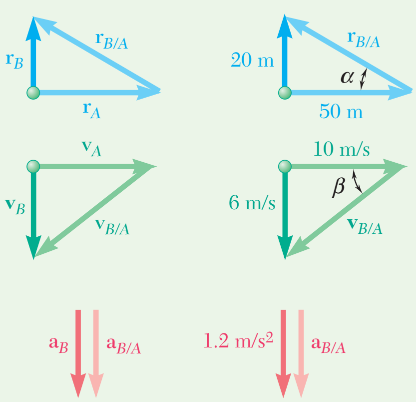


Fig. 2 Vector triangles for position, velocity, and acceleration.

Motion of *B* Relative to *A*. Draw the triangle corresponding to the vector equation $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$ (Fig. 2) and obtain the magnitude and direction of the position vector of B relative to A.

$$r_{B/A} = 53.9\text{ m} \qquad \alpha = 21.8° \qquad \mathbf{r}_{B/A} = 53.9\text{ m} \nwarrow 21.8° \quad \blacktriangleleft$$

Proceeding in a similar fashion (Fig. 2), find the velocity and acceleration of B relative to A. Hence,

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$
$$v_{B/A} = 11.66\text{ m/s} \qquad \beta = 31.0° \qquad \mathbf{v}_{B/A} = 11.66\text{ m/s} \swarrow 31.0° \quad \blacktriangleleft$$
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} \qquad \mathbf{a}_{B/A} = 1.2\text{ m/s}^2 \downarrow \quad \blacktriangleleft$$

REFLECT and THINK: Note that the relative position and velocity of B relative to A change with time; the values given here are only for the moment $t = 5$ s. Rather than drawing triangles, you could have also used vector algebra. When the vectors are at right angles, as in this problem, drawing vector triangles is usually easiest.

Sample Problem 11.15

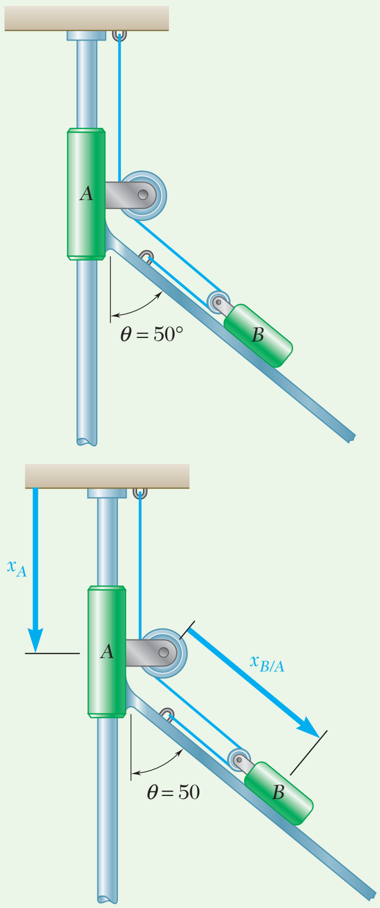


Fig. 1 Position vectors to A and B.

Knowing that at the instant shown cylinder/ramp A has a velocity of 200 mm/s directed down, determine the velocity of block B.

STRATEGY: You have objects connected by cables, so this is a dependent-motion problem. You should define coordinates for each block-object and write a constraint equation for the cable. You will also need to use relative motion, since B slides on A.

MODELING and ANALYSIS: Define position vectors, as shown in Fig. 1.

Constraint Equations. Assuming the cable is inextensible, you can write the length in terms of the coordinates and then differentiate.

The constraint equation for the cable is

$$x_A + 2x_{B/A} = \text{constant}$$

Differentiating this gives

$$v_A = -2v_{B/A} \quad \textbf{(1)}$$

Substituting for v_A gives $v_{B/A} = -100$ mm/s or 100 mm/s up the incline.

Dependent Motion: You know that the direction of $v_{B/A}$ is directed up the incline. Therefore, the relative motion equation relating the velocities of blocks A and B is $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$. You could either draw a vector triangle or use vector algebra. Let's use vector algebra. Using the coordinate system shown in Fig. 2 and substituting in the magnitudes gives

$$(v_B)_x\,\mathbf{i} + (v_B)_y\,\mathbf{j} = (-200 \text{ mm/s})\mathbf{j} + (-100 \text{ mm/s}) \sin 50°\,\mathbf{i} + (100 \text{ mm/s}) \cos 50°\,\mathbf{j}$$

Equating components gives

$$\mathbf{i}\text{: } (v_B)_x = -(100 \text{ mm/s})\sin 50° \quad \rightarrow \quad v_{B_x} = -76.6 \text{ mm/s}$$

$$\mathbf{j}\text{: } (v_B)_y = (-200 \text{ mm/s}) + (100 \text{ mm/s})\cos 50° \quad \rightarrow \quad v_{B_y} = -135.7 \text{ mm/s}$$

Finding the magnitude and direction gives

$$\mathbf{v}_B = 155.8 \text{ mm/s} \measuredangle 60.6° \blacktriangleleft$$

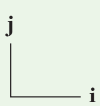


Fig. 2 Coordinates for vector algebra.

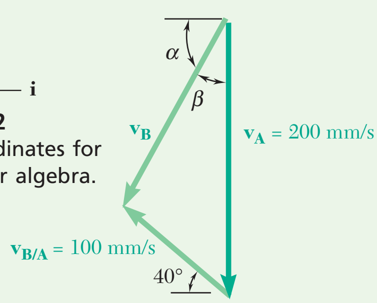


Fig. 3 Vector triangle for velocity of blocks A and B.

REFLECT and THINK: Rather than using vector algebra, you could have also drawn a vector triangle, as shown in Fig. 3. To use this vector triangle, you need to use the law of cosines and the law of sines. Looking at the mechanism, block B should move up the incline if block A moves downward; our mathematical result is consistent with this. It is also interesting to note that, even though B moves up the incline relative to A, block B is actually moving down and to the left, as shown in the calculation here. This occurs because block A is also moving down.

SOLVING PROBLEMS ON YOUR OWN

In the problems for this section, you will analyze the **curvilinear motion** of a particle. The physical interpretations of velocity and acceleration are the same as in the first sections of the chapter, but you should remember that these quantities are vectors. In addition, recall from your experience with vectors in statics that it is often advantageous to express position vectors, velocities, and accelerations in terms of their rectangular scalar components [Eqs. (11.25) through (11.27)].

A. Analyzing the motion of a projectile. Many of the following problems deal with the two-dimensional motion of a projectile where we can neglect the resistance of the air. In Sec. 11.4C, we developed the equations that describe this type of motion, and we observed that the horizontal component of the velocity remains constant (uniform motion), while the vertical component of the acceleration is constant (uniformly accelerated motion). We are able to consider the horizontal and the vertical motions of the particle separately. Assuming that the projectile is fired from the origin, we can write the two equations as

$$x = (v_x)_0 t \qquad y = (v_y)_0 t - \tfrac{1}{2} g t^2$$

1. If you know the initial velocity and firing angle, you can obtain the value of y corresponding to any given value of x (or the value of x for any value of y) by solving one of the previous equations for t and substituting for t into the other equation [Sample Prob. 11.10].

2. If you know the initial velocity and the coordinates of a point of the trajectory and you wish to determine the firing angle α, begin your solution by expressing the components $(v_x)_0$ and $(v_y)_0$ of the initial velocity as functions of α. Then substitute these expressions and the known values of x and y into the previous equations. Finally, solve the first equation for t and substitute that value of t into the second equation to obtain a trigonometric equation in α, which you can solve for that unknown [Sample Prob. 11.11].

B. Solving translational two-dimensional relative-motion problems. You saw in Sec. 11.4D that you can obtain the absolute motion of a particle *B* by combining the motion of a particle *A* and the **relative motion** of *B* with respect to a frame attached to *A* that is in *translation* [Sample Probs. 11.12 and 11.13]. You can then express the velocity and acceleration of *B* as shown in Eqs. (11.32) and (11.33), respectively.

1. To visualize the relative motion of *B* with respect to *A*, imagine that you are attached to particle *A* as you observe the motion of particle *B*. For example, to a passenger in automobile *A* of Sample Prob. 11.14, automobile *B* appears to be heading in a southwesterly direction (*south* should be obvious; *west* is due to the fact that automobile *A* is moving to the east—automobile *B* then appears to travel to the west). Note that this conclusion is consistent with the direction of $\mathbf{v}_{B/A}$.

2. To solve a relative-motion problem, first write the vector equations (11.30), (11.32), and (11.33), which relate the motions of particles *A* and *B*. You may then use either of the following methods.

a. Construct the corresponding vector triangles and solve them for the desired position vector, velocity, and acceleration [Sample Prob. 11.14].

b. Express all vectors in terms of their rectangular components and solve the resulting two independent sets of scalar equations [Sample Prob. 11.15]. If you choose this approach, be sure to select the same positive direction for the displacement, velocity, and acceleration of each particle.

Problems

CONCEPT QUESTIONS

11.CQ3 Two model rockets are fired simultaneously from a ledge and follow the trajectories shown. Neglecting air resistance, which of the rockets will hit the ground first?

a. *A*.
b. *B*.
c. They hit at the same time.
d. The answer depends on *h*.

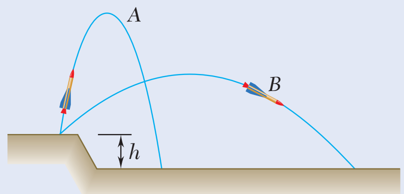


Fig. P6.CQ3

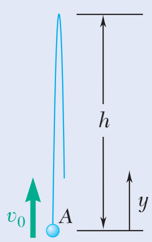


Fig. P6.CQ4

11.CQ4 Ball *A* is thrown straight up. Which of the following statements about the ball are true at the highest point in its path?

a. The velocity and acceleration are both zero.
b. The velocity is zero, but the acceleration is not zero.
c. The velocity is not zero, but the acceleration is zero.
d. Neither the velocity nor the acceleration is zero.

11.CQ5 Ball *A* is thrown straight up with an initial speed v_0 and reaches a maximum elevation *h* before falling back down. When *A* reaches its maximum elevation, a second ball is thrown straight upward with the same initial speed v_0. At what height, *y*, will the balls cross paths?

a. $y = h$
b. $y > h/2$
c. $y = h/2$
d. $y < h/2$
e. $y = 0$

11.CQ6 Two cars are approaching an intersection at constant speeds as shown. What velocity will car *B* appear to have to an observer in car *A*?

a. → **b.** ↘ **c.** ↖ **d.** ↗ **e.** ↙

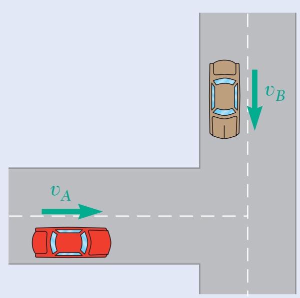
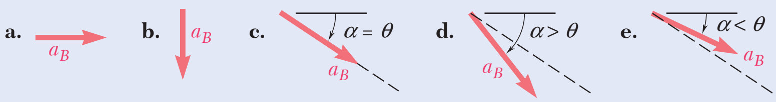


Fig. P6.CQ6

11.CQ7 Blocks A and B are released from rest in the positions shown. Neglecting friction between all surfaces, which figure best indicates the direction α of the acceleration of block B?



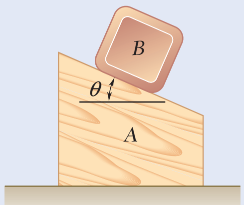


Fig. P6.CQ7

END-OF-SECTION PROBLEMS

11.89 A ball is thrown so that the motion is defined by the equations $x = 5t$ and $y = 2 + 6t - 4.9t^2$, where x and y are expressed in meters and t is expressed in seconds. Determine (*a*) the velocity at $t = 1$ s, (*b*) the horizontal distance the ball travels before hitting the ground.

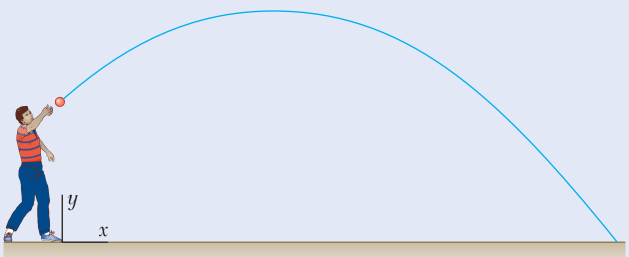


Fig. P11.89

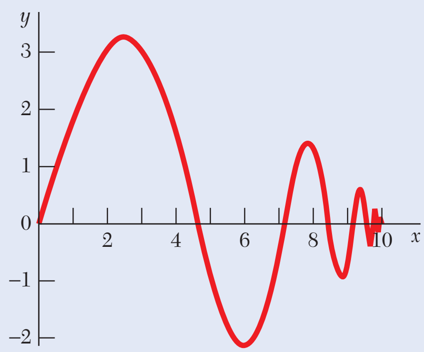


Fig. P11.90

11.90 The motion of a vibrating particle is defined by the position vector $\mathbf{r} = 10(1 - e^{-3t})\mathbf{i} + (4e^{-2t} \sin 15t)\mathbf{j}$, where $\mathbf{r}$ and t are expressed in millimeters and seconds, respectively. Determine the velocity and acceleration when (*a*) $t = 0$, (*b*) $t = 0.5$ s.

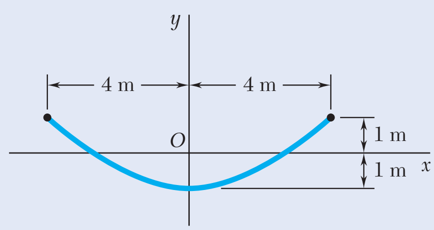


Fig. P11.91

11.91 The motion of a vibrating particle is defined by the position vector $\mathbf{r} = (4 \sin \pi t)\mathbf{i} - (\cos 2\pi t)\mathbf{j}$, where r is expressed in meters and t in seconds. (*a*) Determine the velocity and acceleration when $t = 1$ s. (*b*) Show that the path of the particle is parabolic.

11.92 The motion of a particle is defined by the equations $x = 100t - 50 \sin t$ and $y = 100 - 50 \cos t$, where x and y are expressed in mm and t is expressed in seconds. Sketch the path of the particle for the time interval $0 \leq t \leq 2\pi$, and determine (*a*) the magnitudes of the smallest and largest velocities reached by the particle, (*b*) the corresponding times, positions, and directions of the velocities.

11.93 The damped motion of a vibrating particle is defined by the position vector $\mathbf{r} = x_1[1 - 1/(t + 1)]\mathbf{i} + (y_1 e^{-\pi t/2} \cos 2\pi t)\mathbf{j}$, where t is expressed in seconds. For $x_1 = 30$ mm and $y_1 = 20$ mm, determine the position, the velocity, and the acceleration of the particle when (*a*) $t = 0$, (*b*) $t = 1.5$ s.

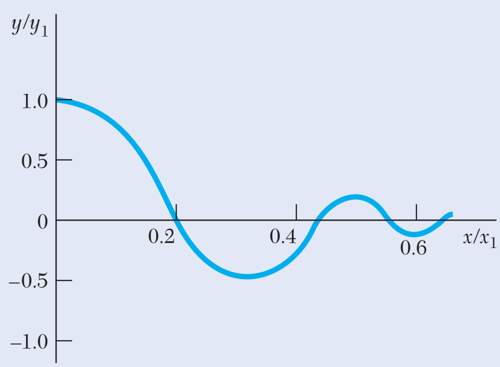


Fig. *P11.93*

11.94 A girl operates a radio-controlled model car in a vacant parking lot. The girl's position is at the origin of the xy coordinate axes, and the surface of the parking lot lies in the x–y plane. The motion of the car is defined by the position vector $\mathbf{r} = (2 + 2t^2)\mathbf{i} + (6 + t^3)\mathbf{j}$ where $\mathbf{r}$ and t are expressed in meters and seconds, respectively. Determine (*a*) the distance between the car and the girl when $t = 2$ s, (*b*) the distance the car traveled in the interval from $t = 0$ to $t = 2$ s, (*c*) the speed and direction of the car's velocity at $t = 2$ s, (*d*) the magnitude of the car's acceleration at $t = 2$ s.

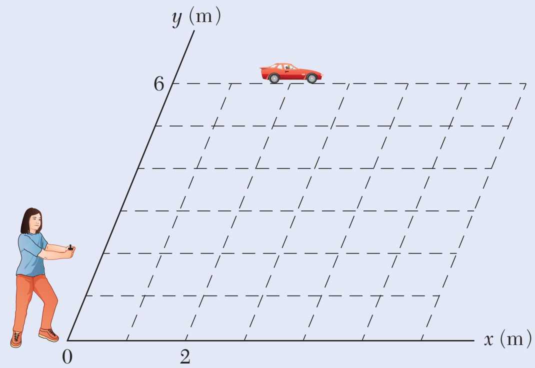


Fig. P11.94

11.95 The three-dimensional motion of a particle is defined by the position vector $\boldsymbol{r} = (Rt \cos \omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt \sin \omega_n t)\mathbf{k}$. Determine the magnitudes of the velocity and acceleration of the particle. (The space curve described by the particle is a conic helix.)

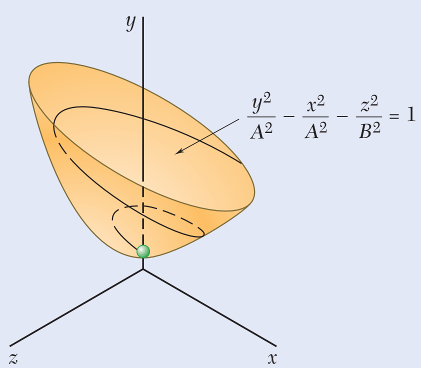


Fig. P11.96

***11.96** The three-dimensional motion of a particle is defined by the position vector $\boldsymbol{r} = (At \cos t)\mathbf{i} + (A\sqrt{t^2 + 1})\mathbf{j} + (Bt \sin t)\mathbf{k}$, where r and t are expressed in meters and seconds, respectively. Show that the curve described by the particle lies on the hyperboloid $(y/A)^2 - (x/A)^2 - (z/B)^2 = 1$. For $A = 3$ and $B = 1$, determine (*a*) the magnitudes of the velocity and acceleration when $t = 0$, (*b*) the smallest nonzero value of t for which the position vector and the velocity are perpendicular to each other.

11.97 An airplane used to drop water on brushfires is flying horizontally in a straight line at 315 km/h at an altitude of 80 m. Determine the distance d at which the pilot should release the water so that it will hit the fire at B.

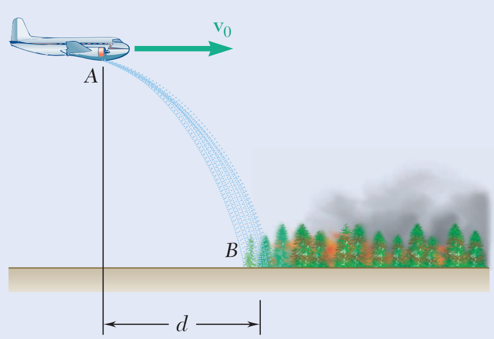


Fig. P11.97

11.98 A ski jumper starts with a horizontal take-off velocity of 25 m/s and lands on a straight landing hill inclined at 30°. Determine (*a*) the time between take-off and landing, (*b*) the length *d* of the jump, (*c*) the maximum vertical distance between the jumper and the landing hill.

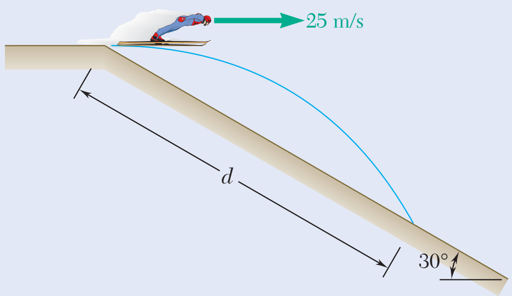


Fig. P11.98

11.99 A baseball pitching machine "throws" baseballs with a horizontal velocity $\mathbf{v}_0$. Knowing that height h varies between 788 mm and 1068 mm, determine (*a*) the range of values of v_0, (*b*) the values of α corresponding to h = 788 mm and h = 1068 mm.

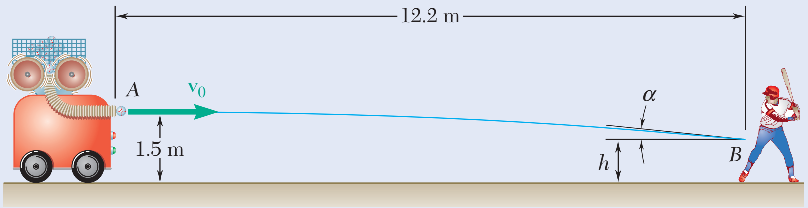


Fig. P11.99

11.100 While delivering newspapers, a girl throws a newspaper with a horizontal velocity $\mathbf{v}_0$. Determine the range of values of v_0 if the newspaper is to land between points B and C.

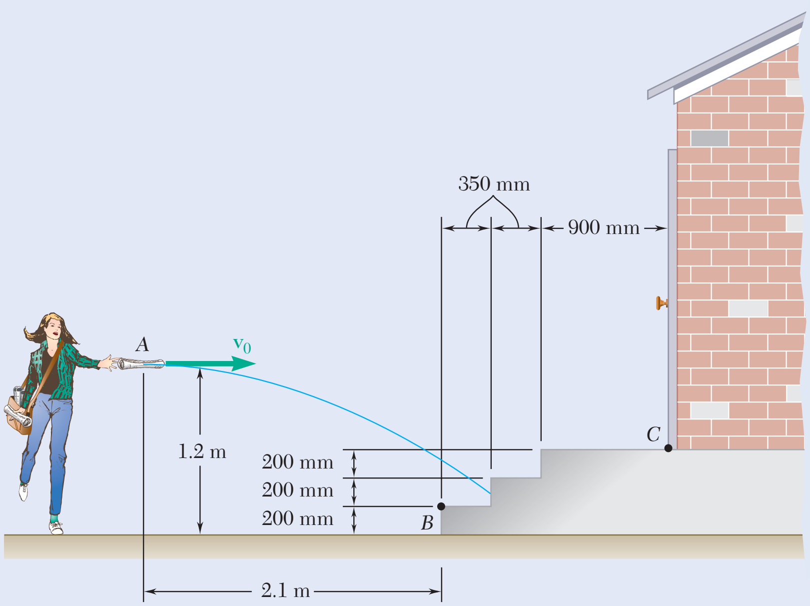


Fig. P11.100

11.101 Water flows from a drain spout with an initial velocity of 0.75 m/s at an angle of 15° with the horizontal. Determine the range of values of the distance d for which the water will enter the trough BC.

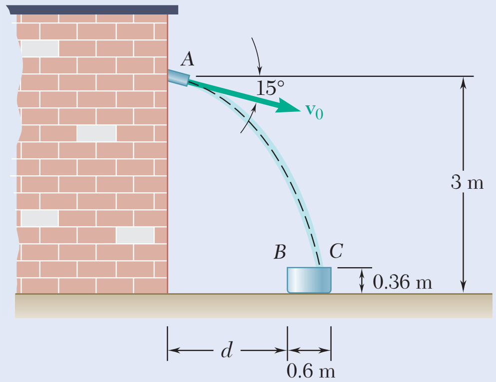


Fig. P11.101

11.102 In slow pitch softball, the underhand pitch must reach a maximum height of between 1.8 m and 3.7 m above the ground. A pitch is made with an initial velocity $\mathbf{v}_0$ with a magnitude of 13 m/s at an angle of 33° with the horizontal. Determine (*a*) if the pitch meets the maximum height requirement, (*b*) the height of the ball as it reaches the batter.

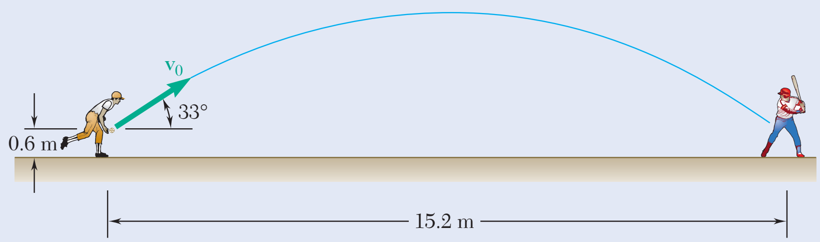


Fig. P11.102

11.103 A volleyball player serves the ball with an initial velocity $\mathbf{v}_0$ of magnitude 13.40 m/s at an angle of 20° with the horizontal. Determine (*a*) if the ball will clear the top of the net, (*b*) how far from the net the ball will land.

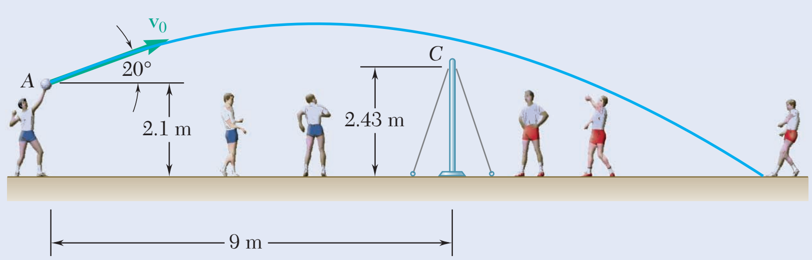


Fig. P11.103

11.104 A golfer hits a golf ball with an initial velocity of 50 m/s at an angle of 25° with the horizontal. Knowing that the fairway slopes downward at an average angle of 5°, determine the distance d between the golfer and point B where the ball first lands.

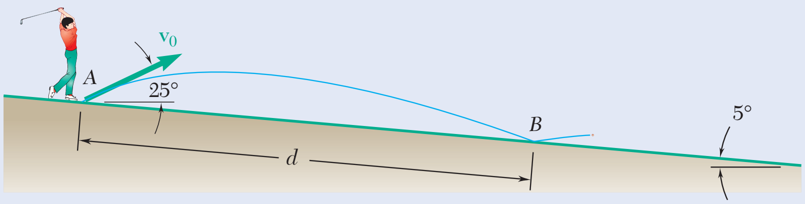


Fig. *P11.104*

11.105 A homeowner uses a snowblower to clear his driveway. Knowing that the snow is discharged at an average angle of 40° with the horizontal, determine the initial velocity v_0 of the snow.

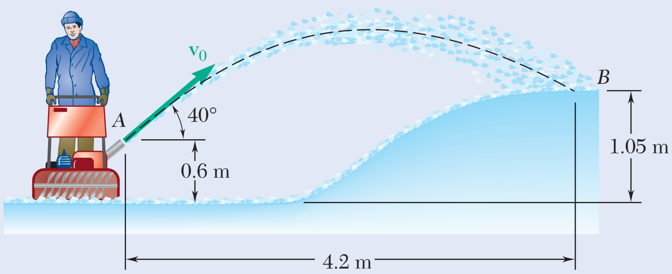


Fig. P11.105

11.106 At halftime of a football game souvenir balls are thrown to the spectators with a velocity $\mathbf{v}_0$. Determine the range of values of v_0 if the balls are to land between points B and C.

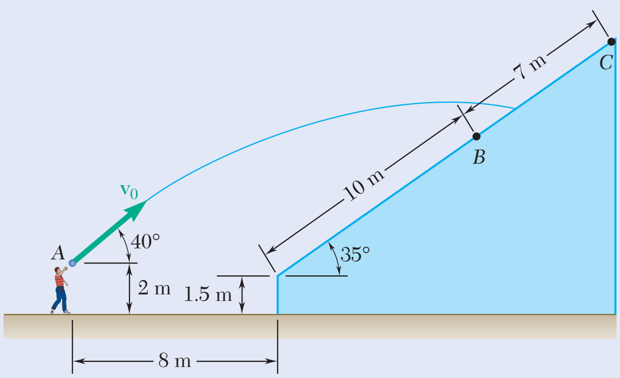


Fig. P11.106

11.107 A basketball player shoots when she is 5 m from the backboard. Knowing that the ball has an initial velocity $\mathbf{v}_0$ at an angle of 30° with the horizontal, determine the value of v_0 when d is equal to (*a*) 225 mm, (*b*) 425 mm.

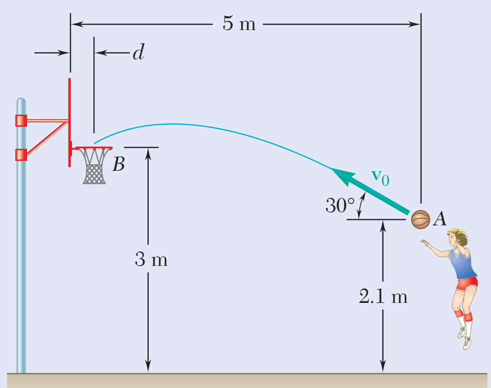


Fig. P11.107

11.108 A tennis player serves the ball at a height $h = 2.5$ m with an initial velocity of $\mathbf{v_0}$ at an angle of 5° with the horizontal. Determine the range of v_0 for which the ball will land in the service area that extends to 6.4 m beyond the net.

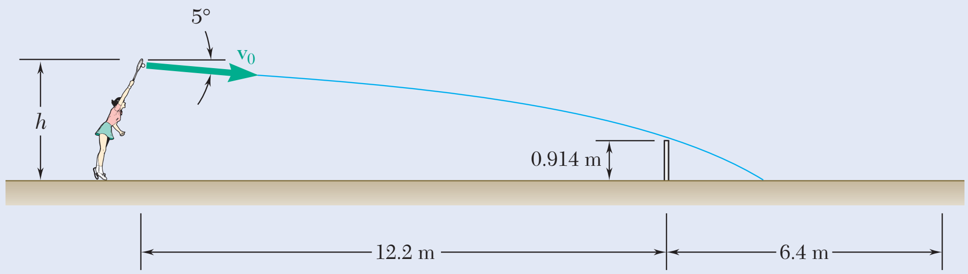


Fig. P11.108

11.109 The nozzle at A discharges cooling water with an initial velocity $\mathbf{v}_0$ at an angle of 6° with the horizontal onto a grinding wheel 350 mm in diameter. Determine the range of values of the initial velocity for which the water will land on the grinding wheel between points B and C.

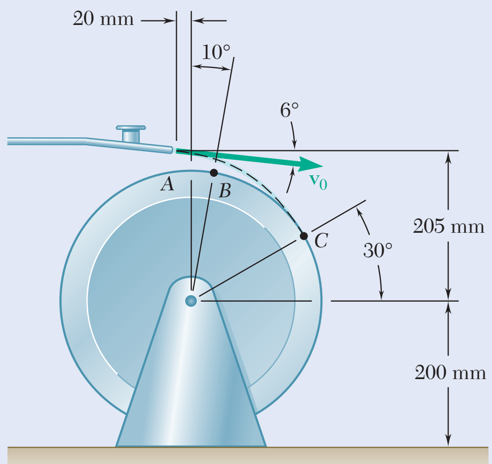


Fig. *P11.109*

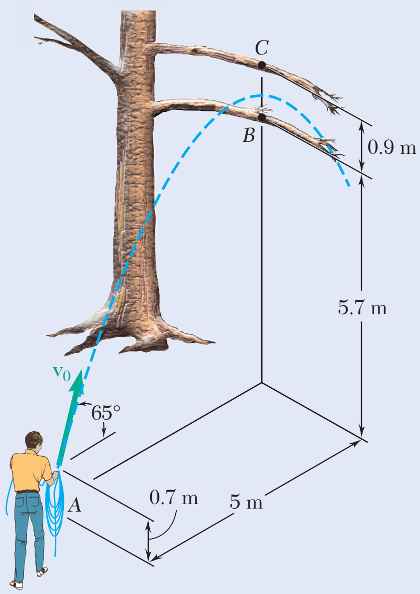


Fig. *P11.110*

11.110 While holding one of its ends, a worker lobs a coil of rope over the lowest limb of a tree. If he throws the rope with an initial velocity $\mathbf{v}_0$ at an angle of 65° with the horizontal, determine the range of values of v_0 for which the rope will go over only the lowest limb.

11.111 The pitcher in a softball game throws a ball with an initial velocity $\mathbf{v}_0$ of 72 km/h at an angle α with the horizontal. If the height of the ball at point B is 0.68 m, determine (*a*) the angle α, (*b*) the angle θ that the velocity of the ball at point B forms with the horizontal.

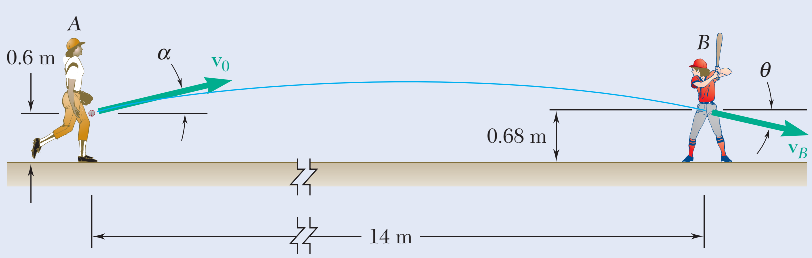


Fig. P11.111

11.112 A model rocket is launched from point A with an initial velocity $\mathbf{v}_0$ of 75 m/s. If the rocket's descent parachute does not deploy and the rocket lands a distance $d = 100$ m from A, determine (*a*) the angle α that $\mathbf{v}_0$ forms with the vertical, (*b*) the maximum height above point A reached by the rocket, (*c*) the duration of the flight.

11.113 The initial velocity $\mathbf{v}_0$ of a hockey puck is 160 km/h. Determine (*a*) the largest value (less than 45°) of the angle α for which the puck will enter the net, (*b*) the corresponding time required for the puck to reach the net.

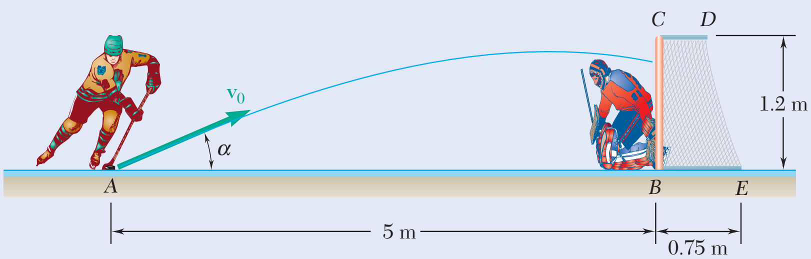


Fig. P11.113

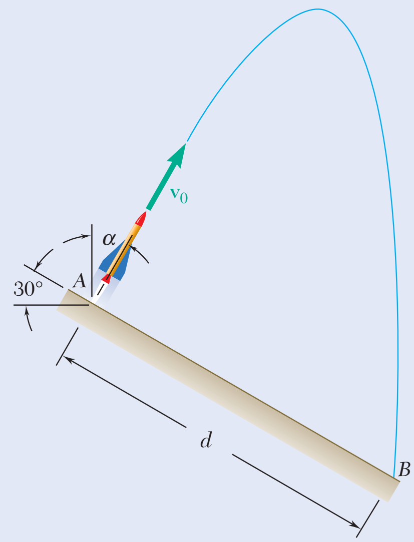


Fig. P11.112

11.114 A worker uses high-pressure water to clean the inside of a long drainpipe. If the water is discharged with an initial velocity $\mathbf{v}_0$ of 11.5 m/s, determine (*a*) the distance d to the farthest point B on the top of the pipe that the worker can wash from his position at A, (*b*) the corresponding angle α.

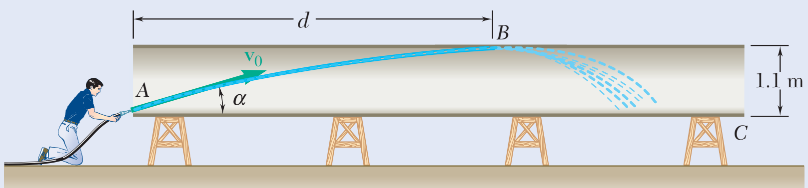


Fig. P11.114

11.115 An oscillating garden sprinkler which discharges water with an initial velocity $\mathbf{v}_0$ of 8 m/s is used to water a vegetable garden. Determine the distance d to the farthest point B that will be watered and the corresponding angle α when (*a*) the vegetables are just beginning to grow, (*b*) the height h of the corn is 1.8 m.

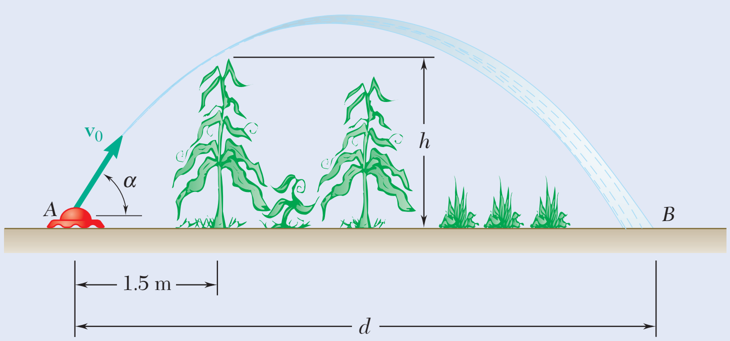


Fig. *P11.115*

***11.116** A ball is dropped onto a step at point A and rebounds with a velocity $\mathbf{v}_0$ at an angle of 15° with the vertical. Determine the value of v_0 knowing that just before the ball bounces at point B its velocity $\mathbf{v}_B$ forms an angle of 12° with the vertical.

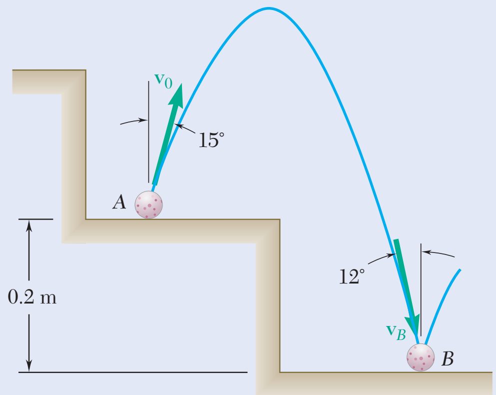


Fig. *P11.116*

11.117 The velocities of skiers A and B are as shown. Determine the velocity of A with respect to B.

Fig. P11.117

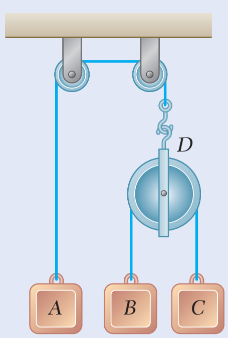


Fig. P11.118

11.118 The three blocks shown move with constant velocities. Find the velocity of each block, knowing that the relative velocity of A with respect to C is 300 mm/s upward and that the relative velocity of B with respect to A is 200 mm/s downward.

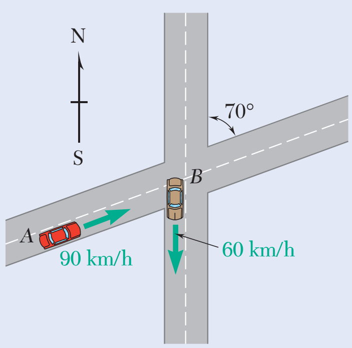


Fig. P11.119

11.119 Three seconds after automobile B passes through the intersection shown, automobile A passes through the same intersection. Knowing that the speed of each automobile is constant, determine (*a*) the relative velocity of B with respect to A, (*b*) the change in position of B with respect to A during a 4-s interval, (*c*) the distance between the two automobiles 2 s after A has passed through the intersection.

11.120 Shore-based radar indicates that a ferry leaves its slip with a velocity $\mathbf{v} = 18$ km/h $\angle 70°$, while instruments aboard the ferry indicate a speed of 18.4 km/h and a heading of 30° west of south relative to the river. Determine the velocity of the river.

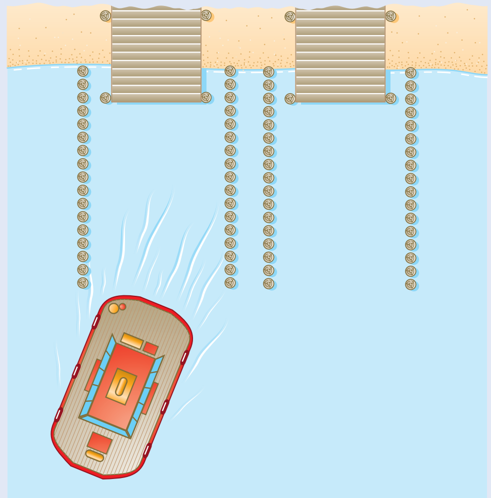


Fig. P11.120

11.121 Airplanes A and B are flying at the same altitude and are tracking the eye of hurricane C. The relative velocity of C with respect to A is $\mathbf{v}_{C/A} = 350$ km/h $\angle 75°$, and the relative velocity of C with respect to B is $\mathbf{v}_{C/B} = 400$ km/h $\angle 40°$. Determine (a) the relative velocity of B with respect to A, (b) the velocity of A if ground-based radar indicates that the hurricane is moving at a speed of 30 km/h due north, (c) the change in position of C with respect to B during a 15-min interval.

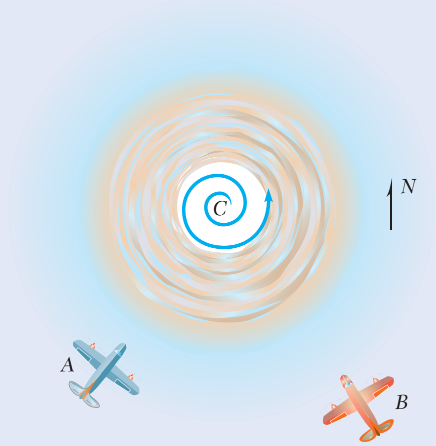


Fig. *P11.121*

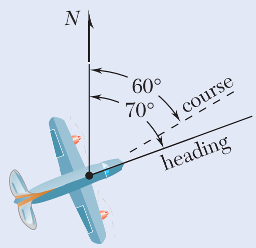


Fig. *P11.122*

11.122 Instruments in an airplane which is in level flight indicate that the velocity relative to the air (airspeed) is 120 km/h and the direction of the relative velocity vector (heading) is 70° east of north. Instruments on the ground indicate that the velocity of the airplane (ground speed) is 110 km/h and the direction of flight (course) is 60° east of north. Determine the wind speed and direction.

11.123 Knowing that the velocity of block B with respect to block A is $\mathbf{v}_{B/A} = 5.6$ m/s $\angle 70°$, determine the velocities of A and B.

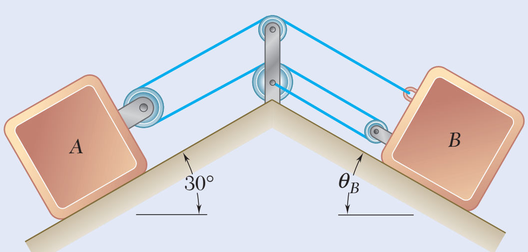


Fig. P11.123

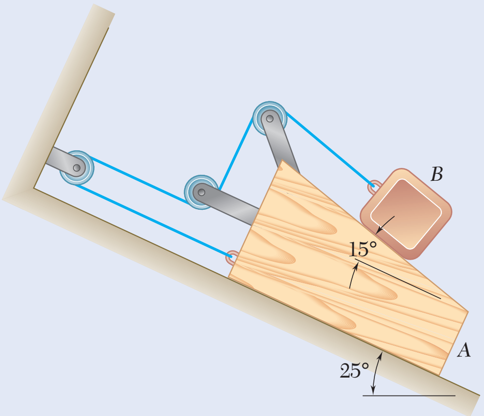


Fig. P11.124

11.124 Knowing that at the instant shown block A has a velocity of 200 mm/s and an acceleration of 150 mm/s^2 both directed down the incline, determine (*a*) the velocity of block B, (*b*) the acceleration of block B.

11.125 A boat is moving to the right with a constant deceleration of 0.3 m/s^2 when a boy standing on the deck D throws a ball with an initial velocity relative to the deck which is vertical. The ball rises to a maximum height of 8 m above the release point and the boy must step forward a distance d to catch it at the same height as the release point. Determine (*a*) the distance d, (*b*) the relative velocity of the ball with respect to the deck when the ball is caught.

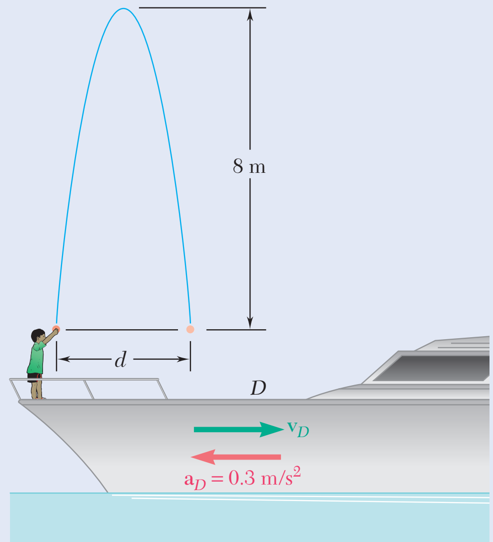


Fig. P11.125

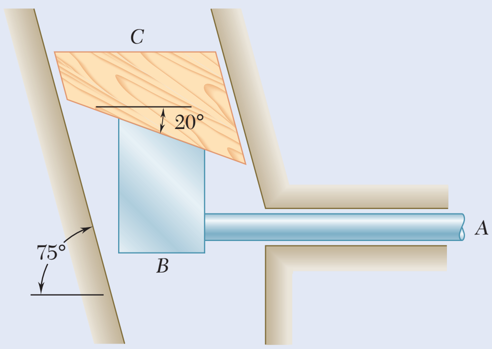


Fig. P11.126

11.126 The assembly of rod A and wedge B starts from rest and moves to the right with a constant acceleration of 2 mm/s^2. Determine (*a*) the acceleration of wedge C, (*b*) the velocity of wedge C when $t = 10$ s.

11.127 Determine the required velocity of the belt B if the relative velocity with which the sand hits belt B is to be (*a*) vertical, (*b*) as small as possible.

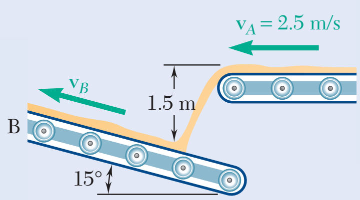


Fig. P11.127

11.128 Conveyor belt A, which forms a 20° angle with the horizontal, moves at a constant speed of 1.2 m/s and is used to load an airplane. Knowing that a worker tosses duffel bag B with an initial velocity of 0.75 m/s at an angle of 30° with the horizontal, determine the velocity of the bag relative to the belt as it lands on the belt.

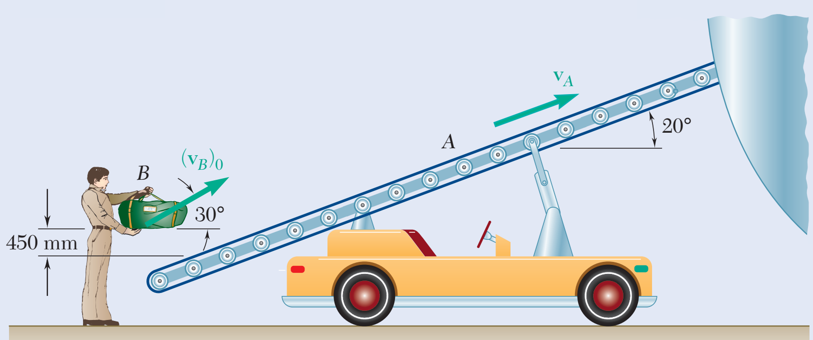


Fig. P11.128

11.129 During a rainstorm the paths of the raindrops appear to form an angle of 30° with the vertical and to be directed to the left when observed from a side window of a train moving at a speed of 15 km/h. A short time later, after the speed of the train has increased to 24 km/h, the angle between the vertical and the paths of the drops appears to be 45°. If the train were stopped, at what angle and with what velocity would the drops be observed to fall?

11.130 Instruments in airplane *A* indicate that, with respect to the air, the plane is headed 30° north of east with an air speed of 480 km/h. At the same time, radar on ship *B* indicates that the relative velocity of the plane with respect to the ship is 416 km/h in the direction 33° north of east. Knowing that the ship is steaming due south at 20 km/h, determine (*a*) the velocity of the airplane, (*b*) the wind speed and direction.

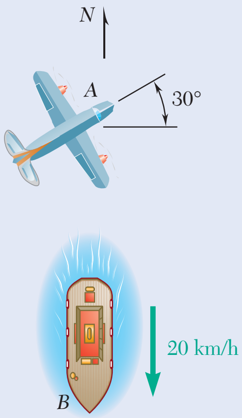


Fig. *P11.130*

11.131 When a small boat travels north at 5 km/h, a flag mounted on its stern forms an angle $\theta = 50°$ with the centerline of the boat as shown. A short time later, when the boat travels east at 20 km/h, angle θ is again 50°. Determine the speed and the direction of the wind.

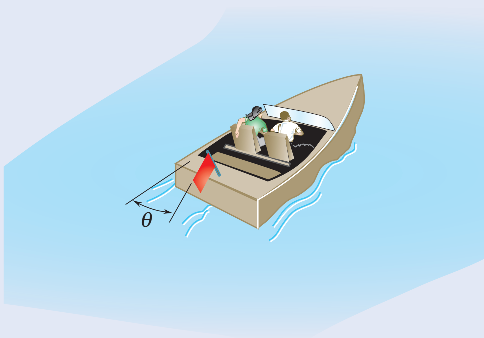


Fig. P11.131

11.132 As part of a department store display, a model train *D* runs on a slight incline between the store's up and down escalators. When the train and shoppers pass point *A*, the train appears to a shopper on the up escalator *B* to move downward at an angle of 22° with the horizontal, and to a shopper on the down escalator *C* to move upward at an angle of 23° with the horizontal and to travel to the left. Knowing that the speed of the escalators is 1 m/s, determine the speed and the direction of the train.

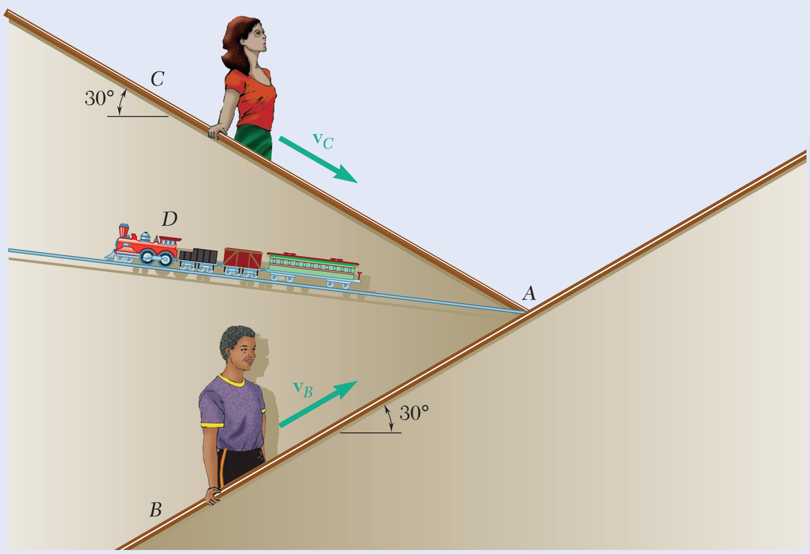


Fig. *P11.132*

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11.5 NON-RECTANGULAR COMPONENTS

Sometimes it is useful to analyze the motion of a particle in a coordinate system that is not rectangular. In this section, we introduce two common and important systems. The first system is based on the path of the particle; the second system is based on the radial distance and angular displacement of the particle.

11.5A Tangential and Normal Components

We saw in Sec. 11.4 that the velocity of a particle is a vector tangent to the path of the particle, but in general, the acceleration is not tangent to the path. It is sometimes convenient to resolve the acceleration into components directed, respectively, along the tangent and the normal to the path of the particle. We will refer to this reference frame as tangential and normal coordinates, which are sometimes called path coordinates.

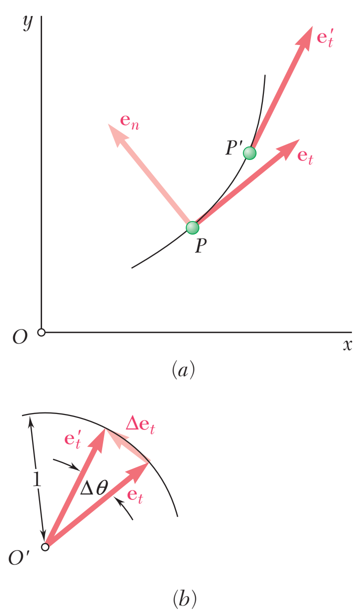


Fig. 11.19 (*a*) Unit tangent vectors for two positions of particle *P*; (*b*) the angle between the unit tangent vectors and their difference $\Delta \mathbf{e}_t$.

Planar Motion of a Particle. First we consider a particle that moves along a curve contained in a plane. Let P be the position of the particle at a given instant. We attach at P a unit vector $\mathbf{e}_t$ tangent to the path of the particle and pointing in the direction of motion (Fig. 11.19*a*). Let $\mathbf{e}'_t$ be the unit vector corresponding to the position P' of the particle at a later instant. Drawing both vectors from the same origin O', we define the vector $\Delta \mathbf{e}_t = \mathbf{e}'_t - \mathbf{e}_t$ (Fig. 11.19*b*). Since $\mathbf{e}_t$ and $\mathbf{e}'_t$ are of unit length, their tips lie on a circle with a radius of 1. Denote the angle formed by $\mathbf{e}_t$ and $\mathbf{e}'_t$ by $\Delta\theta$. Then the magnitude of $\Delta \mathbf{e}_t$ is $2 \sin(\Delta\theta/2)$. Considering now the vector $\Delta \mathbf{e}_t/\Delta\theta$, we note that, as $\Delta\theta$ approaches zero, this vector becomes tangent to the unit circle of Fig. 11.19*b*, i.e., perpendicular to $\mathbf{e}_t$, and that its magnitude approaches

$$\lim_{\Delta\theta \to 0} \frac{2\sin(\Delta\theta/2)}{\Delta\theta} = \lim_{\Delta\theta \to 0} \frac{\sin(\Delta\theta/2)}{\Delta\theta/2} = 1$$

Thus, the vector obtained in the limit is a unit vector along the normal to the path of the particle in the direction toward which $\mathbf{e}_t$ turns. Denoting this vector by $\mathbf{e}_n$, we have

$$\mathbf{e}_n = \lim_{\Delta\theta \to 0} \frac{\Delta \mathbf{e}_t}{\Delta\theta}$$

$$\mathbf{e}_n = \frac{d\mathbf{e}_t}{d\theta} \qquad \textbf{(11.34)}$$

Now, since the velocity $\mathbf{v}$ of the particle is tangent to the path, we can express it as the product of the scalar v and the unit vector $\mathbf{e}_t$. We have

$$\mathbf{v} = v\mathbf{e}_t \qquad \textbf{(11.35)}$$

To obtain the acceleration of the particle, we differentiate Eq. (11.35) with respect to t. Applying the rule for the differentiation of the product of a scalar and a vector function (Sec. 11.4B), we have

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{dv}{dt}\mathbf{e}_t + v\frac{d\mathbf{e}_t}{dt} \tag{11.36}$$

However,

$$\frac{d\mathbf{e}_t}{dt} = \frac{d\mathbf{e}_t}{d\theta}\frac{d\theta}{ds}\frac{ds}{dt}$$

Recall from Eq. (11.15) that $ds/dt = v$, from Eq. (11.34) that $d\mathbf{e}_t/d\theta = \mathbf{e}_n$, and from elementary calculus that $d\theta/ds$ is equal to $1/\rho$, where ρ is the radius of curvature of the path at P (Fig. 11.20). Then we have

$$\frac{d\mathbf{e}_t}{dt} = \frac{v}{\rho}\mathbf{e}_n \tag{11.37}$$

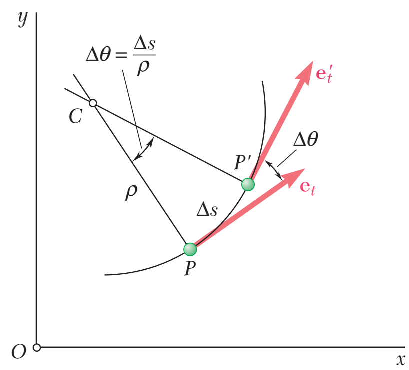


Fig. 11.20 Relationship among $\Delta\theta$, Δs, and ρ. Recall that for a circle, the arc length is equal to the radius multiplied by the angle.

Substituting into Eq. (11.36), we obtain

Acceleration in normal and tangential components

$$\mathbf{a} = \frac{dv}{dt}\mathbf{e}_t + \frac{v^2}{\rho}\mathbf{e}_n \tag{11.38}$$

Thus, the scalar components of the acceleration are

$$a_t = \frac{dv}{dt} \qquad a_n = \frac{v^2}{\rho} \tag{11.39}$$

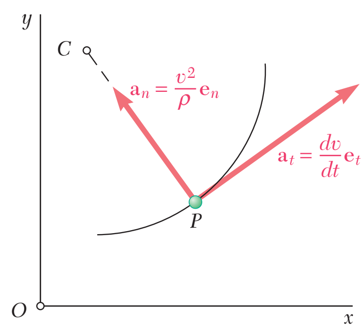


Fig. 11.21 Acceleration components in normal and tangential coordinates; the normal component always points toward the center of curvature of the path.

These relations state that the **tangential component** of the acceleration is equal to the **rate of change of the speed of the particle**, whereas the **normal component** is equal to the **square of the speed divided by the radius of curvature of the path at *P***. For a given speed, the normal acceleration increases as the radius of curvature decreases. If the particle travels in a straight line, then ρ is infinite, and the normal acceleration is zero. If the speed of the particle increases, a_t is positive, and the vector component $\mathbf{a}_t$ points in the direction of motion. If the speed of the particle decreases, a_t is negative, and $\mathbf{a}_t$ points against the direction of motion. The vector component $\mathbf{a}_n$, on the other hand, **is always directed toward the center of curvature *C* of the path** (Fig. 11.21).

We conclude from this discussion that the tangential component of the acceleration reflects a change in the speed of the particle, whereas its normal component reflects a change in the direction of motion of the particle. The acceleration of a particle is zero only if both of its components are zero. Thus, the acceleration of a particle moving with constant speed along a curve is not zero unless the particle happens to pass through a point of inflection of the curve (where the radius of curvature is infinite) or unless the curve is a straight line.

The fact that the normal component of acceleration depends upon the radius of curvature of the particle's path is taken into account in the design of structures or mechanisms as widely different as airplane wings, railroad tracks, and cams. In order to avoid sudden changes in the acceleration of the air particles flowing past a wing, wing profiles are designed without any sudden change in curvature. Similar care is taken in designing

Photo 11.5 The passengers in a train traveling around a curve experience a normal acceleration toward the center of curvature of the path.

railroad curves to avoid sudden changes in the acceleration of the cars (which would be hard on the equipment and unpleasant for the passengers). A straight section of track, for instance, is never directly followed by a circular section. Special transition sections are used to help pass smoothly from the infinite radius of curvature of the straight section to the finite radius of the circular track. Likewise, in the design of high-speed cams (that can be used to transform rotary motion into translational motion), abrupt changes in acceleration are avoided by using transition curves that produce a continuous change in acceleration.

Motion of a Particle in Space. The relations in Eqs. (11.38) and (11.39) still hold in the case of a particle moving along a space curve. However, since an infinite number of straight lines are perpendicular to the tangent at a given point P of a space curve, it is necessary to define more precisely the direction of the unit vector $\mathbf{e}_n$.

Let us consider again the unit vectors $\mathbf{e}_t$ and $\mathbf{e}'_t$ tangent to the path of the particle at two neighboring points P and P' (Fig. 11.22*a*). Again the vector $\Delta\mathbf{e}_t$ represents the difference between $\mathbf{e}_t$ and $\mathbf{e}'_t$ (Fig. 11.22*b*). Let us now imagine a plane through P (Fig. 11.22*c*) parallel to the plane defined by the vectors $\mathbf{e}_t$, $\mathbf{e}'_t$, and $\Delta\mathbf{e}_t$ (Fig. 11.22*b*). This plane contains the tangent to the curve at P and is parallel to the tangent at P'. If we let P' approach P, we obtain in the limit the plane that fits the curve most closely in the neighborhood of P. This plane is called the **osculating plane** at P (from the Latin *osculari,* to kiss). It follows from this definition that the osculating plane contains the unit vector $\mathbf{e}_n$, since this vector represents the limit of the vector $\Delta\mathbf{e}_t/\Delta\theta$. The normal defined by $\mathbf{e}_n$ is thus contained in the osculating plane; it is called the **principal normal** at P. The unit vector $\mathbf{e}_b = \mathbf{e}_t \times \mathbf{e}_n$ that completes the right-handed triad $\mathbf{e}_t$, $\mathbf{e}_n$, and $\mathbf{e}_b$ (Fig. 11.22*c*) defines the **binormal** at P. The binormal is thus perpendicular to the osculating plane. We conclude that the acceleration of the particle at P can be resolved into two components: one along the tangent and the other along the principal normal at P, as indicated in Eq. (11.38). Note that the acceleration has no component along the binormal.

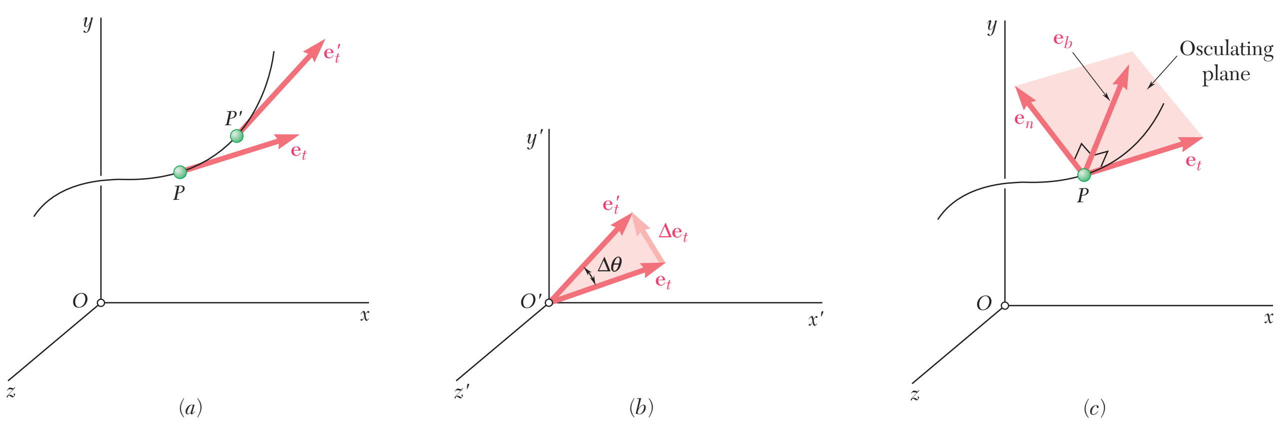


Fig. 11.22 (*a*) Unit tangent vectors for a particle moving in space; (*b*) the plane defined by the unit vectors and the vector difference $\Delta\mathbf{e}_t$; (*c*) the osculating plane contains the unit tangent and principal normal vectors and is perpendicular to the unit binormal vector.

11.5B Radial and Transverse Components

In some situations in planar motion, the position of particle P is defined by its polar coordinates r and θ (Fig. 11.23*a*). It is then convenient to resolve the velocity and acceleration of the particle into components parallel and perpendicular to the radial line OP. These components are called **radial and transverse components**.

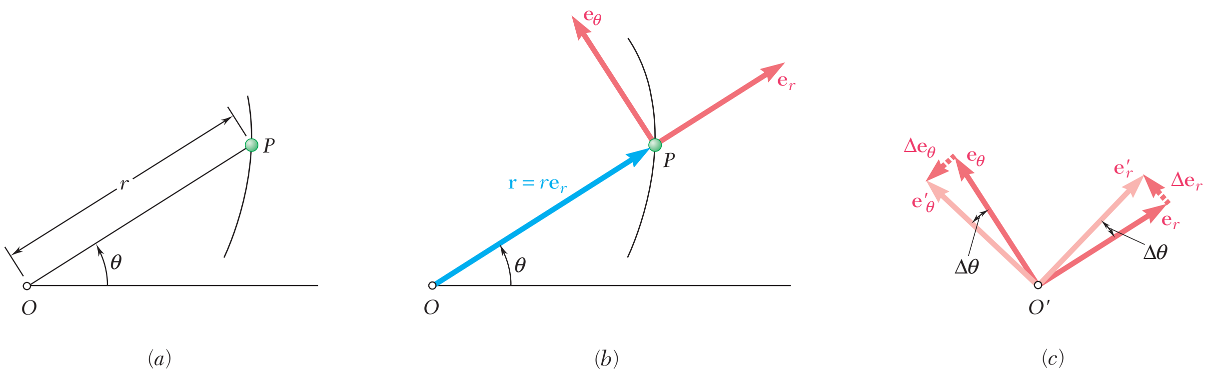


Fig. 11.23 (*a*) Polar coordinates r and θ of a particle at P; (*b*) radial and transverse unit vectors; (*c*) changes of the radial and transverse unit vectors resulting from a change in angle $\Delta\theta$.

We attach two unit vectors, $\mathbf{e}_r$ and $\mathbf{e}_\theta$, at P (Fig. 11.23*b*). The vector $\mathbf{e}_r$ is directed along OP and the vector $\mathbf{e}_\theta$ is obtained by rotating $\mathbf{e}_r$ through 90° counterclockwise. The unit vector $\mathbf{e}_r$ defines the **radial** direction, i.e., the direction in which P would move if r were increased and θ were kept constant. The unit vector $\mathbf{e}_\theta$ defines the **transverse** direction, i.e., the direction in which P would move if θ were increased and r were kept constant. A derivation similar to the one we used in the preceding section to determine the unit vector $\mathbf{e}_t$ leads to the relations

$$\frac{d\mathbf{e}_r}{d\theta} = \mathbf{e}_\theta \qquad \frac{d\mathbf{e}_\theta}{d\theta} = -\mathbf{e}_r \tag{11.40}$$

Here $-\mathbf{e}_r$ denotes a unit vector with a sense opposite to that of $\mathbf{e}_r$ (Fig. 11.23*c*). Using the chain rule of differentiation, we express the time derivatives of the unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$ as

$$\frac{d\mathbf{e}_r}{dt} = \frac{d\mathbf{e}_r}{d\theta}\frac{d\theta}{dt} = \mathbf{e}_\theta\frac{d\theta}{dt} \qquad \frac{d\mathbf{e}_\theta}{dt} = \frac{d\mathbf{e}_\theta}{d\theta}\frac{d\theta}{dt} = -\mathbf{e}_r\frac{d\theta}{dt}$$

or using dots to indicate differentiation with respect to t as

$$\dot{\mathbf{e}}_r = \dot{\theta}\mathbf{e}_\theta \qquad \dot{\mathbf{e}}_\theta = -\dot{\theta}\mathbf{e}_r \tag{11.41}$$

To obtain the velocity $\mathbf{v}$ of particle P, we express the position vector $\mathbf{r}$ of P as the product of the scalar r and the unit vector $\mathbf{e}_r$ and then differentiate with respect to t for

$$\mathbf{v} = \frac{d}{dt}(r\mathbf{e}_r) = \dot{r}\mathbf{e}_r + r\dot{\mathbf{e}}_r$$

Photo 11.6 The foot pedals on an elliptical trainer undergo curvilinear motion.

Using the first of the relations of Eq. (11.41), we can rewrite this as

Velocity in radial and transverse components

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta \tag{11.42}$$

Differentiating again with respect to t to obtain the acceleration, we have

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \ddot{r}\mathbf{e}_r + \dot{r}\dot{\mathbf{e}}_r + \dot{r}\dot{\theta}\mathbf{e}_\theta + r\ddot{\theta}\mathbf{e}_\theta + r\dot{\theta}\dot{\mathbf{e}}_\theta$$

Substituting for $\dot{\mathbf{e}}_r$ and $\dot{\mathbf{e}}_\theta$ from Eq. (11.41) and factoring $\mathbf{e}_r$ and $\mathbf{e}_\theta$, we obtain

Acceleration in radial and transverse components

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \tag{11.43}$$

The scalar components of the velocity and the acceleration in the radial and transverse directions are

$$v_r = \dot{r} \qquad v_\theta = r\dot{\theta} \tag{11.44}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \tag{11.45}$$

It is important to note that a_r is *not* equal to the time derivative of v_r and that a_θ is *not* equal to the time derivative of v_θ.

In the case of a particle moving along a circle with a center O, we have r = constant and $\dot{r} = \ddot{r} = 0$, so the formulas (11.42) and (11.43) reduce, respectively, to

$$\mathbf{v} = r\dot{\theta}\mathbf{e}_\theta \qquad \mathbf{a} = -r\dot{\theta}^2\mathbf{e}_r + r\ddot{\theta}\mathbf{e}_\theta \tag{11.46}$$

Compare this to using tangential and normal coordinates for a particle in a circular path. In this case, the radius of curvature ρ is equal to the radius of the circle r, and we have $\mathbf{v} = v\mathbf{e}_t$ and $\mathbf{a} = \dot{v}\mathbf{e}_t + (v^2/r)\mathbf{e}_n$. Note that $\mathbf{e}_r$ and $\mathbf{e}_n$ point in opposite directions ($\mathbf{e}_n$ inward and $\mathbf{e}_r$ outward).

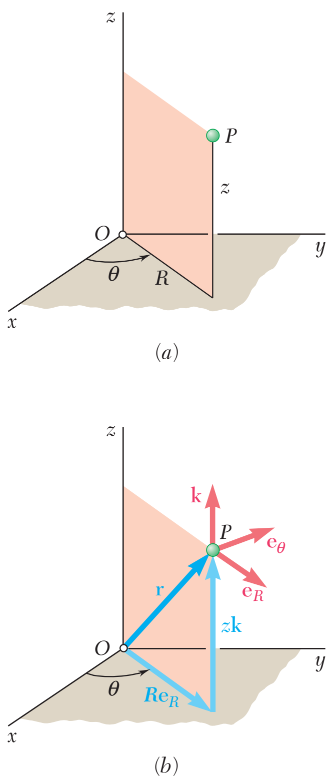


Fig. 11.24 (*a*) Cylindrical coordinates R, θ, and z; (*b*) unit vectors in cylindrical coordinates for a particle in space.

Extension to the Motion of a Particle in Space: Cylindrical Coordinates. Sometimes it is convenient to define the position of a particle P in space by its cylindrical coordinates R, θ, and z (Fig. 11.24*a*). We can then use the unit vectors $\mathbf{e}_R$, $\mathbf{e}_\theta$, and $\mathbf{k}$ shown in Fig. 11.24*b*. Resolving the position vector $\mathbf{r}$ of particle P into components along the unit vectors, we have

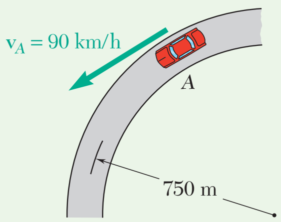
$$\mathbf{r} = R\mathbf{e}_R + z\mathbf{k} \tag{11.47}$$

Observe that $\mathbf{e}_R$ and $\mathbf{e}_\theta$ define the radial and transverse directions in the horizontal xy plane, respectively, and that the vector $\mathbf{k}$, which defines the **axial** direction, is constant in direction as well as in magnitude. Then we can verify that

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{R}\mathbf{e}_R + R\dot{\theta}\mathbf{e}_\theta + \dot{z}\mathbf{k} \tag{11.48}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = (\ddot{R} - R\dot{\theta}^2)\mathbf{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\mathbf{e}_\theta + \ddot{z}\mathbf{k} \tag{11.49}$$

Sample Problem 11.16



A motorist is traveling on a curved section of highway with a radius of 750 m at a speed of 90 km/h. The motorist suddenly applies the brakes, causing the automobile to slow down at a constant rate. If the speed has been reduced to 72 km/h after 8 s, determine the acceleration of the automobile immediately after the brakes have been applied.

STRATEGY: You know the path of the motion, and that the forward speed of the vehicle defines the direction of $\mathbf{e}_t$. Therefore, you can use tangential and normal components.

MODELING and ANALYSIS:

Tangential Component of Acceleration. First express the speeds in m/s.

$$90 \text{ km/h} = \left(90 \frac{\text{km}}{\text{h}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 25 \text{ m/s}$$

$$72 \text{ km/h} = 20 \text{ m/s}$$

Since the automobile slows down at a constant rate, you have the tangential acceleration of

$$a_t = \text{average } a_t = \frac{\Delta v}{\Delta t} = \frac{20 \text{ m/s} - 25 \text{ m/s}}{8 \text{ s}} = -0.625 \text{ m/s}^2$$

Normal Component of Acceleration. Immediately after the brakes have been applied, the speed is still 88 ft/s. Therefore, you have

$$a_n = \frac{v^2}{\rho} = \frac{(25 \text{ m/s})^2}{750 \text{ m}} = 0.833 \text{ m/s}^2$$

Magnitude and Direction of Acceleration. The magnitude and direction of the resultant $\mathbf{a}$ of the components $\mathbf{a}_n$ and $\mathbf{a}_t$ are (Fig. 1)

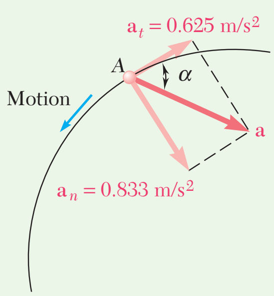


Fig. 1 Acceleration of the car.

$$\tan \alpha = \frac{a_n}{a_t} = \frac{0.833 \text{ m/s}^2}{0.625 \text{ m/s}^2} \qquad \alpha = 53.1° \blacktriangleleft$$

$$a = \frac{a_n}{\sin \alpha} = \frac{0.833 \text{ m/s}^2}{\sin 53.1°} \qquad \mathbf{a} = 1.041 \text{ m/s}^2 \blacktriangleleft$$

REFLECT and THINK: The tangential component of acceleration is opposite the direction of motion, and the normal component of acceleration points to the center of curvature, which is what you would expect for slowing down on a curved path. Attempting to do this problem in Cartesian coordinates is quite difficult.

Sample Problem 11.17

Determine the minimum radius of curvature of the trajectory described by the projectile considered in Sample Prob. 11.10.

STRATEGY: You are asked to find the radius of curvature, so you should use normal and tangential coordinates.

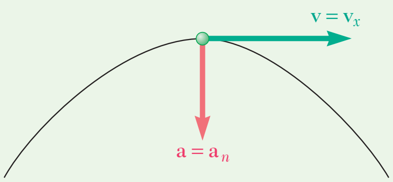


Fig. 1 Acceleration and velocity of the projectile.

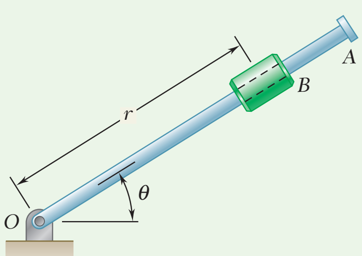
MODELING and ANALYSIS: Since $a_n = v^2/\rho$, you have $\rho = v^2/a_n$. Therefore, the radius is small when v is small or when a_n is large. The speed v is minimum at the top of the trajectory, since $v_y = 0$ at that point; a_n is maximum at that same point, since the direction of the vertical coincides with the direction of the normal (Fig. 1). Therefore, the minimum radius of curvature occurs at the top of the trajectory. At this point, you have

$$v = v_x = 155.9 \text{ m/s} \qquad a_n = a = 9.81 \text{ m/s}^2$$

$$\rho = \frac{v^2}{a_n} = \frac{(155.9 \text{ m/s})^2}{9.81 \text{ m/s}^2} \qquad \rho = 2480 \text{ m} \quad \blacktriangleleft$$

REFLECT and THINK: The top of the trajectory is the easiest point to determine the radius of curvature. At any other point in the trajectory, you need to find the normal component of acceleration. You can do this easily at the top, because you know that the total acceleration is pointed vertically downward and the normal component is simply the component perpendicular to the tangent to the path. Once you have the normal acceleration, it is straightforward to find the radius of curvature if you know the speed.

Sample Problem 11.18



The rotation of the 0.9-m arm OA about O is defined by the relation $\theta = 0.15t^2$, where θ is expressed in radians and t in seconds. Collar B slides along the arm in such a way that its distance from O is $r = 0.9 - 0.12t^2$, where r is expressed in meters and t in seconds. After the arm OA has rotated through 30°, determine (*a*) the total velocity of the collar, (*b*) the total acceleration of the collar, (*c*) the relative acceleration of the collar with respect to the arm.

STRATEGY: You are given information in terms of r and θ, so you should use polar coordinates.

MODELING and ANALYSIS: Model the collar as a particle.

Time t at which $\theta = 30°$. Substitute $\theta = 30° = 0.524$ rad into the expression for θ. You obtain

$$\theta = 0.15t^2 \qquad 0.524 = 0.15t^2 \qquad t = 1.869 \text{ s}$$

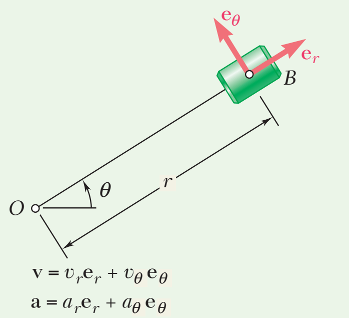


Fig. 1 Radial and transverse coordinates for collar *B*.

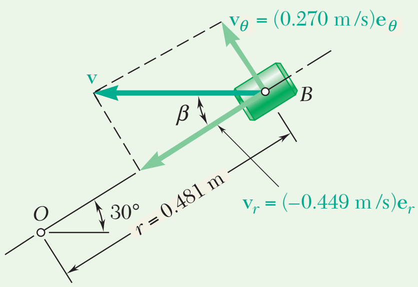


Fig. 2 Velocity of collar *B*.

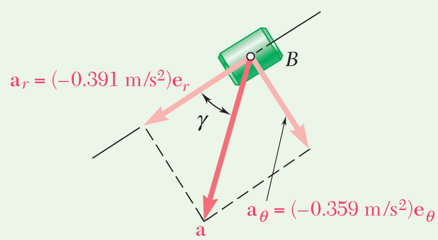


Fig. 3 Acceleration of collar *B*.

Equations of Motion. Substituting $t = 1.869$ s in the expressions for r, θ, and their first and second derivatives, you have

$$r = 0.9 - 0.12t^2 = 0.481 \text{ m} \qquad \theta = 0.15t^2 = 0.524 \text{ rad}$$
$$\dot{r} = -0.24t = -0.449 \text{ m/s} \qquad \dot{\theta} = 0.30t = 0.561 \text{ rad/s}$$
$$\ddot{r} = -0.24 = -0.240 \text{ m/s}^2 \qquad \ddot{\theta} = 0.30 = 0.300 \text{ rad/s}^2$$

a. Velocity of *B*. Using Eqs. (11.44), you can obtain the values of v_r and v_θ when $t = 1.869$ s (Fig. 1).

$$v_r = \dot{r} = -0.449 \text{ m/s}$$
$$v_\theta = r\dot{\theta} = 0.481(0.561) = 0.270 \text{ m/s}$$

Solve the right triangle shown in Fig. 2 to obtain the magnitude and direction of the velocity,

$$v = 0.524 \text{ m/s} \qquad \beta = 31.0° \quad ◀$$

b. Acceleration of *B*. Using Eqs. (11.45), you obtain (Fig. 3)

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$= -0.240 - 0.481(0.561)^2 = -0.391 \text{ m/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
$$= 0.481(0.300) + 2(-0.449)(0.561) = -0.359 \text{ m/s}^2$$
$$a = 0.531 \text{ m/s}^2 \qquad \gamma = 42.6° \quad ◀$$

c. Acceleration of *B* with Respect to Arm *OA*. Note that the motion of the collar with respect to the arm is rectilinear and defined by the coordinate r (Fig. 4). You have

$$a_{B/OA} = \ddot{r} = -0.240 \text{ m/s}^2$$
$$a_{B/OA} = 0.240 \text{ m/s}^2 \text{ toward } O. \quad ◀$$

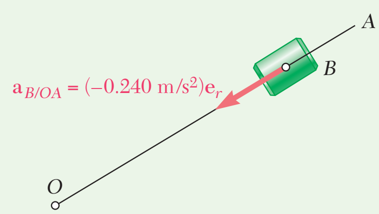
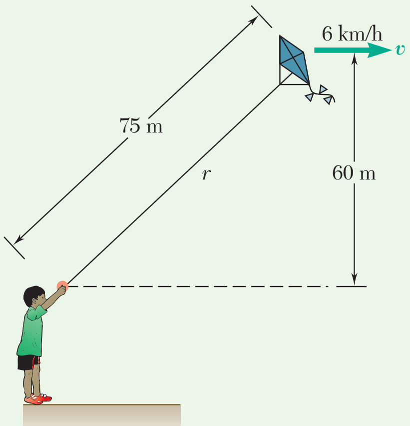


Fig. 4

REFLECT and THINK: You should consider polar coordinates for any kind of rotational motion. They turn this problem into a straightforward solution, whereas any other coordinate system would make this problem much more difficult. One way to make this problem harder would be to ask you to find the radius of curvature in addition to the velocity and acceleration. To do this, you would have to find the normal component of the acceleration; that is, the component of acceleration that is perpendicular to the tangential direction defined by the velocity vector.

Sample Problem 11.19



A boy is flying a kite that is 60 m high with 75 m of cord out. The kite moves horizontally from this position at a constant 6 km/h that is directly away from the boy. Ignoring the sag in the cord, determine how fast the cord is being let out at this instant and how fast this rate is increasing.

STRATEGY: The most natural way to describe the position of the kite is using a radial vector and angle, as shown in Fig. 1. The distance r is changing, so use polar coordinates.

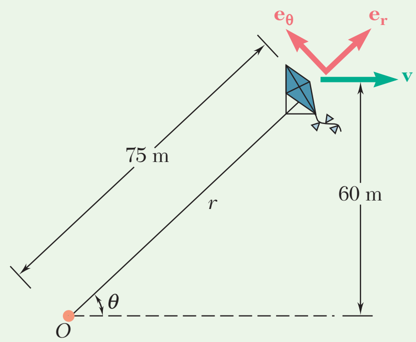


Fig. 1 Radial and transverse coordinates for the kite.

MODELING and ANALYSIS: The angle and the speed of the kite in m/s are found by

$$\theta = \sin^{-1}\left(\frac{60}{75}\right) = 53.13° \quad \text{and} \quad v = 6\left(\frac{\text{km}}{\text{hr}}\right)\left(\frac{\text{hr}}{3600\ \text{s}}\right)\left(\frac{1000\ \text{m}}{\text{km}}\right) = \frac{5}{3}\ \text{m/s}$$

Velocity in Polar Coordinates: You know that in polar coordinates the velocity is $\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_r$. Using Fig. 1, you can resolve the velocity vector into polar coordinates, giving

$$\dot{r} = v\cos\theta = \left(\frac{5}{3}\ \text{m/s}\right)\cos 53.13° \qquad \dot{r} = 1.000\ \text{m/s} \ \blacktriangleleft$$

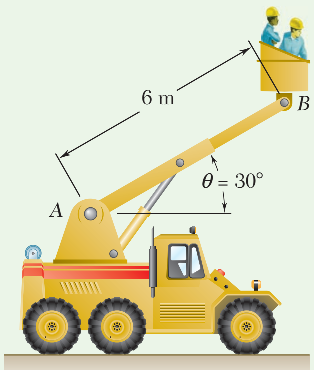
$$r\dot{\theta} = -v\sin\theta \qquad \dot{\theta} = -\frac{v\sin\theta}{r} = -\frac{(5/3\ \text{m/s})\sin 53.13°}{75\ \text{m}} = 0.01778\ \text{rad/s}$$

Acceleration in Polar Coordinates: You know that the acceleration is zero, because the kite is traveling at a constant speed. This means that both components of the acceleration need to be zero. You know the radial component is $a_r = \ddot{r} - r\dot{\theta}^2 = 0$. So

$$\ddot{r} = r\dot{\theta}^2 = (75\ \text{m})(-0.01778\ \text{rad/s})^2 \qquad \ddot{r} = 0.0237\ \text{m/s}^2 \ \blacktriangleleft$$

REFLECT and THINK: When the angle is 90°, then $\dot{r}$ will be zero. When the angle is very small—that is, when the kite is far away—you would expect the cord to increase at a rate of 6 m/s, which is the speed of the kite. Our answer is reasonable since it is between these two limits.

Sample Problem 11.20



At the instant shown, the length of the boom AB is being *decreased* at the constant rate of 0.2 m/s, and the boom is being lowered at the constant rate of 0.08 rad/s. Determine (*a*) the velocity of point B, (*b*) the acceleration of point B.

STRATEGY: Use polar coordinates, since that is the most natural way to describe the position of point B.

MODELING and ANALYSIS: From the problem statement, you know

$$\dot{r} = -0.2 \text{ m/s} \qquad \ddot{r} = 0 \qquad \dot{\theta} = -0.08 \text{ rad/s} \qquad \ddot{\theta} = 0$$

a. Velocity of B. Using Eqs.(11.44), you can determine the values of v_r and v_θ at this instant to be

$$v_r = \dot{r} = -0.2 \text{ m/s}$$
$$v_\theta = r\dot{\theta} = (6 \text{ m})(-0.08 \text{ rad/s}) = -0.48 \text{ m/s}$$

Therefore, you can write the velocity vector as

$$\mathbf{v} = (-0.200 \text{ m/s})\mathbf{e}_r + (-0.480 \text{ m/s})\mathbf{e}_t \quad ◀$$

b. Acceleration of B. Using Eqs. (11.45), you find

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (6 \text{ m})(-0.08 \text{ rad/s})^2 = -0.0384 \text{ m/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.02 \text{ m/s})(-0.08 \text{ rad/s}) = 0.00320 \text{ m/s}^2$$

or

$$\mathbf{a} = (-0.0384 \text{ m/s}^2)\mathbf{e}_r + (0.00320 \text{ m/s}^2)\mathbf{e}_\theta \quad ◀$$

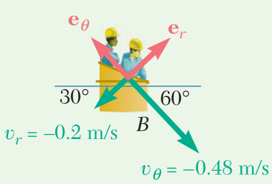


Fig. 1 Velocity of B.

REFLECT and THINK: Once you identify what you are given in the problem statement, this problem is quite straightforward. Sometimes you will be asked to express your answer in terms of a magnitude and direction. The easiest way is to first determine the x and y components and then to find the magnitude and direction. From Fig. 1,

$$\xrightarrow{+}: (v_B)_x = 0.48 \cos 60° - 0.2 \cos 30° = 0.06680 \text{ m/s}$$

$$+\uparrow: (v_B)_y = -0.48 \sin 60° - 0.2 \sin 30° = -0.5157 \text{ m/s}$$

So the magnitude and direction are

$$v_B = \sqrt{0.06680^2 + 0.5157^2}$$

$$= 0.520 \text{ m/s} \qquad \tan\beta = \frac{0.51569}{0.06680}, \quad \beta = 82.6°$$

So, an alternative way of expressing the velocity of B is $\mathbf{v}_B = 0.520$ m/s ∡82.6°

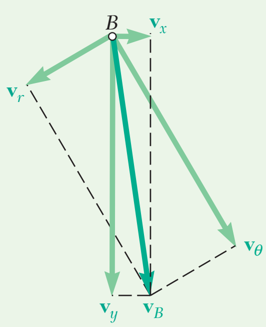


Fig. 2 Resultant velocity of collar B in Cartesian and in radial and transverse coordinates.

You could also find the magnitude and direction of the acceleration if you needed it expressed in this way. It is important to note that no matter what coordinate system we choose, the resultant velocity vector is the same. You can choose to express this vector in whatever coordinate system is most useful. Figure 2 shows the velocity vector $\mathbf{v}_B$ resolved into x and y components and r and θ coordinates.

SOLVING PROBLEMS ON YOUR OWN

In the following problems, you will be asked to express the velocity and the acceleration of particles in terms of either their **tangential and normal components** or their **radial and transverse components**. Although these components may not be as familiar to you as rectangular components, you will find that they can simplify the solution of many problems and that certain types of motion are more easily described when they are used.

1. Using tangential and normal components. These components are most often used when the particle of interest travels along a known curvilinear path or when the radius of curvature of the path is to be determined [Sample Prob. 11.16]. Remember that the unit vector $\mathbf{e}_t$ is tangent to the path of the particle (and thus aligned with the velocity), whereas the unit vector $\mathbf{e}_n$ is directed along the normal to the path and always points toward its center of curvature. It follows that the directions of the two unit vectors are constantly changing as the particle moves.

2. Acceleration in terms of tangential and normal components. We derived in Sec. 11.5A the following equation, which is applicable to both the two-dimensional and the three-dimensional motion of a particle:

$$\mathbf{a} = \frac{dv}{dt}\mathbf{e}_t + \frac{v^2}{\rho}\mathbf{e}_n \qquad \textbf{(11.38)}$$

The following observations may help you in solving the problems of this section.

a. The tangential component of the acceleration measures the rate of change of the speed as $a_t = dv/dt$. It follows that, when a_t is constant, you can use the equations for uniformly accelerated motion with the acceleration equal to a_t. Furthermore, when a particle moves at a constant speed, we have $a_t = 0$, and the acceleration of the particle reduces to its normal component.

b. The normal component of the acceleration is always directed toward the center of curvature of the path of the particle, and its magnitude is $a_n = v^2/\rho$. Thus, you can determine the normal component if you know the speed of the particle and the radius of curvature ρ of the path. Conversely, if you know the speed and normal acceleration of the particle, you can find the radius of curvature of the path by solving this equation for ρ [Sample Prob. 11.17].

3. Using radial and transverse components. These components are used to analyze the planar motion of a particle P when the position of P is defined by its polar coordinates r and θ. As shown in Fig. 11.23, the unit vector $\mathbf{e}_r$, which defines the **radial** direction, is attached to P and points away from the fixed point O, whereas the unit vector $\mathbf{e}_\theta$, which defines the **transverse** direction, is obtained by rotating $\mathbf{e}_r$ *counterclockwise* through 90°. The velocity and acceleration of a particle are expressed in terms of their radial and transverse components in Eqs. (11.42) and (11.43), respectively. Note that the expressions obtained contain the first and second derivatives with respect to t of both coordinates r and θ.

In the problems of this section, you will encounter the following types of problems involving radial and transverse components.

a. Both r and θ are known functions of t. In this case, you compute the first and second derivatives of r and θ and substitute the resulting expressions into Eqs. (11.42) and (11.43).

b. A certain relationship exists between r and θ. First, you should determine this relationship from the geometry of the given system and use it to express r as a function of θ. Once you know the function $r = f(\theta)$, you can apply the chain rule to determine $\dot{r}$ in terms of θ and $\dot{\theta}$, and $\ddot{r}$ in terms of θ, $\dot{\theta}$, and $\ddot{\theta}$:

$$\dot{r} = f'(\theta)\dot{\theta}$$

$$\ddot{r} = f''(\theta)\dot{\theta}^2 + f'(\theta)\ddot{\theta}$$

You can then substitute these expressions into Eqs. (11.42) and (11.43).

c. The three-dimensional motion of a particle, as indicated at the end of Sec. 11.5B, often can be described effectively in terms of the **cylindrical coordinates** R, θ, and z (Fig. 11.24). The unit vectors then should consist of $\mathbf{e}_R$, $\mathbf{e}_\theta$, and **k.** The corresponding components of the velocity and the acceleration are given in Eqs. (11.48) and (11.49). Note that the radial distance R is always measured in a plane parallel to the xy plane, and be careful not to confuse the position vector $\mathbf{r}$ with its radial component $R\mathbf{e}_R$.

Problems

CONCEPT QUESTIONS

11.CQ8 The Ferris wheel is rotating with a constant angular velocity ω. What is the direction of the acceleration of point A?
a. $\rightarrow$ **b.** $\uparrow$ **c.** $\downarrow$ **d.** $\leftarrow$ **e.** The acceleration is zero.

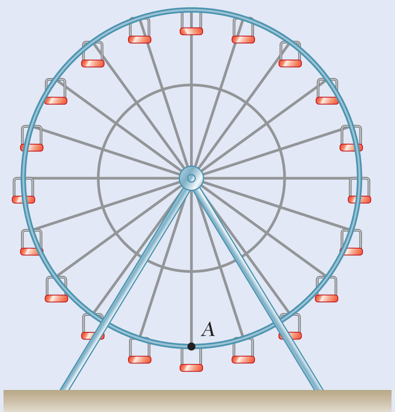


Fig. P11.CQ8

11.CQ9 A race car travels around the track shown at a constant speed. At which point will the race car have the largest acceleration?
a. A. **b.** B. **c.** C. **d.** D. **e.** The acceleration will be zero at all the points.

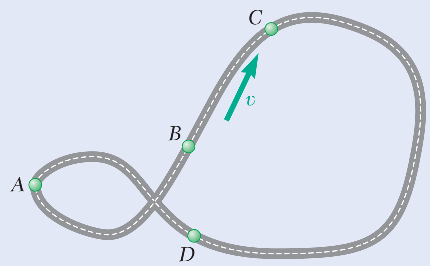


Fig. P11.CQ9

11.CQ10 A child walks across merry-go-round A with a constant speed u relative to A. The merry-go-round undergoes fixed-axis rotation about its center with a constant angular velocity ω counterclockwise. When the child is at the center of A, as shown, what is the direction of his acceleration when viewed from above?
a. $\rightarrow$ **b.** $\leftarrow$ **c.** $\uparrow$ **d.** $\downarrow$ **e.** The acceleration is zero.

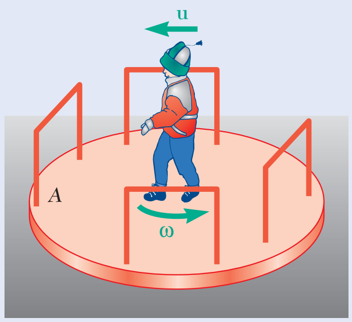


Fig. P11.CQ10

END-OF-SECTION PROBLEMS

11.133 Determine the smallest radius that should be used for a highway if the normal component of the acceleration of a car traveling at 72 km/h is not to exceed 0.8 m/s^2.

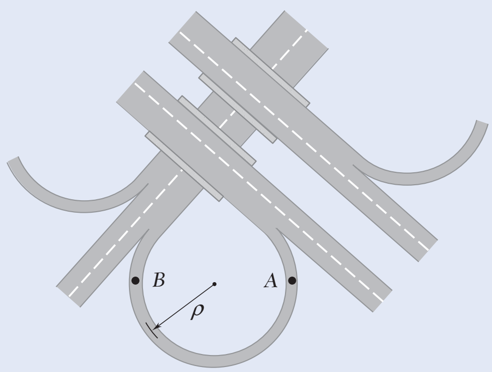


Fig. P11.133

11.134 Determine the maximum speed that the cars of the roller-coaster can reach along the circular portion AB of the track if $\rho = 25$ m and the normal component of their acceleration cannot exceed 3g.

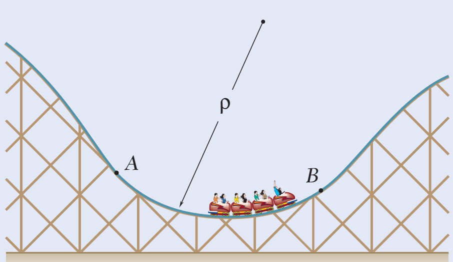


Fig. P11.134

11.135 Human centrifuges are often used to simulate different acceleration levels for pilots and astronauts. Space shuttle pilots typically face inwards towards the center of the gondola in order to experience a simulated 3-g forward acceleration. Knowing that the astronaut sits 5 m from the axis of rotation and experiences 3 g's inward, determine her velocity.

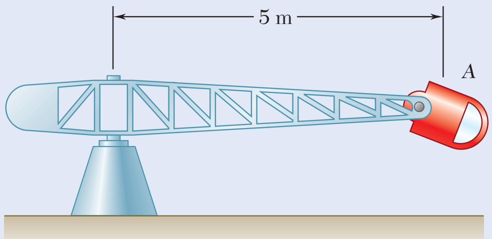


Fig. P11.135

11.136 Pin A, which is attached to link AB, is constrained to move in the circular slot CD. Knowing that at $t = 0$ the pin starts from rest and moves so that its speed increases at a constant rate of 20 mm/s^2, determine the magnitude of its total acceleration when (*a*) $t = 0$, (*b*) $t = 2$ s.

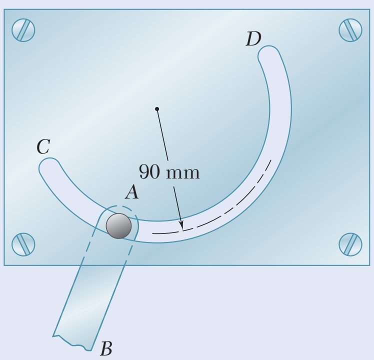


Fig. P11.136

11.137 A monorail train starts from rest on a curve of radius 400 m and accelerates at the constant rate a_t. If the maximum total acceleration of the train must not exceed 1.5 m/s^2, determine (*a*) the shortest distance in which the train can reach a speed of 72 km/h, (*b*) the corresponding constant rate of acceleration a_t.

11.138 A robot arm moves so that P travels in a circle about point B, which is not moving. Knowing that P starts from rest, and its speed increases at a constant rate of 10 mm/s^2, determine (*a*) the magnitude of the acceleration when $t = 4$ s, (*b*) the time for the magnitude of the acceleration to be 80 mm/s^2.

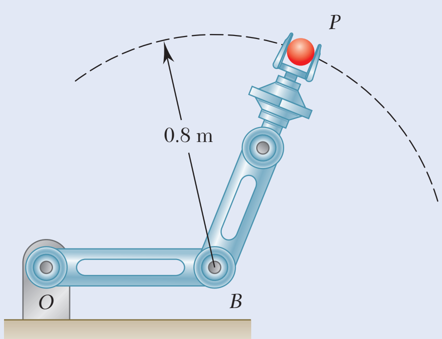


Fig. P11.138

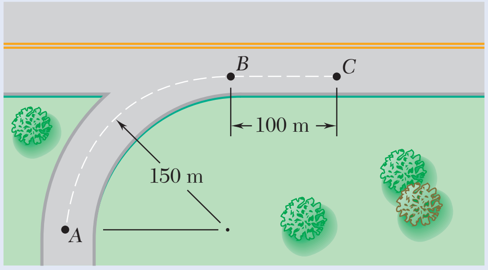


Fig. ***P11.140***

11.139 A monorail train starts from rest on a curve of radius 400 m and accelerates at the constant rate a_t. If the maximum total acceleration of the train must not exceed 1.5 m/s^2, determine (*a*) the shortest distance in which the train can reach a speed of 72 km/h, (*b*) the corresponding constant rate of acceleration a_t.

11.140 A motorist starts from rest at point *A* on a circular entrance ramp when $t = 0$, increases the speed of her automobile at a constant rate and enters the highway at point *B*. Knowing that her speed continues to increase at the same rate until it reaches 100 km/h at point *C*, determine (*a*) the speed at point *B*, (*b*) the magnitude of the total acceleration when $t = 20$ s.

11.141 Race car *A* is traveling on a straight portion of the track while race car *B* is traveling on a circular portion of the track. At the instant shown, the speed of *A* is increasing at the rate of 10 m/s^2, and the speed of *B* is decreasing at the rate of 6 m/s^2. For the position shown, determine (*a*) the velocity of *B* relative to *A*, (*b*) the acceleration of *B* relative to *A*.

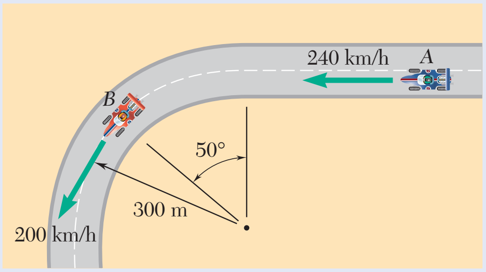


Fig. P11.141

11.142 At a given instant in an airplane race, airplane *A* is flying horizontally in a straight line, and its speed is being increased at the rate of 8 m/s^2. Airplane *B* is flying at the same altitude as airplane *A* and, as it rounds a pylon, is following a circular path of 300-m radius. Knowing that at the given instant the speed of *B* is being decreased at the rate of 3 m/s^2, determine, for the positions shown, (*a*) the velocity of *B* relative to *A*, (*b*) the acceleration of *B* relative to *A*.

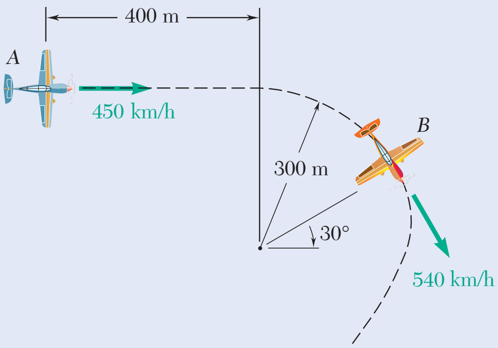


Fig. ***P11.142***

11.143 A race car enters the circular portion of a track that has a radius of 70 m. When the car enters the curve at point *P*, it is travelling with a speed of 120 km/h that is increasing at 5 m/s^2. Three seconds later, determine the *x* and *y* components of velocity and acceleration of the car.

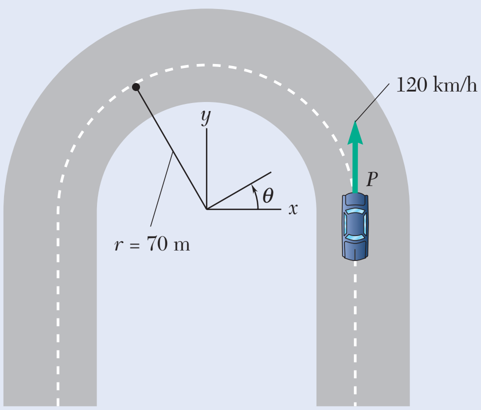


Fig. P11.143

11.144 An airplane flying at a constant speed of 240 m/s makes a banked horizontal turn. What is the minimum allowable radius of the turn if the structural specifications require that the acceleration of the airplane shall never exceed 4 g?

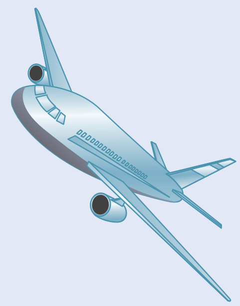


Fig. P11.144

11.145 A golfer hits a golf ball from point A with an initial velocity of 50 m/s at an angle of 25° with the horizontal. Determine the radius of curvature of the trajectory described by the ball (*a*) at point A, (*b*) at the highest point of the trajectory.

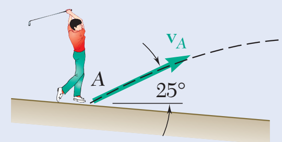


Fig. P11.145

11.146 Three children are throwing snowballs at each other. Child A throws a snowball with a horizontal velocity $\mathbf{v}_0$. If the snowball just passes over the head of child B and hits child C, determine the radius of curvature of the trajectory described by the snowball (*a*) at point B, (*b*) at point C.

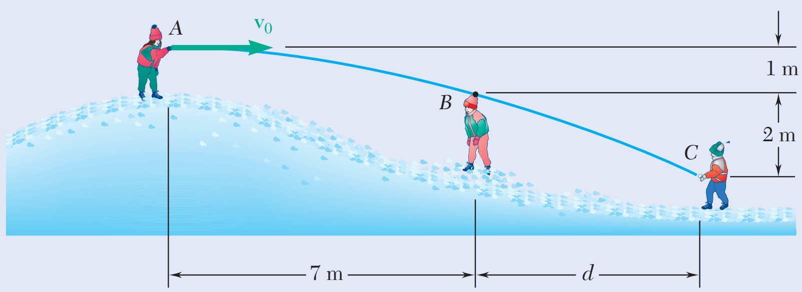


Fig. P11.146

11.147 Coal is discharged from the tailgate A of a dump truck with an initial velocity $\mathbf{v}_A = 2$ m/s ⦣ 50°. Determine the radius of curvature of the trajectory described by the coal (*a*) at point A, (*b*) at the point of the trajectory 1 m below point A.

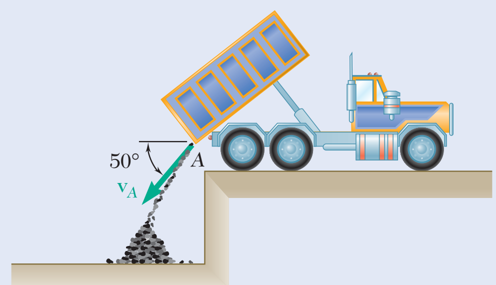


Fig. P11.147

11.148 From measurements of a photograph, it has been found that as the stream of water shown left the nozzle at A, it had a radius of curvature of 25 m. Determine (*a*) the initial velocity $\mathbf{v}_A$ of the stream, (*b*) the radius of curvature of the stream as it reaches its maximum height at B.

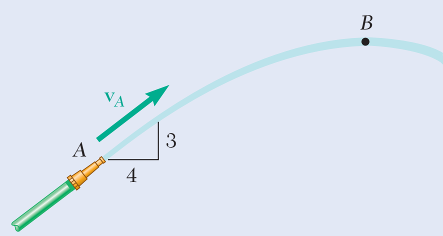


Fig. *P11.148*

11.149 A child throws a ball from point A with an initial velocity $\mathbf{v}_0$ at an angle of 3° with the horizontal. Knowing that the ball hits a wall at point B, determine (*a*) the magnitude of the initial velocity, (*b*) the minimum radius of curvature of the trajectory.

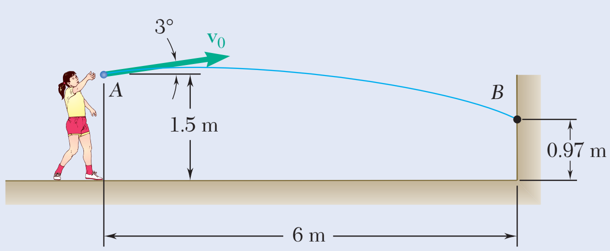


Fig. P11.149

11.150 A projectile is fired from point A with an initial velocity $\mathbf{v}_0$. (*a*) Show that the radius of curvature of the trajectory of the projectile reaches its minimum value at the highest point B of the trajectory. (*b*) Denoting by θ the angle formed by the trajectory and the horizontal at a given point C, show that the radius of curvature of the trajectory at C is $\rho = \rho_{min}/\cos^3\theta$.

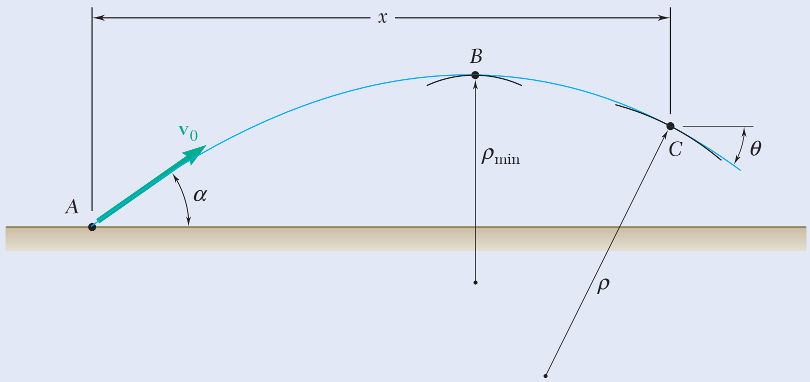


Fig. *P11.150*

***11.151** Determine the radius of curvature of the path described by the particle of Prob. 11.95 when $t = 0$.

***11.152** Determine the radius of curvature of the path described by the particle of Prob. 11.96 when $t = 0$, $A = 3$, and $B = 1$.

11.153 and 11.154 A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to $g(R/r)^2$, where g is the acceleration of gravity at the surface of the planet, R is the radius of the planet, and r is the distance from the center of the planet to the satellite. Knowing that the diameter of the sun is 1.39 Gm and that the acceleration of gravity at its surface is 274 m/s^2, determine the radius of the orbit of the indicated planet around the sun assuming that the orbit is circular.

11.153 Earth: $(v_{mean})_{orbit} = 107$ Mm/h.
11.154 Saturn: $(v_{mean})_{orbit} = 34.7$ Mm/h.

11.155 through *11.157* Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 160 km above the surface of the planet. (See information given in Probs. 11.153–11.154.)

11.155 Venus: $g = 8.53$ m/s^2, $R = 6161$ km.
11.156 Mars: $g = 3.83$ m/s^2, $R = 3332$ km.
11.157 Jupiter: $g = 26.0$ m/s^2, $R = 69{,}893$ km.

11.158 A satellite will travel indefinitely in a circular orbit around the earth if the normal component of its acceleration is equal to $g(R/r)^2$, where $g = 9.81$ m/s^2, R = radius of the earth = 6370 km, and r = distance from the center of the earth to the satellite. Assuming that the orbit of the moon is a circle with a radius of 384×10^3 km, determine the speed of the moon relative to the earth.

11.159 Knowing that the radius of the earth is 6370 km, determine the time of one orbit of the Hubble Space Telescope if the telescope travels in a circular orbit 590 km above the surface of the earth. (See information given in Probs. 11.153–11.154.)

11.160 Satellites A and B are traveling in the same plane in circular orbits around the earth at altitudes of 180 and 300 km, respectively. If at $t = 0$ the satellites are aligned as shown and knowing that the radius of the earth is $R = 6370$ km, determine when the satellites will next be radially aligned. (See information given in Probs. 11.153–11.154.)

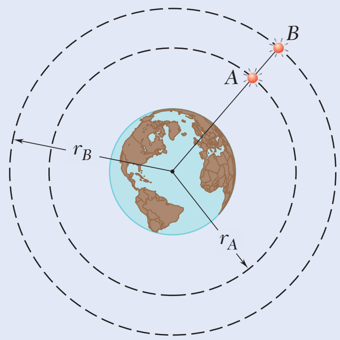


Fig. *P11.160*

11.161 The oscillation of rod OA about O is defined by the relation $\theta = (2/\pi)(\sin \pi t)$, where θ and t are expressed in radians and seconds, respectively. Collar B slides along the rod so that its distance from O is $r = \dfrac{625}{(t+4)}$ where r and t are expressed in mm and seconds, respectively. When $t = 1$ s, determine (*a*) the velocity of the collar, (*b*) the acceleration of the collar, (*c*) the acceleration of the collar relative to the rod.

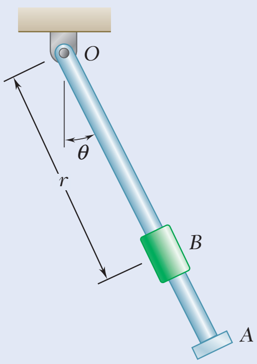


Fig. P11.161

11.162 The path of a particle P is a limaçon. The motion of the particle is defined by the relations $r = b(2 + \cos \pi t)$ and $\theta = \pi t$ where t and θ are expressed in seconds and radians, respectively. Determine (*a*) the velocity and the acceleration of the particle when $t = 2$ s, (*b*) the value of θ for which the magnitude of the velocity is maximum.

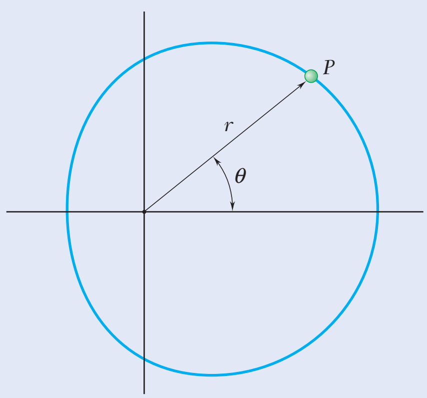


Fig. P11.162

11.163 During a parasailing ride, the boat is traveling at a constant 30 km/hr with a 200-m long tow line. At the instant shown, the angle between the line and the water is 30° and is increasing at a constant rate of 2°/s. Determine the velocity and acceleration of the parasailer at this instant.

Fig. P11.163

11.164 Pin P is attached to BC and slides freely in the slot of OA. Determine the rate of change $\dot{\theta}$ of the angle θ, knowing that BC moves at a constant speed v_0. Express your answer in terms of v_0, h, β, and θ.

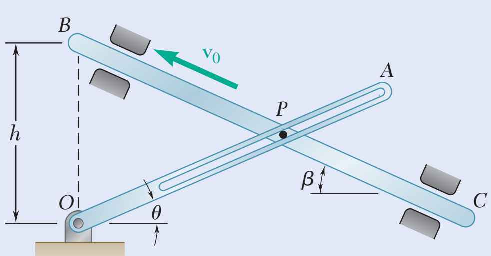


Fig. P11.164

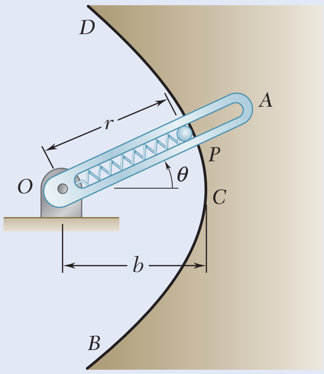


Fig. P11.165

11.165 As rod OA rotates, pin P moves along the parabola BCD. Knowing that the equation of this parabola is $r = 2b/(1 + \cos\theta)$ and that $\theta = kt$, determine the velocity and acceleration of P when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

11.166 The pin at B is free to slide along the circular slot DE and along the rotating rod OC. Assuming that the rod OC rotates at a constant rate $\dot{\theta}$, (*a*) show that the acceleration of pin B is of constant magnitude, (*b*) determine the direction of the acceleration of pin B.

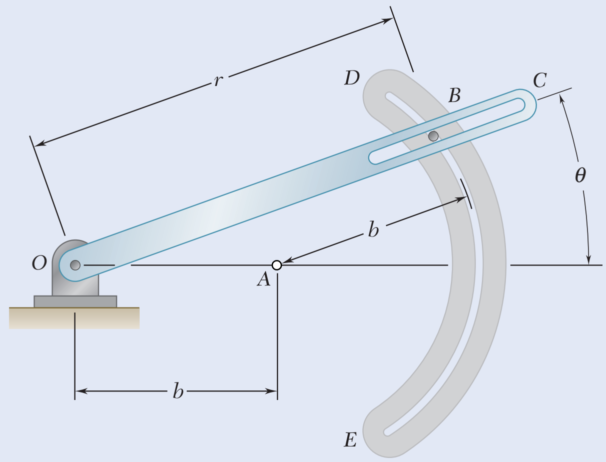


Fig. P11.166

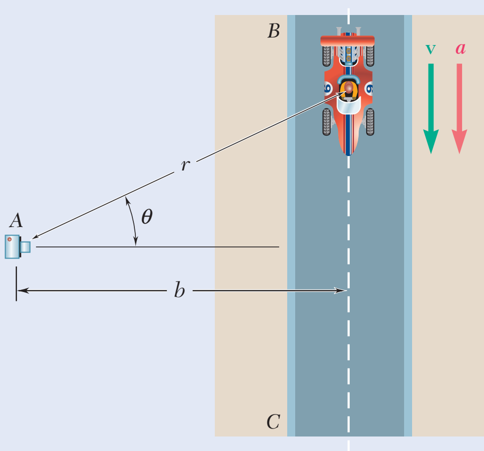


Fig. *P11.167*

11.167 To study the performance of a race car, a high-speed camera is positioned at point A. The camera is mounted on a mechanism which permits it to record the motion of the car as the car travels on straightaway BC. Determine (*a*) the speed of the car in terms of b, θ, and $\dot{\theta}$, (*b*) the magnitude of the acceleration in terms of b, θ, $\dot{\theta}$, and $\ddot{\theta}$.

11.168 After taking off, a helicopter climbs in a straight line at a constant angle β. Its flight is tracked by radar from point A. Determine the speed of the helicopter in terms of d, β, θ, and $\dot{\theta}$.

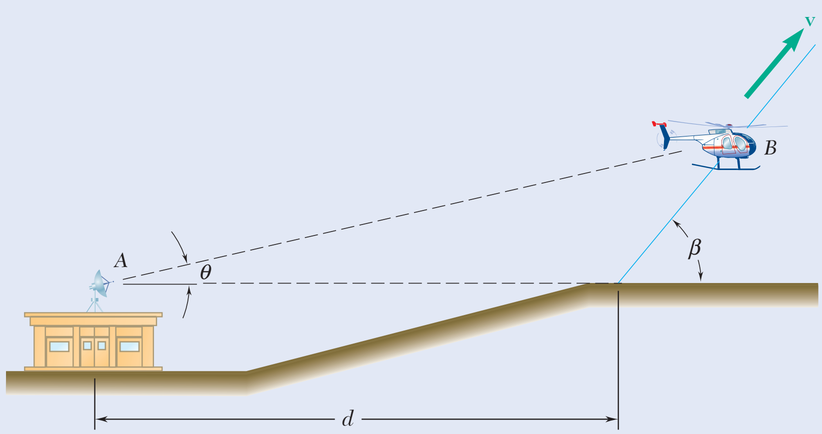


Fig. *P11.168*

11.169 At the bottom of a loop in the vertical plane an airplane has a horizontal velocity of 150 m/s and is speeding up at a rate of 25 m/s^2. The radius of curvature of the loop is 2000 m. The plane is being tracked by radar at O. What are the recorded values of $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$ for this instant?

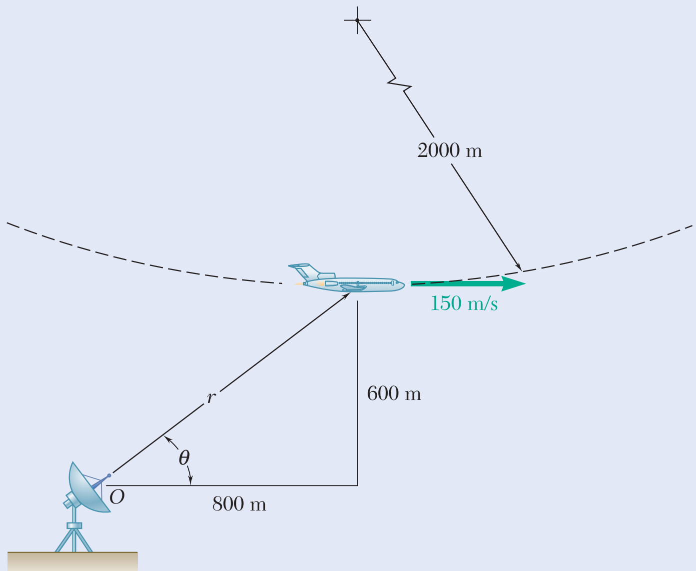


Fig. P11.169

11.170 Pin C is attached to rod BC and slides freely in the slot of rod OA which rotates at the constant rate ω. At the instant when $\beta = 60°$, determine (*a*) $\dot{r}$ and $\dot{\theta}$, (*b*) $\ddot{r}$ and $\ddot{\theta}$. Express your answers in terms of d and ω.

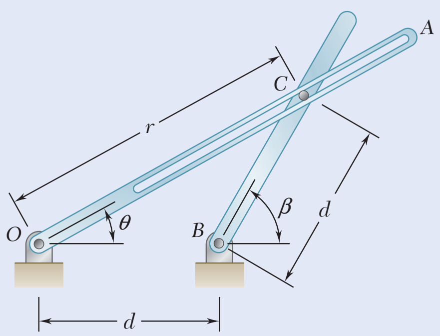


Fig. P11.170

11.171 For the race car of Prob. 11.167, it was found that it took 0.5 s for the car to travel from the position $\theta = 60°$ to the position $\theta = 35°$. Knowing that $b = 25$ m, determine the average speed of the car during the 0.5-s interval.

11.172 For the helicopter of Prob. 11.168, it was found that when the helicopter was at B, the distance and the angle of elevation of the helicopter were $r = 1000$ m and $\theta = 20°$, respectively. Four seconds later, the radar station sighted the helicopter at $r = 1100$ m and $\theta = 23.1°$. Determine the average speed and the angle of climb β of the helicopter during the 4-s interval.

11.173 and 11.174 A particle moves along the spiral shown. Determine the magnitude of the velocity of the particle in terms of b, θ, and $\dot{\theta}$.

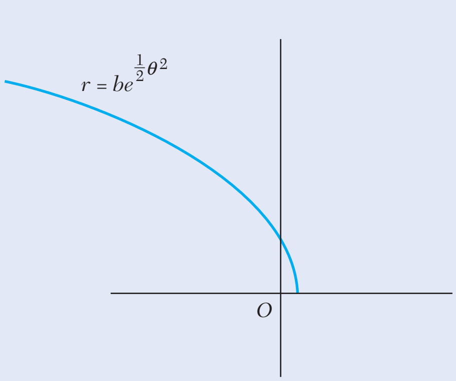


Fig. P11.173 and P11.175

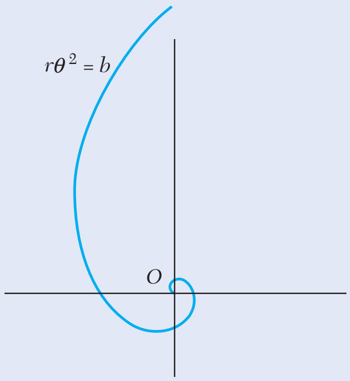


Fig. P11.174 and P11.176

11.175 and 11.176 A particle moves along the spiral shown. Knowing that $\dot{\theta}$ is constant and denoting this constant by ω, determine the magnitude of the acceleration of the particle in terms of b, θ, and $\dot{\theta}$.

11.177 The motion of a particle on the surface of a right circular cylinder is defined by the relations $R = A$, $\theta = 2\pi t$, and $z = B \sin 2\pi nt$, where A and B are constants and n is an integer. Determine the magnitudes of the velocity and acceleration of the particle at any time t.

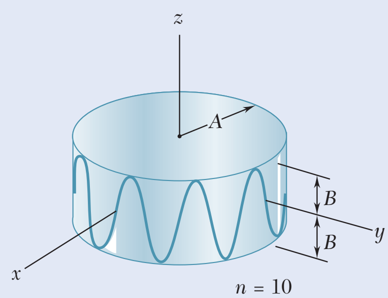


Fig. P11.177

11.178 Show that $\dot{r} = h\dot{\phi} \sin \theta$ knowing that at the instant shown, step AB of the step exerciser is rotating counterclockwise at a constant rate $\dot{\phi}$.

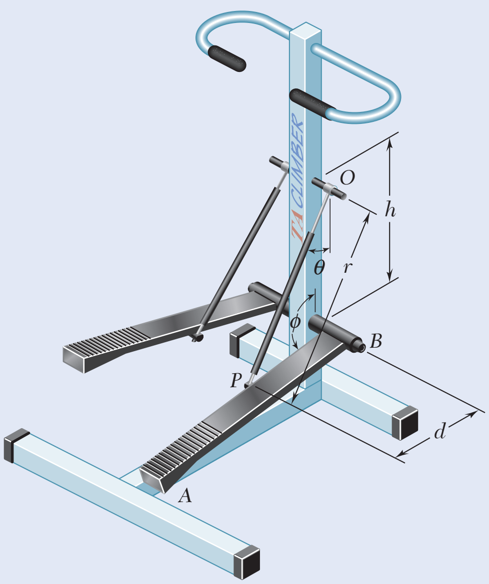


Fig. P11.178

11.179 The three-dimensional motion of a particle is defined by the relations $R = A(1 - e^{-t})$, $\theta = 2\pi t$, and $z = B(1 - e^{-t})$. Determine the magnitudes of the velocity and acceleration when (*a*) $t = 0$, (*b*) $t = \infty$.

***11.180** For the conic helix of Prob. 11.95, determine the angle that the osculating plane forms with the y axis.

***11.181** Determine the direction of the binormal of the path described by the particle of Prob. 11.96 when (*a*) $t = 0$, (*b*) $t = \pi/2$ s.

Review and Summary

Position Coordinate of a Particle in Rectilinear Motion

In the first half of this chapter, we analyzed the **rectilinear motion of a particle**, i.e., the motion of a particle along a straight line. To define the position P of the particle on that line, we chose a fixed origin O and a positive direction (Fig. 11.25). The distance x from O to P, with the appropriate sign, completely defines the position of the particle on the line and is called the **position coordinate** of the particle [Sec. 11.1A].

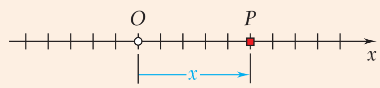


Fig. 11.25

Velocity and Acceleration in Rectilinear Motion

The **velocity** v of the particle was shown to be equal to the time derivative of the position coordinate x, so

$$v = \frac{dx}{dt} \tag{11.1}$$

And we obtained the **acceleration** a by differentiating v with respect to t, as

$$a = \frac{dv}{dt} \tag{11.2}$$

or

$$a = \frac{d^2x}{dt^2} \tag{11.3}$$

We also noted that a could be expressed as

$$a = v\frac{dv}{dx} \tag{11.4}$$

We observed that the velocity v and the acceleration a are represented by algebraic numbers that can be positive or negative. A positive value for v indicates that the particle moves in the positive direction, and a negative value shows that it moves in the negative direction. A positive value for a, however, may mean that the particle is truly accelerated (i.e., moves faster) in the positive direction or that it is decelerated (i.e., moves more slowly) in the negative direction. A negative value for a is subject to a similar interpretation [Sample Prob. 11.1].

Determination of the Velocity and Acceleration by Integration

In most problems, the conditions of motion of a particle are defined by the type of acceleration that the particle possesses and by the initial conditions [Sec. 11.1B]. Then we can obtain the velocity and position of the particle by integrating two of the equations (11.1) to (11.4). The selection of these equations depends upon the type of acceleration involved [Sample Probs. 11.2 through 11.4].

Uniform Rectilinear Motion

Two types of motion are frequently encountered. **Uniform rectilinear motion** [Sec. 11.2A], in which the velocity v of the particle is constant, is described by

$$x = x_0 + vt \tag{11.5}$$

Uniformly Accelerated Rectilinear Motion

Uniformly accelerated rectilinear motion [Sec. 11.2B], in which the acceleration a of the particle is constant, is described by

$$v = v_0 + at \tag{11.6}$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \tag{11.7}$$

$$v^2 = v_0^2 + 2a(x - x_0) \tag{11.8}$$

Relative Motion of Two Particles

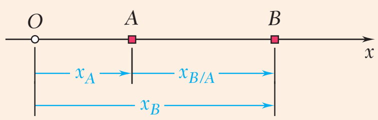


Fig. 11.26

When two particles A and B (such as two aircraft) move, we may wish to consider the **relative motion** of B with respect to A [Sec. 11.2C]. Denoting the **relative position coordinate** of B with respect to A by $x_{B/A}$ (Fig. 11.26), we have

$$x_B = x_A + x_{B/A} \tag{11.9}$$

Differentiating Eq. (11.9) twice with respect to t, we obtained successively

$$v_B = v_A + v_{B/A} \tag{11.10}$$

$$a_B = a_A + a_{B/A} \tag{11.11}$$

where $v_{B/A}$ and $a_{B/A}$ represent, respectively, the **relative velocity** and the **relative acceleration** of B with respect to A.

Dependent Motion

When several blocks are **connected by inextensible cords**, it is possible to write a linear relation between their position coordinates. We can then write similar relations between their velocities and between their accelerations, which we can use to analyze their motion [Sample Probs. 11.7 and 11.8].

Graphical Solutions

It is sometimes convenient to use a **graphical solution** for problems involving the rectilinear motion of a particle [Sec. 11.3]. The graphical solution most commonly used involves the x–t, v–t, and a–t curves [Sample Prob. 11.10]. It was shown at any given time t that

$$v = \text{slope of } x\text{–}t \text{ curve}$$

$$a = \text{slope of } v\text{–}t \text{ curve}$$

Also, over any given time interval from t_1 to t_2, we have

$$v_2 - v_1 = \text{area under } a\text{–}t \text{ curve}$$

$$x_2 - x_1 = \text{area under } v\text{–}t \text{ curve}$$

Position Vector and Velocity in Curvilinear Motion

In the second half of this chapter, we analyzed the **curvilinear motion of a particle**, i.e., the motion of a particle along a curved path. We defined the position P of the particle at a given time [Sec. 11.4A] by the **position vector r** joining the O of the coordinates and point P (Fig. 11.27). We defined the **velocity v** of the particle by the relation

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \tag{11.14}$$

The velocity is a **vector tangent to the path of the particle** with a magnitude v (called the **speed** of the particle) equal to the time derivative of the length s of the arc described by the particle. Thus,

$$v = \frac{ds}{dt} \tag{11.15}$$

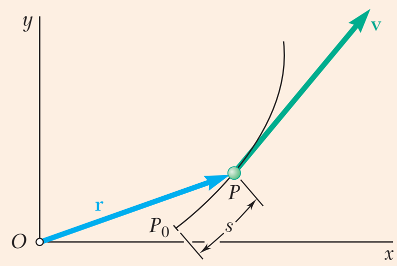


Fig. 11.27

Acceleration in Curvilinear Motion

We defined the **acceleration a** of the particle by the relation

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} \tag{11.17}$$

and we noted that, in general, *the acceleration is not tangent to the path of the particle*.

Derivative of a Vector Function

Before proceeding to the consideration of the components of velocity and acceleration, we reviewed the formal definition of the derivative of a vector function and established a few rules governing the differentiation of sums and products of vector functions. We then showed that the rate of change of a vector is the same with respect both to a fixed frame and to a frame in translation [Sec. 11.4B].

Rectangular Components of Velocity and Acceleration

Denoting the rectangular coordinates of a particle P by x, y, and z, we found that the rectangular components of the velocity and acceleration of P equal, respectively, the first and second derivatives with respect to t of the corresponding coordinates. Thus,

$$v_x = \dot{x} \qquad v_y = \dot{y} \qquad v_z = \dot{z} \tag{11.28}$$

$$a_x = \ddot{x} \qquad a_y = \ddot{y} \qquad a_z = \ddot{z} \tag{11.29}$$

Component Motions

When the component a_x of the acceleration depends only upon t, x, and/or v_x; when, similarly, a_y depends only upon t, y, and/or v_y; and a_z upon t, z, and/or v_z, Eq. (11.29) can be integrated independently. The analysis of the given curvilinear motion then reduces to the analysis of three independent rectilinear component motions [Sec. 11.4C]. This approach is particularly effective in the study of the motion of projectiles [Sample Probs. 11.10 and 11.11].

Relative Motion of Two Particles

For two particles A and B moving in space (Fig. 11.28), we considered the relative motion of B with respect to A, or more precisely, with respect to a moving frame attached to A and in translation with A [Sec. 11.4D]. Denoting the **relative position vector** of B with respect to A by $\mathbf{r}_{B/A}$ (Fig. 11.28), we have

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A} \tag{11.30}$$

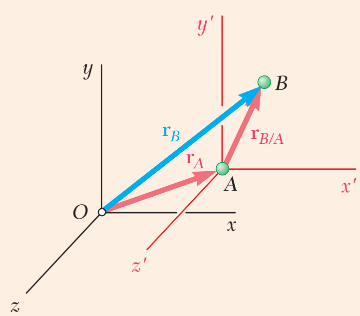


Fig. 11.28

Denoting the **relative velocity** and the **relative acceleration** of B with respect to A by $\mathbf{v}_{B/A}$ and $\mathbf{a}_{B/A}$, respectively, we also showed that

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \tag{11.32}$$

and

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} \tag{11.33}$$

Tangential and Normal Components

It is sometimes convenient to resolve the velocity and acceleration of a particle P into components other than the rectangular x, y, and z components. For a particle P moving along a path contained in a plane, we attached to P unit vectors $\mathbf{e}_t$ tangent to the path and $\mathbf{e}_n$ normal to the path and directed toward the center of curvature of the path [Sec. 11.5A]. We then express the velocity and acceleration of the particle in terms of tangential and normal components. We have

$$\mathbf{v} = v\mathbf{e}_t \tag{11.35}$$

and

$$\mathbf{a} = \frac{dv}{dt}\mathbf{e}_t + \frac{v^2}{\rho}\mathbf{e}_n \tag{11.38}$$

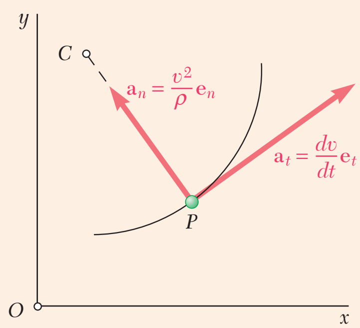


Fig. 11.29

where v is the speed of the particle and ρ is the radius of curvature of its path [Sample Probs. 11.16, ,and 11.17]. We observed that, while the velocity $\mathbf{v}$ is directed along the tangent to the path, the acceleration $\mathbf{a}$ consists of a component $\mathbf{a}_t$ directed along the tangent to the path and a component $\mathbf{a}_n$ directed toward the center of curvature of the path (Fig. 11.29).

Motion Along a Space Curve

For a particle P moving along a space curve, we defined the plane that most closely fits the curve in the neighborhood of P as the **osculating plane**. This plane contains the unit vectors $\mathbf{e}_t$ and $\mathbf{e}_n$ that define the tangent and principal normal to the curve, respectively. The unit vector $\mathbf{e}_b$, which is perpendicular to the osculating plane, defines the **binormal**.

Radial and Transverse Components

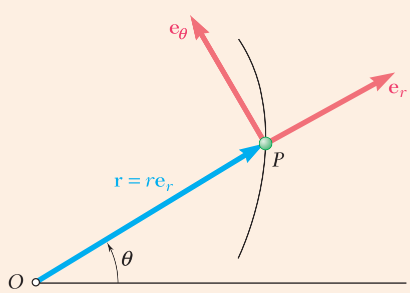


Fig. 11.30

When the position of a particle P moving in a plane is defined by its polar coordinates r and θ, it is convenient to use radial and transverse components directed, respectively, along the position vector $\mathbf{r}$ of the particle and in the direction obtained by rotating $\mathbf{r}$ through 90° counterclockwise [Sec. 11.5B]. We attached to P unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$ directed in the radial and transverse directions, respectively (Fig. 11.30). We then expressed the velocity and acceleration of the particle in terms of radial and transverse components as

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta \tag{11.42}$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \tag{11.43}$$

where dots are used to indicate differentiation with respect to time. The scalar components of the velocity and acceleration in the radial and transverse directions are therefore

$$v_r = \dot{r} \qquad v_\theta = r\dot{\theta} \tag{11.44}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \tag{11.45}$$

It is important to note that a_r is *not* equal to the time derivative of v_r and that a_θ is *not* equal to the time derivative of v_θ [Sample Probs. 11.18, 11.19, and 11.20].

This chapter ended with a discussion of the use of cylindrical coordinates to define the position and motion of a particle in space.

Review Problems

11.182 The motion of a particle is defined by the relation $x = 2t^3 - 15t^2 + 24t + 4$, where x and t are expressed in meters and seconds, respectively. Determine (*a*) when the velocity is zero, (*b*) the position and the total distance traveled when the acceleration is zero.

11.183 A drag car starts from rest and moves down the racetrack with an acceleration defined by $a = 50 - 10t$, where a and t are in m/s^2 and seconds, respectively. After reaching a speed of 125 m/s, a parachute is deployed to help slow down the dragster. Knowing that this deceleration is defined by the relationship $a = -0.02v^2$, where v is the velocity in m/s, determine (*a*) the total time from the beginning of the race until the car slows back down to 10 m/s, (*b*) the total distance the car travels during this time.

11.184 A particle moves in a straight line with the acceleration shown in the figure. Knowing that the particle starts from the origin with $v_0 = -2$ m/s, (*a*) construct the v–t and x–t curves for $0 < t < 18$ s, (*b*) determine the position and the velocity of the particle and the total distance traveled when $t = 18$ s.

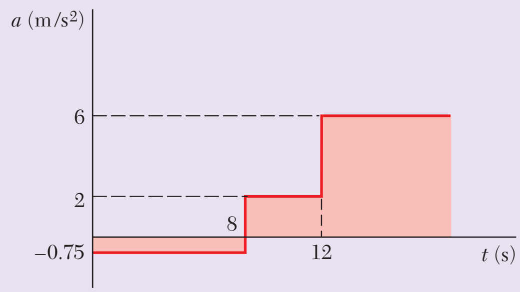


Fig. *P11.184*

11.185 The velocities of commuter trains A and B are as shown. Knowing that the speed of each train is constant and that B reaches the crossing 10 min after A passed through the same crossing, determine (*a*) the relative velocity of B with respect to A, (*b*) the distance between the fronts of the engines 3 min after A passed through the crossing.

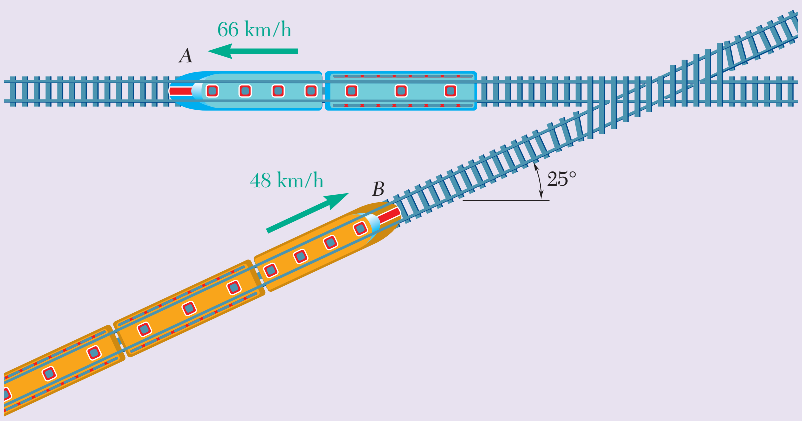


Fig. P11.185

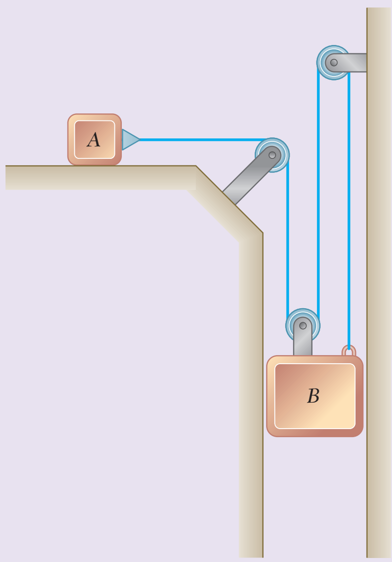


Fig. *P11.186*

11.186 Block B starts from rest and moves downward with a constant acceleration. Knowing that after slider block A has moved 400 mm is velocity is 4 m/s, determine (*a*) the acceleration of A and B, (*b*) the velocity and change in position of B after 2 s.

11.187 Collar A starts from rest at $t = 0$ and moves downward with a constant acceleration of 175 mm/s^2. Collar B moves upward with a constant acceleration, and its initial velocity is 200 mm/s. Knowing that collar B moves through 500 mm between $t = 0$ and $t = 2$ s, determine (*a*) the accelerations of collar B and block C, (*b*) the time at which the velocity of block C is zero, (*c*) the distance through which block C will have moved at that time.

11.188 A golfer hits a ball with an initial velocity of magnitude v_0 at an angle α with the horizontal. Knowing that the ball must clear the tops of two trees and land as close as possible to the flag, determine v_0 and the distance d when the golfer uses (*a*) a six-iron with $\alpha = 31°$, (*b*) a five-iron with $\alpha = 27°$.

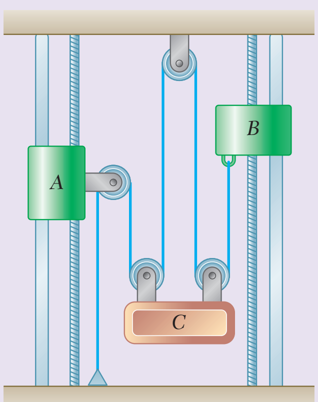


Fig. P11.187

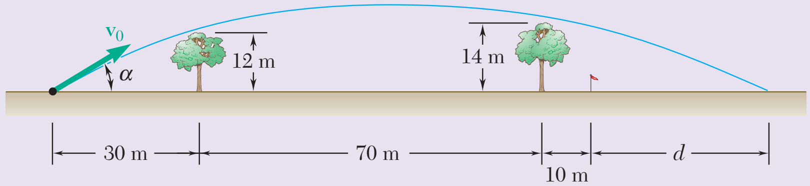


Fig. P11.188

11.189 As the truck shown begins to back up with a constant acceleration of 1.2 m/s^2, the outer section B of its boom starts to retract with a constant acceleration of 0.48 m/s^2 relative to the truck. Determine (*a*) the acceleration of section B, (*b*) the velocity of section B when $t = 2$ s.

11.190 A velodrome is a specially designed track used in bicycle racing that has constant radius curves at each end. Knowing that a rider starts from rest $a_t = (11.46 - 0.01878v^2)$ m/s^2, determine her acceleration at point B.

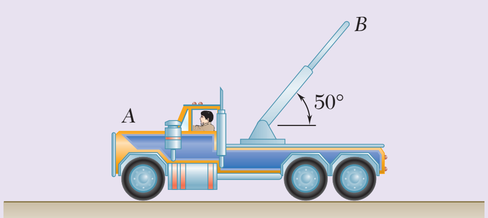


Fig. P11.189

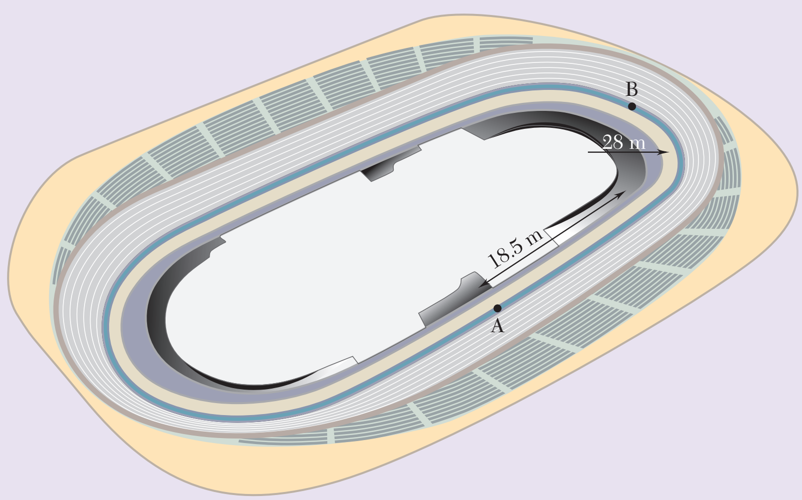


Fig. P11.190

11.191 Sand is discharged at A from a conveyor belt and falls onto the top of a stockpile at B. Knowing that the conveyor belt forms an angle $\alpha = 25°$ with the horizontal, determine (*a*) the speed v_0 of the belt, (*b*) the radius of curvature of the trajectory described by the sand at point B.

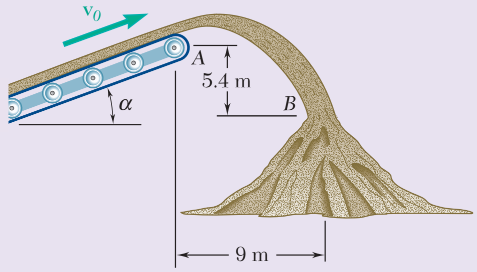


Fig. P11.191

11.192 The end point B of a boom is originally 5 m from fixed point A when the driver starts to retract the boom with a constant radial acceleration of $\ddot{r} = -1.0\text{ m/s}^2$ and lower it with a constant angular acceleration $\ddot{\theta} = -0.5$ rad/s rad/s². At $t = 2$ s, determine (*a*) the velocity of point B, (*b*) the acceleration of point B, (*c*) the radius of curvature of the path.

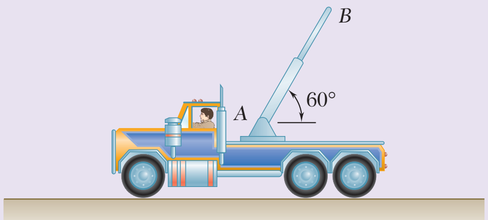


Fig. *P11.192*

11.193 A telemetry system is used to quantify kinematic values of a ski jumper immediately before she leaves the ramp. According to the system $r = 150$ m, $\dot{r} = -31.5\text{ m/s}$, $\ddot{r} = -3\text{ m/s}^2$, $\theta = 25°$, $\dot{\theta} = 0.07\text{ rad/s}$, $\ddot{\theta} = 0.06\text{ rad/s}^2$. Determine (*a*) the velocity of the skier immediately before she leaves the jump, (*b*) the acceleration of the skier at this instant, (*c*) the distance of the jump d neglecting lift and air resistance.

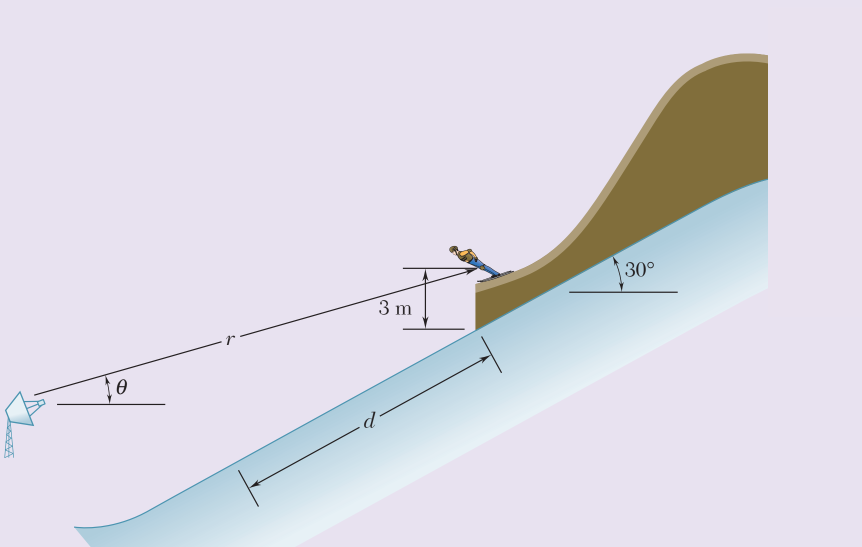


Fig. *P11.193*