## 14.5 Geometry of Fuzzy Sets

As shown in Fig. 14.4, a fuzzy set can be given an interesting geometrical interpretation. Consider a universe of two objects, $A$ and $B$. In this geometry, we assume that each axis describes the degrees of membership of objects $A$ and $B$ respectively in a set $P$ which can be either classical or fuzzy.

The geometric interpretation of a fuzzy set is an outcome of Bart Kosko's Ph.D. dissertation "Foundations of Fuzzy Estimation Theory", University of California, Irvine, 1987. A very accessible treatment of the subject (as well as other related topics) can be found in Kosko's books [323, 324], from where this material is derived.

Corners of the unit square represent the classical power set $2^{\mathcal{X}}$. Fuzzy sets are the points that lie within the square.

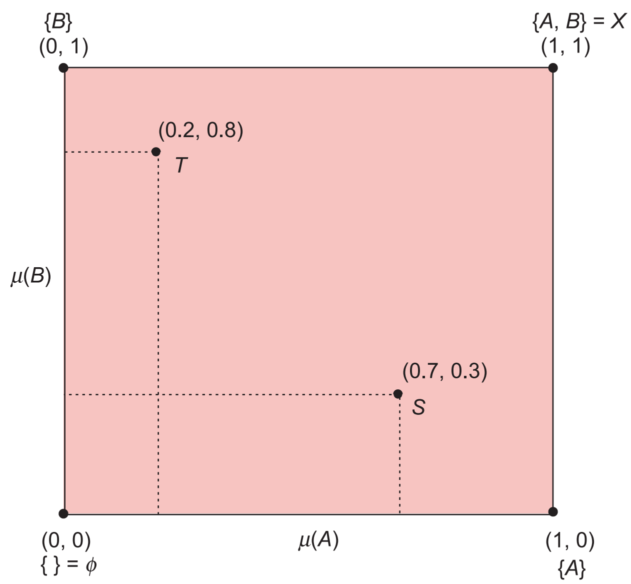


**Fig. 14.4** The geometry of fuzzy sets

In the geometrical representation of Fig. 14.4, coordinate (0,0) is interpreted as $A/0 + B/0$, implying that neither $A$ nor $B$ are members of $P$. In other words, the origin represents $P = \phi$ or the null set. Coordinate (1,1) is interpreted as $A/1 + B/1$, where both $A$ and $B$ are members of $P$, and this point therefore represents the universal set $P = \{A, B\} = \mathcal{X}$. Based on this reasoning, coordinates (1,0) and (0,1) are interpreted as representing the singleton sets {A} and {B} respectively. It is important to understand that the classical power set $2^{\mathcal{X}} = \{\{\}, \{A\}, \{B\}, \{A, B\}\}$ is represented by the four corners of the unit square and is denoted by $\mathbb{B}^2$.

In contrast, the point $S$ in Fig. 14.4 with coordinates (0.7, 0.3) represents a fuzzy set $S = A/0.7 + B/0.3$ with the connotation: $A$ and $B$ are members of $S$ to degrees 0.7 and 0.3 respectively. In the present example, the points that lie within the square denoted by $\mathbb{I}^2 = [0, 1] \times [0, 1]$, represent different fuzzy sets. The set of all such possible fuzzy sets—the infinite points in $\mathbb{I}^2$—comprise the fuzzy power set $\mathcal{F}^{\mathcal{X}}$. Note that since $\mathbb{B}^2 \subset \mathbb{I}^2$, classical sets are special cases of fuzzy sets.