Bode Plot

Bode plot consists of two graphs, one is logarithm of magnitude of $F(j\omega)$ and the other is phase angle of $F(j\omega)$, both plotted against frequency in logarithmic scale.

The standard representation of the logarithmic magnitude of $F(j\omega)$ is $20 \log |F(j\omega)|$, where the base of logarithm is 10. The unit of magnitude $20 \log |F(j\omega)|$ is decibel, abbreviated as db. The curves are normally drawn on semilog paper using log scale for frequency and linear scale for magnitude in db and phase in degrees. The main advantage of using logarithmic plot is that multiplication of magnitudes can be converted into addition.

In Bode plot, frequency ratios are expressed in terms of octaves or decades. An octal is a frequency band from ω_1 to $2\,\omega_1$, where ω_1 is any frequency. A decade is a frequency band from ω_1 to $10\,\omega_1$. On the logarithmic scale of semilog paper, any given frequency ratio can be represented by same horizontal distance. For example, the horizontal distances from $\omega = 1$ to $\omega = 10$ is equal to that from $\omega = 5$ to $\omega = 50$.

Consider a transfer function $F(s)$ given by,

$$F(s) = \frac{H\,s(s+a)}{(s+b)(s^2+\alpha\;s+\beta^2)}$$

$$= \frac{H \cdot a \cdot s\left(1+\dfrac{s}{a}\right)}{b \cdot \beta^2 \cdot \left(1+\dfrac{s}{b}\right)\left(1+\dfrac{\alpha}{\beta^2}s+\dfrac{1}{\beta^2}s^2\right)}$$

$$= \frac{K s\left(1+\frac{s}{a}\right)}{\left(1+\frac{s}{b}\right)\left(1+\frac{\alpha}{\beta^2}s+\frac{1}{\beta^2}s^2\right)}$$

Putting $s = j\omega$,

$$F(j\omega) = \frac{K\, j\omega\left(1+\frac{j\omega}{a}\right)}{\left(1+\frac{j\omega}{b}\right)\left[\left(1-\frac{\omega^2}{\beta^2}\right)+j\frac{\alpha\,\omega}{\beta^2}\right]}$$

The magnitude of $F(j\omega)$ in db is written as,

$$20\log|F(j\omega)| = 20\log K + 20\log|j\omega| + 20\log\left|1+\frac{j\omega}{a}\right| - 20\log\left|1+\frac{j\omega}{b}\right| - 20\log\left|\left(\frac{1-\omega^2}{\beta^2}\right) + j\,\frac{\alpha\,\omega}{\beta^2}\right|$$

The phase angle of $F(j\omega)$ is written as,

$$\phi(\omega) = \angle F(j\omega) = \angle K + \angle j\omega + \angle(1+\frac{j\omega}{a}) - \angle(1+\frac{j\omega}{b}) - \angle\left[\left(1-\frac{\omega^2}{\beta^2}\right)+j\frac{\alpha\,\omega}{\beta^2}\right]$$

The basic factors that frequently occur in any function $F(j\omega)$ are,

(a) Constant K

(b) Root at the origin, $j\omega$

(c) Simple real root, $1 + \frac{j\omega}{a}$

(d) Complex conjugate root $\left[\left(1-\frac{\omega^2}{\beta^2}\right)+j\frac{\alpha\,\omega}{\beta^2}\right]$

If these factors are in the numerator, their magnitudes in db and phase angle in degrees carry positive signs. If these factors belong to the denominator, their magnitudes in db and phase angle in degrees carry negative signs.

(1) **Constant K:** $F(j\omega) = K$

The magnitude of K in db is given by,

$$20\log|F(j\omega)| = 20\log K = M \text{ db}$$

M is positive if $K > 1$ and negative if $K < 1$. Thus, the magnitude plot for constant K is a straight line at the magnitude of 20 log K db.

The phase angle $\phi(\omega)$ is either $0°$ or $-180°$ depending on whether K is positive or negative. The magnitude and phase for constant K are shown in Fig. 1(a).

(2) **Factor jω:** $F(j\omega) = j\omega$

The magnitude of jω in db is given by,

$$20 \log |F(j\omega)| = 20 \log |j\omega| = 20 \log \omega \text{ db}$$

Thus, the magnitude plot for jω is a straight line with slope of 20 db/decade passing through 0 db at $\omega = 1$. The phase angle $\phi(\omega)$ of jω is given by,

$$\phi(\omega) = 90°$$

For factor $(j\omega)^n$, magnitude in db is given by,

$$20 \log |(j\omega)^n| = n \times 20 \log |j\omega|$$
$$= 20\, n \log \omega \text{ db}$$

Thus, magnitude plot of $(j\omega)^n$ is a straight line with slope of 20 n db/decade passing through 0 db at $\omega = 1$. The phase angle of $(j\omega)^n$ is equal to 90° n for all ω. The magnitude plot and phase plots for $(j\omega)^n$ are shown in Fig. 1(b).

(3) **Factor $\left(1 + \frac{j\omega}{a}\right)$:** $F(j\omega) = 1 + \frac{j\omega}{a}$

The magnitude of $F(j\omega)$ in db is given by,

$$20 \log |F(j\omega)| = 20 \log \left|1 + \frac{j\omega}{a}\right|$$
$$= 20 \log \sqrt{1 + \frac{\omega^2}{a^2}} \text{ db}$$

For low frequencies i.e. $\frac{\omega}{a} << 1$,

$$20 \log |F(j\omega)| = 20 \log 1 = 0 \text{ db}$$

For high frequencies i.e. $\frac{\omega}{a} >> 1$,

$$20 \log |F(j\omega)| = 20 \log \frac{\omega}{a} \text{ db}$$

Thus, magnitude plot can be approximated by two straight line asymptotes, one a straight line at 0 db for the frequency range $0 < \omega < a$ and other a straight line with slope of 20 db/dec for frequency range $a < \omega < \infty$. The frequency at which the two asymptotes meet is called corner or break frequency. The phase angle of $\left(1 + \frac{j\omega}{a}\right)$ is given by,

$$\phi(\omega) = \tan^{-1} \frac{\omega}{a}$$

At zero frequency, the phase angle is 0°. At the corner frequency i.e. at $\omega = a, \phi(\omega) = \tan^{-1} \frac{\omega}{a} = 45°$ At infinity, the phase angle is 90°. Thus, phase angle varies from 0° to 90°.

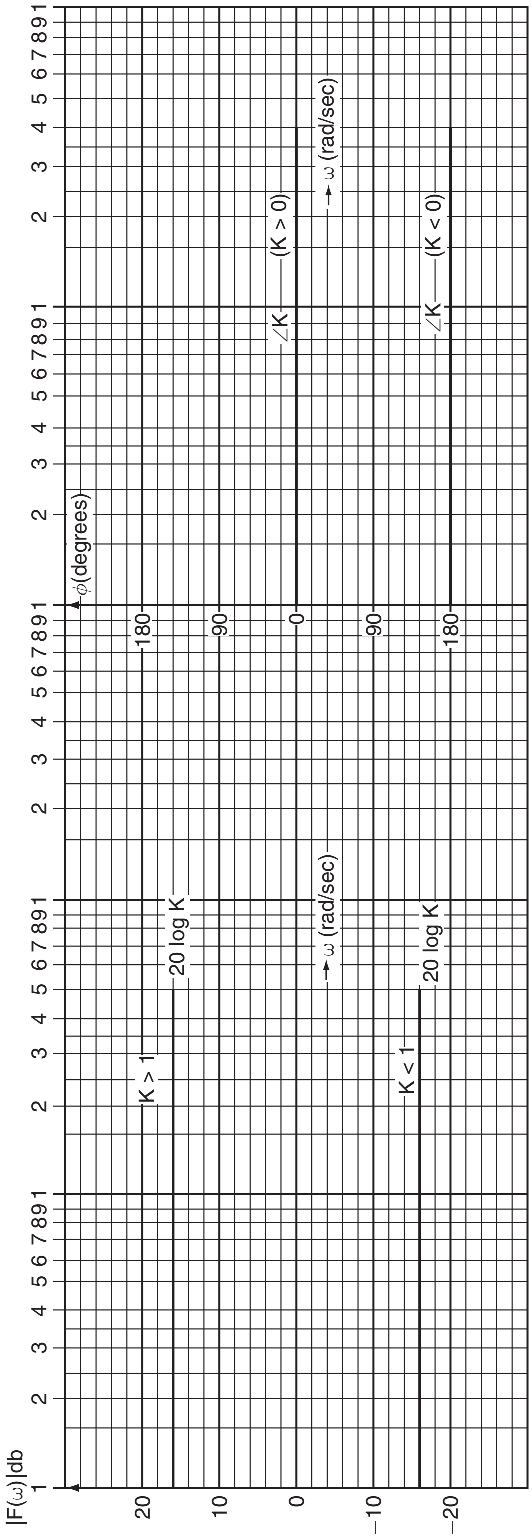

|F(ω)|db
φ(degrees)
→ ω (rad/sec)
20 log K
K > 1
K < 1
∠K (K > 0)
∠K (K < 0)
20
10
0
–10
–20
180
90
0
90
180

Fig. 1(a)

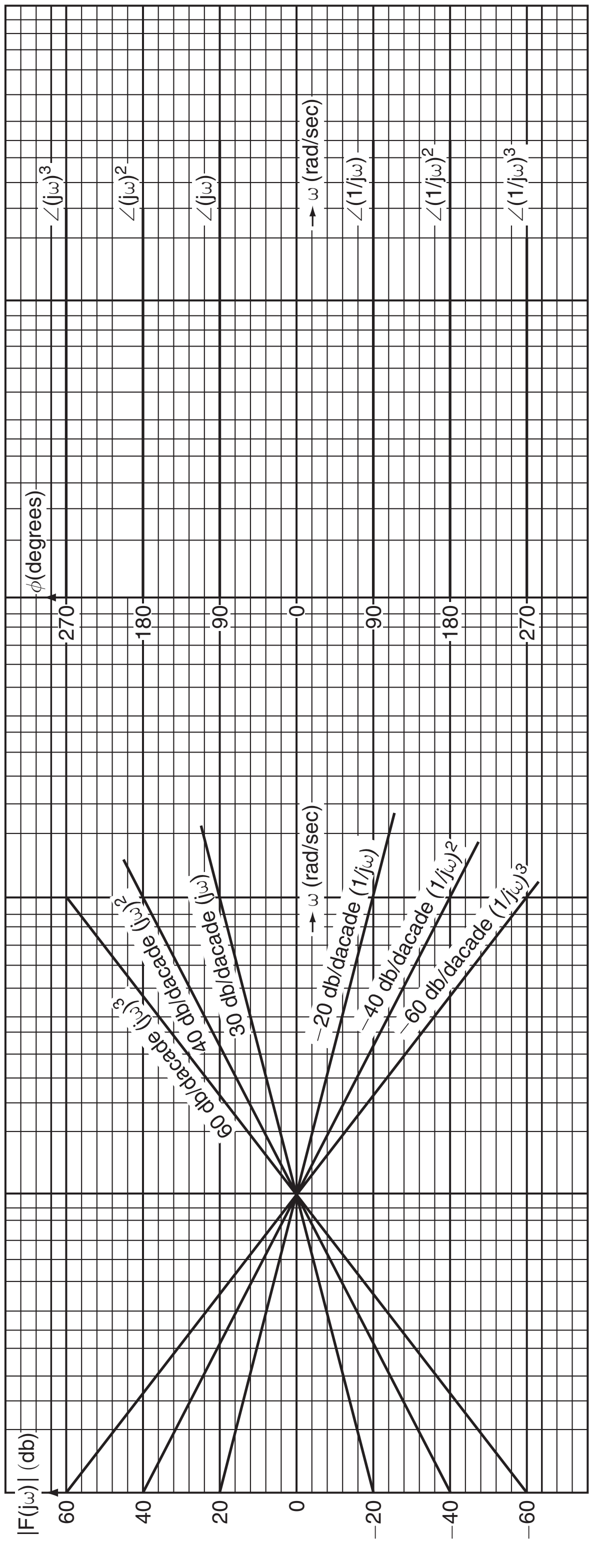

|F(jω)| (db)
φ(degrees)
→ ω (rad/sec)
∠(jω)³
∠(jω)²
∠(jω)
∠(1/jω)
∠(1/jω)²
∠(1/jω)³
60 db/dacade (jω)³
40 db/dacade (jω)²
30 db/dacade (jω)
–20 db/dacade (1/jω)
–40 db/dacade (1/jω)²
–60 db/dacade (1/jω)3
60
40
20
0
–20
–40
–60
270
180
90
0
90
180
270

Fig. 1(b)

Similarly for the factor $(1 + \frac{j\omega}{a})^n$, the magnitude plot can be approximated by two straight line asymptotes, one a straight line at 0 db for $0 < \omega < a$ and other a straight line with slope of 20 n db/decade for frequency range $a < \omega < \infty$. The phase angle will be n times $\tan^{-1}\frac{\omega}{a}$.

The magnitude and phase plots for function $(1 + j\omega)$ and $\frac{1}{(1+j\omega)}$ shown in Fig. 2.

(4) **Quadratic factor** $\left[\left(1 - \frac{\omega^2}{\beta^2}\right) + j\frac{\alpha\,\omega}{\beta^2}\right]$**:** $F(j\omega) = 1 - \frac{\omega^2}{\beta^2} + j\frac{\alpha\omega}{\beta^2}$

The magnitude of $F(j\omega)$ in db is given by,

$$20\log|F(j\omega)| = 20\log\left|1 - \frac{\omega^2}{\beta^2} + j\frac{\alpha\,\omega}{\beta^2}\right|$$

$$= 20\log\sqrt{\left(1 - \frac{\omega^2}{\beta^2}\right)^2 + \left(\frac{\alpha\,\omega}{\beta^2}\right)^2}$$

For low frequencies i.e. $\frac{\omega}{\beta} << 1$,

$20\log|F(j\omega)| = 20\log 1 = 0$ db

For high frequencies i.e. $\frac{\omega}{\beta} >> 1$,

$$20\log|F(j\omega)| = 20\log\frac{\omega^2}{\beta^2}$$

$$= 40\log\frac{\omega}{\beta}\,\text{db}$$

Thus, magnitude plot can be approximated by two straight line asymptotes, one a straight line at 0 db for the frequency range $0 < \omega < \beta$ and other a straight line with slope of 40 db/decade. For frequency range $\beta < \omega < \infty$, the corner frequency is at $\omega = \beta$.

The phase angle of $F(j\omega)$ is given by,

$$\phi(\omega) = \tan^{-1}\frac{\frac{\alpha\omega}{\beta^2}}{1 - \frac{\omega^2}{\beta^2}}$$

$$= \tan^{-1}\left(\frac{\alpha\omega}{\beta^2 - \omega^2}\right)$$

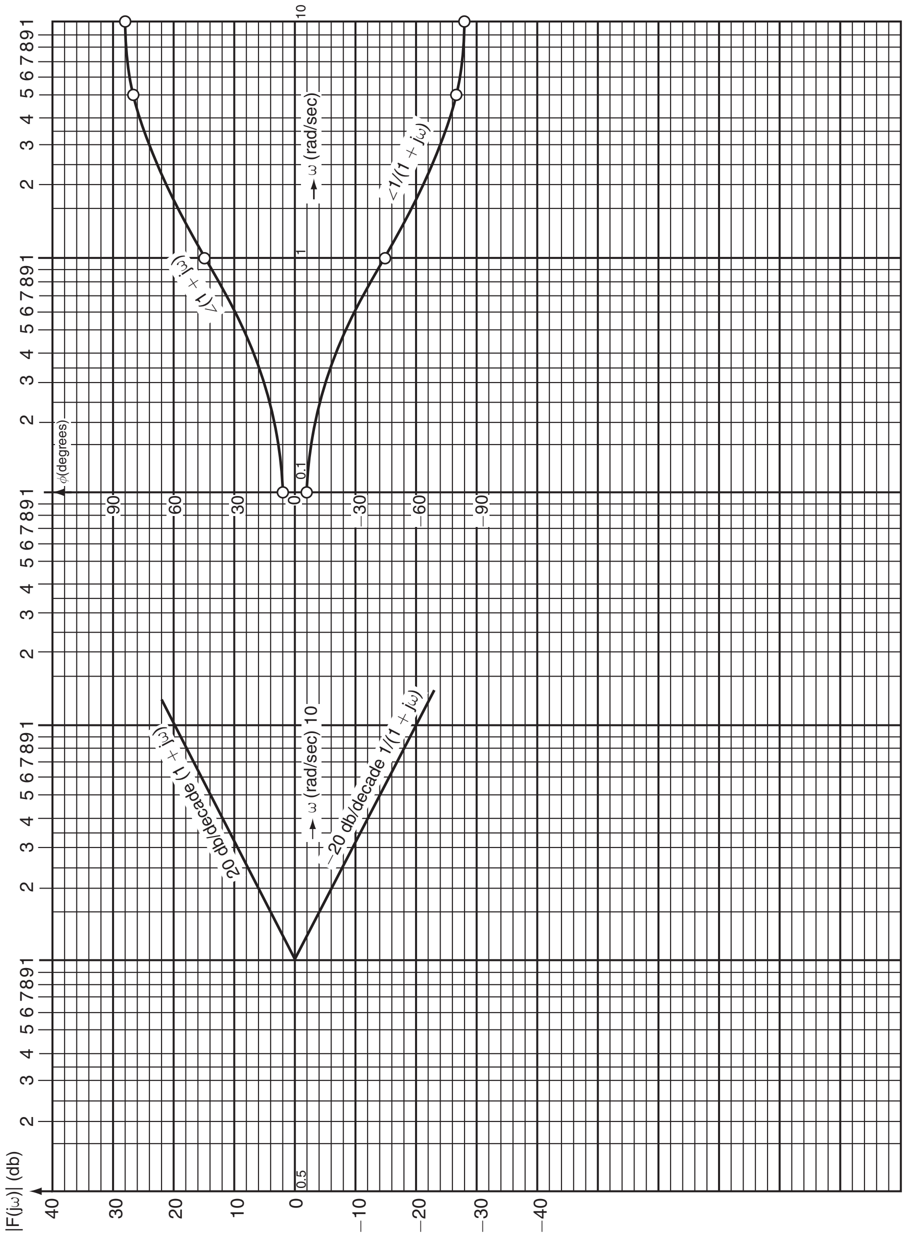


Fig. 2

GAIN MARGIN AND PHASE MARGIN

Gain Margin: It is the factor by which the gain can be increased to bring the system to the verge of instability. Gain margin is defined as the reciprocal of the gain at the frequency at which the phase angle becomes $-180°$. The frequency at which the phase angle is $-180°$ is called phase cross over frequency.

$$\text{Gain margin} = \frac{1}{|F(j\omega)|}$$

In terms of decibel,

Gain margin (db) = $-20 \log |F(j\omega)|$

Phase Margin: It is that amount of additional phase lag at the gain crossover frequency required to being the system to the verge of instability. The gain cross over frequency is the frequency at which $|F(j\omega)|$, the magnitude of the function, is unity. The phase margin is 180° plus the phase angle of the transfer function at the gain cross over frequency.

$$\text{Phase margin} = 180^\circ + \phi$$

1. *Draw the Bode plot for the function*

$$F(s) = \frac{10(s+10)}{s(s+2)(s+5)}$$

Calculate gain margin and phase margin.

Step 1: Write F(s) in standard form.

$$F(s) = \frac{10\times10\left(1+\frac{s}{10}\right)}{2\times5\times s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{5}\right)}$$

$$= \frac{10\left(1+\frac{s}{10}\right)}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{5}\right)}$$

Putting $s = j\omega$,

$$F(j\omega) = \frac{10\left(1+\frac{j\omega}{10}\right)}{j\omega\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{5}\right)}$$

$$20 \log|F(j\omega)| = 20 \log 10 - 20 \log|j\omega| - 20 \log\left|1+\frac{j\omega}{2}\right| - 20 \log\left|1+\frac{j\omega}{5}\right| + 20 \log\left|1+\frac{j\omega}{10}\right|$$

Step 2: Write down each factor in order of their occurrence as frequency increase in the table.

No.	Factor	Corner frequency	Magnitude characteristic
1	10	–	Straight line of magnitude 20 log 10 = 20 db
2	$\frac{1}{j\omega}$	–	Straight line of slope −20 db/decade passing through 0 db at $\omega = 1$.
3	$\frac{1}{1+\frac{j\omega}{2}}$	2	Straight line of 0 db for $\omega < 2$, straight line of slope −20 db/decade for $\omega > 2$.
4	$\frac{1}{1+\frac{j\omega}{5}}$	5	Straight line of 0 db for $\omega < 5$, straight line of slope −20 db/decade for $\omega > 5$.
5	$1+\frac{j\omega}{10}$	10	Straight line of 0 db for $\omega < 10$, straight line of slope 20 db/decade for $\omega > 10$.

Step 3: Draw all the factors clearly on semilog paper.

Step 4: Add all the factors in following manner given below.

(i) We start with left most point. The factor 10 raises the magnitude curve of factor $\frac{1}{j\omega}$ by the amount, 20 log 10 = 20 db. It shifts the plot of $\frac{1}{j\omega}$ to 40 db with same slope – 20 db/dec.

(ii) Let us now add the plot of the factor $\frac{1}{\left(1+\frac{j\omega}{2}\right)}$ corresponding to the lowest corner frequency $\omega = 2$. Since this factor contributes 0 db for $\omega < 2$, the resultant plot upto $\omega = 2$ is same as that of the combination of 10 and $\frac{1}{j\omega}$. From $\omega > 2$, this factor contributes – 20 db/decade such that resultant plot of these three factors is the straight line of slope (– 20) + (– 20) = – 40 db/decade upto next corner frequency $\omega = 5$.

(iii) Above $\omega = 5$, the factor $\frac{1}{\left(1+\frac{j\omega}{5}\right)}$ is effective. This gives rise to a straight line of slope – 20 db/decade for $\omega > 5$, which when added results in a straight line with a slope of (– 40) + (– 20) = – 60 db/decade from $\omega = 5$ to next corner frequency $\omega = 10$.

(iv) Above $\omega = \frac{5}{10}$, the plot of $\left(1+\frac{j\omega}{10}\right)$ is to be added. This factor gives rise to a straight line of slope 20 db/decade for $\omega > 10$, which when added results in a straight line having a slope of $(-60) + 20 = -40$ db/decade from $\omega = 10$ to $\omega = \infty$.

Step 5: Draw the phase plot with the help of table drawn for phase angle $\phi(\omega)$.

$$\phi(\omega) = 0 - 90° - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{5} + \tan^{-1}\frac{\omega}{10}$$

$$= \tan^{-1}\frac{\omega}{10} - \left(\tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{5} + 90°\right)$$

ω	$\tan^{-1}\frac{\omega}{2}$	$\tan^{-1}\frac{\omega}{5}$	$\tan^{-1}\frac{\omega}{10}$	$\phi(\omega)$
0.1	2.86°	1.15°	0.57°	−93.44°
1	26.57°	11.31°	5.71°	− 122.17°
2	45°	21.8°	11.31°	− 145.49°
5	68.19°	45°	26.57°	− 176.62°
10	78.69°	63.43°	45°	− 187.12°
100	88.85°	87.14°	84.29°	− 181.7°

Magnitude and phase plots, drawn on semilog paper is shown in Fig. 3.

Phase Margin: Unity gain occurs at $\omega = 4.6$ rad/sec. This is gain cross over frequency. Phase corresponding to $\omega = 4.6$ rad/sec is $-171°$.

Phase margin $= 180° + \phi = 180° - 171° = 9°$

Gain Margin: Phase plot has phase of $-180°$ at $\omega = 6$ rad/sec

At $\omega_p = 6$ rad/sec, gain margin = 6 db

2. *Sketch the Bode plot for the following transfer function,*

$$F(s) = \frac{20\,s}{(s+1)\,(s+10)}$$

Step 1: Write F(s) in standard form.

$$F(s) = \frac{20\text{ s}}{10\,(1+s)\left(1+\frac{s}{10}\right)}$$

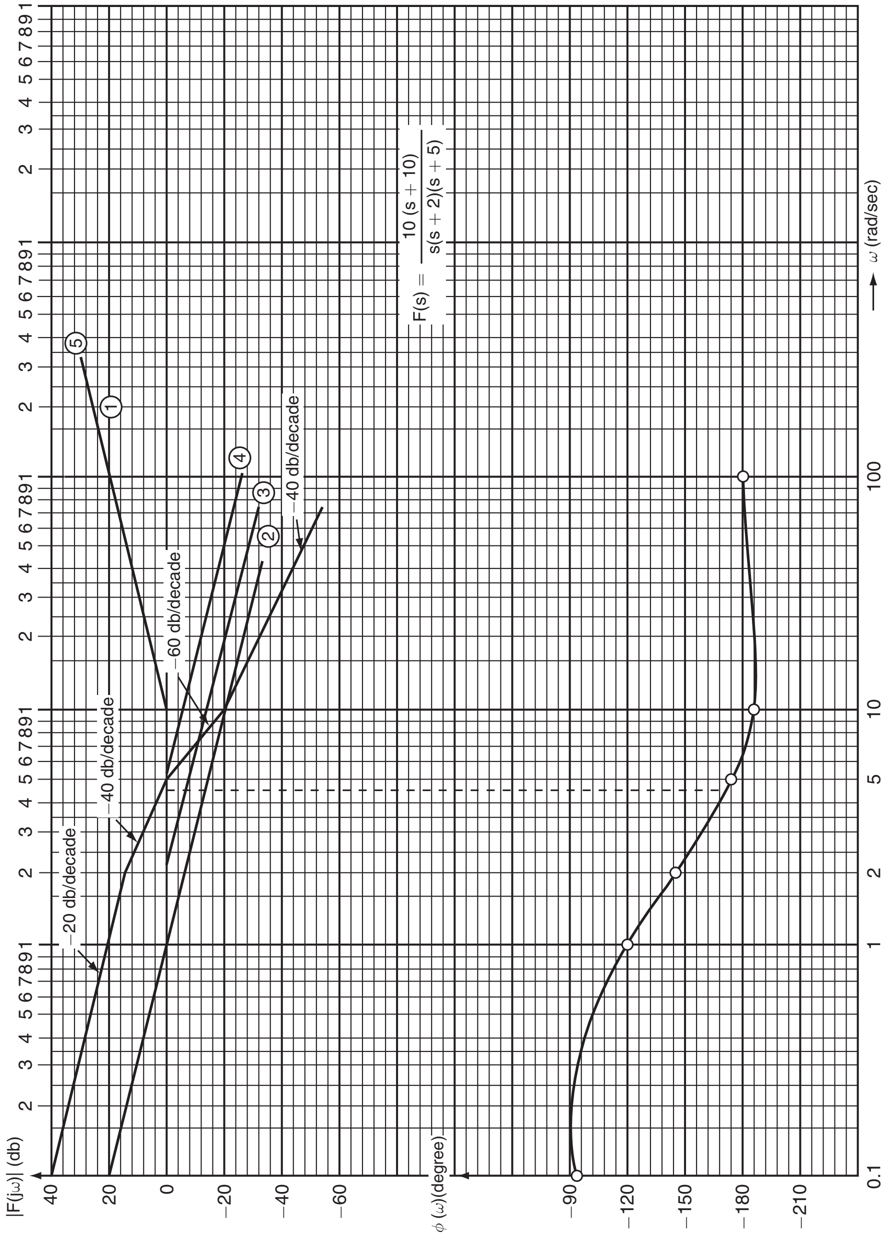


Fig. 3

$$= \frac{2\,s}{(1+s)\left(1+\dfrac{s}{10}\right)}$$

Putting s = jω,

$$F(j\omega) = \frac{2\,j\omega}{(1+j\omega)\left(1+\frac{j\omega}{10}\right)}$$

$$20\log|F(j\omega)| = 20\log 2 + 20\log|j\omega| - 20\log|1+j\omega| - 20\log\left|\left(1+\frac{j\omega}{10}\right)\right|$$

Step 2: Write down each factor in the table.

No.	Factor	Corner frequency	Magnitude characteristic
1	2	–	Straight line of magnitude 20 log 2 = 6.02 db
2	$j\omega$	–	Straight line of slope 20 db/decade passing through 0 db at $\omega = 1$.
3	$\frac{1}{1+j\omega}$	1	Straight line of 0 db for $\omega < 1$, straight line of slope -20 db/decade for $\omega > 1$.
4	$\frac{1}{1+\frac{j\omega}{10}}$	10	Straight line of 0 db for $\omega < 10$, straight line of slope -20 db/decade for $\omega > 10$.

Step 3: Draw all the factors clearly on semilog paper.

Step 4: Add all the factors in the following manner given below:

(i) We start with left most point. The factor 2 raises the magnitude curve of factor $j\omega$ by the amount 20 log 2 = 6.02 db. Hence plot of $j\omega$ starts with the point -14 db approximately having same slope 20 db/decade.

(ii) Now plot of the factor $\frac{1}{1+j\omega}$ corresponding to corner frequency $\omega = 1$ is added. Since this factor contributes 0 db for $\omega < 1$, the resultant plot upto $\omega = 1$ is same as that of the combination of 2 and $j\omega$. From $\omega = 1$, this factor contributes -20 db/decade such that resultant plot of these three factors is the straight line of slope $(-20) + 20 = 0$ db/decade upto next corner frequency $\omega = 10$.

(iii) Above $\omega = 10$, the plot of $\frac{1}{1+\frac{j\omega}{10}}$ is to be added. This factor gives rise to a straight line of slope -20 db/decade for $\omega > 10$, which when added results in a straight line having a slope of $0 + (-20) = -20$ db/decade from $\omega = 10$ to $\omega = \infty$.

Step 5: Draw the phase plot with the help of table for $\phi(\omega)$. For any arbitrary value of ω.

$$\phi(\omega) = 0 + 90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{10}$$

$$= 90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{10}$$

ω	$\tan^{-1}\omega$	$\tan^{-1}\frac{\omega}{10}$	$\phi(\omega)$
0.1	5.71°	0.57°	83.72°
1	45°	5.71°	39.29°
5	78.69°	26.57°	–15.26°
10	84.29°	45°	–39.29°
20	87.14°	63.43°	–60.57°

Magnitude and phase plots are shown in Fig. 4.

3. *Draw the Bode plot for the function*

$$F(s) = \frac{4\left(1+\frac{s}{2}\right)}{s^2\left(1+\frac{s}{8}\right)\left(1+\frac{s}{10}\right)}$$

Calculate gain margin and phase margin.

Step 1: Write F(s) in standard form.

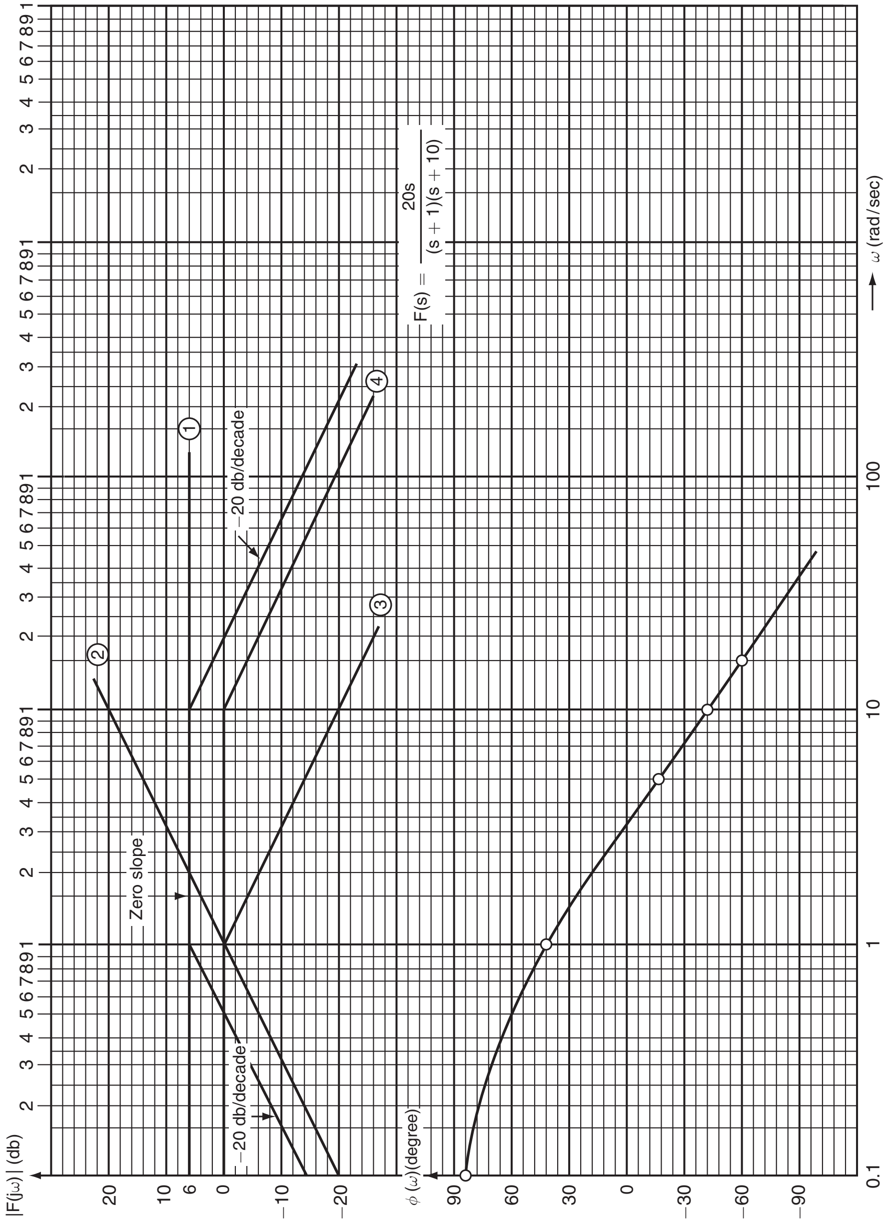
$$F(s) = \frac{4\left(1+\frac{s}{2}\right)}{s^2\left(1+\frac{s}{8}\right)\left(1+\frac{s}{10}\right)}$$

Putting $s = j\omega$,

$$F(j\omega) = \frac{4\left(1+\frac{j\omega}{2}\right)}{(j\omega)^2 + \left(1+\frac{j\omega}{8}\right)\left(1+\frac{j\omega}{10}\right)}$$

$$20\log|F(j\omega) = 20\log 4 - 20\log|(j\omega)^2| + 20\log\left|\left(1+\frac{j\omega}{2}\right)\right| - 20\log\left|\left(1+\frac{j\omega}{8}\right)\right|$$

$$- 20\log\left|\left(1+\frac{j\omega}{10}\right)\right|$$



|F(jω)| (db)
φ (ω) (degree)
ω (rad/sec)
F(s) = 20s / (s + 1)(s + 10)
Zero slope
−20 db/decade
−20 db/decade

Fig. 4

Step 2: Write down each factor in the table.

No.	Factor	Corner frequency	Magnitude characteristic
1	4	–	Straight line of magnitude 20 log 4 = 12.02 db.
2	$\frac{1}{(j\omega)^2}$	–	Straight line of slope −40 db/decade passing through 0 db at $\omega = 1$.
3	$1 + \frac{j\omega}{2}$	2	Straight line of 0 db for $\omega < 2$, straight line of 20 db/decade for $\omega > 2$.
4	$\frac{1}{1+\frac{j\omega}{8}}$	8	Straight line of db for $\omega < 8$, straight line of −20 db/decade for $\omega > 8$.
5	$\frac{1}{1+\frac{j\omega}{10}}$	10	Straight line of 0 db for $\omega < 10$, straight line of −20 db/decade for $\omega > 10$.

Step 3: Draw all the factors clearly.

Step 4: Add all the factors in the same manner as done in the previous problems to get magnitude plot.

Step 5: Draw the phase plot with the help of table for $\phi(\omega)$ for any arbitrary values of ω.

$$\phi(\omega) = -180^\circ + \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{8} - \tan^{-1}\frac{\omega}{10}$$

ω	$\tan^{-1}\frac{\omega}{2}$	$\tan^{-1}\frac{\omega}{8}$	$\tan^{-1}\frac{\omega}{10}$	$\phi(\omega)$
0.1	2.86°	0.716°	0.57°	−178.43°
1	26.56°	7.13°	5.71°	−166.28°
2	45°	14.04°	11.31°	−160.35°
8	75.96°	45°	38.66°	−187.7°
10	78.69°	51.34°	45°	−197.65°
100	88.85°	85.43°	89.43°	−266.01°

Magnitude and phase plots are shown in Fig. 5.

Phase Margin: Unity gain occurs at $\omega = 2$ rad/sec. This is gain cross over frequency. Corresponding phase at $\omega = 2$ is −166.28°.

Phase margin = $180 + \phi = 180^\circ + (-166.28^\circ) = 13.72^\circ$.

Gain Margin: Phase plot has phase of −180° at $\omega = 6.6$ rad/sec. This is phase cross over frequency.

Gain margin = 10 db.

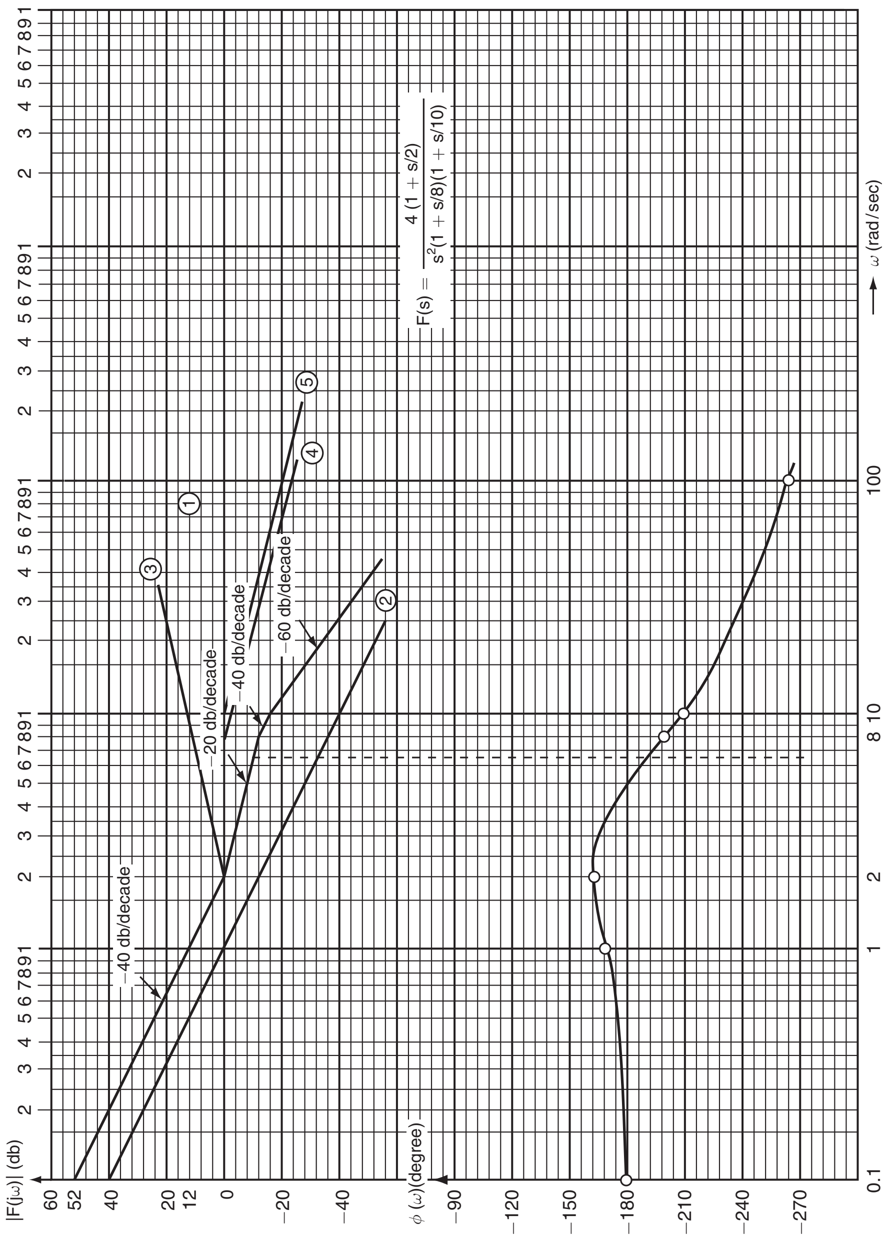


Fig. 5

4. *Sketch the Bode plot for following function,*

$$F(s) = \frac{200(s+2)}{s(s^2 + 10s + 100)}$$

Step 1: Write F(s) in standard form.

$$F(s) = \frac{200 \times 2\left(1+\frac{s}{2}\right)}{100 \times s\left(1+\frac{s}{10}+\frac{s^2}{100}\right)}$$

Putting s = jω,

$$F(j\omega) = \frac{4\left(1+\frac{j\omega}{2}\right)}{j\omega\left[\left(1-\frac{\omega^2}{100}\right)+\frac{j\omega}{10}\right]}$$

$$20 \log |F(j\omega)| = 20 \log 4 - 20 \log |j\omega| + 20 \log \left|\left(1+\frac{j\omega}{2}\right)\right| - 20 \log \left|\left(1-\frac{\omega^2}{100}\right)+\frac{j\omega}{10}\right|$$

Step 2: Write down each factor in the table.

No.	Factor	Corner frequency	Magnitude characteristic
1	4	–	Straight line of magnitude 20 log 4 = 6.02 db.
2	$\frac{1}{j\omega}$	–	Straight line of slope − 20 db/decade passing through 0 db at ω = 1.
3	$\left(1+\frac{j\omega}{2}\right)$	2	Straight line of 0 db for ω < 2, straight line of slope 20 db/decade for ω > 2.
4	$\frac{1}{\left(1-\frac{\omega^2}{100}\right)+\frac{j\omega}{10}}$	10	Straight line of 0 db for ω < 10, straight line of slope − 40 db/decade for ω >10.

Step 3: Draw all the factors clearly.

Step 4: Add all the factors in the same manner as done in the previous problems to get magnitude plot.

Step 5: Draw the phase plot with the help of table for ϕ(ω) for any values of ω.

$$\phi(\omega) = 0 - 90° + \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{0.1\,\omega}{1-\frac{\omega^2}{100}}$$

$$= -90° + \tan^{-1}\frac{\omega}{2} - \tan^{-1}\left(\frac{10\omega}{100-\omega^2}\right)$$

ω	$\tan^{-1}\frac{\omega}{2}$	$\tan^{-1}\frac{10\omega}{100-\omega^2}$	$\phi(\omega)$
0.1	2.86°	0.57°	– 87.71°
1	26.57°	5.77°	– 69.20°
2	45°	11.77°	– 56.77°
5	68.20°	33.69°	– 55.49°
10	78.69°	90°	– 101.32°
100	88.85°	174.22°	– 175.68°

Magnitude and phase plots are shown in Fig. 6

5. *Construct the Bode plot for the function*

$$F(s) = \frac{4}{(1+s)\left(1+\frac{s}{3}\right)^2}$$

Step 1: Write F(s) in standard form.

$$F(s) = \frac{4}{(1+s)\left(1+\frac{s}{3}\right)^2}$$

Putting $s = j\omega$,

$$F(j\omega) = \frac{4}{(1+j\omega)\left(1+\frac{j\omega}{3}\right)^2}$$

$$20\log|F(j\omega)| = 20\log 4 - 20\log|1+j\omega| - 20\log\left|\left(1+\frac{j\omega}{3}\right)^2\right|$$

Step 2: Write down each factor in the table.

No.	Factor	Corner frequency	Magnitude characteristic
1	4	–	Straight line of magnitude 20 log 4 = 12.04 db.
2	$\frac{1}{1+j\omega}$	1	Straight line of 0 db for $\omega < 1$, straight line of slope – 20 db/decade for $\omega > 1$.
3	$\frac{1}{\left(1+\frac{j\omega}{3}\right)^2}$	3	Straight line of 0 db for $\omega < 3$, straight line of slope – 40db/decade for $\omega > 3$.

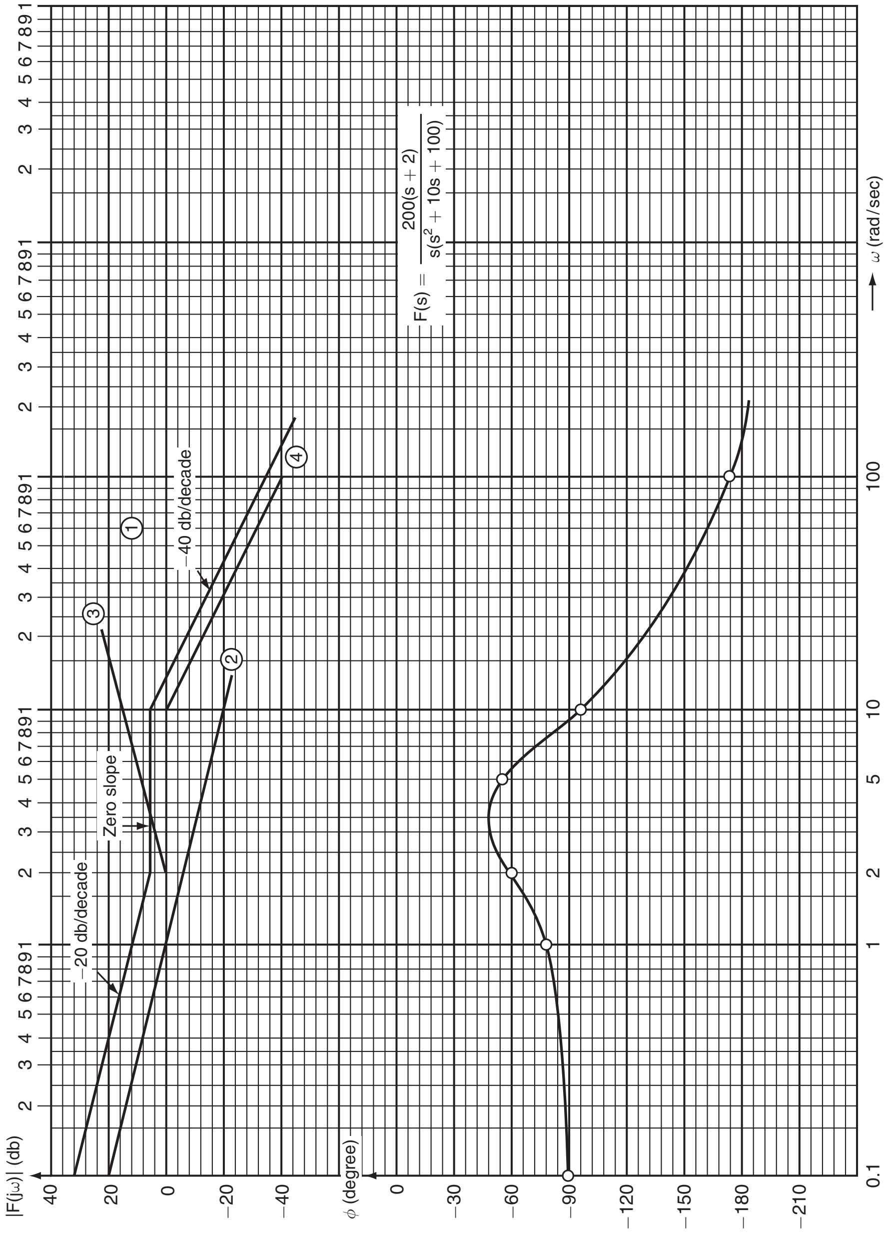


Fig. 6

Step 3: Draw all the factors clearly.

Step 4: Add all the factors to get magnitude plot.

Step 5: Draw the phase plot with the help of table for $\phi(\omega)$ for any value of ω.

$$\phi(\omega) = \ \phi \tan^{-1}\omega - 2\tan^{-1}\frac{\omega}{3}.$$

ω	$\tan^{-1}\omega$	$2\tan^{-1}\frac{\omega}{3}$	$\phi(\omega)$
0.1	5.71°	3.78°	– 9.49°
1	45°	36.53°	– 81.53°
3	71.57°	89.42°	– 160.99°
5	78.69°	117.56°	–196.25°
10	84.29°	146.28°	– 230.57°
20	87.14°	162.77°	– 249.91°

Magnitude and phase plots are shown in Fig. 7.

6. *Determine the transfer function for the asymptotic bode plot shown in Fig. 8.*

In this fig. slope changes at ω = 1,10,100,1000 rad/sec.

Corner frequency (rad/sec):1,10,100,1000

Corner Frequency	Change in slope	Term in transfer function
1	– 20 –0 = –20 db/dec	$\frac{1}{\left(1+\frac{s}{1}\right)}$
10	– 40 – (–20) = – 20 db/dec	$\frac{1}{\left(1+\frac{s}{10}\right)}$
100	0 – (–40) = 40 db/dec	$\left(1+\frac{s}{100}\right)^2$
1000	20 – 0 = 20 db/dec	$\left(1+\frac{s}{1000}\right)$

Hence transfer function can be written as,

$$F(S) = \frac{K\left(1+\frac{s}{100}\right)^2\left(1+\frac{s}{1000}\right)}{(1+s)\left(1+\frac{s}{10}\right)}$$

The constant can be evaluated as,

$20 \log K = 40$

$K = 100$

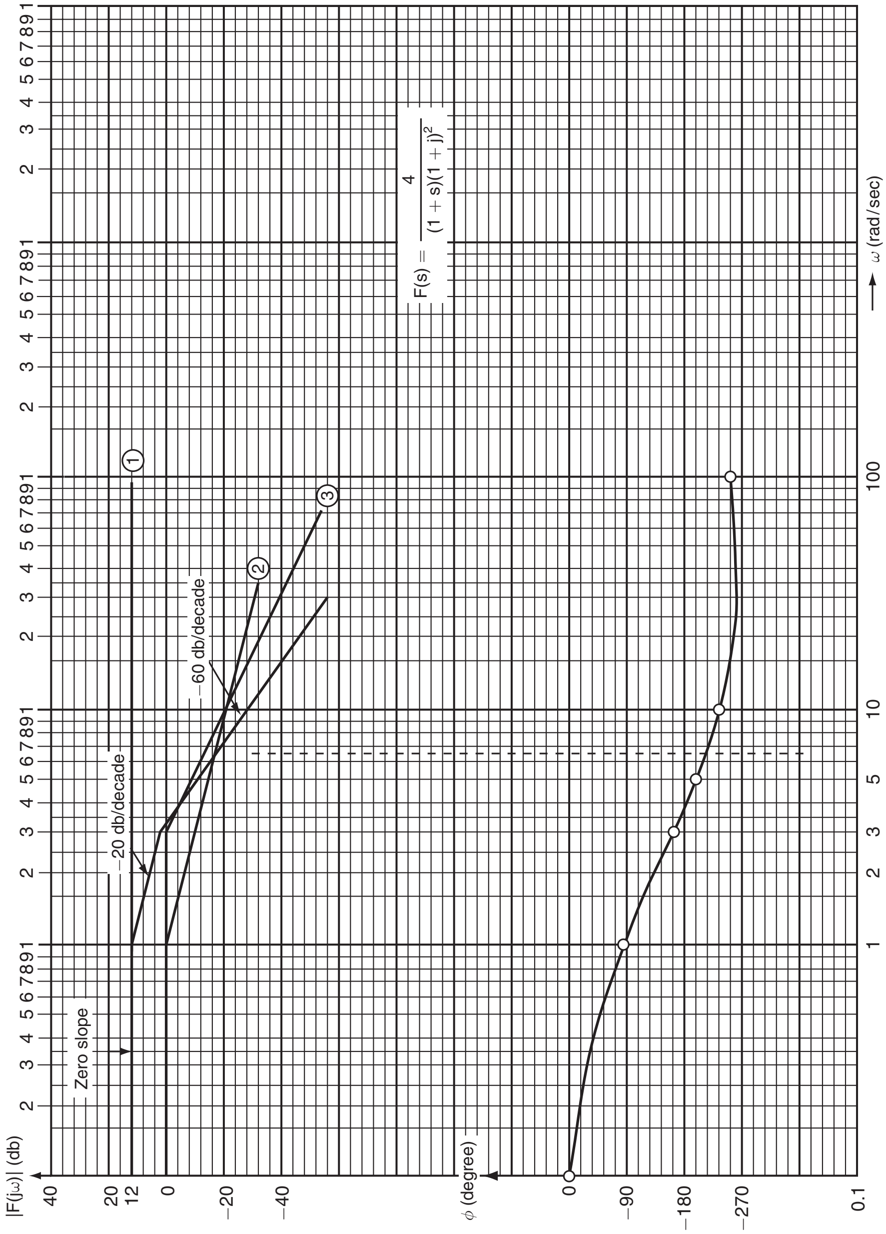


Fig. 7

$$F(s) = \frac{100\left(1+\frac{s}{100}\right)^2\left(1+\frac{s}{1000}\right)}{(1+s)\left(1+\frac{s}{10}\right)}$$

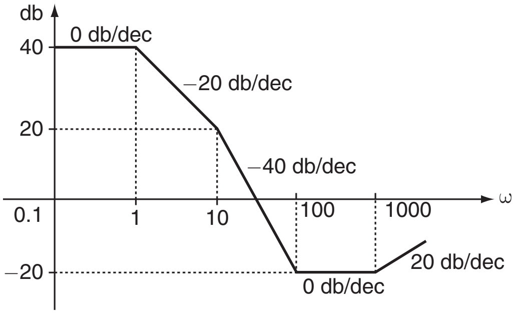


Fig. 8

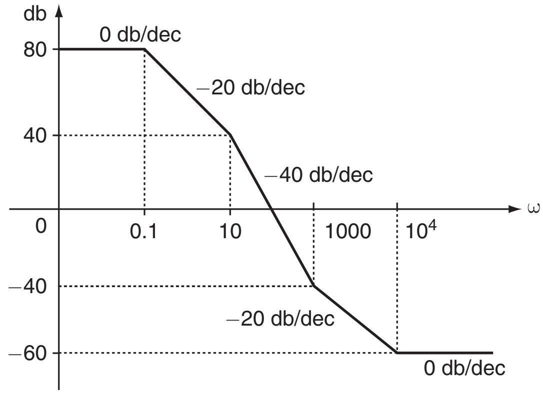


Fig. 9

7. *Determine the transfer function for the asymptotic bode plot shown in Fig. 9.*

In this fig. slope changes at $\omega = 0.1, 10, 1000, 10^4$ rad/sec.

Corner frequency (rad/sec): $0.1, 10, 1000, 10^4$

Corner Frequency	Change in slope	Term in transfer function
0.1	– 20 – 0 = –20 db/dec	$\frac{1}{\left(1+\frac{s}{0.1}\right)}$
10	–40 – (–20) = –20 db/dec	$\frac{1}{\left(1+\frac{s}{10}\right)}{}_{s}$
1000	–20 –(–40) = 20 db/dec	$\left(1+\frac{s}{1000}\right)$
10^4	0 – (– 20) = 20 db/dec	$\left(1+\frac{s}{10^4}\right)$

Hence, transfer function can be written as,

$$F(s) = \frac{K\left(1+\frac{s}{1000}\right)\left(1+\frac{s}{10^4}\right)}{\left(1+\frac{s}{0.1}\right)\left(1+\frac{s}{10}\right)}$$

The constant K can be evaluated as,

20 log K = 80

K = 10000

$$F(s) = \frac{10000\left(1+\frac{s}{1000}\right)\left(1+\frac{s}{10^4}\right)}{\left(1+\frac{s}{0.1}\right)\left(1+\frac{s}{10}\right)}$$

8. *Determine the transfer function for the asymptotic bode plot shown in Fig. 10.*

In this fig. slope changes at ω = 2, 5, 10 rad/sec

Corner frequency (rad/sec) = 2, 5, 10

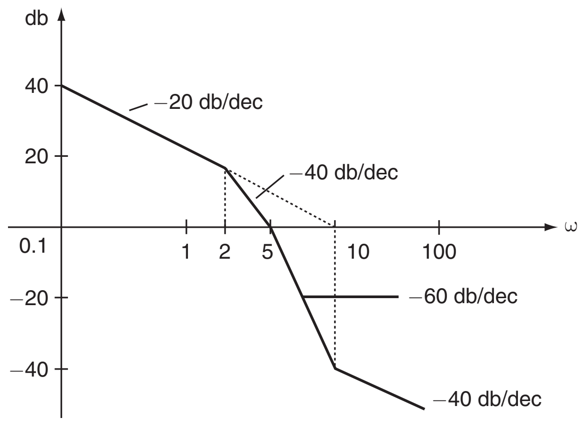


Fig. 10

Corner Frequency	Change in slope	Term in transfer function
2	– 40 –(–20) = – 20 db/dec	$\frac{1}{\left(1+\frac{s}{2}\right)}$
5	–60 – (–40) = – 20 db/dec	$\frac{1}{\left(1+\frac{s}{5}\right)}$
10	–40 –(–60) = 20 db/dec	$\left(1+\frac{s}{10}\right)$

Since low frequency asymplote has a slope of − 20 db/dec, it indicates the presence of a term $\frac{K}{s}$ in the transfer function.

$$F(s) = \frac{K\left(1+\frac{s}{10}\right)}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{5}\right)}$$

The frequency at which the asymptute (extended if necessary) intersects the 0 db line numerically represents the value of K.

Here, low frequency asymptute intersects the 0 db axis at $\omega = 10$.

$$F(s) = \frac{10\left(1+\frac{s}{10}\right)}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{2}\right)}$$